Modelling in Stochastic HYPE

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Outline

Introduction

- behaviours to be included
  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
  - stochastic behaviour: exponentially-distributed durations

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  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
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- process algebra approach
  - formal languages for expressing concurrency
  - compositional semantics
  - notions of equivalence
  - lift properties to language level: well-behaved HYPE models

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  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
  - stochastic behaviour: exponentially-distributed durations
- process algebra approach
  - formal languages for expressing concurrency
  - compositional semantics
  - notions of equivalence
  - lift properties to language level: well-behaved HYPE models
- extension of HYPE process algebra
  - only instantaneous and continuous behaviour

Motivation

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  - monolithic ODEs in the syntax
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Language considerations: ODEs versus flows

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- flows in stochastic HYPE ($W_j \subseteq \mathcal{V}$)

$$\begin{array}{lcl} A_1 & \stackrel{def}{=} & \ldots \quad (\iota_1, r_1, I_1(\mathcal{W}_1)) \quad \ldots \\ \vdots & \vdots & \qquad\qquad \vdots \\ A_n & \stackrel{def}{=} & \ldots \quad (\iota_n, r_n, I_n(\mathcal{W}_n)) \quad \ldots \end{array}$$

Language considerations: ODEs versus flows

- notation: $\mathcal{V}$, a set of continuous variables
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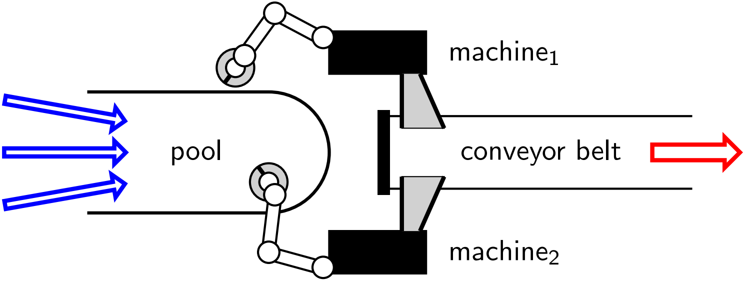
$$\begin{array}{ccccc} A_1 & \stackrel{def}{=} & \ldots & (\iota_1, r_1, I_1(\mathcal{W}_1)) & \ldots \\ \vdots & \vdots & & \vdots & \\ A_n & \stackrel{def}{=} & \ldots & (\iota_n, r_n, I_n(\mathcal{W}_n)) & \ldots \end{array}$$

and $$\frac{dV}{dt} = \sum\{r_j.I_j(\mathcal{W}_j) \mid iv(\iota_j) = V, \ldots\}$$

Assembly system



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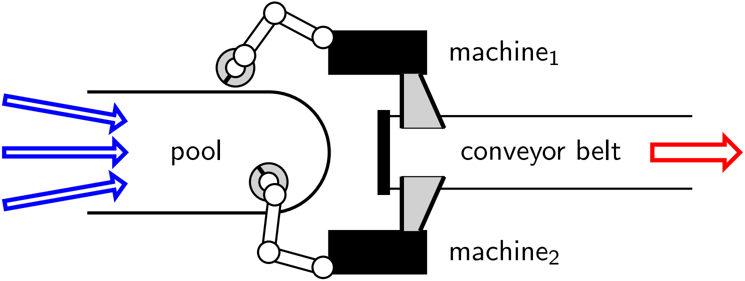

- continuous variables
  - individual items in pool: P
  - assembled items at start of conveyor belt: B
  - power consumption of machine$_i$: W_i
  - timers: T_i, T

Stochastic HYPE model

Stochastic HYPE model

uncontrolled system

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big)$$

Stochastic HYPE model

uncontrolled system

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie}$$

Stochastic HYPE model

uncontrolled system controllers/sequencers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.\big(Con_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} Con_m\big)$$

