How restrictive is the current action decomposition property for compression bisimulation?

Vashti Galpin
LFCS, University of Edinburgh

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Bio-PEPA

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- some success

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- basic syntax, excludes locations, transportation and events

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  - counts can be converted to levels

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$$((\alpha,\kappa)\downarrow S)(\,\ell\,) \xrightarrow{(\alpha,[S:\,\downarrow\,(\ell,\kappa)])}_c S(\,\ell-\kappa\,) \qquad \kappa \leq \ell \leq N_S$$

$$((\alpha,\kappa)\uparrow S)(\,\ell\,) \xrightarrow{(\alpha,[S:\,\uparrow\,(\ell,\kappa)])}_c S(\,\ell+\kappa\,) \qquad 0 \leq \ell \leq N_S-\kappa$$

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Bio-PEPA semantics (continued)

- Constant

$$\frac{S(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')}{C(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')} \quad C \stackrel{def}{=} S$$

Bio-PEPA semantics (continued)

- Constant

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- Choice

$$\frac{S_1(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')}{(S_1 + S_2)(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')} \quad \frac{S_2(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')}{(S_1 + S_2)(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')}$$

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- Cooperation for $\alpha \notin L$

$$\frac{P \xrightarrow{(\alpha,w)}_c P'}{P \underset{L}{\bowtie} Q \xrightarrow{(\alpha,w)}_c P' \underset{L}{\bowtie} Q} \quad \frac{Q \xrightarrow{(\alpha,w)}_c Q'}{P \underset{L}{\bowtie} Q \xrightarrow{(\alpha,w)}_c P \underset{L}{\bowtie} Q'}$$

Bio-PEPA semantics (continued)

- Cooperation for $\alpha \in L$

$$\frac{P \xrightarrow{(\alpha,\, v\ )}_c P' \quad Q \xrightarrow{(\alpha,\, u\ )}_c Q'}{P \underset{L}{\bowtie} Q \xrightarrow{(\alpha,\, v :: u\ )}_c P' \underset{L}{\bowtie} Q'} \quad \alpha \in L$$

Bio-PEPA semantics (continued)

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- quantitative, only consider α

Example: reaction with enzyme, max level 3

- state vector (S, E, SE, P) and $N_S = N_E = N_{SE} = N_P = 3$

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$$\begin{tikzcd}
(3,3,0,0) \arrow[r, shift left, "\alpha"] & (2,2,1,0) \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & (1,1,2,0) \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & (0,0,3,0) \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & (2,3,0,1) \arrow[r, shift left, "\alpha"] & (1,2,1,1) \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & (0,1,2,1) \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & (1,3,0,2) \arrow[r, shift left, "\alpha"] & (0,2,1,2) \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & (0,3,0,3)
\end{tikzcd}$$

Example: reaction with enzyme, max level 7

- state vector $S\ E\ SE\ P$ and $N_S = N_E = N_{SE} = N_P = 7$

Example: reaction with enzyme, max level 7

- state vector $S\ E\ SE\ P$ and $N_S = N_E = N_{SE} = N_P = 7$

$$
\begin{tikzcd}
7700 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 6610 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 5520 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 4430 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 3340 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 2250 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 1160 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0070 \arrow[d, "\gamma"] \\
 & 6701 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 5611 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 4521 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 3431 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 2341 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 1251 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0161 \arrow[d, "\gamma"] \\
 & & 5702 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 4612 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 3522 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 2432 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 1342 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0252 \arrow[d, "\gamma"] \\
 & & & 4703 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 3613 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 2523 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 1433 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0342 \arrow[d, "\gamma"] \\
 & & & & 3704 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 2614 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 1524 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0614 \arrow[d, "\gamma"] \\
 & & & & & 2705 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 1615 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] \arrow[d, "\gamma"] & 0525 \arrow[d, "\gamma"] \\
 & & & & & & 1706 \arrow[r, shift left, "\alpha"] \arrow[r, <-, shift right, "\beta"'] & 0616 \arrow[d, "\gamma"] \\
 & & & & & & & 0707
\end{tikzcd}
$$

