A process algebra for the modelling of hybrid systems

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Outline

Introduction

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Hybrid systems

- discrete behaviour
- continuous behaviour, expressed as ODEs

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Hybrid automata

- well known
- graphical rather than textual
- not very compositional

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Process algebras for hybrid systems

- compositional language
- semantic equivalences

Motivation

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Other hybrid process algebras

- ACP^{srt}_{hs} – Bergstra and Middelburg
- HyPA – Cuijpers and Reniers
- hybrid χ – van Beek *et al*
- ϕ-calculus – Rounds and Song

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Comparison

differences: syntax, semantics, discontinuous behaviour, flow-determinism, theoretical results, tools – Khadim

similarity: require full description of dynamic behaviour of subcomponents, ODEs appear in syntax

Monolithic ODEs

Monolithic ODEs

Heater system with temperature limit

$$\theta \equiv (T_B^{\bullet} = {}^{\bullet}T_B) \qquad \psi \equiv (T_B = 25)$$

$$\begin{aligned}
\text{Start} &\stackrel{\mathit{def}}{=} (T_B = T_0) \mathrel{\wedge\!\!\blacktriangle} \text{Off12} \\
\text{Off12} &\stackrel{\mathit{def}}{=} (\dot{T}_B = -T_B) \curvearrowright\!\!\blacktriangledown\, \sigma^*_{\text{rel}}(\theta \ulcorner\!\!\blacktriangledown\, (\mathit{on}_1 \cdot \text{On1} + \mathit{on}_2 \cdot \text{On2})) \\
\text{On1} &\stackrel{\mathit{def}}{=} (T_B \leq 25 \wedge \dot{T}_B = -T_B + 0.5r_1) \\
&\quad \curvearrowright\!\!\blacktriangledown\, \sigma^*_{\text{rel}}((\theta \ulcorner\!\!\blacktriangledown\, \mathit{on}_2 \cdot \text{On12}) + (\psi :\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\, \mathit{off}_1 \cdot \text{Off12}))) \\
\text{On2} &\stackrel{\mathit{def}}{=} (T_B \leq 25 \wedge \dot{T}_B = -T_B + 0.5r_2) \\
&\quad \curvearrowright\!\!\blacktriangledown\, \sigma^*_{\text{rel}}((\theta \ulcorner\!\!\blacktriangledown\, \mathit{on}_1 \cdot \text{On12}) + (\psi :\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\, \mathit{off}_2 \cdot \text{Off12}))) \\
\text{On12} &\stackrel{\mathit{def}}{=} (T_B \leq 25 \wedge \dot{T}_B = -T_B + 0.5(r_1 + r_2)) \\
&\quad \curvearrowright\!\!\blacktriangledown\, \sigma^*_{\text{rel}}(\psi :\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\, (\mathit{off}_1 \cdot \text{On2} + \mathit{off}_2 \cdot \text{On1})))
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\mathrm{On2} &\stackrel{def}{=} (T_B \leq 25 \wedge {\color{red}\dot{T}_B = -T_B + 0.5r_2}) \\
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\end{aligned}$$

More monolithic ODEs

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Elements of orbiter system

$$
\begin{aligned}
DOD &\stackrel{def}{=} (dK/dt = -K) \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\mathrm{rel}}\Big((\theta \ulcorner\!\!\blacktriangledown\ light \cdot LOD) \\
&\qquad\qquad + ((K \le t_2) :\!\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ on \cdot DND))\Big) \\
LOD &\stackrel{def}{=} (dK/dt = r_s - K) \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\mathrm{rel}}\Big((\theta \ulcorner\!\!\blacktriangledown\ dark \cdot DOD) \\
&\qquad\qquad + ((K \ge t_3) :\!\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ up \cdot LOU))\Big) \\
DND &\stackrel{def}{=} (dK/dt = r_h - K) \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\mathrm{rel}}\Big((\theta \ulcorner\!\!\blacktriangledown\ light \cdot LND) \\
&\qquad\qquad + ((K \ge t_1) :\!\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ off \cdot DND))\Big) \\
LOU &\stackrel{def}{=} (dK/dt = r_s - r_d - K) \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\mathrm{rel}}\Big((\theta \ulcorner\!\!\blacktriangledown\ dark \cdot DOU) \\
&\qquad\qquad + ((K \le t_4) :\!\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ down \cdot LOD))\Big)
\end{aligned}
$$

