Stochastic hybrid modelling with composition of flows

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Introduction

- stochastic hybrid process algebra
  - discrete behaviour
  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, using exponential distribution
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  - discrete behaviour
  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, using exponential distribution
- why use this process algebra
  - compositionality of flows
  - ODEs generated by model
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  - discrete behaviour
  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, using exponential distribution
- why use this process algebra
  - compositionality of flows
  - ODEs generated by model
- outline
  - stochastic HYPE
  - semantics given by TDSHA
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  - equivalent behaviour
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Assembly system

pool

machine$_1$

conveyor belt

machine$_2$

continuous variables

items in pool: $P$

items at start of conveyor belt: $B$

power consumption of machine $i$: $W_i$

timers: $T_i$, $T_V$

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Assembly system

- continuous variables
  - items in pool: $P$
  - items at start of conveyor belt: $B$
  - power consumption of machine$_i$: $W_i$
  - timers: $T_i, T$

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Uncontrolled components

\[ Feed_i \overset{def}{=} \text{init} : (p_i, arrivals_i, const).Feed_i + \text{full} : (p_i, 0, const).Feed_i \]
Uncontrolled components

\[
\text{Feed}_i \overset{\text{def}}{=} \text{init} : (p_i, \text{arrivals}_i, \text{const}).\text{Feed}_i + \text{full} : (p_i, 0, \text{const}).\text{Feed}_i
\]
Uncontrolled components

\[
Feed_i \overset{\text{def}}{=} \begin{array}{l}
\text{init} : (p_i, \text{arrivals}_i, \text{const}) . Feed_i \\
\text{full} : (p_i, 0, \text{const}) . Feed_i
\end{array}
\]
Uncontrolled components

\[ Feed_i \overset{\text{def}}{=} \text{init} : (p_i, \text{arrivals}_i, \text{const}).Feed_i + \text{full} : (p_i, 0, \text{const}).Feed_i \]

\[ Output \overset{\text{def}}{=} \text{init} : (b, \text{departures}, \text{const}).Output + \text{full} : (b, 0, \text{const}).Output \]
Uncontrolled components

\begin{align*}
  \text{Feed}_i \overset{\text{def}}{=} & \text{init} : (p_i, \text{arrivals}_i, \text{const}).\text{Feed}_i + \\
  \text{full} : & (p_i, 0, \text{const}).\text{Feed}_i \\

  \text{Output} \overset{\text{def}}{=} & \text{init} : (b, \text{departures}, \text{const}).\text{Output} + \\
  \text{full} : & (b, 0, \text{const}).\text{Output} \\

  \text{Machine}_i(W_i) \overset{\text{def}}{=} & \text{init} : (w_i, wa_i, \text{linear}(W_i)).\text{Machine}_i(W_i) + \\
  \text{prep} : & (w_i, 0, \text{const}).\text{Machine}_i(W_i) + \\
  \text{take}_i : & (w_i, wt_i, \text{linear}(W_i)).\text{Machine}_i(W_i) + \\
  \text{assem}_i : & (w_i, wa_i, \text{linear}(W_i)).\text{Machine}_i(W_i)
\end{align*}
Uncontrolled components

\[
\text{Feed}_i \overset{\text{def}}{=} \text{init} : (p_i, \text{arrivals}_i, \text{const}).\text{Feed}_i + \\
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\]

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\text{Output} \overset{\text{def}}{=} \text{init} : (b, \text{departures}, \text{const}).\text{Output} + \\
\text{full} : (b, 0, \text{const}).\text{Output}
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\text{Machine}_i(W_i) \overset{\text{def}}{=} \text{init} : (w_i, wa_i, \text{linear}(W_i)).\text{Machine}_i(W_i) + \\
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\text{assem}_i : (w_i, wa_i, \text{linear}(W_i)).\text{Machine}_i(W_i)
\]

\[
\text{Timer}_i \overset{\text{def}}{=} \text{init} : (t_i, 0, \text{const}).\text{Timer}_i + \\
\text{take}_i : (t_i, 1, \text{const}).\text{Timer}_i + \\
\text{assem}_i : (t_i, 0, \text{const}).\text{Timer}_i
\]
Uncontrolled components

