How to use HYPE to model hybrid systems: a railway crossing example

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Outline

Introduction

- hybrid systems
  - discrete behaviour
  - continuous behaviour, expressed as ODEs

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- hybrid automata
  - well known, graphical
  - somewhat compositional

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- hybrid systems
  - discrete behaviour
  - continuous behaviour, expressed as ODEs
- hybrid automata
  - well known, graphical
  - somewhat compositional
- process algebras for hybrid systems
  - textual
  - compositional
  - often require monolithic ODEs in syntax
  - HYPE: more fine-grained, individual flows

Train gate system

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- standard example for hybrid systems modelling

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- track to be modelled
  - starts at least 1500m before crossing
  - ends 100m after crossing

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  - first: 1000m before crossing sends *approach* signal to controller
  - second: 100m after crossing sends *exit* signal to controller

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  - first: 1000m before crossing sends *approach* signal to controller
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- train speed
  - between 48m/s and 52m/s before first sensor
  - between 40m/s and 52m/s after first sensor

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- train speed
  - between 48m/s and 52m/s before first sensor
  - between 40m/s and 52m/s after first sensor
- gate
  - opens and closes at 20°/s

Train gate system (continued)

- controller
  - on receiving *approach* signal, takes up to 5 seconds to send *lower* signal to gate
  - on receiving *exit* signal, takes up to 5 seconds to send *raise* signal to gate

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  - on receiving *approach* signal, takes up to 5 seconds to send *lower* signal to gate
  - on receiving *exit* signal, takes up to 5 seconds to send *raise* signal to gate
- initial conditions
  - train starts at 1400m before crossing, gate is open (at 90°)

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  - train starts at 1400m before crossing, gate is open (at 90°)
- safety property
  - when the train is 100m before the gate, the gate is closed

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- initial conditions
  - train starts at 1400m before crossing, gate is open (at 90°)
- safety property
  - when the train is 100m before the gate, the gate is closed
- assumptions for HYPE model, worst case
  - trains start at 1500m before crossing
  - signals from controller take 5 seconds to be received by gate
  - trains travel at 52m/s

HYPE syntax

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- events: instantaneous, discrete changes

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  $\underline{\text{a}} \in \mathcal{E}$

- activities: influences on continuous aspect of system, flows

  $\alpha \in \mathcal{A} \qquad \alpha(\mathcal{W}) = (\iota, r, I(\mathcal{W}))$

  - influence name $\iota \in IN$
  - rate $r \in \mathbb{R}$
  - influence type $I(\mathcal{W})$ with $[\![I(\mathcal{W})]\!] = f(\mathcal{W})$ with $\mathcal{W} \subseteq \mathcal{V}$

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  - influence type $I(\mathcal{W})$ with $[\![I(\mathcal{W})]\!] = f(\mathcal{W})$ with $\mathcal{W} \subseteq \mathcal{V}$
- well-defined subcomponent: $\underline{\mathrm{a}}_i \neq \underline{\mathrm{a}}_j$ for $i \neq j$

$$C_s(\mathcal{W}) \stackrel{def}{=} \sum_{i=1}^{n} \underline{\mathrm{a}}_i : (\iota, r_i, I_i(\mathcal{W})).C_s(\mathcal{W}) + \underline{\mathrm{init}} : (\iota, r, I(\mathcal{W})).C_s(\mathcal{W})$$

Continuous aspects of train gate system

- train

$$Train \stackrel{def}{=} \underline{\mathrm{init}}:(d, r_{tr}, c).Train$$

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- continuous aspects of gate

$$\begin{aligned} Gate \stackrel{def}{=} \; & \underline{\text{lower}}:(g, -r_{gt}, c).Gate + \underline{\text{closed}}:(g, 0, c).Gate \; + \\ & \underline{\text{raise}}:(g, r_{gt}, c).Gate + \underline{\text{open}}:(g, 0, c).Gate \; + \\ & \underline{\text{init}}:(g, 0, c).Gate \end{aligned}$$

