Comparison of non-interleaving semantic equivalences

Vashti Galpin

vcg@dcs.ed.ac.uk

Department of Computer Science
University of Edinburgh
Scotland

Outline

- recap of interleaving semantics
- location equivalence
- local/global cause equivalence
- other non-interleaving equivalences
- comparison of semantic equivalences
- conclusions

Process domains

- Petri nets
- event structures
- labelled transition systems

Labelled transition system $(\mathcal{S}, \mathcal{A}, \{\stackrel{a}{\rightarrow} \subseteq \mathcal{S} \times \mathcal{S} \mid a \in \mathcal{A}\})$

- $\mathcal{S}$ – set of states
- $\mathcal{A}$ – set of transition labels, actions
- relations $\stackrel{a}{\rightarrow}$ describe which transitions occur between states.
- write $s \stackrel{a}{\rightarrow} s'$ for $(s, s') \in \stackrel{a}{\rightarrow}$
- no structure on states or actions – pure labelled transition system
- structured states or actions – modified labelled transition system

Strong bisimulation and strong bisimilarity (Milner)

A **strong bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ such that $(S, T) \in \mathcal{R}$ if for all $a \in \mathcal{A}$

whenever $S \stackrel{a}{\rightarrow} S'$, then there exists $T' \in \mathcal{S}$ such that $T \stackrel{a}{\rightarrow} T'$ and $(S', T') \in \mathcal{R}$

Strong bisimilarity $\sim$ is the union of all strong bisimulations and is the largest strong bisimulation

Two processes are **strongly bisimilar** if they occur as a pair in a strong bisimulation

Weak bisimulation and observation equivalence (Milner)

Write $\Longrightarrow$ for $(\xrightarrow{\tau})^n, n \geq 0$

Write $\overset{a}{\Longrightarrow}$ for $\Longrightarrow\xrightarrow{a}\Longrightarrow$

Write $\overset{m}{\Longrightarrow}$ where $m = a_1.a_2.\ldots.a_k, k \geq 0$ for $\overset{a_1}{\Longrightarrow}\overset{a_2}{\Longrightarrow}\cdots\overset{a_k}{\Longrightarrow}$

Consider the labelled transition system $(\mathcal{S}, \mathcal{A}^*, \{\overset{m}{\Longrightarrow} \subseteq \mathcal{S} \times \mathcal{S} \mid m \in \mathcal{A}^*\})$

A **(weak) bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ such that $(S,T) \in \mathcal{R}$ if for all $m \in \mathcal{M}^*$

whenever $S \overset{m}{\Longrightarrow} S'$, then there exists $T' \in \mathcal{S}$ such that $T \overset{m}{\Longrightarrow} T'$ and $(S', T') \in \mathcal{R}$

Observation equivalence $\approx$ is the union of all weak bisimulations and is the largest weak bisimulation

Two processes are **observation equivalent** if they occur as a pair in a weak bisimulation

CCS syntax

$$P ::= nil \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$$

- $\alpha \in Act = \{a, b, c, \ldots, \overline{a}, \overline{b}, \overline{c}, \ldots\} \cup \tau$
- $L \subset \mathcal{L} = \{a, b, c, \ldots, \overline{a}, \overline{b}, \overline{c}, \ldots\}$
- f, relabelling function such that $f(\overline{\ell}) = \overline{f(\ell)}$ and $f(\tau) = \tau$
- $\mathcal{P}$ denotes the set of processes generated by this syntax

Operational semantics for CCS

(T1)	$\alpha.P \stackrel{\alpha}{\to} P$	$\alpha \in Act$	
(T2)	$P \stackrel{\alpha}{\to} P'$	implies	$P + Q \stackrel{\alpha}{\to} P'$ $Q + P \stackrel{\alpha}{\to} P'$
(T3)	$P \stackrel{\alpha}{\to} P'$	implies	$P \mid Q \stackrel{\alpha}{\to} P' \mid Q$ $Q \mid P \stackrel{\alpha}{\to} Q \mid P'$
(T4)	$P \stackrel{a}{\to} P', Q \stackrel{\overline{a}}{\to} Q'$	implies	$P \mid Q \stackrel{\tau}{\to} P' \mid Q'$
(T5)	$P \stackrel{\alpha}{\to} P'$	implies	$P[f] \stackrel{f(\alpha)}{\to} P'[f]$
(T6)	$P \stackrel{\alpha}{\to} P'$	implies	$P\backslash L \stackrel{\alpha}{\to} P'\backslash L \quad \alpha, \overline{\alpha} \notin L$

Then the operational semantics generate the following labelled transition systems:

- $(\mathcal{P}, Act, \{\stackrel{a}{\to} \mid m \in Act\})$
- $(\mathcal{P}, \mathcal{L}^*, \{\stackrel{m}{\Longrightarrow} \mid m \in \mathcal{L}^*\})$

where the transition relations are the least relations that satisfy the operational rules T1–T6.