\[
\begin{align*}
Feed_i & \overset{\text{def}}{=} \text{init} : (p_i, \text{arrivals}_i, \text{const}).Feed_i + \\
& \quad \text{full} : (p_i, 0, \text{const}).Feed_i \\
Output & \overset{\text{def}}{=} \text{init} : (b, \text{departures}, \text{const}).Output + \\
& \quad \text{full} : (b, 0, \text{const}).Output \\
Machine_i(W_i) & \overset{\text{def}}{=} \text{init} : (w_i, wa_i, \text{linear}(W_i)).Machine_i(W_i) + \\
& \quad \text{prep} : (w_i, 0, \text{const}).Machine_i(W_i) + \\
& \quad \text{take}_i : (w_i, wt_i, \text{linear}(W_i)).Machine_i(W_i) + \\
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& \quad \text{assem}_i : (t_i, 0, \text{const}).Timer_i
\end{align*}
\]
Mapping of influences, event conditions, influence types

\[ iv(p_i) = P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i \]
Mapping of influences, event conditions, influence types

\[ iv(p_i) = P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i \]

\[ ec(init) = (true, \quad P' = P_0 \land T_i' = 0 \land W_i' = 10 \land B' = B_0) \]
Mapping of influences, event conditions, influence types

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\[ ec(init) = (true, \quad P' = P_0 \land T'_i = 0 \land W'_i = 10 \land B' = B_0) \]

\[ ec(full) = (B \geq B_f, \quad true) \]

\[ ec(take_i) = (P \geq n_i, \quad P' = P' - n_i \land T'_i = 0) \]

\[ ec(assem_i) = (T_i \geq time_i, \quad B' = B + m_i) \]

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Mapping of influences, event conditions, influence types

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Mapping of influences, event conditions, influence types

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\[ ec(prep) = (prepare, \quad true) \]

\[ \lbrack\text{const}\rbrack = 1 \quad \lbrack\text{linear}(X)\rbrack = X \]
Uncontrolled system, controlled system, controller example

\[
\text{Sys} \overset{\text{def}}{=} (\text{Feed}_1 \bowtie \text{Feed}_2 \bowtie \text{Feed}_3) \bowtie \text{Output} \\
\quad (\text{Timer}_1 \bowtie \text{Machine}_1(W_1)) \bowtie \text{Output} \\
\quad (\text{Timer}_2 \bowtie \text{Machine}_2(W_2))
\]
Uncontrolled system, controlled system, controller example

\[
Sys \overset{\text{def}}{=} (Feed_1 \bowtie Feed_2 \bowtie Feed_3) \bowtie\bowtie \\
Output \bowtie\bowtie \\
(Timer_1 \bowtie Machine_1(W_1)) \bowtie\bowtie \\
(Timer_2 \bowtie Machine_2(W_2))
\]

\[
Assembler_j \overset{\text{def}}{=} Sys \bowtie init.Con_j
\]
Uncontrolled system, controlled system, controller example

\[ \text{Sys} \overset{\text{def}}{=} (\text{Feed}_1 \bowtie \text{Feed}_2 \bowtie \text{Feed}_3) \bowtie \text{Output} \]
\[ = (\text{Timer}_1 \bowtie \text{Machine}_1(W_1)) \bowtie (\text{Timer}_2 \bowtie \text{Machine}_2(W_2)) \]

\[ \text{Assembler}_j \overset{\text{def}}{=} \text{Sys} \bowtie \text{init}.\text{Con}_j \]

\[ \text{AOff}_i \overset{\text{def}}{=} \text{prep}.\text{AOn}_i \]
\[ \text{AOn}_i \overset{\text{def}}{=} \text{take}_i.\text{AProc}_i \]
\[ \text{AProc}_i \overset{\text{def}}{=} \text{assem}_i.\text{AOff}_i \]

\[ \text{FC} \overset{\text{def}}{=} \text{full}.0 \]
Stochastic HYPE semantics

- two equivalent semantics
- TDSHA: transition-driven stochastic hybrid automata
  ⊆ piecewise deterministic Markov processes
Stochastic HYPE semantics