More monolithic ODEs

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DOD &\stackrel{def}{=} (dK/dt = -K) \curvearrowright \sigma^*_{\mathrm{rel}}\Big((\theta \Rsh \mathit{light} \cdot LOD) \\
&\qquad + ((K \leq t_2) :\longrightarrow (\theta \Rsh \mathit{on} \cdot DND))\Big) \\
LOD &\stackrel{def}{=} (dK/dt = r_s - K) \curvearrowright \sigma^*_{\mathrm{rel}}\Big((\theta \Rsh \mathit{dark} \cdot DOD) \\
&\qquad + ((K \geq t_3) :\longrightarrow (\theta \Rsh \mathit{up} \cdot LOU))\Big) \\
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&\qquad + ((K \geq t_1) :\longrightarrow (\theta \Rsh \mathit{off} \cdot DND))\Big) \\
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\end{aligned}
$$

A different approach

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- process algebra for hybrid systems

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- not stochastic (yet)

A different approach

HYPE

- process algebra for hybrid systems
- individual additive flows
- more fine-grained approach
- separation of modelling concerns
- influence of continuous semantics of PEPA
- mapping to hybrid automata
- not stochastic (yet)
- running example: temperature control for an orbiter

HYPE actions

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Events

instantaneous, discrete changes

$\underline{a} \in \mathcal{E}$

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Activities or influences

influences on continuous aspects, flows

$\alpha \in \mathcal{A}$

$$\alpha(\vec{X}) = (\iota, r, I(\vec{X}))$$

influence name rate influence type

with $[\![I(\vec{X})]\!] = f(\vec{X})$

where $\vec{X}$ is a formal parameter.

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HYPE uncontrolled system

HYPE uncontrolled system

Subcomponents

$S ::= \underline{\mathsf{a}} : \alpha . C_s \mid S + S \qquad \underline{\mathsf{a}} \in \mathcal{E}, \alpha \in \mathcal{A}$

$C_s(\vec{X}) \stackrel{def}{=} S$

HYPE uncontrolled system

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Components

$P ::= C(\vec{X}) \mid P \underset{L}{\bowtie} P \qquad L \subseteq \mathcal{E}$

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HYPE uncontrolled system

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Uncontrolled system

$\Sigma ::= C(\vec{V}) \mid \Sigma \underset{L}{\bowtie} \Sigma \qquad L \subseteq \mathcal{E}$

HYPE controlled system

HYPE controlled system

Controller

$$M ::= \underline{\mathrm{a}}.M \mid 0 \mid M + M \qquad \underline{\mathrm{a}} \in \mathcal{E}$$

$$Con ::= M \mid Con \underset{L}{\bowtie} Con \qquad L \subseteq \mathcal{E}$$

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Controlled system

$ConSys ::= \Sigma \underset{L}{\bowtie} Con \qquad L \subseteq \mathcal{E}$

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Well-defined HYPE system

$\mathcal{C}_s(\vec{X}) \stackrel{def}{=} \underline{a}_1 : \alpha_1.\mathcal{C}_s(\vec{X}) + \ldots + \underline{a}_n : \alpha_n.\mathcal{C}_s(\vec{X}) \quad \underline{a}_i \neq \underline{a}_j$

$\underline{\mathrm{init}} : (\iota, \_, \_)$ appears exactly once

$\underline{a} : (\iota, \_, \_)$ appears at most once

synchronisation on shared events

Orbiter Temperature Control in HYPE

Orbiter Temperature Control in HYPE

Passing of time

$$\mathrm{Time} \stackrel{\mathit{def}}{=} \underline{\mathrm{light}}:(t, 1, \mathit{const}).\mathrm{Time} + \underline{\mathrm{dark}}:(t, 1, \mathit{const}).\mathrm{Time} + \underline{\mathrm{init}}:(t, 1, \mathit{const}).\mathrm{Time}$$

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Effect of sun

$$\text{Sun} \stackrel{\mathit{def}}{=} \underline{\text{light}}:(s, r_s, \mathit{const}).\text{Sun} + \underline{\text{dark}}:(s, 0, \mathit{const}).\text{Sun} + \underline{\text{init}}:(s, 0, \mathit{const}).\text{Sun}$$

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Continual cooling

$$\mathrm{Cool}(X) \stackrel{\mathit{def}}{=} \underline{\mathrm{init}}\!:\!(c, -1, \mathit{linear}(X)).\mathrm{Cool}(X)$$