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- aim for a bisimulation-style equivalence
- bisimilarity, $P \sim Q$ if
  1. $P \xrightarrow{(\alpha,v)}_c P'$, $Q \xrightarrow{(\alpha,u)}_c Q'$ and $P' \sim Q'$
  2. $Q \xrightarrow{(\alpha,u)}_c Q'$, $P \xrightarrow{(\alpha,v)}_c P'$ and $P' \sim Q'$

Equivalence definition

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- base equivalence on standard bisimilarity
- compression bisimilarity, $P \doteqdot Q$ if $[P] \sim [Q]$, namely if
  1. $[P] \overset{\alpha}{\hookrightarrow} [P']$, $[Q] \overset{\alpha}{\hookrightarrow} [Q']'$ and $[P'] \sim [Q']$
  2. $[Q] \overset{\alpha}{\hookrightarrow} [Q']$, $[P] \overset{\alpha}{\hookrightarrow} [P']'$ and $[P'] \sim [Q']$

Equivalence illustrated

- $B \stackrel{\mathit{def}}{=} (\alpha, 3)\downarrow B \,+\, (\beta, 4)\uparrow B \,+\, (\gamma, 1)\uparrow B$

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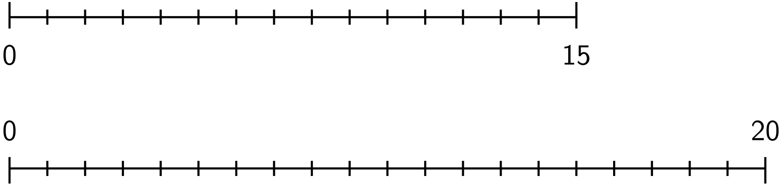

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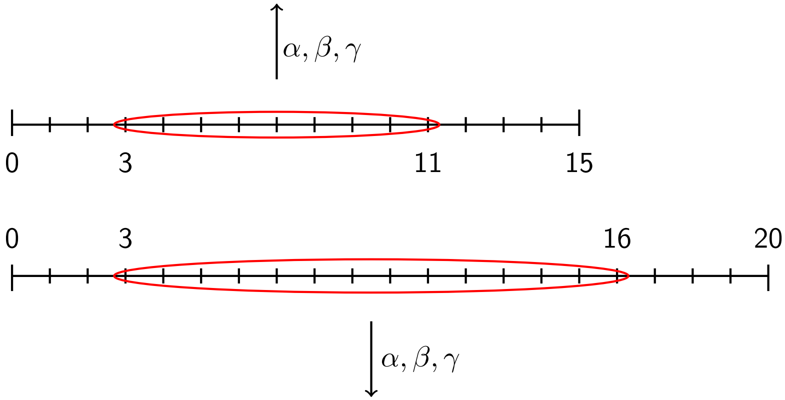

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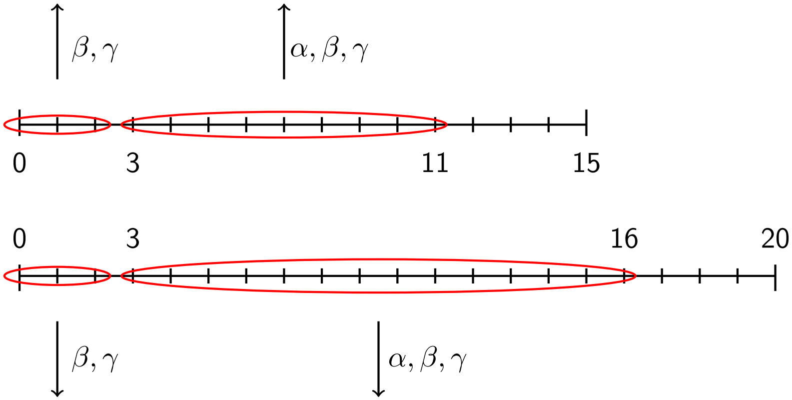