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- two timers

$$\begin{aligned} Timer_L \stackrel{def}{=} \; & \underline{\text{appr}}:(t_L, 1, const).Timer_L + \underline{\text{lower}}:(t_L, 0, const).Timer_L + \\ & \underline{\text{init}}:(t_L, 0, const).Timer_L \\ Timer_R \stackrel{def}{=} \; & \underline{\text{exit}}:(t_R, 1, const).Timer_R + \underline{\text{raise}}:(t_R, 0, const).Timer_R + \\ & \underline{\text{init}}:(t_R, 0, const).Timer_R \end{aligned}$$

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- controller: $M ::= \underline{\mathrm{a}}.M \mid 0 \mid M + M \qquad \underline{\mathrm{a}} \in \mathcal{E}$

  $Con ::= M \mid Con \underset{L}{\bowtie} Con \qquad L \subseteq \mathcal{E}$

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- controlled system: $ConSys ::= \Sigma \underset{L}{\bowtie} \underline{\mathrm{init}}.Con \qquad L \subseteq \mathcal{E}$
- HYPE model: $(ConSys, \mathcal{V}, IN, IT, \mathcal{E}, \mathcal{A}, \mathrm{ec}, \mathrm{iv}, EC, ID)$
  - IN influence names, IT influence types
  - $\mathrm{ec} : \mathcal{E} \to EC$, associates events with event conditions
  - EC, event conditions, (activation condition, reset)
  - $\mathrm{iv} : IN \to \mathcal{V}$, associates influence names with variables
  - ID influence descriptions, $[\![I(\vec{X})]\!] = f(\vec{X})$,

Discrete aspects of the train gate system

- sequencing of train events

$$Seq_a \stackrel{def}{=} \underline{\text{appr}}.Seq_p \quad Seq_p \stackrel{def}{=} \underline{\text{pass}}.Seq_e \quad Seq_e \stackrel{def}{=} \underline{\text{exit}}.Seq_a$$

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- gate internal controller

$$\begin{aligned}
GC_o &\stackrel{def}{=} \underline{\text{raise}}.GC_o + \underline{\text{lower}}.GC_l \\
GC_l &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_l + \underline{\text{closed}}.GC_c \\
GC_c &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_c \\
GC_r &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_l + \underline{\text{open}}.GC_o
\end{aligned}$$

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- event conditions

$$\mathrm{ec}(\underline{\text{appr}}) = (D = -1000,\ T_L' = 0) \qquad \mathrm{ec}(\underline{\text{pass}}) = (D = 0, \mathit{true})$$

$$\mathrm{ec}(\underline{\text{exit}}) = (D = 100,\ T_R' = 0 \wedge D' = -1500)$$

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- event conditions

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\mathrm{ec}(\underline{\text{appr}}) &= (D = -1000, T'_L = 0) \qquad & \mathrm{ec}(\underline{\text{pass}}) &= (D = 0, \mathit{true}) \\
\mathrm{ec}(\underline{\text{exit}}) &= (D = 100, T'_R = 0 \wedge D' = -1500) & &
\end{aligned}$$

$$\begin{aligned}
\mathrm{ec}(\underline{\text{lower}}) &= (T_L = 5, T'_L = 0) \qquad & \mathrm{ec}(\underline{\text{closed}}) &= (G = 0, \mathit{true}) \\
\mathrm{ec}(\underline{\text{raise}}) &= (T_R = 5, T'_R = 0) & \mathrm{ec}(\underline{\text{open}}) &= (G = 90, \mathit{true})
\end{aligned}$$

Discrete aspects of the train gate system (continued)

- system controller

$$
\begin{aligned}
Con_a &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_a \\
Con_l &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_l + \underline{\text{lower}}.Con_e \\
Con_e &\stackrel{def}{=} \underline{\text{appr}}.Con_e + \underline{\text{exit}}.Con_r \\
Con_r &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_r + \underline{\text{raise}}.Con_a
\end{aligned}
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Con_r &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_r + \underline{\text{raise}}.Con_a
\end{aligned}
$$