Orbiter Temperature Control in HYPE (continued)

Orbiter Temperature Control in HYPE (continued)

Effect of heater

$$\begin{aligned}\mathrm{Heat} \stackrel{\mathit{def}}{=} &\ \underline{\mathrm{on}}\!:\!(h, r_h, \mathit{const}).\mathrm{Heat} + \underline{\mathrm{off}}\!:\!(h, 0, \mathit{const}).\mathrm{Heat} + \\ &\ \underline{\mathrm{init}}\!:\!(h, 0, \mathit{const}).\mathrm{Heat}\end{aligned}$$

Orbiter Temperature Control in HYPE (continued)

Effect of heater

$$\mathrm{Heat} \stackrel{def}{=} \underline{\mathrm{on}}:(h, r_h, const).\mathrm{Heat} + \underline{\mathrm{off}}:(h, 0, const).\mathrm{Heat} + \underline{\mathrm{init}}:(h, 0, const).\mathrm{Heat}$$

Effect of shade

$$\mathrm{Shade} \stackrel{def}{=} \underline{\mathrm{up}}:(d, -r_d, const).\mathrm{Shade} + \underline{\mathrm{down}}:(h, 0, const).\mathrm{Shade} + \underline{\mathrm{init}}:(d, 0, const).\mathrm{Shade}$$

Orbiter Temperature Control in HYPE (continued)

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Uncontrolled system

$$\mathrm{Sys} \stackrel{\mathit{def}}{=} (\mathrm{Heat} \underset{\{\underline{\mathrm{init}}\}}{\bowtie} \mathrm{Shade}) \underset{\{\underline{\mathrm{init}}\}}{\bowtie} (\mathrm{Cool}(K) \underset{\{\underline{\mathrm{init}}\}}{\bowtie} \mathrm{Sun} \underset{\{\underline{\mathrm{init}},\underline{\mathrm{light}},\underline{\mathrm{dark}}\}}{\bowtie} \mathrm{Time})$$

Orbiter Temperature Control in HYPE (continued)

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Controllers

$Con_h \stackrel{def}{=} \underline{\mathrm{on}}.\underline{\mathrm{off}}.Con_h$
$Con_d \stackrel{def}{=} \underline{\mathrm{up}}.\underline{\mathrm{down}}.Con_d$
$Con_s \stackrel{def}{=} \underline{\mathrm{light}}.\underline{\mathrm{dark}}.Con_s$

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$Con \stackrel{def}{=} Con_h \underset{\emptyset}{\bowtie} Con_d \underset{\emptyset}{\bowtie} Con_s$

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Controlled system

$OTC \stackrel{def}{=} Sys \underset{M}{\bowtie} \underline{init}.Con \quad M = \{\underline{init}, \underline{on}, \underline{off}, \underline{up}, \underline{down}, \underline{light}, \underline{dark}\}$

HYPE syntax (continued)

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HYPE model

$(ConSys, \mathcal{V}, \mathcal{X}, IN, IT, \mathcal{E}, \mathcal{A}, ec, iv, EC, ID)$

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$\mathcal{V}$, actual variables; $\mathcal{X}$, formal variables

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EC, event conditions, (activation condition, reset)

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ID, influence descriptions, $[\![I(\vec{X})]\!] = f(\vec{X})$

Orbiter Temperature Control in HYPE (continued)

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Remaining definitions

$V = \{T, K\}$

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$ec(\underline{\text{init}}) = (true, (T = 0 \wedge K = k_0))$
$ec(\underline{\text{light}}) = (T = 12, true) \quad ec(\underline{\text{dark}}) = (T = 24, T' = 0)$

Orbiter Temperature Control in HYPE (continued)

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$[\![const]\!] = 1 \quad [\![linear(X)]\!] = X$

Operational semantics

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Definitions

state: $\sigma : IN \rightarrow \mathbb{R} \times IT$ configuration: $\langle ConSys, \sigma \rangle$

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Change identifying function: $\Gamma : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \to \mathcal{S}$

$$(\Gamma(\sigma, \tau, \tau'))(\iota) = \begin{cases} \tau(\iota) & \text{if } \sigma(\iota) = \tau'(\iota) \\ \tau'(\iota) & \text{if } \sigma(\iota) = \tau(\iota) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Operational semantics (continued)

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Prefix with influence:
$$\frac{}{\langle \underline{a}:(\iota, r, I).E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma[\iota \mapsto (r, I)]\rangle}$$