- two equivalent semantics
- TDSHA: transition-driven stochastic hybrid automata \(\subseteq\) piecewise deterministic Markov processes
- first: compositional mapping to TDSHA using product
Stochastic HYPE semantics

- two equivalent semantics
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  \[ \subseteq \text{piecewise deterministic Markov processes} \]
- first: compositional mapping to TDSHA using product
- second: generation of LTS mapped to TDSHA
Stochastic HYPE semantics

- two equivalent semantics
- TDSHA: transition-driven stochastic hybrid automata
  \[ \subseteq \text{piecewise deterministic Markov processes} \]
- first: compositional mapping to TDSHA using product
- second: generation of LTS mapped to TDSHA
  - structured operational semantics
  - event labelled transition system over configurations
  - configuration: \( \langle \text{Sys} \, \bowtie \, \text{Con}, \sigma \rangle \)
  - state: \( \sigma : \text{influence} \mapsto (\text{influence strength, influence type}) \)
Stochastic HYPE semantics

- two equivalent semantics
- TDSHA: transition-driven stochastic hybrid automata ⊆ piecewise deterministic Markov processes
- first: compositional mapping to TDSHA using product
- second: generation of LTS mapped to TDSHA
  - structured operational semantics
  - event labelled transition system over configurations
  - configuration: \( \langle \text{Sys} \otimes \text{Con}, \sigma \rangle \)
  - state: \( \sigma : \text{influence} \mapsto (\text{influence strength, influence type}) \)
- configurations are mapped to modes
- states are mapped to continuous transitions giving ODEs

\[
\frac{dV}{dt} = \sum \{ r[I(\vec{W})] \mid \text{iv}(\nu) = V, \sigma_1(\nu) = (r, I(\vec{W})) \}
\]
Simulation of assembly system

Sys \bowtie \text{init.}(A\text{Off}_1 \parallel A\text{Off}_2)

(arrivals_i=20, departures=-0.1, time_i=2, prepare=0.6, n_i=100, m_i=2, wt_i=0.01, wa_i=0.06)
Simulation of assembly system

Sys \downarrow \text{init.}(A\text{Off}_1 \parallel A\text{Off}_2 \parallel FC)

(arrivals_i=20, departures=-0.1, time_i=2, prepare=0.6, n_i=100, m_i=2, wt_i=0.01, wa_i=0.06)
Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states
Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states
  given an equivalence relation $B \subseteq C \times C$
Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to \( \equiv \) over states
  given an equivalence relation \( B \subseteq C \times C \)
  then for all \( (P, Q) \in B, \sigma \equiv \tau, C \in (\mathcal{F}/B)/ \equiv, \)
Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states

  given an equivalence relation $B \subseteq C \times C$

then for all $(P, Q) \in B$, $\sigma \equiv \tau$, $C \in (\mathcal{F}/B)/ \equiv$,

1. for all $\overline{a} \in \mathcal{E}_d$, whenever

\[
\langle P, \sigma \rangle \xrightarrow{\overline{a}} \langle P', \sigma' \rangle \in C, \ \exists \langle Q', \tau' \rangle \in C \text{ with } \langle Q, \tau \rangle \xrightarrow{\overline{a}} \langle Q', \tau' \rangle
\]

\[
\langle Q, \tau \rangle \xrightarrow{\overline{a}} \langle Q', \tau' \rangle \in C, \ \exists \langle P', \sigma' \rangle \in C \text{ with } \langle P, \sigma \rangle \xrightarrow{\overline{a}} \langle P', \sigma' \rangle.
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Equivalence semantics for stochastic HYPE

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  given an equivalence relation $B \subseteq C \times C$

  then for all $(P, Q) \in B$, $\sigma \equiv \tau$, $C \in (\mathcal{F}/B)/\equiv$,

1. for all $\underline{a} \in \mathcal{E}_d$, whenever
   
   $\langle P, \sigma \rangle \overset{\underline{a}}{\rightarrow} \langle P', \sigma' \rangle \in C$, $\exists \langle Q', \tau' \rangle \in C$ with $\langle Q, \tau \rangle \overset{\underline{a}}{\rightarrow} \langle Q', \tau' \rangle$
   
   $\langle Q, \tau \rangle \overset{\underline{a}}{\rightarrow} \langle Q', \tau' \rangle \in C$, $\exists \langle P', \sigma' \rangle \in C$ with $\langle P, \sigma \rangle \overset{\underline{a}}{\rightarrow} \langle P', \sigma' \rangle$.