Stochastic HYPE model

uncontrolled system — controllers/sequencers

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well-defined subcomponent

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

Stochastic HYPE model

uncontrolled system

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subcomponents are parameterised by variables

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events can be instantaneous: $\underline{\mathrm{a}}_j$

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events can be instantaneous: $\underline{\mathrm{a}}_j$

events can be stochastic: $\overline{\mathrm{a}}_j$

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influences are defined by a triple

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uncontrolled system $\qquad\qquad$ controllers/sequencers

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initial event and influence required

Uncontrolled system

$$\begin{aligned} \mathit{Machine}_i(W_i) \stackrel{\mathit{def}}{=}\ & \underline{\mathrm{init}} : (w_i, wa_i, \mathit{linear}(W_i)).\mathit{Machine}_i(W_i) + \\ & \overline{\mathrm{prep}}_i : (w_i, 0, \mathit{const}).\mathit{Machine}_i(W_i) + \\ & \underline{\mathrm{take}}_i : (w_i, wt_i, \mathit{linear}(W_i)).\mathit{Machine}_i(W_i) + \\ & \underline{\mathrm{assem}}_i : (w_i, wa_i, \mathit{linear}(W_i)).\mathit{Machine}_i(W_i) \end{aligned}$$

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\end{aligned}
$$

$$
\begin{aligned}
\mathit{Timer}_i \stackrel{\mathit{def}}{=}\ & \underline{\mathrm{init}} : (t_i, 0, \mathit{const}).\mathit{Timer}_i + \\
& \underline{\mathrm{take}}_i : (t_i, 1, \mathit{const}).\mathit{Timer}_i + \\
& \underline{\mathrm{assem}}_i : (t_i, 0, \mathit{const}).\mathit{Timer}_i
\end{aligned}
$$

Uncontrolled system (continued)

$$\begin{aligned} \mathit{Feed}_i \stackrel{\mathit{def}}{=} \ & \underline{\mathrm{init}} : (p_i, \mathit{arrivals}, \mathit{const}).\mathit{Feed}_i + \\ & \underline{\mathrm{full}} : (p_i, 0, \mathit{const}).\mathit{Feed}_i \end{aligned}$$

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$$\begin{aligned} \mathit{Output} &\stackrel{\mathit{def}}{=} \underline{\mathrm{init}} : (b, \mathit{departures}, \mathit{const}).\mathit{Output} + \\ &\quad \underline{\mathrm{full}} : (b, 0, \mathit{const}).\mathit{Output} \end{aligned}$$

Uncontrolled system (continued)

$$\begin{aligned} Feed_i &\stackrel{def}{=} \underline{\mathrm{init}} : (p_i, arrivals, const).Feed_i + \\ &\quad\; \underline{\mathrm{full}} : (p_i, 0, const).Feed_i \end{aligned}$$

$$\begin{aligned} Output &\stackrel{def}{=} \underline{\mathrm{init}} : (b, departures, const).Output + \\ &\quad\; \underline{\mathrm{full}} : (b, 0, const).Output \end{aligned}$$

$$\begin{aligned} Sys \stackrel{def}{=} \; & (Feed_1 \underset{*}{\bowtie} Feed_2 \underset{*}{\bowtie} Feed_3) \quad \underset{*}{\bowtie} \\ & Output \quad \underset{*}{\bowtie} \\ & (Timer_1 \underset{*}{\bowtie} Machine_1(W_1)) \quad \underset{*}{\bowtie} \\ & (Timer_2 \underset{*}{\bowtie} Machine_2(W_2)) \end{aligned}$$

Stochastic HYPE model

uncontrolled system

controllers/sequencers

$$\left(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\right) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.\left(Con_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} Con_m\right)$$

well-defined subcomponent

$$C(\mathcal{V}) \stackrel{\mathit{def}}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

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events have event conditions: guards/durations and resets

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events have event conditions: guards/durations and resets

$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V}))$ with $g : \mathbb{R}^{|\mathcal{V}|} \to \{true, false\}$ instantaneous

