Modelling network performance with a spatial stochastic process algebra

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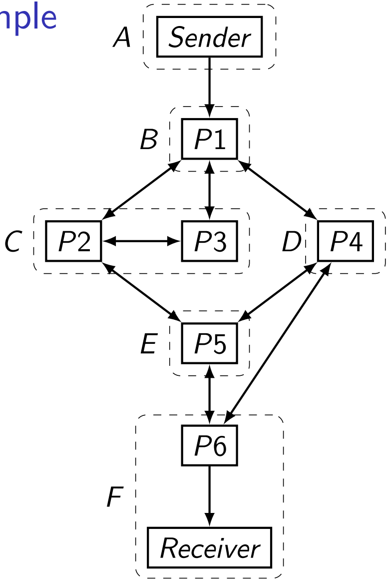
  - biological models – BioAmbients, attributed π-calculus

Outline

Motivating example



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- new ingredient: general notion of location

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  - weights modify rates on actions between locations

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- other rules defined in the obvious manner

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  - then apply to concrete process algebra

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  - each sequential component must have single fixed location
  - communication must be pairwise and directional

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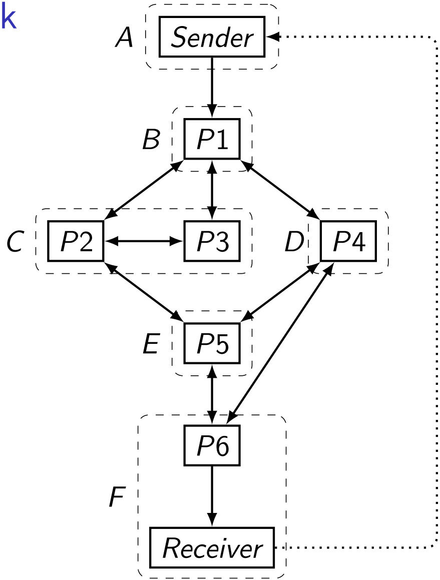
$$rsync(P_1, P_2, L_1, L_2, r_1, r_2)$$
$$= \begin{cases} \dfrac{r_1}{r_\alpha(P_1)} \dfrac{r_2}{r_\alpha(P_2)} \min(r_\alpha(P_1), r_\alpha(P_2)) \cdot w((\ell_1, \ell_2)) & \\ \qquad \text{if } L_1 = \{\ell_1\}, L_2 = \{\ell_2\}, (\ell_1, \ell_2) \in E & \\ \perp \qquad \text{otherwise} & \end{cases}$$

Example network



PEPA model

$$\begin{aligned}
Sender@A &\stackrel{def}{=} (prepare, \rho).Sending@A \\
Sending@A &\stackrel{def}{=} \textstyle\sum_{i=1}^{6} (c_{Si}, r_S).(ack, r_{ack}).Sender@A
\end{aligned}$$

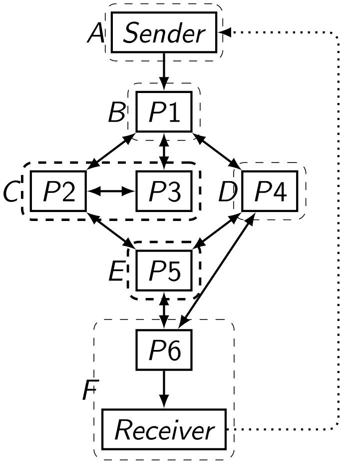
$$\begin{aligned}
Receiver@F &\stackrel{def}{=} \textstyle\sum_{i=1}^{6} (c_{iR}, r_6).Receiving@F \\
Receiving@F &\stackrel{def}{=} (consume, \gamma).(ack, r_{ack}).Receiver@F
\end{aligned}$$

$$\begin{aligned}
P_i@\ell_i &\stackrel{def}{=} (c_{Si}, \top).Q_i@\ell_i + \textstyle\sum_{j=1, j\neq i}^{6} (c_{ji}, r).Q_i@\ell_i \\
Q_i@\ell_i &\stackrel{def}{=} (c_{iR}, \top).P_i@\ell_i + \textstyle\sum_{j=1, j\neq i}^{6} (c_{ij}, r).P_i@\ell_i
\end{aligned}$$

$$\begin{aligned}
Network \stackrel{def}{=} \; & (Sender@A \underset{*}{\bowtie} (P1@B \underset{*}{\bowtie} (P2@C \underset{*}{\bowtie} (P3@C \underset{*}{\bowtie} \\
& (P4@D \underset{*}{\bowtie} (P5@E \underset{*}{\bowtie} (P6@F \underset{*}{\bowtie} Receiver@F)))))))
\end{aligned}$$

