Stochastic HYPE
a stochastic hybrid process algebra

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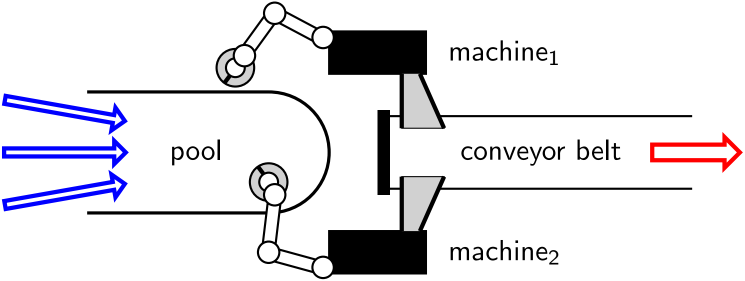
Assembly system



Outline

Introduction

- behaviours to be included
  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
  - stochastic behaviour: exponentially-distributed durations

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- process algebra approach
  - formal languages for expressing concurrency
  - compositional semantics
  - notions of equivalence

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  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
  - stochastic behaviour: exponentially-distributed durations
- process algebra approach
  - formal languages for expressing concurrency
  - compositional semantics
  - notions of equivalence
- HYPE
  - only discrete and continuous behaviour
  - operational semantics define labelled transition system
  - mapping from labelled transition system to hybrid automaton

Motivation

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- flows in stochastic HYPE ($W_j \subseteq \mathcal{V}$)

$$\begin{array}{lllll} A_1 & \stackrel{\mathit{def}}{=} & \ldots & (\iota_1, r_1, I_1(\mathcal{W}_1)) & \ldots \\ \vdots & \vdots & & \vdots & \\ A_n & \stackrel{\mathit{def}}{=} & \ldots & (\iota_n, r_n, I_n(\mathcal{W}_n)) & \ldots \end{array}$$

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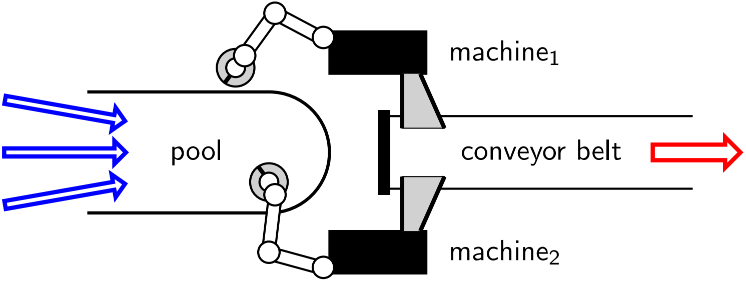
and $$\frac{dV}{dt} = \sum\{r_j.I_j(\mathcal{W}_j) \mid iv(\iota_j) = V, \dots\}$$

Assembly system



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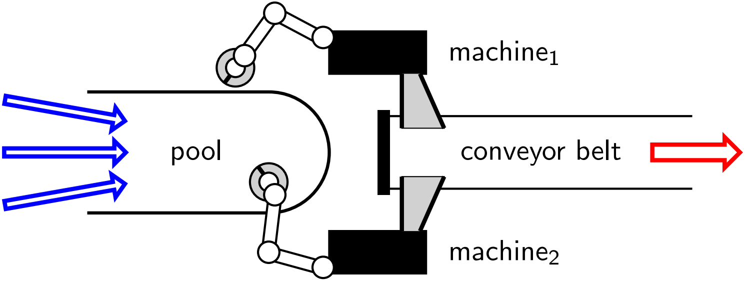

- continuous variables
  - individual items in pool: P
  - assembled items at start of conveyor belt: B
  - power consumption of machine$_i$: W_i
  - timers: T_i, T

Stochastic HYPE model

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uncontrolled system

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big)$$

Stochastic HYPE model

uncontrolled system

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie}$$

Stochastic HYPE model

uncontrolled system controllers/sequencers

$$\left(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\right) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.\left(Con_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} Con_m\right)$$

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well-defined subcomponent

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

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uncontrolled system | controllers/sequencers

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subcomponents are parameterised by variables

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events have event conditions: guards/durations and resets

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events have event conditions: guards/durations and resets

$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V}))$ with $g : \mathbb{R}^{|\mathcal{V}|} \rightarrow \{true, false\}$ discrete

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uncontrolled system controllers/sequencers

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$$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V})) \text{ with } g : \mathbb{R}^{|\mathcal{V}|} \to \{true, false\} \qquad \text{discrete}$$

$$ec(\overline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V})) \text{ with } f : \mathbb{R}^{|\mathcal{V}|} \to [0, \infty) \qquad \text{stochastic}$$