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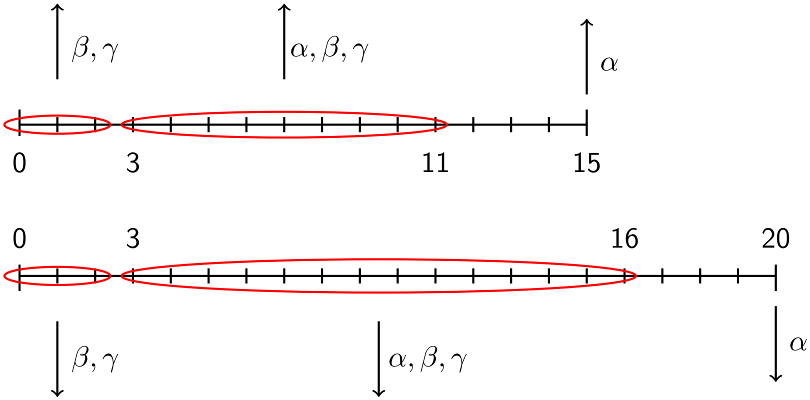

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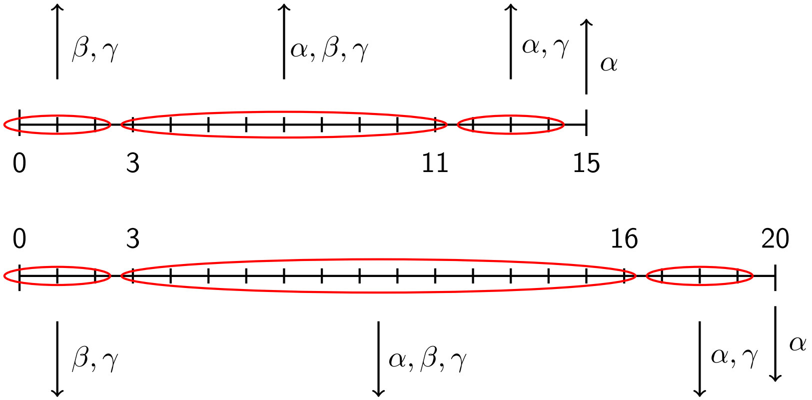

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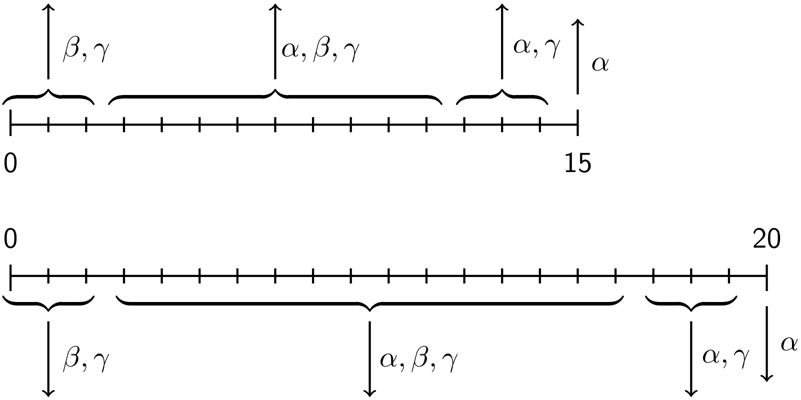

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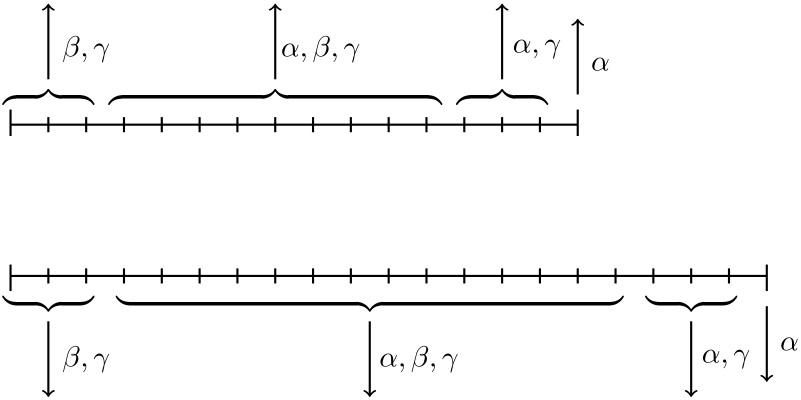