2. for all $\overline{a} \in \mathcal{E}_s$, $r(\langle P, \sigma \rangle, \overline{a}, C) = r(\langle Q, \tau \rangle, \overline{a}, C)$. 

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Equivalence semantics for stochastic HYPE

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  given an equivalence relation $B \subseteq C \times C$

  then for all $(P, Q) \in B$, $\sigma \equiv \tau$, $C \in (\mathcal{F}/B)/\equiv$,

  1. for all $\bar{a} \in \mathcal{E}_d$, whenever
      $\langle P, \sigma \rangle \xrightarrow{\bar{a}} \langle P', \sigma' \rangle \in C$, $\exists \langle Q', \tau' \rangle \in C$ with $\langle Q, \tau \rangle \xrightarrow{\bar{a}} \langle Q', \tau' \rangle$
      $\langle Q, \tau \rangle \xrightarrow{\bar{a}} \langle Q', \tau' \rangle \in C$, $\exists \langle P', \sigma' \rangle \in C$ with $\langle P, \sigma \rangle \xrightarrow{\bar{a}} \langle P', \sigma' \rangle$.

  2. for all $\bar{a} \in \mathcal{E}_s$, $r(\langle P, \sigma \rangle, \bar{a}, C) = r(\langle Q, \tau \rangle, \bar{a}, C)$.

- notation: $P \sim_{\equiv} Q$
Equivalence semantics for TDSHA

- TDSHA labelled bisimulation
Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation \( B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2}) \)
Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation \( B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2}) \)

  then for all \( ((q_1, x_1), (q_2, x_2)) \in B \)
Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation \( B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2}) \)

then for all \( ((q_1, x_1), (q_2, x_2)) \in B \)

- \( \text{out}_1(x_1) = \text{out}_2(x_2) \)
- exit rates of \( q_1 \) and \( q_2 \) must be equal
- disjunction of guards must evaluate to the same for \( x_1 \) and \( x_2 \)
- disjunction of guards must become true at the same time
- for all \( a \in \mathcal{E}_d \), one step priorities must match
- for all \( \overline{a} \in \mathcal{E}_s \), one step probabilities must match
Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation \( B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2}) \)

  then for all \(((q_1, x_1), (q_2, x_2)) \in B\)

    - \(\text{out}_1(x_1) = \text{out}_2(x_2)\)
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    - disjunction of guards must become true at the same time
    - for all \(\bar{a} \in \mathcal{E}_d\), one step priorities must match
    - for all \(\bar{a} \in \mathcal{E}_s\), one step probabilities must match

  notation: \(\mathcal{T}_1 \sim_\ell \mathcal{T}_2\)
Results

- $\sim \equiv$ is a congruence (under certain conditions on $\equiv$)
Results

- $\sim^\equiv$ is a congruence (under certain conditions on $\equiv$)

- if $Con_1 \sim^\equiv Con_2$ then $Sys \& init.Con_1 \sim^\equiv Sys \& init.Con_2$
Results

- $\sim^\equiv$ is a congruence (under certain conditions on $\equiv$)
- if $Con_1 \sim^\equiv Con_2$ then $Sys \triangleright\leftarrow init. Con_1 \sim^\equiv Sys \triangleright\leftarrow init. Con_2$
- if $P_1 \sim^\dot\equiv P_2$ then $\mathcal{T}(P_1) \sim^T_{\ell} \mathcal{T}(P_2)$
Results

- $\sim_{\equiv}$ is a congruence (under certain conditions on $\equiv$)

- if $Con_1 \sim_{\equiv} Con_2$ then $Sys \ast \text{init} . Con_1 \sim_{\equiv} Sys \ast \text{init} . Con_2$