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uncontrolled system

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$$\underset{*}{\bowtie}$$

controllers/sequencers

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influences are defined by a triple

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influence names are mapped to variables

$$iv(\iota_j) \in \mathcal{V}$$

Uncontrolled system

$$
\begin{aligned}
Machine_i(W_i) \stackrel{def}{=}\ & \underline{\mathrm{init}} : (w_i, wa_i,\ linear(W_i)).Machine_i(W_i) + \\
& \overline{\mathrm{prep}}_i : (w_i, 0, const).Machine_i(W_i) + \\
& \underline{\mathrm{take}}_i : (w_i, wt_i,\ linear(W_i)).Machine_i(W_i) + \\
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$$

$$
\begin{aligned}
\mathit{Timer}_i \;&\stackrel{\mathit{def}}{=}\; \underline{\mathrm{init}} : (t_i, 0, \mathit{const}).\mathit{Timer}_i + \\
&\quad \underline{\mathrm{take}}_i : (t_i, 1, \mathit{const}).\mathit{Timer}_i + \\
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\end{aligned}
$$

Uncontrolled system (continued)

$$\begin{aligned} \mathit{Feed}_i & \stackrel{\mathit{def}}{=} \underline{\mathrm{init}} : (p_i, \mathit{arrivals}_i, \mathit{const}).\mathit{Feed}_i + \\ & \quad \underline{\mathrm{full}} : (p_i, 0, \mathit{const}).\mathit{Feed}_i \end{aligned}$$

Uncontrolled system (continued)

$$
\begin{array}{rcl}
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\end{array}
$$

$$
\begin{array}{rcl}
\mathit{Output} & \stackrel{\mathit{def}}{=} & \underline{\mathrm{init}} : (b, \mathit{departures}, \mathit{const}).\mathit{Output} + \\
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\end{array}
$$

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\end{aligned}
$$

$$
\begin{aligned}
\mathit{Sys} \stackrel{\mathit{def}}{=} & (\mathit{Feed}_1 \underset{*}{\bowtie} \mathit{Feed}_2 \underset{*}{\bowtie} \mathit{Feed}_3) \underset{*}{\bowtie} \\
& \mathit{Output} \underset{*}{\bowtie} \\
& (\mathit{Timer}_1 \underset{*}{\bowtie} \mathit{Machine}_1(W_1)) \underset{*}{\bowtie} \\
& (\mathit{Timer}_2 \underset{*}{\bowtie} \mathit{Machine}_2(W_2))
\end{aligned}
$$

Mapping of influences, event conditions, influence types

$$iv(p_i) \;=\; P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i$$

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$$ec(\underline{\mathrm{init}}) \;=\; (true, \qquad P' = P_0 \wedge T_i' = 0 \wedge W_i' = 10 \wedge B' = B_0)$$

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$$\begin{aligned}
ec(\underline{\mathrm{init}}) &= (\mathit{true}, & & P' = P_0 \wedge T_i' = 0 \wedge W_i' = 10 \wedge B' = B_0) \\[2ex]
ec(\underline{\mathrm{full}}) &= (B \geq B_f, & & \mathit{true}) \\
ec(\underline{\mathrm{take}}_i) &= (P \geq n_i, & & P' = P - n_i \wedge T_i' = 0) \\
ec(\underline{\mathrm{assem}}_i) &= (T_i \geq \mathit{atime}_i, & & B' = B + m_i)
\end{aligned}$$

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 & & & \\
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\end{aligned}$$

$$[\![const]\!] = 1 \qquad [\![linear(X)]\!] = X$$

Stochastic HYPE model

uncontrolled system — controllers/sequencers

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controller grammar

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$$M ::= \mathrm{a}.M \mid 0 \mid M + M$$

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Stochastic HYPE model

uncontrolled system | controllers/sequencers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.\big(\mathit{Con}_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} \mathit{Con}_m\big)$$

controller grammar

$$M ::= \mathrm{a}.M \mid 0 \mid M + M$$
$$\mathit{Con} ::= M \mid \mathit{Con} \underset{*}{\bowtie} \mathit{Con}$$

Controllers and controlled system

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\begin{aligned}
AOff_i &\stackrel{\mathit{def}}{=} \overline{\mathrm{prep}}_i.AOn_i \\
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Assembler \stackrel{def}{=} Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.(AOff_1 \parallel AOff_2 \parallel FC)
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Transition-driven stochastic hybrid automata

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- stochastic transitions
  - source mode, target mode, event name
  - rate: function defining speed of transition
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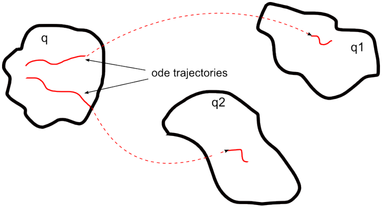
- product of TDSHAs
  - pairs of modes and union of variables
  - combining transitions
    (with conditions on resets and initial values)