Prefix without influence:
$$\frac{}{\langle \underline{a}.E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma\rangle}$$

Choice:
$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle} \qquad \frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}$$

Constant:
$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle A, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}(A \stackrel{def}{=} E)$$

Operational semantics (continued)

Prefix with influence:
$$\frac{}{\langle \underline{a}:(\iota, r, I).E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma[\iota \mapsto (r, I)]\rangle}$$

Prefix without influence:
$$\frac{}{\langle \underline{a}.E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma\rangle}$$

Choice:
$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle} \qquad \frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}$$

Constant:
$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle A, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}(A \stackrel{def}{=} E)$$

Operational semantics (continued)

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Parallel without synchronisation:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E \underset{M}{\bowtie} F, \sigma\rangle \xrightarrow{\underline{a}} \langle E' \underset{M}{\bowtie} F, \sigma'\rangle} \quad \underline{a} \notin M$$

$$\frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E \underset{M}{\bowtie} F, \sigma\rangle \xrightarrow{\underline{a}} \langle E \underset{M}{\bowtie} F', \sigma'\rangle} \quad \underline{a} \notin M$$

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$\underline{a} \in M, \Gamma$ defined

Operational semantics (continued)

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Orbiter Temperature Control – transition derivation

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Orbiter Temperature Control – labelled transition system

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8 configurations represent all possibilities with 8 distinct states

$$
\begin{aligned}
\sigma_0 &= \{h \mapsto (0, const),\ d \mapsto (0, const),\ s \mapsto (0, const)\} \cup D\\
\sigma_1 &= \{h \mapsto (0, const),\ d \mapsto (0, const),\ s \mapsto (r_s, const)\} \cup D\\
\sigma_2 &= \{h \mapsto (0, const),\ d \mapsto (-r_d, const),\ s \mapsto (0, const)\} \cup D\\
\sigma_3 &= \{h \mapsto (0, const),\ d \mapsto (-r_d, const),\ s \mapsto (r_s, const)\} \cup D\\
\sigma_4 &= \{h \mapsto (r_h, const),\ d \mapsto (0, const),\ s \mapsto (0, const)\} \cup D\\
\sigma_5 &= \{h \mapsto (r_h, const),\ d \mapsto (0, const),\ s \mapsto (r_s, const)\} \cup D\\
\sigma_6 &= \{h \mapsto (r_h, const),\ d \mapsto (-r_d, const),\ s \mapsto (0, const)\} \cup D\\
\sigma_7 &= \{h \mapsto (r_h, const),\ d \mapsto (-r_d, const),\ s \mapsto (r_s, const)\} \cup D
\end{aligned}
$$

where $D = \{c \mapsto (-1, linear(K)),\ t \mapsto (1, k)\}$

Hybrid semantics

Hybrid semantics

Extracting ODEs from states

each state σ in the lts of CS provides an ODE for each $V \in \mathcal{V}$

$$CS_\sigma = \left\{ \text{ODE for variable } V \;\middle|\; V \in \mathcal{V} \right\}$$

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Constructing an ODE for state σ

$$\frac{dV}{dt} = \sum \left\{ r[\![I(\vec{W})]\!] \;\middle|\; iv(\iota) = V \text{ and } \sigma(\iota) = (r, I(\vec{W})) \right\}$$

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- determine from σ its rate and influence type

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- determine from σ its rate and influence type
- multiply its rate and influence function together

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- for any influence name associated with V
- determine from σ its rate and influence type
- multiply its rate and influence function together
- sum these over all associated influence names

Orbiter Temperature Control – ODEs

Orbiter Temperature Control – ODEs

Sun shining and shade up

Consider the state

$$\begin{aligned}\sigma_3 \quad = \quad & \{h \mapsto (0, \mathit{const}),\ d \mapsto (-r_d, \mathit{const}), \\ & s \mapsto (r_s, \mathit{const}),\ c \mapsto (-1, \mathit{linear}(K)),\ t \mapsto (1, k)\}\end{aligned}$$

Orbiter Temperature Control – ODEs

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ODEs in state σ_3

$$\frac{dT}{dt} = 1 \qquad \frac{dK}{dt} = r_s - r_d - K$$

Hybrid automata

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- $(V, E, \mathbf{X}, \mathcal{E}, \mathit{flow}, \mathit{init}, \mathit{inv}, \mathit{event}, \mathit{jump}, \mathit{reset}, \mathit{urgent})$