Two CCS terms can be compared for bisimilarity or observation equivalence

Both these equivalences obey the Expansion Law, for example:

$$a.nil \mid b.nil \approx a.b.nil + b.a.nil$$

Non-interleaving equivalences are those equivalences under which the Expansion Law does not hold.

Equivalences based on location (Boudol, Castellani, Hennessy & Kiehn)

Consider the location transition system:

$$(\mathcal{S}, \mathcal{A}, Loc, \{\overset{a}{\underset{u}{\Longrightarrow}} \subseteq \mathcal{S} \times \mathcal{S} \mid a \in \mathcal{A}, u \in Loc^*\} \cup \overset{\tau}{\Longrightarrow})$$

where is Loc is a set of locations disjoint from A.

A **location bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ such that $(S, T) \in \mathcal{R}$ iff

1. whenever $S \Longrightarrow S'$ then there exists $T' \in \mathcal{S}$ such that $T \Longrightarrow T'$ and $(S', T') \in \mathcal{R}$
2. whenever $S \overset{a}{\underset{u}{\Longrightarrow}} S'$ then there exists $T' \in \mathcal{S}$ such that $T \overset{a}{\underset{u}{\Longrightarrow}} T'$ and $(S', T') \in \mathcal{R}$.

Location equivalence is defined to be the largest location bisimulation

Syntax for CCS with locations

$$P ::= nil \mid u :: P \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$$

- $u \in Loc^*$
- $\mathcal{P}_{Loc}$ denotes the set of processes generated by this syntax

Operational semantics for CCS with locations

(LT1) $a.P \xrightarrow[l]{a} l :: P \quad a \in \mathcal{L}, \quad l \in Loc$

OR

(LT1$_l$) $a.P \xrightarrow[u]{a} u :: P \quad a \in \mathcal{L}, \quad u \in Loc^*$

(LT2) $P \xrightarrow[u]{a} P'$ implies $v :: P \xrightarrow[vu]{a} v :: P'$

(LT3) $P \xrightarrow[u]{a} P'$ implies $P + Q \xrightarrow[u]{a} P'$
$Q + P \xrightarrow[u]{a} P'$

(LT4) $P \xrightarrow[u]{a} P'$ implies $P \mid Q \xrightarrow[u]{a} P' \mid Q$
$Q \mid P \xrightarrow[u]{a} Q \mid P'$

(LT5) $P \xrightarrow[u]{a} P'$ implies $P[f] \xrightarrow[u]{f(a)} P'[f]$

(LT6) $P \xrightarrow[u]{a} P'$ implies $P \backslash L \xrightarrow[u]{a} P' \backslash L \quad a, \overline{a} \notin L$

Write $\overset{a}{\underset{u}{\Longrightarrow}}$ for $\Longrightarrow\xrightarrow[u]{a}\Longrightarrow$

Consider the two location transition systems

- $LTS = (\mathcal{P}_{Loc}, \mathcal{L}, Loc, \{\overset{a}{\underset{u}{\Longrightarrow}} \mid a \in \mathcal{L}, u \in Loc^+\} \cup \overset{\tau}{\Longrightarrow})$
  defined using LT1–LT6 plus the τ transitions defined by T1–T6
- $LTS_l = (\mathcal{P}_{Loc}, \mathcal{L}, Loc, \{\overset{a}{\underset{u}{\Longrightarrow}} \mid a \in \mathcal{L}, u \in Loc^*\} \cup \overset{\tau}{\Longrightarrow})$
  defined using LT1$_l$ and LT2–LT6 plus the τ transitions defined by T1–T6

Use $\approx_l$ to denote location equivalence over LTS—**location equivalence**

Use $\approx_{ll}$ to denote location equivalence over LTS_l—**loose location equivalence**

Example

$$a.nil \mid b.nil \not\approx_{l_b} a.b.nil + b.a.nil$$

Consider the following transitions for $l, m \in Loc$

$$(a.nil \mid b.nil) \overset{a}{\underset{l}{\Longrightarrow}} (l :: nil \mid b.nil) \overset{b}{\underset{m}{\Longrightarrow}} (l :: nil \mid m :: nil)$$

whereas

$$a.b.nil + b.a.nil \overset{a}{\underset{l}{\Longrightarrow}} l :: b.nil \overset{b}{\underset{lm}{\Longrightarrow}} l :: m :: nil$$

It can be shown that $\approx_l \subset \approx_{ll}$

$(a.c.nil|\bar{c}.b.nil)\backslash\{c\} \not\approx_l (a.(c.nil + b.nil)|\bar{c}.b.nil)\backslash\{c\}$

Local and global cause equivalence (Kiehn)

Consider the local/global cause transition system:

$$(\mathcal{S}, \mathcal{A}, \mathcal{C}, \{\overset{a}{\underset{A,B,l}{\Longrightarrow}} \subseteq \mathcal{S} \times \mathcal{S} \mid a \in \mathcal{A}, l \in \mathcal{C}, A, B \subseteq \mathcal{C}\} \cup \overset{\tau}{\Longrightarrow})$$

where $\mathcal{C}$ is a set of causes disjoint from $\mathcal{A}$.