- additional items

$$
\begin{array}{lclclcl}
\mathrm{iv}(g) & = & G & \quad & \mathrm{iv}(t_L) & = & T_L \\
\mathrm{iv}(d) & = & D & \quad & \mathrm{iv}(t_R) & = & T_R
\end{array}
$$

Discrete aspects of the train gate system (continued)

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\mathrm{iv}(d) & = & D & \quad \mathrm{iv}(t_R) & = & T_R & \quad [\![c]\!] & = & 1
\end{array}$$

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\end{aligned}$$

- additional items

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\mathrm{iv}(g) & = & G & \mathrm{iv}(t_L) & = & T_L & & & \\
\mathrm{iv}(d) & = & D & \mathrm{iv}(t_R) & = & T_R & [\![c]\!] & = & 1
\end{array}$$

$$\mathrm{ec}(\underline{\text{init}}) = (true, G' = 90 \wedge D' = -1400 \wedge T_L' = 0 \wedge T_R' = 0)$$

Discrete aspects of the train gate system (continued)

- system controller

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\end{aligned}
$$

- additional items

$$
\begin{array}{rclrclrcl}
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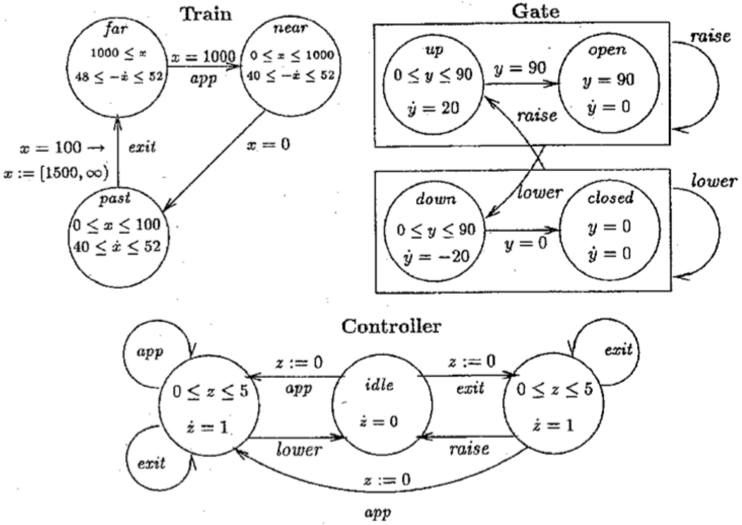
\mathrm{iv}(d) & = & D & \mathrm{iv}(t_R) & = & T_R & [\![c]\!] & = & 1
\end{array}
$$

$$
\mathrm{ec}(\underline{\text{init}}) = (true, G' = 90 \wedge D' = -1400 \wedge T_L' = 0 \wedge T_R' = 0)
$$

- controlled system

$$
(Gate \underset{*}{\bowtie} Train \underset{*}{\bowtie} Timer_L \underset{*}{\bowtie} Timer_R) \underset{*}{\bowtie} \underline{\text{init}}.((Con_a \underset{*}{\bowtie} GC_o) \underset{*}{\bowtie} Seq_a)
$$

Train gate system as a hybrid automata product



Operational semantics

- state: $\sigma : \mathit{IN} \rightarrow (\mathbb{R} \times \mathit{IT})$

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- configuration: $\left\langle ConSys, \sigma \right\rangle$
- labelled transition system: $(\mathcal{F}, \mathcal{E}, \to \subseteq \mathcal{F} \times \mathcal{E} \times \mathcal{F})$
- updating function: $\sigma[\iota \mapsto (r, I)]$

$$\sigma[\iota \mapsto (r, I)](x) = \begin{cases} (r, I) & \text{if } x = \iota \\ \sigma(x) & \text{otherwise} \end{cases}$$

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- change identifying function: $\Gamma : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \to \mathcal{S}$

$$(\Gamma(\sigma, \tau, \tau'))(\iota) = \begin{cases} \tau(\iota) & \text{if } \sigma(\iota) = \tau'(\iota) \\ \tau'(\iota) & \text{if } \sigma(\iota) = \tau(\iota) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Operational semantics (cont.)