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  - associated ODEs: $\dot{\mathbf{X}} = \mathit{flow}(v)$
  - initial conditions: $\mathit{init}(v)$
  - invariants: $\mathit{inv}(v)$

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  - initial conditions: $\mathit{init}(v)$
  - invariants: $\mathit{inv}(v)$
- (control) switches: $e \in E$
  - events: $\mathit{event}(e) \in \mathcal{E}$
  - predicate on $\mathbf{X}$: $\mathit{jump}(e)$
  - predicate on $\mathbf{X} \cup \mathbf{X}'$: $\mathit{reset}(e)$
  - boolean: $\mathit{urgent}(e)$

HYPE model to hybrid automaton

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- modes V: set of reachable configurations

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- if $v_j = \langle P_j, \sigma_j \rangle$ then
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- let e be an edge associated with $\underline{a}$ and let $ec(\underline{a}) = (act_{\underline{a}}, res_{\underline{a}})$

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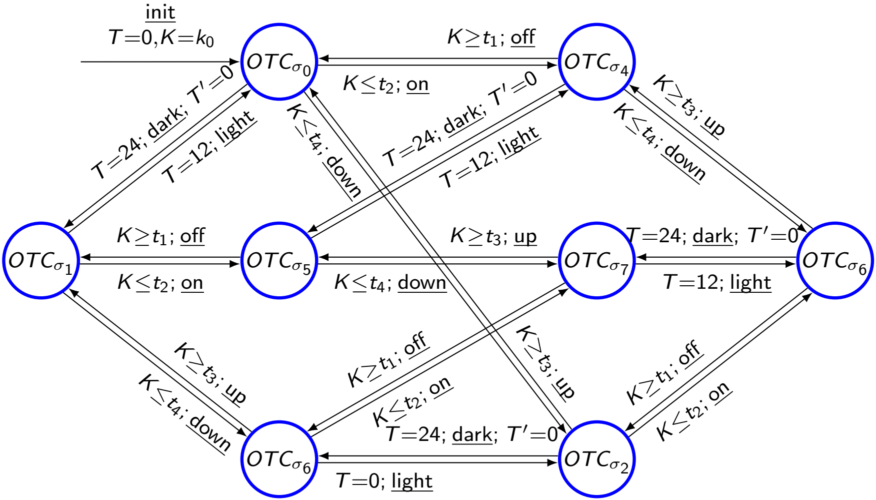
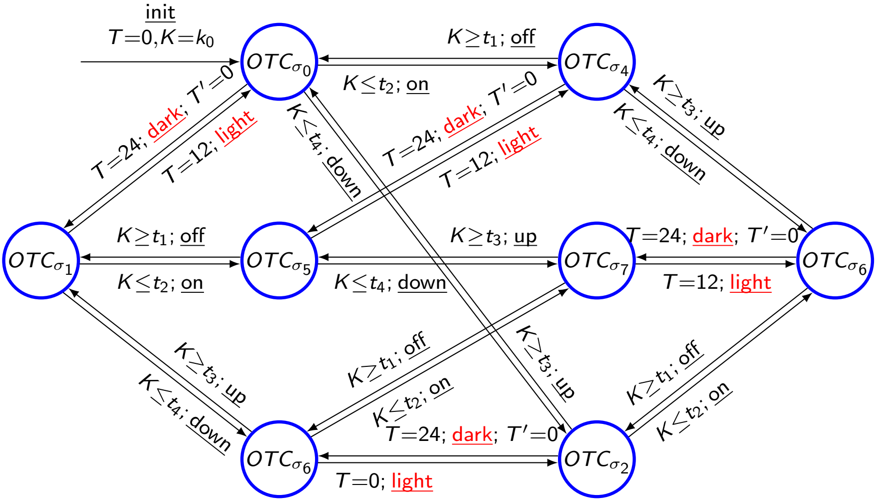
    else $\mathit{jump}(e) = \mathit{true}$ and $\mathit{urgent}(e) = \mathit{false}$
- $\mathit{init}(v) = \begin{cases} \mathit{res}_{\underline{\mathrm{init}}} & \text{if } v = \langle P, \sigma \rangle \text{ with primes removed} \\ \mathit{false} & \mathit{otherwise} \end{cases}$