A **local/global cause bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ such that $(S, T) \in \mathcal{R}$ iff

1. whenever $S \Longrightarrow S'$ then there exists $T' \in \mathcal{S}$ such that $T \Longrightarrow T'$ and $(S', T') \in \mathcal{R}$
2. whenever $S \overset{a}{\underset{A,B,l}{\Longrightarrow}} S'$ then there exists $T' \in \mathcal{S}$ such that $T \overset{a}{\underset{A,B,l}{\Longrightarrow}} T'$ and $(S', T') \in \mathcal{R}$.

Local/global cause equivalence $\approx_{lg}$ is defined to be the largest local/global cause bisimulation

Local cause equivalence $\approx_{gc}$ can be defined by requiring the first sets and the current causes to be equal

Global cause equivalence $\approx_{lc}$ can be defined by requiring the second sets and the current causes to be equal

Syntax for CCS with local/global causes

$$P ::= nil \mid l :: P \mid X :: P \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$$

- $l \in \mathcal{C}, X \subset \mathcal{C}$
- $\mathcal{P}_{\mathcal{LG}}$ denotes the set of processes generated by this syntax

Operational semantics for CCS with local/global causes

(LG1)	$a.P \xrightarrow[\emptyset,\emptyset,l]{a} l :: P$		$a \in \mathcal{L}, \quad l \in \mathcal{C}$
(LG2)	$P \xrightarrow[A,B,l]{a} P'$	implies	$k :: P \xrightarrow[A\cup\{k\},B\cup\{k\},l]{a} k :: P'$
(LG3)	$P \xrightarrow[A,B,l]{a} P'$	implies	$X :: P \xrightarrow[A,B\cup X,l]{a} X :: P'$
(LG4)	$P \xrightarrow[A,B,l]{a} P'$	implies	$P + Q \xrightarrow[A,B,l]{a} P'$ $Q + P \xrightarrow[A,B,l]{a} P'$
(LG5)	$P \xrightarrow[A,B,l]{a} P'$	implies	$P \mid Q \xrightarrow[A,B,l]{a} P' \mid Q$ $Q \mid P \xrightarrow[A,B,l]{a} Q \mid P'$
(LG6)	$P \xrightarrow[A,B,l]{a} P'$	implies	$P[f] \xrightarrow[A,B,l]{f(a)} P'[f]$
(LG7)	$P \xrightarrow[A,B,l]{a} P'$	implies	$P \backslash L \xrightarrow[A,B,l]{a} P' \backslash L \quad a, \overline{a} \notin L$
(LG8)	$P \xrightarrow[A,B,l]{a} P'$, $Q \xrightarrow[A',B',l']{\overline{a}} Q'$	imply	$P \mid Q \xrightarrow{\tau} P'[l \mapsto B'] \mid Q'[l' \mapsto B]$

Let $\xRightarrow[A,B,l]{a}$ be $\Longrightarrow\xrightarrow[A,B,l]{a}\Longrightarrow$

Consider the local/global cause transition system

$$(P_{\mathcal{LG}}, Act, \mathcal{C}, \{\xRightarrow[A,B,l]{a} \mid a \in \mathcal{L}, l \in \mathcal{C}, A, B \subseteq \mathcal{C}\} \cup \xRightarrow{\tau})$$

defined by LG1–LG8

We can consider the three equivalences, $\approx_{lg}$, $\approx_{gc}$ and $\approx_{lc}$ over this transition system.