Prefix with influence:

$$\frac{}{\langle \underline{a}:(\iota, r, I).E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma[\iota \mapsto (r, I)]\rangle}$$

Prefix without influence:

$$\frac{}{\langle \underline{a}.E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma\rangle}$$

Choice:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle} \qquad \frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}$$

Constant:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle A, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}(A \stackrel{def}{=} E)$$

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Operational semantics (cont.)

Parallel without synchronisation:

$$\frac{\langle E, \sigma \rangle \xrightarrow{\underline{a}} \langle E', \sigma' \rangle}{\langle E \underset{M}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E' \underset{M}{\bowtie} F, \sigma' \rangle} \quad \underline{a} \notin M$$

$$\frac{\langle F, \sigma \rangle \xrightarrow{\underline{a}} \langle F', \sigma' \rangle}{\langle E \underset{M}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E \underset{M}{\bowtie} F', \sigma' \rangle} \quad \underline{a} \notin M$$

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$\underline{a} \in M, \Gamma$ defined

Operational semantics (cont.)

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$\underline{a} \in M, \Gamma$ defined

Hybrid semantics

- extract ODEs from each state σ for each configuration

$$CS_\sigma = \left\{ \text{ODE for variable } V \;\middle|\; V \in \mathcal{V} \right\} \text{ where}$$

$$\frac{dV}{dt} = \sum \left\{ r[\![\mathcal{W}]\!] \;\middle|\; \mathrm{iv}(\iota) = V \text{ and } \sigma(\iota) = (r, \mathcal{W}) \right\}$$

  - for any influence name associated with V
  - determine from σ its rate and influence type
  - multiply its rate and influence function together
  - sum these over all associated influence names

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  - for any influence name associated with V
  - determine from σ its rate and influence type
  - multiply its rate and influence function together
  - sum these over all associated influence names
- map labelled transition system to hybrid automaton
  - configurations are modes, transitions are edges
  - ODEs are flows at configurations

Train gate system

- configurations have the form

$$\Big\langle \big(Gate \underset{*}{\bowtie} Train \underset{*}{\bowtie} Timer_L \underset{*}{\bowtie} Timer_R\big) \underset{*}{\bowtie} \underline{\mathrm{init}}.\big((Con_x \underset{*}{\bowtie} GC_y) \underset{*}{\bowtie} Seq_z\big), \sigma \Big\rangle$$

Train gate system

- configurations have the form

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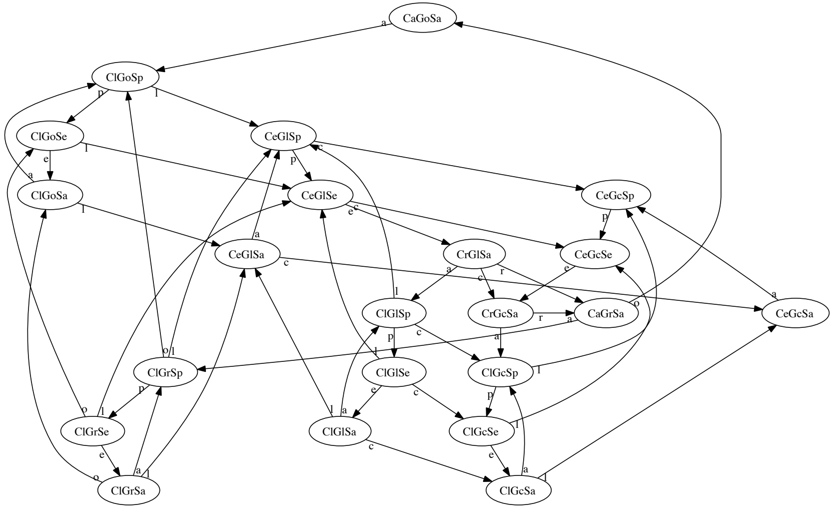
- first configuration has the following ODEs

$$\frac{dD}{dt} = r_{tr} \qquad \frac{dG}{dt} = 0 \qquad \frac{dT_L}{dt} = 0 \qquad \frac{dT_R}{dt} = 0$$