Stochastic HYPE model

uncontrolled system

$$\big(C_1(\mathcal{V}) \bowtie_{*} \cdots \bowtie_{*} C_n(\mathcal{V})\big)$$

$$\bowtie_{*}$$

controllers/sequencers

$$\underline{\mathrm{init}}.\big(Con_1 \bowtie_{L_2} \cdots \bowtie_{L_m} Con_m\big)$$

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events have event conditions: guards/durations and resets

$$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V})) \text{ with } g : \mathbb{R}^{|\mathcal{V}|} \to \{true, false\} \quad \text{instantaneous}$$

$$ec(\overline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V})) \text{ with } f : \mathbb{R}^{|\mathcal{V}|} \to [0, \top) \quad \text{stochastic}$$

Stochastic HYPE model

uncontrolled system

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controllers/sequencers

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influence names are mapped to variables

$$iv(\iota_j) \in \mathcal{V}$$

Mapping of influences, event conditions, influence types

$$ec(\underline{\mathrm{init}}) \;=\; (\mathit{true}, \qquad P' = P_0 \wedge T_i' = 0 \wedge W_i' = 10 \wedge B' = B_0)$$

Mapping of influences, event conditions, influence types

$$
\begin{aligned}
ec(\underline{\text{init}}) &= (true, & P' = P_0 \wedge T_i' = 0 \wedge W_i' = 10 \wedge B' = B_0)
\end{aligned}
$$

$$
\begin{aligned}
ec(\underline{\text{full}}) &= (B \geq B_f, & true) \\
ec(\underline{\text{take}}_i) &= (P \geq n_i, & P' = P - n_i \wedge T_i' = 0) \\
ec(\underline{\text{assem}}_i) &= (T_i \geq atime_i, & B' = B + m_i)
\end{aligned}
$$

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\\
ec(\overline{\text{prep}}_i) &= (prepare, && true)
\end{aligned}
$$

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$$
iv(p_i) = P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i
$$

Mapping of influences, event conditions, influence types

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$$iv(p_i) = P \quad iv(b) = B \quad iv(w_i) = W_i \quad iv(t_i) = T_i$$

$$[\![const]\!] = 1 \qquad [\![linear(X)]\!] = X$$

Stochastic HYPE model

uncontrolled system — controllers/sequencers

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Stochastic HYPE model

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Semantics for stochastic HYPE models

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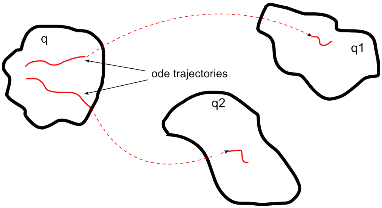
  subset of piecewise deterministic Markov processes



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  - structural operational semantics define labelled transition system which is mapped TDSHA

$$\left(\frac{dV}{dt}\right)_{\sigma} = \sum \left\{ r \cdot [\![I(\overrightarrow{W})]\!] \;\middle|\; iv(\iota) = V, \sigma(\iota) = (r, I(\overrightarrow{W})) \right\}$$

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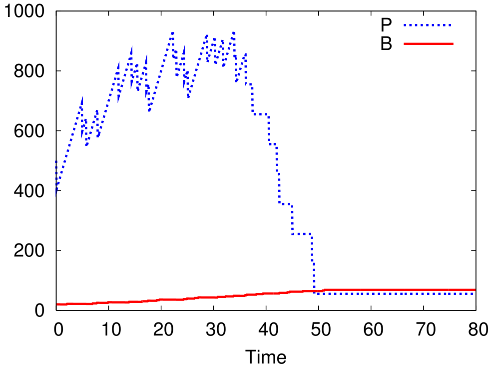
- Γ is defined for all well-defined stochastic HYPE models
  - syntactic restrictions on influences and events

