Stochastic HYPE: modelling stochastic hybrid systems

Vashti Galpin
Laboratory for Foundations of Computer Science
University of Edinburgh

Joint work with Jane Hillston (University of Edinburgh)
and Luca Bortolussi (University of Trieste)

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Introduction

- stochastic hybrid process algebra
  - discrete behaviour
  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, here using exponential distribution

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- why use a process algebra
  - compositional
  - language allows for abstract reasoning

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  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, here using exponential distribution
- why use a process algebra
  - compositional
  - language allows for abstract reasoning
- outline
  - HYPE and stochastic HYPE
  - train gate example
  - checking for infinite discrete behaviour
  - I-graphs and acyclicity

Stochastic HYPE model

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components

$$\left(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\right)$$

Stochastic HYPE model

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$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie}$$

Stochastic HYPE model

$$\overbrace{\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big)}^{\text{components}} \quad \underset{*}{\bowtie} \quad \overbrace{\big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)}^{\text{controllers}}$$

Stochastic HYPE model

components controllers

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well-defined component

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

Stochastic HYPE model

components — controllers

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components are parameterised by variables

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events have event conditions: guards and resets

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$\mathrm{ec}(\underline{a}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V}))$ discrete events

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$\mathrm{ec}(\overline{\mathrm{a}}_j) = (r, \mathcal{V} = f'(\mathcal{V}))$ stochastic events

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$$\alpha_j = (\iota_j, r_j, I(\mathcal{V}))$$

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influence names are mapped to variables

$$\mathrm{iv}(\iota_j) \in \mathcal{V}$$

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controller grammar

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$$M ::= \mathrm{a}.M \mid 0 \mid M + M$$

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  - subset of piecewise deterministic Markov processes (PDMP)

Train gate system

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  - first: 1000m before crossing sends *approach* signal to controller
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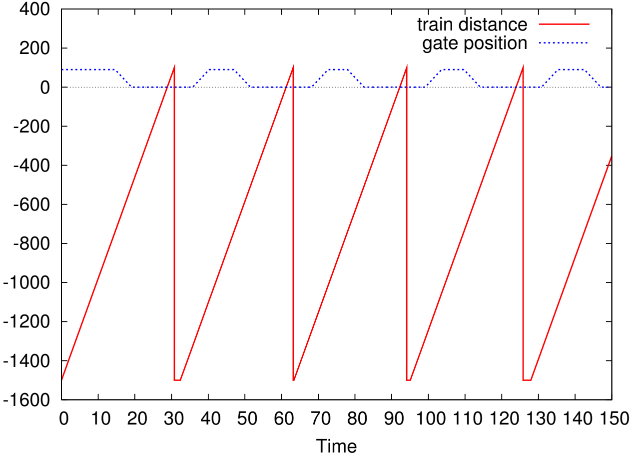
- two sensors
  - first: 1000m before crossing sends *approach* signal to controller
  - second: 100m after crossing sends *exit* signal to controller
- gate controller takes time to respond to signals
- gate opens and closes at fixed speed
- safety property
  - when the train is 100m before the gate, the gate is closed

Train gate system



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- instantaneous Zeno behaviour
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- want to check for this behaviour abstractly
  - without investigating the full behaviour of a model
  - consider controller and event conditions
  - combine these and ignore continuous behaviour

Avoiding instantaneous Zeno behaviour

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  - update of vector by event determines which other events have been activated or disabled by this event
  - include all controller states, all events, activation vector of 1's
- gives an overapproximation, ignores initial conditions

Controllers for train gate system

- system controller

$$
\begin{aligned}
Con_a &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_a \\
Con_l &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_l + \underline{\text{lower}}.Con_e \\
Con_e &\stackrel{def}{=} \underline{\text{appr}}.Con_e + \underline{\text{exit}}.Con_r \\
Con_r &\stackrel{def}{=} \underline{\text{appr}}.Con_l + \underline{\text{exit}}.Con_r + \underline{\text{raise}}.Con_a
\end{aligned}
$$

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\end{aligned}
$$

- gate internal controller

$$
\begin{aligned}
GC_o &\stackrel{def}{=} \underline{\text{raise}}.GC_o + \underline{\text{lower}}.GC_l \\
GC_l &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_l + \underline{\text{closed}}.GC_c \\
GC_c &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_c \\
GC_r &\stackrel{def}{=} \underline{\text{raise}}.GC_r + \underline{\text{lower}}.GC_l + \underline{\text{open}}.GC_o
\end{aligned}
$$