Labelled transition system of train gate system



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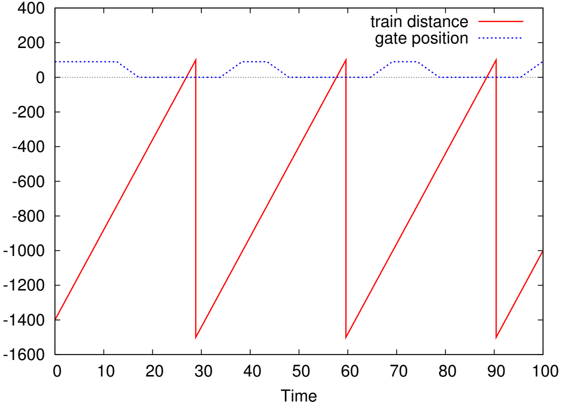

Gate closes when train is 505.99m before crossing

Modification for faster trains

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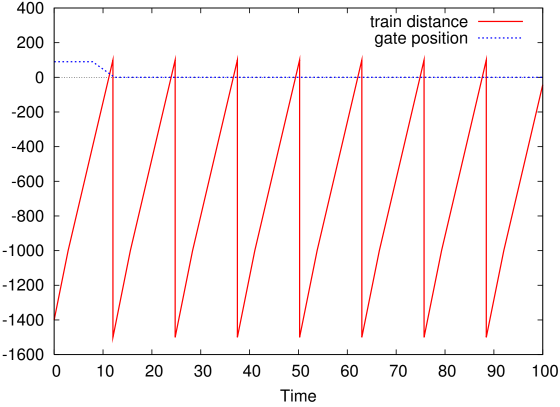
$$\frac{dD}{dt} = r_{tr} - r_{sl}$$

Modified train gate system



parameter modifications: $r_{tr} = 140, \ r_{sl} = 20$

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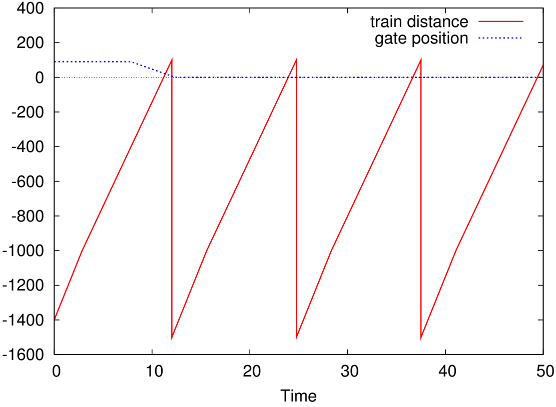

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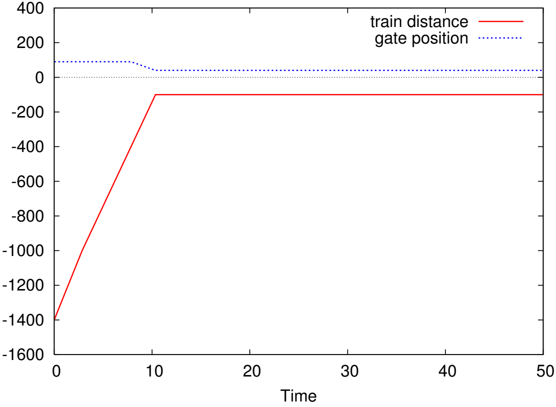
Train gate system with failure event