Simulation of assembly system using SimHyA



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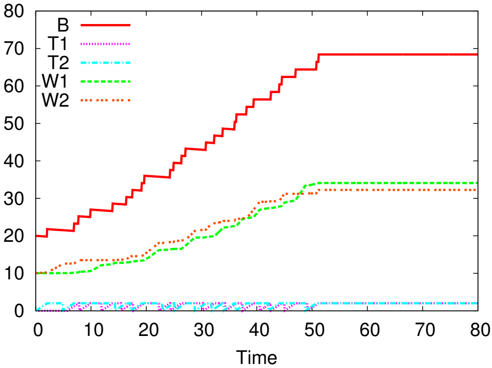

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Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states

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Results applied to assembly system

- $ABOff$: single controller of two machines
  - can prove that $AOff_1 \parallel AOff_2 \sim^{=} ABOff$
  - hence using congruence

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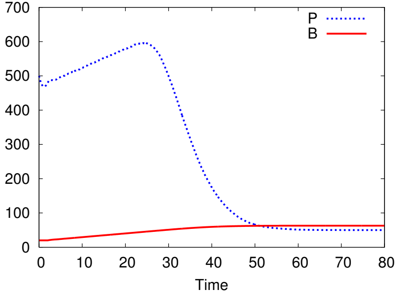
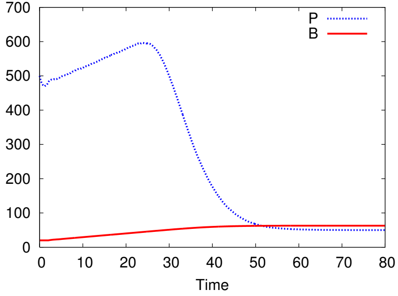
- let $\doteq$ be an equivalence over states that ensures that sums over influences that map to the same variable are equal
  - for all $V \in \mathcal{V}$, $\sum\{r \cdot [\![I(\overrightarrow{W})]\!] \mid iv(\iota) = V, \sigma(\iota) = (r, I(\overrightarrow{W}))\}$ $= \sum\{r \cdot [\![I(\overrightarrow{W})]\!] \mid iv(\iota) = V, \tau(\iota) = (r, I(\overrightarrow{W}))\}$
  - define a single feed subcomponent with an influence rate three times the rate of an original feed subcomponent
  - $Sys \underset{*}{\bowtie} \underline{\mathrm{init}}.Con \sim^{\doteq} Sys_{SF} \underset{*}{\bowtie} \underline{\mathrm{init}}.Con$

Two equivalent controllers





$Sys \bowtie_{*} \underline{\text{init}}.((AOff_1 \parallel AOff_2 \parallel FC)$ $\quad$ $Sys \bowtie_{*} \underline{\text{init}}.(ABOff \parallel FC)$

averages of 5000 simulations

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- biological systems
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- Zebranet simulation: MSc dissertation of Cheng Feng
  - opportunistic networking based on animal movement
  - syntactic extension for repeated components/controllers
  - two-dimensional movement and energy consumption

Conclusions

- stochastic HYPE
  - process algebra for stochastic hybrid systems
  - semantics given by TDSHA and PDMPs
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- well-behaved stochastic HYPE models
  - contain no instantaneous Zeno behaviour
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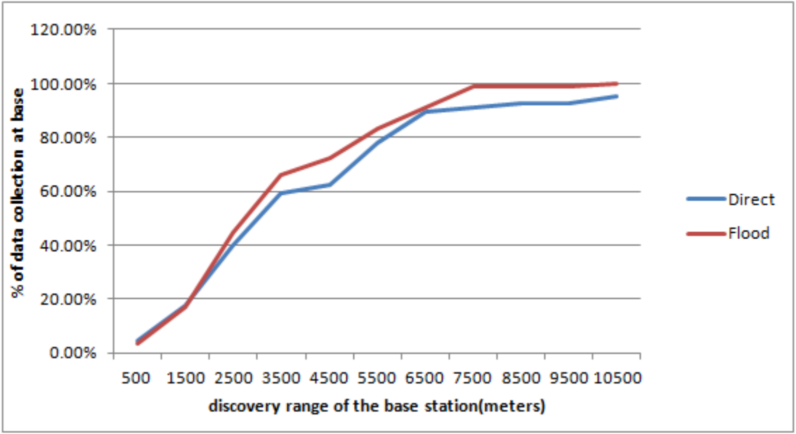
- semantic equivalences