Orbiter Temperature Control – hybrid automaton



Orbiter Temperature Control – hybrid automaton



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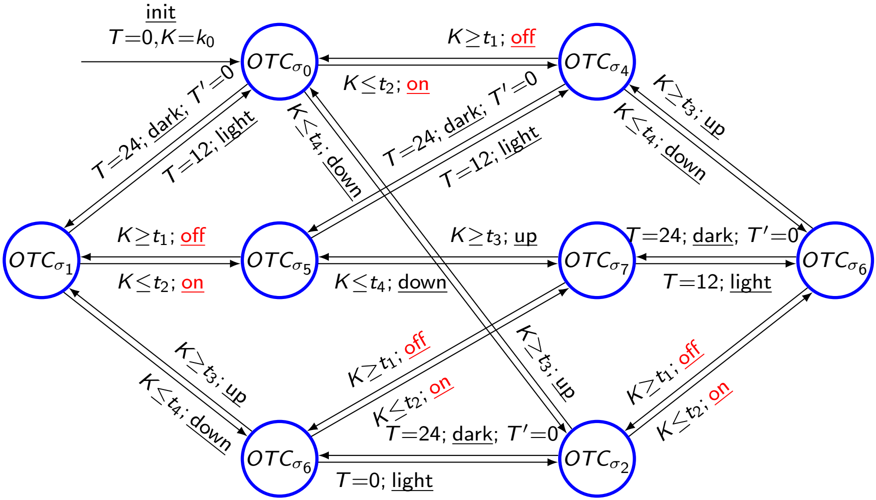

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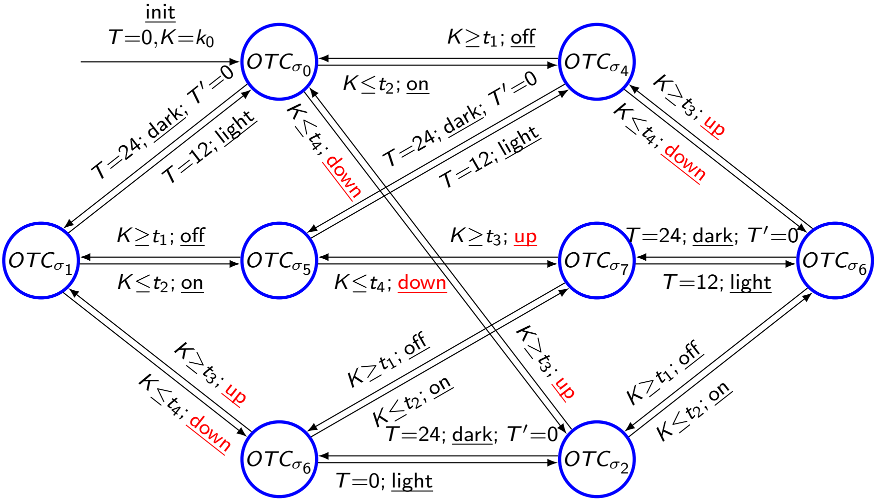

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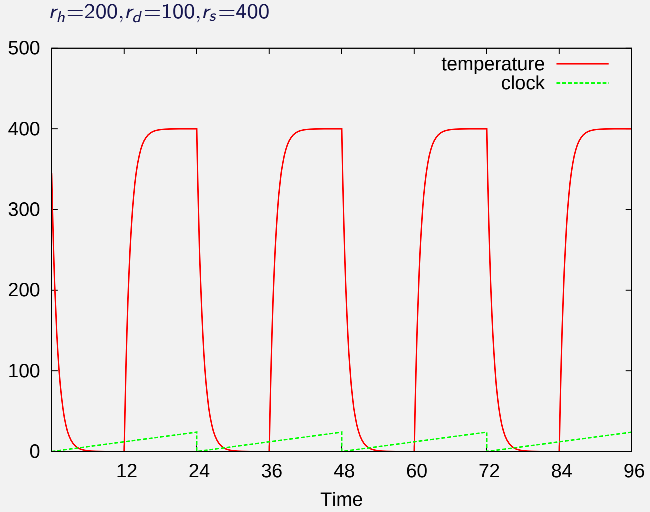