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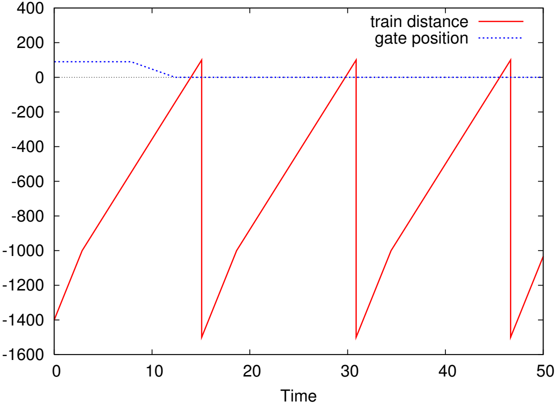
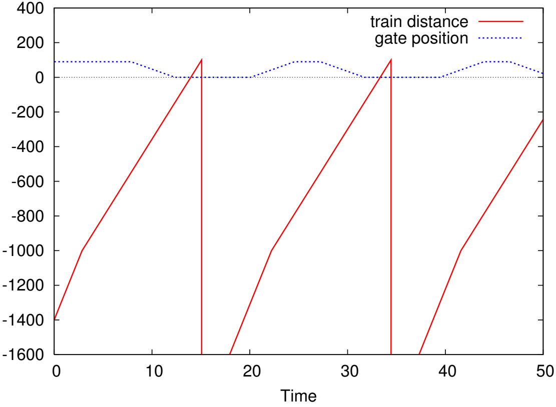
Train gate system: slower speed



parameter modifications: $r_{tr} = 140$, $r_{sl} = 50$

Train gate system: larger orginating distance



parameter modifications: $r_{tr} = 140$, $r_{sl} = 50$, $D = -2000$

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- apply result to train gate controller

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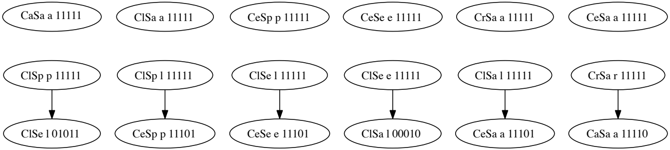

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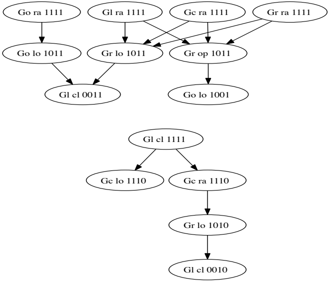

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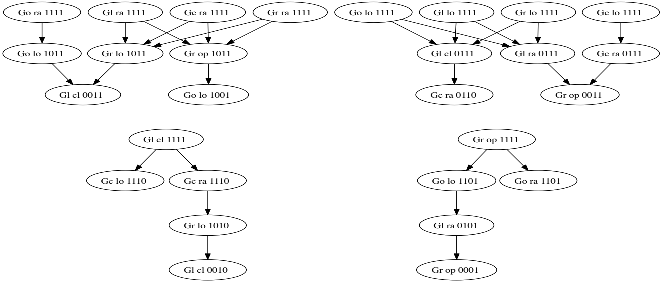

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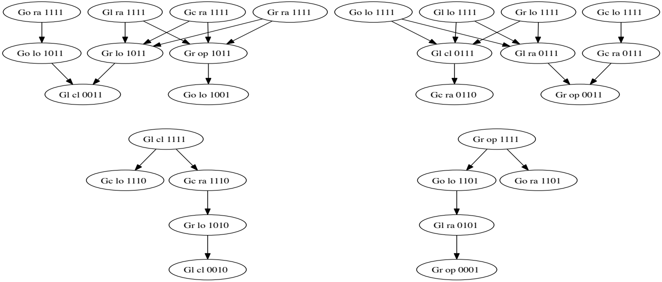

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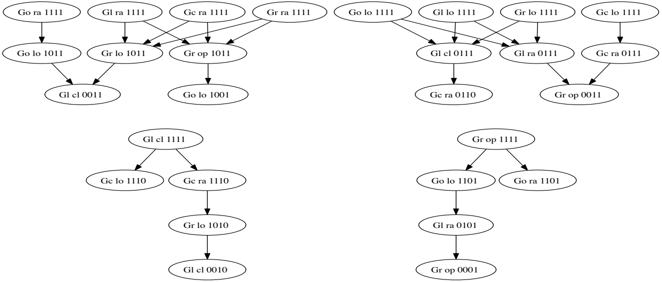

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- exclusion of infinite behaviour at a time instant
  - instantaneous Zeno behaviour
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Thank you

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- basic idea: how does the activation condition and reset of one event affect whether another event can occur immediately

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- construct graph from every valid controller state and event pair together with vector consisting of ones
  - abstracts from initial conditions