Thank you

Data collected by protocol



Transition-driven stochastic hybrid automata

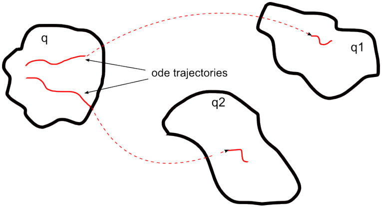
- semantics of stochastic HYPE models
- TDSHA: transition-driven stochastic hybrid automata
  $\subseteq$ PDMP: piecewise deterministic Markov processes
- set of modes, Q and set of continuous variables, $\mathbf{X}$
- instantaneous transitions
  - source mode, target mode, event name
  - guard: activation condition over variables
  - reset: function determining new values of variables
  - priority/weight: to resolve non-determinism
- stochastic transitions
  - source mode, target mode, event name
  - rate: function defining speed of transition
  - guard: activation condition over variables
  - reset: function determining new values of variables

Transition-driven stochastic hybrid automata (continued)

- continuous transitions (flows)
  - source mode
  - vector specifying variables involved
  - Lipschitz continuous function
- continuous behaviour in a mode
  - consider all continuous transitions in that mode
  - trajectory is given by solution of $d\mathbf{X}/dt = \sum s \cdot f(\mathbf{X})$
- instantaneous behaviour: fire when guard becomes true
- stochastic behaviour: fire according to rate
- product of TDSHAs
  - pairs of modes and union of variables
  - combining transitions
    (with conditions on resets and initial values)

Piecewise deterministic Markov processes

- class of stochastic processes
- continuous trajectories over subsets of $\mathbb{R}^{|\mathbf{X}|}$
- instantaneous jumps at boundaries of regions
- stochastic jumps when guards are true



- jumps to boundaries are prohibited

Two equivalent semantics

- compositional mapping to TDSHA
  - define TDSHA for each subcomponent (no event conditions)
  - define TDSHA for each sequential controller
  - use TDSHA product to compose into TDSHA of whole model
- mapping from LTS to TDSHA
  - event labelled transition system over configurations
  - configuration: $\langle \mathit{Sys} \bowtie_{*} \mathit{Con}, \sigma \rangle$
  - state: σ : influence $\mapsto$ (influence strength, influence type)
  - configurations are mapped to modes
  - states giving ODEs which become continuous transitions

$$\left(\frac{dV}{dt}\right)_{\sigma} = \sum \left\{ r \cdot [\![I(\overrightarrow{W})]\!] \;\middle|\; iv(\iota) = V, \sigma(\iota) = (r, I(\overrightarrow{W})) \right\}$$

Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states (for models that only differ in their controlled systems)

  given an equivalence relation $B \subseteq \mathcal{C} \times \mathcal{C}$

  then for all $(P, Q) \in B$, $\sigma \equiv \tau$, $C \in (\mathcal{F}/B)/\equiv$,

  1. for all $\underline{\mathrm{a}} \in \mathcal{E}_d$, whenever
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- notation: $P \sim^{\equiv} Q$
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Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation $B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2})$

  then for all $((q_1, \mathbf{x}_1), (q_2, \mathbf{x}_2)) \in B$

  - $\text{out}_1(\mathbf{x}_1) = \text{out}_2(\mathbf{x}_2)$
  - exit rates of q_1 and q_2 must be equal
  - disjunction of guards must evaluate to the same for $\mathbf{x}_1$ and $\mathbf{x}_2$
  - disjunction of guards must become true at the same time
  - for all $\underline{\mathrm{a}} \in \mathcal{E}_d$, one step priorities must match
  - for all $\overline{\mathrm{a}} \in \mathcal{E}_s$, one step probabilities must match
- notation: $\mathcal{T}_1 \sim_T^{\ell} \mathcal{T}_2$