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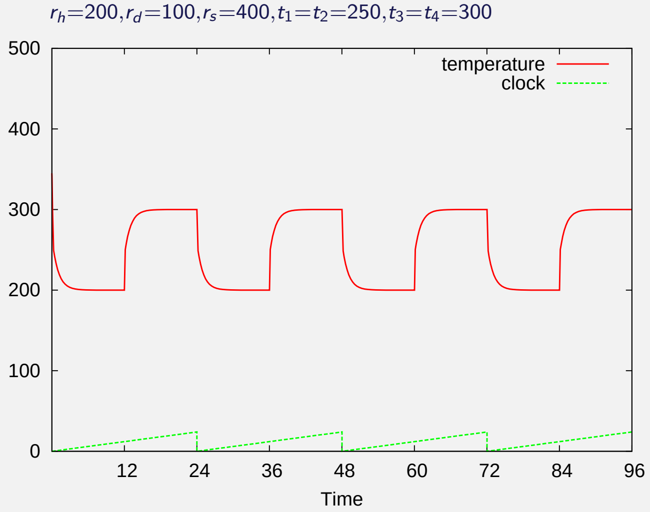

Equivalence semantics

Equivalence semantics

System bisimulation

relation B if for all $(P, Q) \in B$ whenever

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System bisimilar

$P \sim_s Q$ if in a system bisimulation

Results

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Theorem 1

$\sim_s$ *is a congruence for all operators*

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if uncontrolled systems Σ_1 *and* Σ_2 *have the same prefixes then* $\Sigma_1 \underset{L}{\bowtie} \underline{init}.Con \sim_s \Sigma_2 \underset{L}{\bowtie} \underline{init}.Con$*, assuming well-defined systems*

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Theorem 3

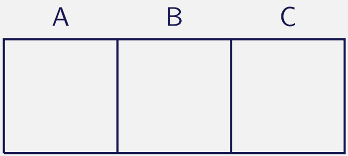
if $P \sim_s Q$ then $P_\sigma = Q_\sigma$ for all σ, assuming well-defined systems

in other words, bisimilar well-defined models have the same ODEs

Heater example

Heater example

Three rooms



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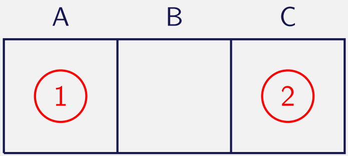

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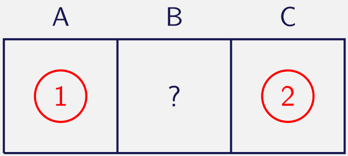

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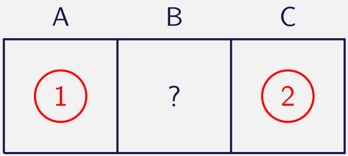

Room

$R_B(T) \stackrel{def}{=} \underline{\mathrm{init}}:(t_{0,B}, -1, linear(T)).R_B(T)$

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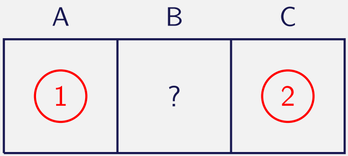

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Heater

$$H_{i,x,B} \stackrel{def}{=} \underline{\text{off}}_i : (t_{i,B}, 0, c).H_{i,x,B} + \underline{\text{on}}_i : (t_{i,B}, r_i, c_{\psi(x,B)}).H_{i,x,B} + \underline{\text{init}} : (t_{i,B}, 0, c).H_{i,x,B}$$

Heater example (continued)

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System

$$Sys \stackrel{def}{=} (H_{1,A,B} \underset{\{\underline{\mathrm{init}}\}}{\bowtie} H_{2,C,B}) \underset{\{\underline{\mathrm{init}}\}}{\bowtie} R_B(T_B)$$

$$Con \stackrel{def}{=} Con_1 \underset{\emptyset}{\bowtie} Con_2 \qquad Con_i \stackrel{def}{=} \underline{\mathrm{on}}_i.\underline{\mathrm{off}}_i.Con_i$$

$$HSys \stackrel{def}{=} Sys \underset{M}{\bowtie} \underline{\mathrm{init}}.Con \qquad M = \{\underline{\mathrm{init}}, \underline{\mathrm{on}}_1, \underline{\mathrm{off}}_1, \underline{\mathrm{on}}_2, \underline{\mathrm{off}}_2\}$$

Heater example (continued)

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HYPE model

$\mathcal{V} = \{T_B\}$ with $iv(t_{i,B}) = T_B$

$ec(\underline{\text{off}}_i) = ((T_B = 25), (T'_B = T_B))$

$ec(\underline{\text{on}}_i) = (\perp, true)$

$ec(\underline{\text{init}}) = (true, (T'_B = T_0))$

$[\![const_{adj}]\!] = 0.5 \qquad [\![const]\!] = 1 \qquad [\![linear(X)]\!] = X$

