Stochastic hybrid modelling with composition of flows

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8 September 2012

Introduction

- stochastic hybrid process algebra
  - discrete behaviour
  - continuous behaviour, expressed as ODEs
  - stochastic behaviour, using exponential distribution

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- why use this process algebra
  - compositionality of flows
  - ODEs generated by model

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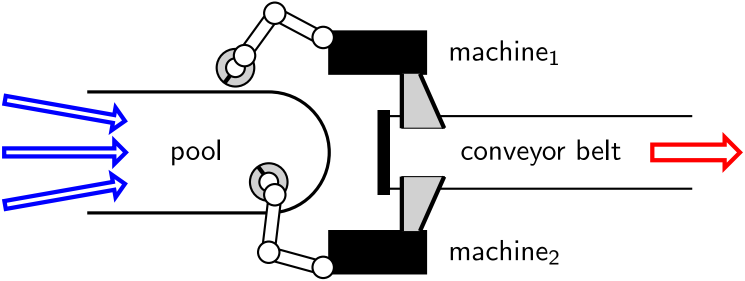
  - compositionality of flows
  - ODEs generated by model
- outline
  - stochastic HYPE
  - semantics given by TDSHA
  - assembly system example
  - equivalent behaviour
  - results

Assembly system



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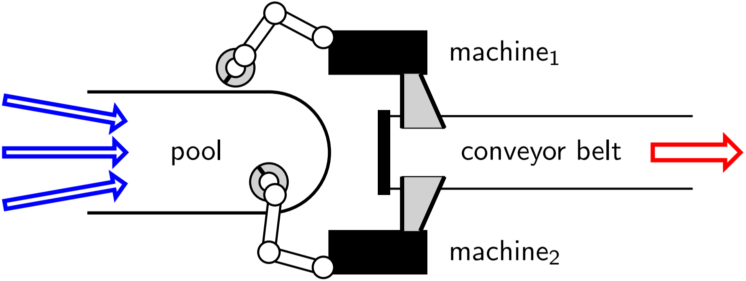

- continuous variables
  - items in pool: P
  - items at start of conveyor belt: B
  - power consumption of machine$_i$: W_i
  - timers: T_i, T

Uncontrolled components

$$
\begin{aligned}
\mathit{Feed}_i \quad \stackrel{\mathit{def}}{=} \quad & \underline{\mathrm{init}} : (p_i, \mathit{arrivals}_i, \mathit{const}).\mathit{Feed}_i + \\
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\begin{aligned}
Machine_i(W_i) &\stackrel{def}{=} \underline{\mathrm{init}} : (w_i, wa_i, linear(W_i)).Machine_i(W_i) + \\
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Mapping of influences, event conditions, influence types

$$iv(p_i) = P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i$$

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ec(\underline{\mathrm{full}}) &= (B \geq B_f, & &\mathit{true}) \\
ec(\underline{\mathrm{take}}_i) &= (P \geq n_i, & &P' = P' - n_i \wedge T_i' = 0) \\
ec(\underline{\mathrm{assem}}_i) &= (T_i \geq \mathit{time}_i, & &B' = B + m_i)
\end{aligned}$$

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$$ec(\overline{\text{prep}}) = (prepare, \quad true)$$

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$$[\![\mathit{const}]\!] \;=\; 1 \qquad [\![\mathit{linear}(X)]\!] \;=\; X$$

Uncontrolled system, controlled system, controller example

$$
\begin{array}{lcll}
Sys & \stackrel{\mathit{def}}{=} & (Feed_1 \underset{*}{\bowtie} Feed_2 \underset{*}{\bowtie} Feed_3) & \underset{*}{\bowtie} \\
 & & \ Output & \underset{*}{\bowtie} \\
 & & (Timer_1 \underset{*}{\bowtie} Machine_1(W_1)) & \underset{*}{\bowtie} \\
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$$
\mathit{Assembler}_j \stackrel{\mathit{def}}{=} \mathit{Sys} \underset{*}{\bowtie} \underline{\mathrm{init}}.\mathit{Con}_j
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$$
Assembler_j \stackrel{def}{=} Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.Con_j
$$

$$
\begin{aligned}
AOff_i &\stackrel{def}{=} \overline{\mathrm{prep}}.AOn_i \\
AOn_i &\stackrel{def}{=} \underline{\mathrm{take}}_i.AProc_i \\
AProc_i &\stackrel{def}{=} \underline{\mathrm{assem}}_i.AOff_i
\end{aligned}
$$

$$
FC \stackrel{def}{=} \underline{\mathrm{full}}.0
$$

Stochastic HYPE semantics

- two equivalent semantics
- TDSHA: transition-driven stochastic hybrid automata
  $\subseteq$ piecewise determinstic Markov processes

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- first: compositional mapping to TDSHA using product
- second: generation of LTS mapped to TDSHA
  - structured operational semantics
  - event labelled transition system over configurations
  - configuration: $\langle \mathit{Sys} \underset{*}{\bowtie} \mathit{Con}, \sigma \rangle$
  - state: σ : influence $\mapsto$ (influence strength, influence type)