Controllers for train gate system

- sequencing of train travel

$$Seq_a \stackrel{def}{=} \underline{\mathrm{appr}}.Seq_p \qquad Seq_p \stackrel{def}{=} \underline{\mathrm{pass}}.Seq_e$$
$$Seq_e \stackrel{def}{=} \underline{\mathrm{exit}}.Seq_f \qquad Seq_f \stackrel{def}{=} \overline{\mathrm{wait}}.Seq_a$$

Controllers for train gate system

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- controller states

$Con_x \underset{*}{\bowtie} GC_y \underset{*}{\bowtie} Seq_z$ will be written as $CxGySz$

Controllers for train gate system

- sequencing of train travel

  $Seq_a \stackrel{def}{=} \underline{\text{appr}}.Seq_p \qquad Seq_p \stackrel{def}{=} \underline{\text{pass}}.Seq_e$

  $Seq_e \stackrel{def}{=} \underline{\text{exit}}.Seq_f \qquad Seq_f \stackrel{def}{=} \overline{\text{wait}}.Seq_a$

- controller states

  $Con_x \bowtie_* GC_y \bowtie_* Seq_z$ will be written as $CxGySz$

- continuous part of the system is

  $Gate \bowtie_* Train \bowtie_* Timer_L \bowtie_* Timer_R$

  and remains unchanged by the occurrence of events because of form of well-defined components

Event conditions

- 8 events (other then $\underline{\text{init}}$)

$$
\begin{array}{rcll}
\mathrm{ec}(\underline{\text{appr}}) & = & (D = -1000, & T_L' = 0) \\
\mathrm{ec}(\underline{\text{pass}}) & = & (D = 0, & \mathit{true}) \\
\mathrm{ec}(\underline{\text{exit}}) & = & (D = 100, & T_R' = 0 \wedge D' = -1500) \\
\mathrm{ec}(\overline{\text{wait}}) & = & (\mathit{delay}, & \mathit{true}) \\
\mathrm{ec}(\underline{\text{lower}}) & = & (T_L = 5, & T_L' = 0) \\
\mathrm{ec}(\underline{\text{raise}}) & = & (T_R = 5 & T_R' = 0) \\
\mathrm{ec}(\underline{\text{closed}}) & = & (G = 0, & \mathit{true}) \\
\mathrm{ec}(\underline{\text{open}}) & = & (G = 90, & \mathit{true})
\end{array}
$$

Event conditions

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\begin{aligned}
\mathrm{ec}(\underline{\text{appr}}) &= (D = -1000, & T'_L = 0)\\
\mathrm{ec}(\underline{\text{pass}}) &= (D = 0, & \mathit{true})\\
\mathrm{ec}(\underline{\text{exit}}) &= (D = 100, & T'_R = 0 \wedge D' = -1500)\\
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\mathrm{ec}(\underline{\text{lower}}) &= (T_L = 5, & T'_L = 0)\\
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\mathrm{ec}(\underline{\text{closed}}) &= (G = 0, & \mathit{true})\\
\mathrm{ec}(\underline{\text{open}}) &= (G = 90, & \mathit{true})
\end{aligned}
$$

- let $b_1 \ldots b_8$ be a binary vector where each element is associated with an event using the above ordering

Instantaneous activation graph

- $(CeGlSp, \underline{\text{closed}}, 11110111) \rightarrow (CeGcSp, \underline{\text{pass}}, 11110110)$
  - transition in controller, $CeGlSp \xrightarrow{\text{closed}} CeGcSp$
  - in source node, all events are enabled except lower
  - in target node, lower and open are disabled

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  - transition in controller, $CeGlSp \xrightarrow{\underline{\text{closed}}} CeGcSp$
  - in source node, all events are enabled except lower
  - in target node, lower and open are disabled
- closed disables open
  - closed occurs when $G = 0$ and G is not reset by closed
  - hence immediately after closed, $G = 0$
  - open occurs when $G = 90$ so disabled immediately after closed

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  - in source node, all events are enabled except <u>lower</u>
  - in target node, <u>lower</u> and <u>open</u> are disabled
- <u>closed</u> disables <u>open</u>
  - <u>closed</u> occurs when $G = 0$ and G is not reset by <u>closed</u>
  - hence immediately after <u>closed</u>, $G = 0$
  - <u>open</u> occurs when $G = 90$ so disabled immediately after <u>closed</u>
- <u>closed</u> does not enable <u>lower</u>