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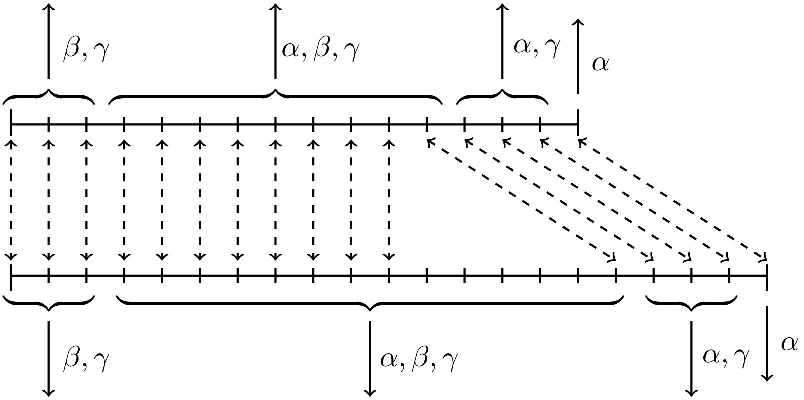

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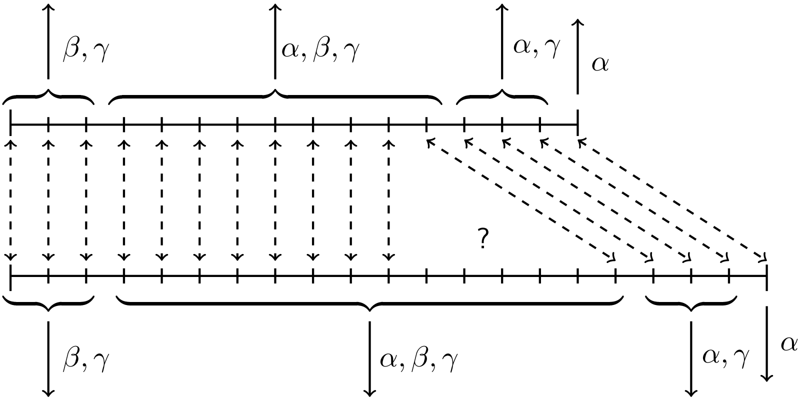

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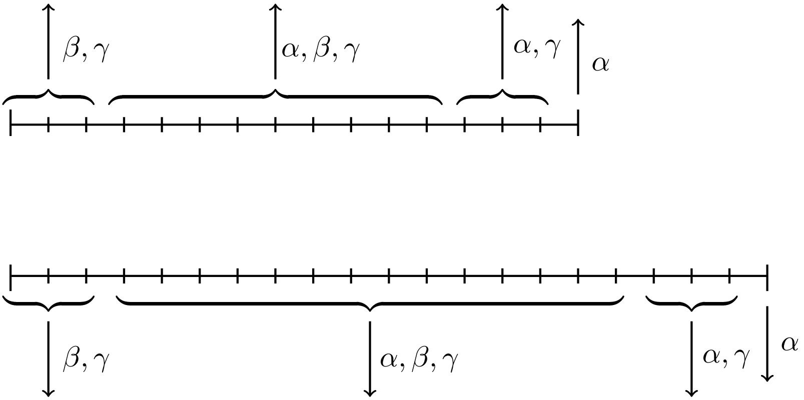