Heater system as a hybrid automata

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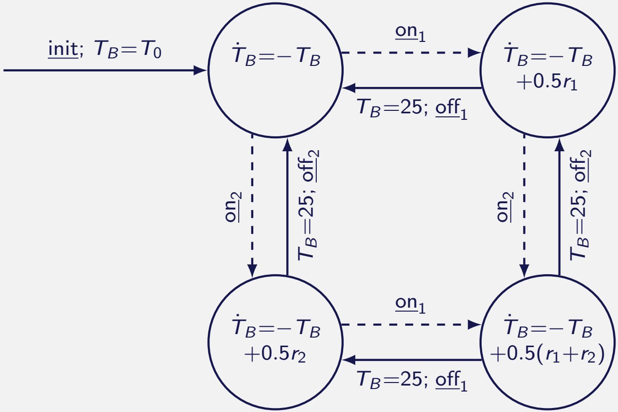

Different heater systems

Different heater systems

Heater 1 in Room A and Heater 2 in Room C

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Comparison

- Sys and Sys' have the same prefixes
- by Theorem 2, $HSys \sim_s HSys'$
- by Theorem 3, $HSys$ and $HSys'$ have the same ODEs

Other equivalences

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bisimulation for ACP^{srt}_{hs}

defined over pairs $(\langle P, \sigma\rangle, \langle Q, \sigma\rangle)$
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Theorem 4

over HYPE models, all three equivalences are the same

A more flexible equivalence

A more flexible equivalence

System equivalence with respect to $\equiv$

relation B if for all $(P, Q) \in B$ whenever

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Theorem 5

if $\equiv$ preserves updating then system equivalence with respect to $\equiv$ is a congruence

Comparison with hybrid automata

Comparison with hybrid automata

Hybrid automata synchronised product: $H_1 \times H_2$

union of variables, product of vertices
product of edges if they have the same event label
functions on vertices: *flow*, *init*, *inv* defined by conjunction
functions on edges: *event*, *jump*, *reset*, *urgent* depend on edge type

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HYPE model synchronisation: $P_1 \otimes P_2$

appropriate synchronisation of P_1 and P_2
union of variables, union of events, union of activities
union of influence names with $IN_1 \cap IN_2 = \emptyset$
union of influence types with matching definition if in $ID_1 \cap ID_2$
function on influence names: *iv* defined as union
function on events: *ec* defined as conjunction if shared,

Comparison results

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Motivation

how compositional is each formalism?

assess at level of model composition

Definition 6

Let $\mathcal{H}(P)$ be the HA obtained from HYPE model P

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Theorem 7

If no variables are shared, then $\mathcal{H}(P_1 \otimes P_2) = \mathcal{H}(P_1) \times \mathcal{H}(P_2)$

Modelling expressiveness

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Theorem 8

HYPE model synchronisation is more expressive than hybrid automata synchronised product if there are shared variables

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For HYPE models, as long as influence names disjoint, synchronisation is defined $\square$

Modelling expressiveness

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Implications

With HA product, only new variables can be added
With HYPE model synchronisation, new influences on existing variables can be added

Adding stochastic behaviour

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Stochastic HYPE: syntax

remove non-urgency $\perp$ in event conditions

replace with rate

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Stochastic HYPE: semantics

piecewise deterministic Markov processes (PDMP)
complex and rarely used
transition-driven stochastic hybrid automata (Bortolussi)

Continuous, discrete and stochastic

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Transition-driven stochastic hybrid automata

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- V, finite set of control modes

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- mapping to PDMPs

Stochastic HYPE

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Work in progress

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- modify HYPE definition

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- use TDSHAs as underlying semantics

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Definition 9

Let $\mathcal{T}(P)$ be the TDSHA obtained from HYPE model P

Stochastic HYPE

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Definition 9

Let $\mathcal{T}(P)$ be the TDSHA obtained from HYPE model P

Preliminary result

for a non-stochastic HYPE model P without non-urgent conditions
$\mathcal{T}(P) = \mathcal{H}(P)$

Conclusion

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Summary

- HYPE for modelling hybrid systems
- use of additive flows to obtain ODEs compositionally
- now modelling continuous, discrete and stochastic behaviour

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Main results

- congruence of semantic equivalences
- system bisimilar HYPE models gives identical ODEs
- combining HYPE models is more expressive than combining HAs

Thank you