- if $P_1 \sim^\dagger P_2$ then $T(P_1) \sim_T^\ell T(P_2)$

- application to assembly system
Results

- $\sim_\equiv$ is a congruence (under certain conditions on $\equiv$)
- if $Con_1 \sim_\equiv Con_2$ then $Sys \bowtie \text{init.} Con_1 \sim_\equiv Sys \bowtie \text{init.} Con_2$
- if $P_1 \sim_\hat{\cdot} P_2$ then $\mathcal{T}(P_1) \sim^L_T \mathcal{T}(P_2)$

application to assembly system

- $\sim_\equiv$: two controllers versus single controller
Results

- $\sim \equiv$ is a congruence (under certain conditions on $\equiv$)

- if $\text{Con}_1 \sim \equiv \text{Con}_2$ then $\text{Sys} \star \text{init.} \text{Con}_1 \sim \equiv \text{Sys} \star \text{init.} \text{Con}_2$

- if $P_1 \sim \hat{=} P_2$ then $\mathcal{T}(P_1) \sim_T \mathcal{T}(P_2)$

- application to assembly system
  - $\sim \equiv$: two controllers versus single controller
  - $\sim \hat{=}$: multiple feeds versus one feed with sum of rates of feeds
Results

- $\sim^\equiv$ is a congruence (under certain conditions on $\equiv$)

- if $\text{Con}_1 \sim^\equiv \text{Con}_2$ then $\text{Sys} \oslash \text{init.} \text{Con}_1 \sim^\equiv \text{Sys} \oslash \text{init.} \text{Con}_2$

- if $P_1 \sim^{\ddagger} P_2$ then $\mathcal{T}(P_1) \sim^\ell_T \mathcal{T}(P_2)$

- application to assembly system
  - $\sim^\equiv$: two controllers versus single controller
  - $\sim^{\ddagger}$: multiple feeds versus one feed with sum of rates of feeds
  - $\ll^{\ddagger}$: pair of timers versus a single timer
  - $\sim^\ell_T$: pair of timers versus a single timer
Two equivalent controllers

\[ \text{Sys} \otimes \text{init.}(A\text{Off}_1 \parallel A\text{Off}_2 \parallel FC) \]

\[ \text{Sys} \otimes \text{init.}(A\text{BOff} \parallel FC) \]

(arrivals\(_i=20\), departures\(_i=-0.1\), time\(_i=2\), prepare\(_i=0.6\), n\(_i=100\), m\(_i=2\), wt\(_i=0.01\))
Conclusions

- stochastic HYPE
  - process algebra for stochastic hybrid systems
  - extension of HYPE
  - semantics given by TDSHA
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- equivalent behaviour
  - stochastic HYPE: equivalence with abstraction over states
  - TDSHA: equivalence based on modes and variable values
Conclusions

- stochastic HYPE
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  - stochastic HYPE: equivalence with abstraction over states
  - TDSHA: equivalence based on modes and variable values

- results
  - congruence and corollary about equivalent controllers
  - relationship between two equivalences
Thank you
Stochastic HYPE model
Stochastic HYPE model

$$\text{components}$$

$$(C_1(\mathcal{V}) \otimes \cdots \otimes C_n(\mathcal{V}))$$
Stochastic HYPE model

components

\((C_1(V) \bowtie \cdots \bowtie C_n(V))\)
Stochastic HYPE model

components

\((C_1(V) \circ \cdots \circ C_n(V))\)

controllers

\((Con_1 \circ \cdots \circ Con_m)\)
Stochastic HYPE model

components

\((C_1(\mathcal{V}) \oslash \cdots \oslash C_n(\mathcal{V})) \oslash (Con_1 \oslash \cdots \oslash Con_m)\)

well-defined component

\[ C(\mathcal{V}) \overset{\text{def}}{=} \sum_{j} a_j : \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V}) \]
Stochastic HYPE model

components

\((C_1(V) \bowtie \cdots \bowtie C_n(V))\) \bowtie (Con_1 \bowtie \cdots \bowtie Con_m)\)