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  - transition in controller, $CeGlSp \xrightarrow{\underline{\text{closed}}} CeGcSp$
  - in source node, all events are enabled except lower
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- closed disables open
  - closed occurs when $G = 0$ and G is not reset by closed
  - hence immediately after closed, $G = 0$
  - open occurs when $G = 90$ so disabled immediately after closed
- closed does not enable lower
- lower disables itself: $\mathrm{ec}(\underline{\text{lower}}) = (T_L = 5, T_L' = 0)$

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- $\overline{\text{wait}}$ disables all events because it has duration

Instantaneous activation graph

- $(CeGlSp, \underline{\text{closed}}, 11110111) \rightarrow (CeGcSp, \underline{\text{pass}}, 11110110)$
  - transition in controller, $CeGlSp \xrightarrow{\underline{\text{closed}}} CeGcSp$
  - in source node, all events are enabled except <u>lower</u>
  - in target node, <u>lower</u> and <u>open</u> are disabled
- <u>closed</u> disables <u>open</u>
  - <u>closed</u> occurs when $G = 0$ and G is not reset by <u>closed</u>
  - hence immediately after <u>closed</u>, $G = 0$
  - <u>open</u> occurs when $G = 90$ so disabled immediately after <u>closed</u>
- <u>closed</u> does not enable <u>lower</u>
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- $\overline{\text{wait}}$ disables all events because it has duration
- straightforward to determine event updates

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- apply results to train gate controller
- use compositional results rather than working with all controllers

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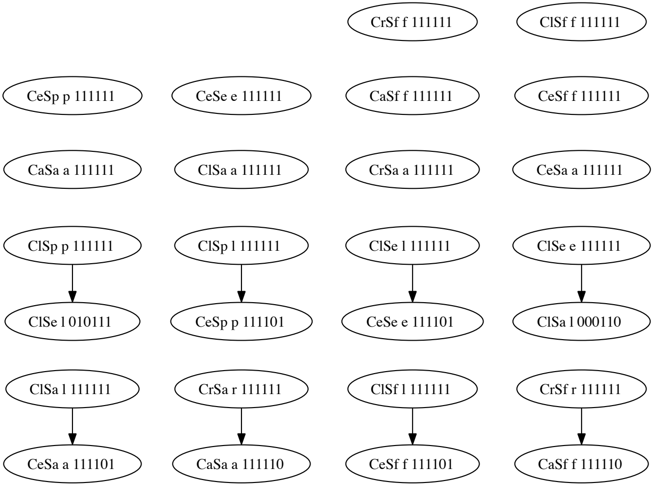
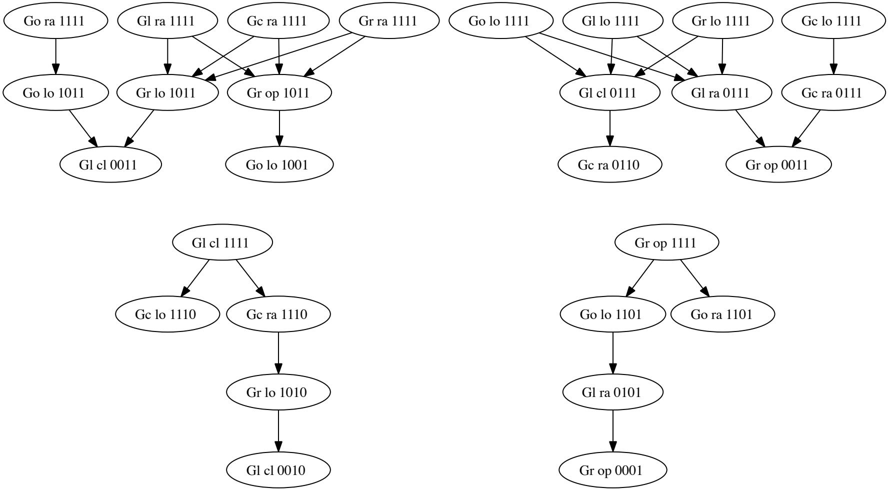
  - $(Con_l, \underline{\text{appr}}, 10001) \rightarrow (Con_l, \underline{\text{appr}}, 10001)$
- consider I-graphs of $Con_a \underset{*}{\bowtie} Seq_a$ and $Gate_a$

I-graph of system controller and sequencer



I-graph of gate controller



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- hence $Con_a \underset{*}{\bowtie} Seq_a \underset{*}{\bowtie} GC_o$ is well-behaved
- hence train gate model can be mapped successfully to a piecewise deterministic Markov process

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- exclusion of infinite behaviour at a time instant
  - instantaneous Zeno behaviour
  - well-behaved stochastic HYPE models
  - construct I-graph from controller and event conditions
  - overapproximation of behaviour
  - check for acyclicity
  - abstract from continuous behaviour

Thank you