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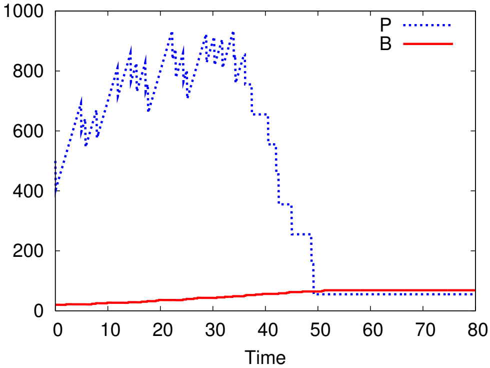
  - configuration: $\langle Sys \underset{*}{\bowtie} Con, \sigma \rangle$
  - state: σ : influence $\mapsto$ (influence strength, influence type)
  - configurations are mapped to modes
  - states are mapped to continuous transitions giving ODEs

$$\frac{dV}{dt} = \sum \{ r [\![I(\overrightarrow{W})]\!] \mid iv(\iota) = V, \sigma_1(\iota) = (r, I(\overrightarrow{W})) \}$$

Simulation of assembly system



$$Sys \bowtie_{*} \underline{\mathrm{init}}.(AOff_1 \parallel AOff_2)$$

$(arrivals_i{=}20,\ departures{=}-0.1,\ time_i{=}2,\ prepare{=}0.6,\ n_i{=}100,\ m_i{=}2,\ wt_i{=}0.01,\ wa_i{=}0.06)$

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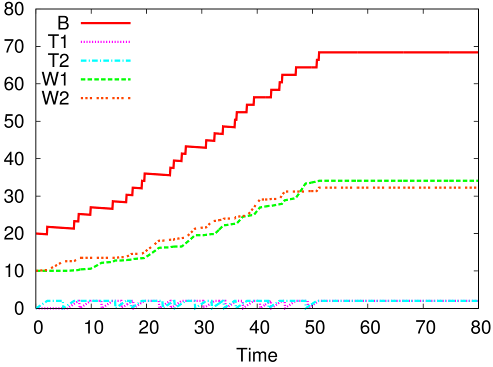

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Equivalence semantics for stochastic HYPE

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  - exit rates of q_1 and q_2 must be equal
  - disjunction of guards must evaluate to the same for x_1 and x_2
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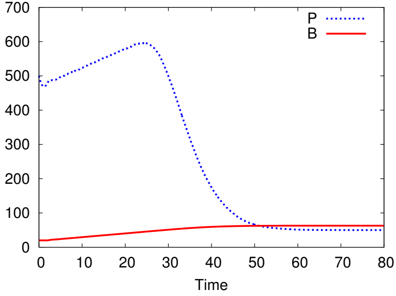
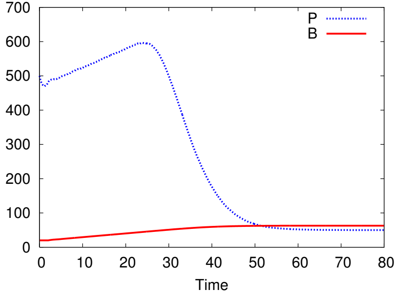
  - $\sim^{\ell}_{T}$: pair of timers versus a single timer

Two equivalent controllers





$Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.((AOff_1 \parallel AOff_2 \parallel FC)$ $\qquad$ $Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.(ABOff \parallel FC)$

$(arrivals_i{=}20,\ departures{=}{-}0.1,\ time_i{=}2,\ prepare{=}0.6,\ n_i{=}100,\ m_i{=}2,\ wt_i{=}0.01)$

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- results
  - congruence and corollary about equivalent controllers
  - relationship between two equivalences

Thank you

Stochastic HYPE model

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components

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big)$$

Stochastic HYPE model

components

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie}$$

Stochastic HYPE model

components controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

Stochastic HYPE model

components controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V}) \big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m \big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

Stochastic HYPE model

components $\qquad\qquad\qquad\qquad\qquad\qquad$ controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

components are parameterised by variables

Stochastic HYPE model

components controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

Stochastic HYPE model

components $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ controllers

$$\big(C_1(\mathcal{V}) \bowtie_* \cdots \bowtie_* C_n(\mathcal{V})\big) \quad \bowtie_* \quad \big(Con_1 \bowtie_* \cdots \bowtie_* Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

events have event conditions: guards and resets

Stochastic HYPE model

components	controllers

$$\big(C_1(\mathcal{V}) \bowtie_* \cdots \bowtie_* C_n(\mathcal{V})\big) \quad \bowtie_* \quad \big(Con_1 \bowtie_* \cdots \bowtie_* Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

events have event conditions: guards and resets

$ec(\underline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V}))$ discrete events

Stochastic HYPE model

components controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

events have event conditions: guards and resets

$ec(\underline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V}))$ discrete events

$ec(\overline{\mathrm{a}}_j) = (r, \mathcal{V} = f'(\mathcal{V}))$ stochastic events

Stochastic HYPE model

components $\qquad\qquad\qquad\qquad$ controllers

$$\left(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\right) \quad \underset{*}{\bowtie} \quad \left(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\right)$$

well-defined component

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

Stochastic HYPE model

components | controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

influences are defined by a triple

Stochastic HYPE model

components controllers

$$\left(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\right) \quad \underset{*}{\bowtie} \quad \left(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\right)$$

well-defined component

$$C(\mathcal{V}) \overset{def}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

influences are defined by a triple

$$\alpha_j = (\iota_j, r_j, I(\mathcal{V}))$$

Stochastic HYPE model

components controllers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \big(Con_1 \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} Con_m\big)$$

well-defined component

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j . C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha . C(\mathcal{V})$$

influences are defined by a triple

$$\alpha_j = (\iota_j, r_j, I(\mathcal{V}))$$

influence names are mapped to variables

$$iv(\iota_j) \in \mathcal{V}$$