Piecewise deterministic Markov processes

- class of stochastic processes
- continuous trajectories over subsets of $\mathbb{R}^{|\mathbf{X}|}$
- instantaneous jumps at boundaries of regions
- stochastic jumps when guards are true



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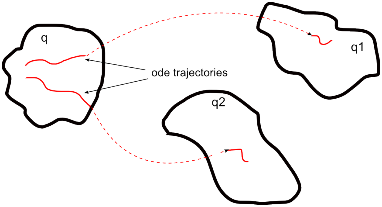

- jumps to boundaries are prohibited

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$\underline{a} \in M, \Gamma$ defined

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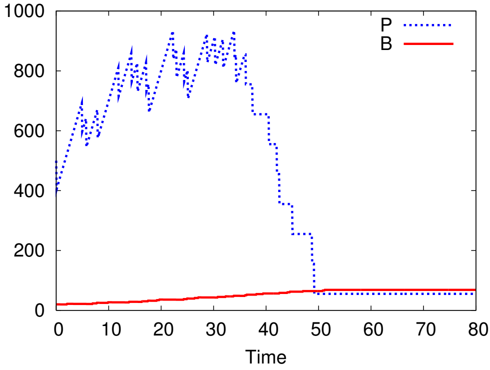
- Γ is defined for all well-defined stochastic HYPE models
  - syntactic restrictions on influences and events

Simulation of assembly system using SimHyA



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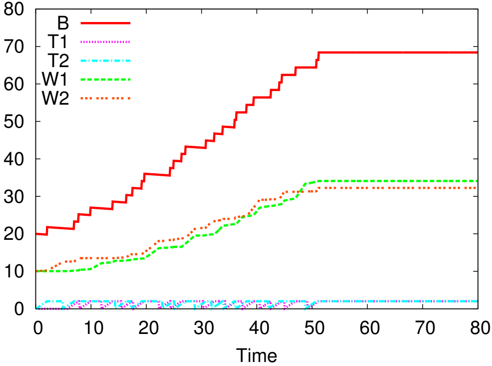

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- well-behaved results for overapproximations and compositions

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- no cycle implies no instantaneous Zeno behaviour
- I-graph construction is not always necessary

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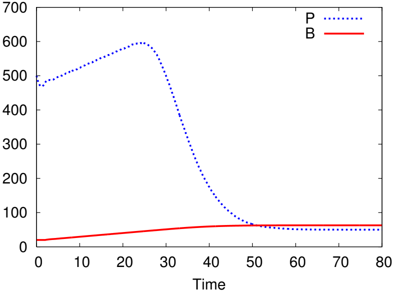
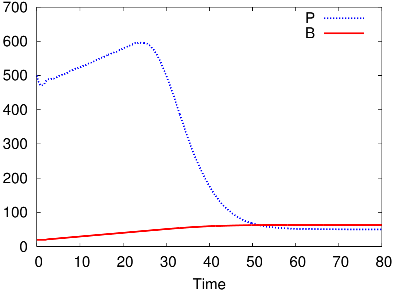
Two equivalent controllers





$Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.((AOff_1 \parallel AOff_2 \parallel FC)$

$Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.(ABOff \parallel FC)$

averages of 5000 simulations

$(arrivals_i{=}20,\ departures{=}-0.1,\ atime_i{=}2,\ prepare{=}0.6,\ n_i{=}100,\ m_i{=}2,\ wt_i{=}0.01)$

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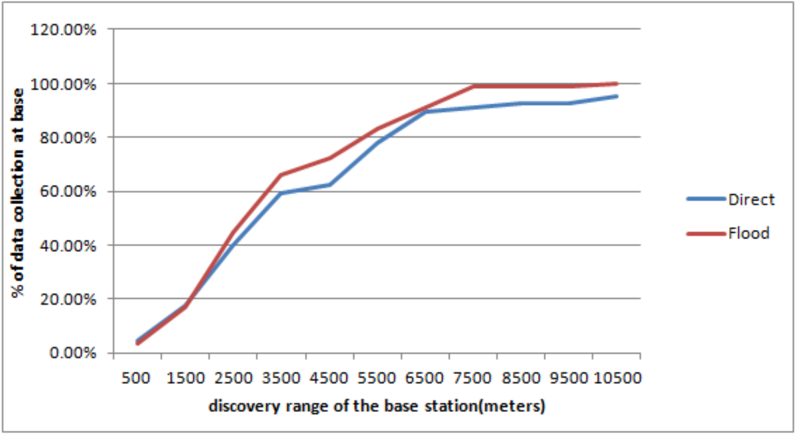

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Thank you