well-defined component

\[ C(V) \overset{\text{def}}{=} \sum_j a_j : \alpha_j \cdot C(V) + \text{init} : \alpha \cdot C(V) \]

components are parameterised by variables

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Stochastic HYPE model

components

\[(C_1(\mathcal{V}) \star \cdots \star C_n(\mathcal{V})) \star (\text{Con}_1 \star \cdots \star \text{Con}_m)\]

well-defined component

\[C(\mathcal{V}) \overset{\text{def}}{=} \sum_j a_j : \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V})\]
Stochastic HYPE model

components
\((C_1(\mathcal{V}) \circ \cdots \circ C_n(\mathcal{V}))\)

controllers
\((\text{Con}_1 \circ \cdots \circ \text{Con}_m)\)

well-defined component
\(C(\mathcal{V}) \overset{\text{def}}{=} \sum_j \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V})\)

events have event conditions: guards and resets
Stochastic HYPE model

\[
\begin{align*}
\text{components} & \quad \text{controllers} \\
(C_1(V) \pipe \cdots \pipe C_n(V)) & \quad (Con_1 \pipe \cdots \pipe Con_m)
\end{align*}
\]

well-defined component

\[
C(V) \overset{\text{def}}{=} \sum_j \alpha_j \cdot C(V) + \text{init} : \alpha \cdot C(V)
\]

events have event conditions: guards and resets

\[
ec(\alpha_j) = (f(V), V' = f'(V))
\]

discrete events
Stochastic HYPE model

components

\[
(C_1(\mathcal{V}) \star \cdots \star C_n(\mathcal{V})) \star \quad (\text{Con}_1 \star \cdots \star \text{Con}_m)
\]

well-defined component

\[
C(\mathcal{V}) \overset{\text{def}}{=} \sum_j a_j : \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V})
\]

events have event conditions: guards and resets

\[
ec(\alpha_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V})) \quad \text{discrete events}
\]

\[
ec(\bar{\alpha}_j) = (r, \mathcal{V} = f'(\mathcal{V})) \quad \text{stochastic events}
\]
Stochastic HYPE model

\[
\begin{align*}
\text{components} & \quad \text{controllers} \\
\left( C_1(V) \mathbin{\star} \cdots \mathbin{\star} C_n(V) \right) & \mathbin{\star} \quad \left( Con_1 \mathbin{\star} \cdots \mathbin{\star} Con_m \right)
\end{align*}
\]

well-defined component
\[
C(V) \overset{\text{def}}{=} \sum_j a_j : \alpha_j \cdot C(V) + \text{init} : \alpha \cdot C(V)
\]
Stochastic HYPE model

components

\((C_1(\mathcal{V}) \otimes \cdots \otimes C_n(\mathcal{V}))\)

controllers

\((\text{Con}_1 \otimes \cdots \otimes \text{Con}_m)\)

well-defined component

\[ C(\mathcal{V}) \overset{\text{def}}{=} \sum_j a_j : \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V}) \]

influences are defined by a triple
**Stochastic HYPE model**

components

\((C_1(\mathcal{V}) \oplus \cdots \oplus C_n(\mathcal{V})) \oplus \cdots \oplus (Con_1 \oplus \cdots \oplus Con_m)\)

controllers

well-defined component

\[ C(\mathcal{V}) \overset{\text{def}}{=} \sum_{j} a_j : \alpha_j \cdot C(\mathcal{V}) + \text{init} : \alpha \cdot C(\mathcal{V}) \]

influences are defined by a triple

\[ \alpha_j = (\iota_j, r_j, I(\mathcal{V})) \]
Stochastic HYPE model

components

\[(C_1(V) \oplus \cdots \oplus C_n(V)) \oplus (Con_1 \oplus \cdots \oplus Con_m)\]

well-defined component

\[C(V) \overset{def}{=} \sum_j a_j : \alpha_j \cdot C(V) + \text{init} : \alpha \cdot C(V)\]

influences are defined by a triple

\[\alpha_j = (\iota_j, r_j, I(V))\]

influence names are mapped to variables

\[iv(\iota_j) \in V\]