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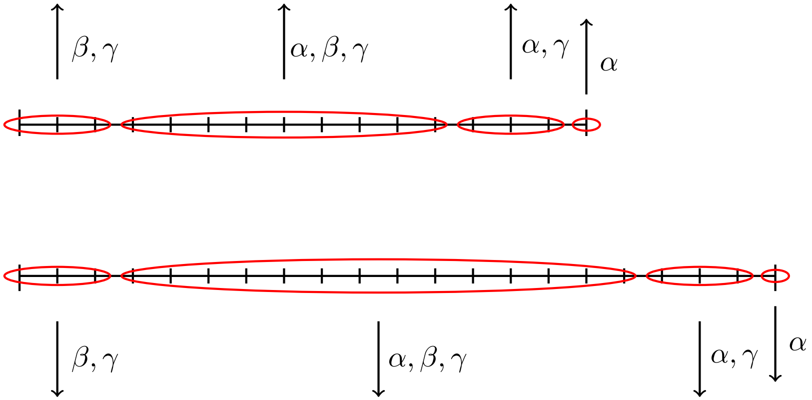

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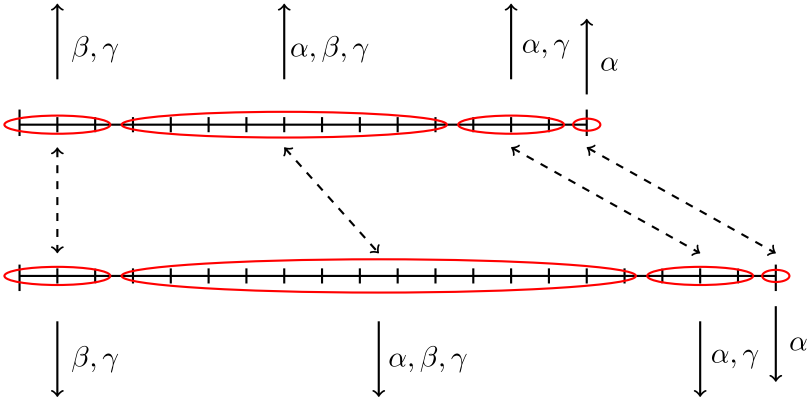

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- if $P_1 \doteq P_2$, $Q_1 \doteq Q_2$ and $P_1 \underset{L}{\bowtie} Q_1$ and $P_2 \underset{L}{\bowtie} Q_2$ have CADP then $P_1 \underset{L}{\bowtie} Q_1 \doteq P_2 \underset{L}{\bowtie} Q_2$

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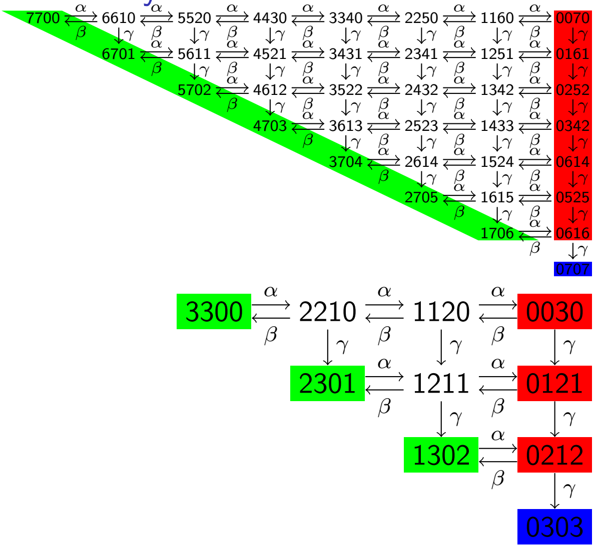
- for a well-defined Bio-PEPA system, $P^n \doteq P^m$ if they have CADP and $n, m \geq k_\downarrow + \max\{k_\downarrow, k_\uparrow\} + k_\uparrow$

Example: reaction with enzyme



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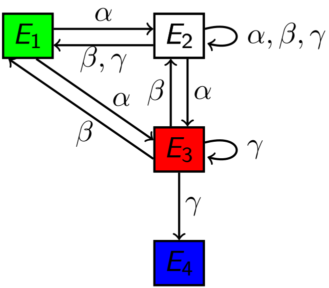