Results

- $\sim^{\equiv}$ is a congruence (under certain conditions on $\equiv$)
- if $Con_1 \sim^{\equiv} Con_2$ then $Sys \bowtie_{*} \underline{\mathrm{init}}.Con_1 \sim^{\equiv} Sys \bowtie_{*} \underline{\mathrm{init}}.Con_2$
- *additively equivalent*: $\sigma \doteq \tau$ iff for all $V \in \mathcal{V}$ and $f(\mathcal{W})$

$$\mathrm{sum}(\sigma, V, f(\mathcal{W})) = \mathrm{sum}(\tau, V, f(\mathcal{W}))$$

where $\mathrm{sum}(\sigma, V, f(W)) \; =$

$$\sum \{\!| \, r \mid iv(\iota) = V, \sigma(\iota) = (r, I(W)), f(\mathcal{W}) = [\![I(W)]\!] \, |\!\}$$

- $P_1 \sim^{\doteq} P_2$ implies $\mathcal{T}(P_1) \sim^{\ell}_{T} \mathcal{T}(P_2)$

Results applied to assembly system

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- hence using congruence

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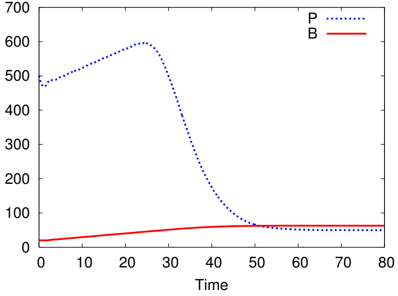
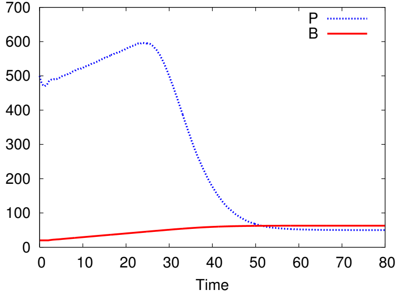
- define a single feed subcomponent with $iv(p) = P$

$$\begin{aligned} SFeed \stackrel{def}{=} & \ \underline{\text{init}} : (p, \textstyle\sum_{k=1}^{3} arrivals_i, const).SFeed + \\ & \ \underline{\text{full}} : (p, 0, const).SFeed \end{aligned}$$

- Sys_{SF} has $(Feed_1 \underset{*}{\bowtie} Feed_2 \underset{*}{\bowtie} Feed_3)$ replaced with $SFeed$
- then $Sys \underset{*}{\bowtie} \underline{\text{init}}.Con \sim^{\doteq} Sys_{SF} \underset{*}{\bowtie} \underline{\text{init}}.Con$

Two equivalent controllers





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Results applied to assembly system (continued)

- does $\mathcal{T}(P_1) \sim_T^\ell \mathcal{T}(P_2)$ imply $P_1 \sim^{\doteq} P_2$?
- no, consider two different assembly system models
- M: individual timers T_i that are set to zero as assembly starts, with guards to check whether $T_i \geq atime_i$
- M': single timer T whose value is stored in S_i as assembly starts, with guards to check whether $T \geq S_i + atime_i$
- can show that at the TDSHA level, $\mathcal{T}(M) \sim_T^\ell \mathcal{T}(M')$
- but $M \not\sim^{\doteq} M'$ since $\underline{\mathrm{take}}_i$ and $\underline{\mathrm{assem}}_i$ have different event conditions in M_1 and M_2 so definition does not apply
- correct definition of bisimilarity?