Graphs

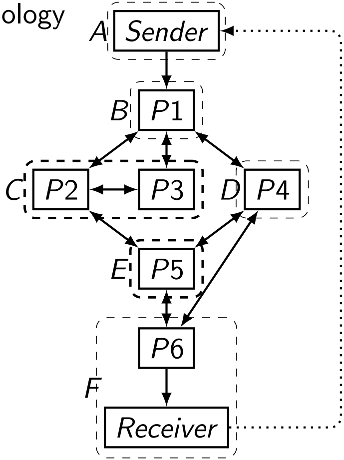
- rates: $r = r_R = r_S = 10$



Graphs

- rates: $r = r_R = r_S = 10$
- the weighted graph G describes the topology

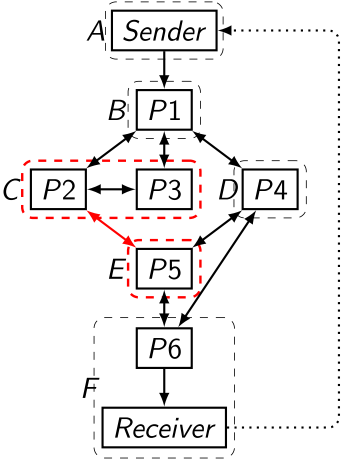
	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		1	
D		1			1	1
E			1	1		1
F	1			1	1	1



Graphs

- G_1 represents heavy traffic between C and E

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		0.1	
D		1			1	1
E			0.1	1		1
F	1			1	1	1



Graphs

- G_2 represents no connectivity between C and E

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		0	
D		1			1	1
E			0	1		1
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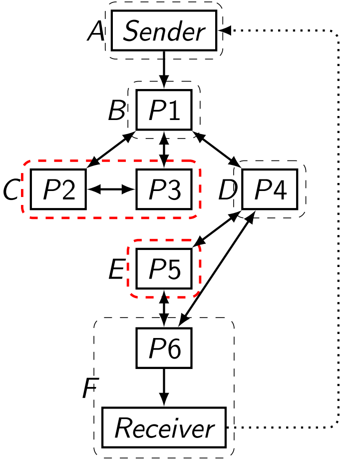
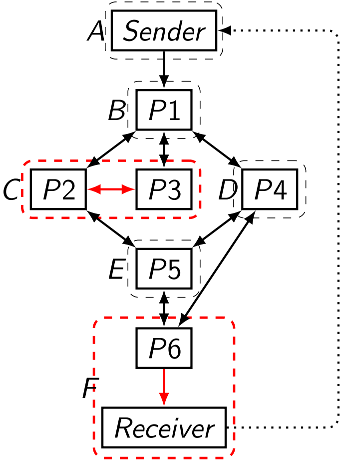


Graphs

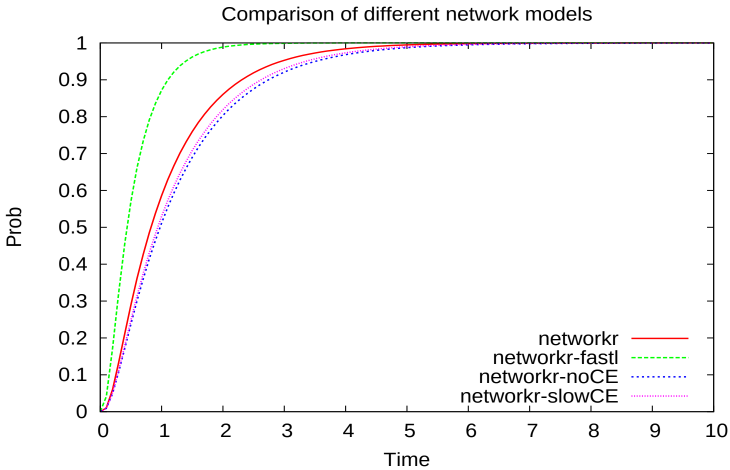
- G_3 represents high connectivity between colocated processes

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	10		1	
D		1			1	1
E			1	1		1
F	1			1	1	10



Analysis

- cumulative density function of passage time



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  - translation into PEPA
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- conclusions
  - presentation of a very general stochastic process algebra with locations
  - use for modelling network performance in a general way

Thank you

This research was funded by the EPSRC SIGNAL Project