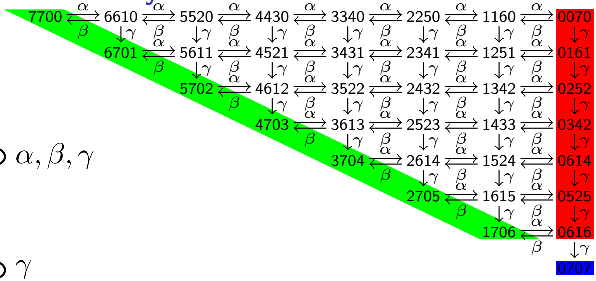

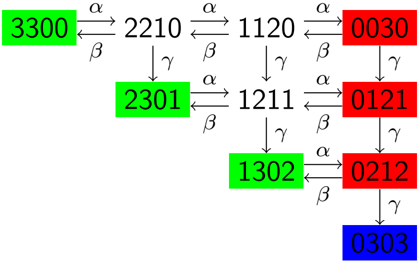

CADP

- CADP: current action decomposition property

$$(P' \underset{L}{\bowtie} Q', P'' \underset{L}{\bowtie} Q'') \in \mathcal{H} \text{ with } P' \underset{L}{\bowtie} Q', P'' \underset{L}{\bowtie} Q'' \in ds(P \underset{L}{\bowtie} Q)$$

$$\Rightarrow \quad (P', P'') \in \mathcal{H} \text{ and } (Q', Q'') \in \mathcal{H}$$

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  - ignore non-violations
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  - consider $\alpha \notin L$ and $\alpha \in L$

Actions not in the cooperation set

- we have two basic cases (excluding symmetry)

$$
\begin{array}{rlrl}
P_1 \underset{L}{\bowtie} Q_1 & \xrightarrow{(\alpha,.)}_c & P_2 \underset{L}{\bowtie} Q_2 & \xrightarrow{(\alpha,.)}_c \\
P_1 & \not\xrightarrow{(\alpha,.)}_c & P_2 & \xrightarrow{(\alpha,.)}_c \\
Q_1 & \xrightarrow{(\alpha,.)}_c & Q_2 & \xrightarrow{(\alpha,.)}_c
\end{array}
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- systems that violate CADP in this manner are not reasonable Bio-PEPA models
- therefore we can ignore them as unimportant

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Example of CADP violation

- consider two species

$$A \stackrel{def}{=} (\alpha, 2) \downarrow A + (\delta, 1) \downarrow A$$
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  - neither can perform α but both can perform δ
  - $A(2)$ can perform α
  - $A(1)$ cannot perform α
- what does the transition system look like when $N_A = 4$ and $N_B = 1$?

Transition system

$$
\begin{tikzcd}
40 \arrow[r, "\delta"] \arrow[d, "\alpha"] & 30 \arrow[r, "\delta"] \arrow[d, "\alpha"] & 20 \arrow[r, "\delta"] \arrow[d, "\alpha"] & 10 \arrow[r, "\delta"] & 00 \\
\boxed{21} \arrow[r, "\delta"] & \boxed{11} \arrow[r, "\delta"] & 01 & &
\end{tikzcd}
$$

- $A(2) \underset{\alpha}{\bowtie} B(1)$, $A(1) \underset{\alpha}{\bowtie} B(1)$ can do nothing
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- Does this look different if $N_B = 2$
- Yes!
- $A(2) \underset{\alpha}{\bowtie} B(1)$, $A(1) \underset{\alpha}{\bowtie} B(1)$ no longer have the same actions

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  - the ability of a reaction to occur is determined by the availability of reactants
  - how should creation and degradation be treated?
- do constrained and/or full expressible systems have CADP?

Example revisited

$$
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- $B(1)$ can perform α, $B(2)$ cannot
- can δ be constrained?

A more complex example

- consider three species with $N_A = 4$, $N_B = N_C = 2$

$$
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A &\stackrel{\text{def}}{=} (\alpha, 2) \downarrow A + (\delta, 1) \downarrow A \\
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- back to the drawing board

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  - ongoing investigation
  - understanding the relationship between species levels
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Thank you