Example

$$a.nil \mid b.nil \not\approx_{lg} a.b.nil + b.a.nil$$

Consider the following transitions for $l, m \in \mathcal{C}$

$$a.nil \mid b.nil \xRightarrow[\emptyset,\emptyset,l]{a} l :: nil \mid b.nil \xRightarrow[\emptyset,\emptyset,m]{b} l :: nil \mid m :: nil$$

whereas

$$a.b.nil + b.a.nil \xRightarrow[\emptyset,\emptyset,l]{a} l :: b.nil \xRightarrow[\{l\},\{l\},m]{b} l :: m :: nil$$

Example

$$(a.c.nil \mid \bar{c}.b.nil)\backslash\{c\} \approx_{gc} a.b.nil$$

Consider the following transitions for $l, m \in \mathcal{C}$

$$(a.c.nil \mid \bar{c}.b.nil)\backslash\{c\} \xRightarrow[\emptyset,\emptyset,l]{a} (l :: c.nil \mid \bar{c}.b.nil)\backslash\{c\} \xRightarrow{\tau} (l :: \emptyset :: nil \mid \{l\} :: b.nil)\backslash\{c\}$$

$$\xRightarrow[\emptyset,\{l\},m]{b} (l :: \emptyset :: nil \mid \{l\} :: m :: nil)\backslash\{c\}$$

and

$$a.b.nil \xRightarrow[\emptyset,\emptyset,l]{a} l :: b.nil \xRightarrow[\{l\},\{l\},m]{b} l :: m :: nil$$

Other non-interleaving equivalences

- causal bisimilarity (Darondeau & Degano)

  $$P \xrightarrow{\langle a,B\rangle} P'$$

- distributed bisimulation equivalence (Castellani & Hennessy)

  $$P \xrightarrow{a} \langle P', P''\rangle$$

- refine equivalence/ST-equivalence (Hennessy)

  $$a.P \xrightarrow{s(a_i)} f(a_i).P \quad \text{and} \quad f(a_i).P \xrightarrow{f(a_i)} P$$

- read/write equivalence (Priami & Yankelvich)

  $$(a.c.b \mid d.\bar{c}.e)\backslash\{c\} \not\approx_{rw} (a.\bar{c}.b \mid d.c.e)\backslash\{c\}$$

- equivalences defined on proved transition systems

Comparison

- Why?
  - to determine the relationship between different equivalences
  - to determine which equivalence to use in a given situation
- How?
  - in terms of CCS processes
  - in terms of labelled transition systems
  - by determining which properties hold under a specific equivalence

Comparison in terms of CCS processes

$\approx$	observation equivalence
$\approx_d$	distributed bisimulation equivalence
$\approx_l$	location equivalence
$\approx_{ll}$	loose location equivalence
$\approx_l^s$	static location equivalence
$\approx_c$	causal bisimilarity
$\approx_{lc}$	local cause equivalence
$\approx_{gc}$	global cause equivalence
$\approx_{lg}$	local/global cause equivalence
$\approx_{rw}$	read/write equivalence
$\approx_{ST}$	ST-equivalence

Comparison in terms of labelled transition system

- Disadvantages of comparison in terms of CCS processes
- More general approach to modified labelled transition systems
  - union
  - general labelled transition system
  - parameterised labelled transition system

Comparison in terms of properties

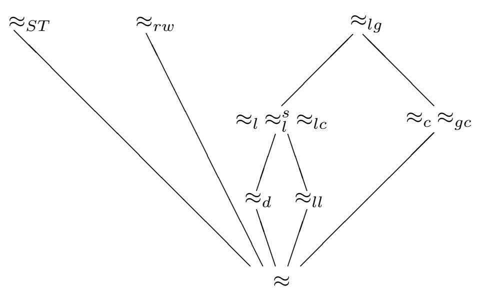
- Local deadlock
- An equivalence $\asymp$ is said to distinguish

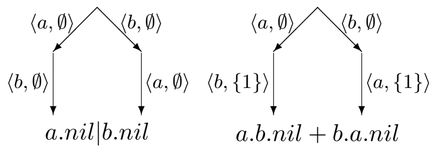
location	iff	$(a.c.b \mid d.\bar{c}.e) \setminus c \not\asymp (a.c.e \mid d.\bar{c}.b) \setminus c$
read-write causality	iff	$(a.c.b \mid d.\bar{c}.e) \setminus c \not\asymp (a.\bar{c}.b \mid d.c.e) \setminus c$
concurrency	iff	$a \mid b \not\asymp a.b + b.a$

- $\approx_l$, $\approx_l'$ and $\approx_l^s$ all distinguish location, but not read-write causality
- $\approx_{rw}$ distinguishes read-write causality, but not location
- $\approx_c$ doesn't distinguish location or read-write causality
- All equivalences shown previously except $\approx$, distinguish concurrency.

Conclusions

- Different approaches to defining non-interleaving equivalences
- Different approaches to comparing non-interleaving equivalences




$\langle a, \emptyset\rangle$
$\langle b, \emptyset\rangle$
$\langle b, \emptyset\rangle$
$\langle a, \emptyset\rangle$
$a.nil|b.nil$
$\langle a, \emptyset\rangle$
$\langle b, \emptyset\rangle$
$\langle b, \{1\}\rangle$
$\langle a, \{1\}\rangle$
$a.b.nil + b.a.nil$