Comparing non-interleaving equivalences on labelled transition systems

Vashti Galpin

vcg@dcs.ed.ac.uk

Department of Computer Science

University of Edinburgh

Outline of seminar

1. Background
   - CCS and observation equivalence
   - Non-interleaving equivalences
2. Comparison of equivalences—why and how
3. Comparison in terms of CCS processes
4. Comparison in terms of transition systems
5. Conclusions and further work

CCS (Milner)

- Syntax

$$P ::= nil \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$$

- $\alpha \in Act = \{a, b, c, \ldots, \overline{a}, \overline{b}, \overline{c}, \ldots\} \cup \tau$
- $L \subset \mathcal{L} = \{a, b, c, \ldots, \overline{a}, \overline{b}, \overline{c}, \ldots\}$
- f, relabelling function such that $f(\overline{\ell}) = \overline{f(\ell)}$ and $f(\tau) = \tau$
- $\mathcal{P}_{CCS}$ denotes the set of processes generated by this syntax

Operational semantics for CCS

(T1)	$\alpha.P \xrightarrow{\alpha} P$	$\alpha \in Act$	
(T2)	$P \xrightarrow{\alpha} P'$	implies	$P + Q \xrightarrow{\alpha} P'$ $Q + P \xrightarrow{\alpha} P'$
(T3)	$P \xrightarrow{\alpha} P'$	implies	$P \mid Q \xrightarrow{\alpha} P' \mid Q$ $Q \mid P \xrightarrow{\alpha} Q \mid P'$
(T4)	$P \xrightarrow{a} P'$, $Q \xrightarrow{\overline{a}} Q'$	implies	$P \mid Q \xrightarrow{\tau} P' \mid Q'$
(T5)	$P \xrightarrow{\alpha} P'$	implies	$P[f] \xrightarrow{f(\alpha)} P'[f]$
(T6)	$P \xrightarrow{\alpha} P'$	implies	$P\backslash L \xrightarrow{\alpha} P'\backslash L \quad \alpha, \overline{\alpha} \notin L$

Observation equivalence

Define $\Longrightarrow = (\stackrel{\tau}{\longrightarrow})^n, n \geq 0$, and $\stackrel{\alpha}{\Longrightarrow} = \Longrightarrow\stackrel{\alpha}{\longrightarrow}\Longrightarrow$

A **(weak) bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{P}_{CCS} \times \mathcal{P}_{CCS}$ such that $(P, Q) \in \mathcal{R}$ if

1. whenever $P \stackrel{\tau}{\longrightarrow} P'$, then there exists $Q' \in \mathcal{P}_{CCS}$ such that $Q \Longrightarrow Q'$ and $(P', Q') \in \mathcal{R}$, and
2. for all $a \in \mathcal{L}$, whenever $P \stackrel{a}{\longrightarrow} P'$, then there exists $Q' \in \mathcal{P}_{CCS}$ such that $Q \stackrel{a}{\Longrightarrow} Q'$ and $(P', Q') \in \mathcal{R}$

Observation equivalence $\approx$ is the union of all weak bisimulations and is the largest weak bisimulation

Two CCS terms can be shown to be observation equivalent, by finding a weak bisimulation that contains them as a pair.

Observation equivalence obey the Expansion Law, for example:

$$a.nil \mid b.nil \approx a.b.nil + b.a.nil$$

Non-interleaving equivalences are those equivalences under which the Expansion Law does not hold.

CCS with locations (Boudol, Castellani, Hennessy & Kiehn)

- Syntax

$$P ::= nil \mid u :: P \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$$

- $u \in Loc^*$
- $\mathcal{P}_{Loc}$ denotes the set of processes generated by this syntax

Operational semantics for CCS with locations

(LT1)	$a.P \xrightarrow[l]{a} l :: P$	$a \in \mathcal{L}$,	$l \in Loc$
(LT2)	$P \xrightarrow[u]{a} P'$	implies	$v :: P \xrightarrow[vu]{a} v :: P'$
(LT3)	$P \xrightarrow[u]{a} P'$	implies	$P + Q \xrightarrow[u]{a} P'$ $Q + P \xrightarrow[u]{a} P'$
(LT4)	$P \xrightarrow[u]{a} P'$	implies	$P \mid Q \xrightarrow[u]{a} P' \mid Q$ $Q \mid P \xrightarrow[u]{a} Q \mid P'$
(LT5)	$P \xrightarrow[u]{a} P'$	implies	$P[f] \xrightarrow[u]{f(a)} P'[f]$
(LT6)	$P \xrightarrow[u]{a} P'$	implies	$P\backslash L \xrightarrow[u]{a} P'\backslash L \quad a, \overline{a} \notin L$

Location equivalence

Define $\overset{a}{\underset{u}{\Longrightarrow}} = \Longrightarrow \overset{a}{\underset{u}{\longrightarrow}} \Longrightarrow$

A **location bisimulation** is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{P}_{Loc} \times \mathcal{P}_{Loc}$ such that $(P, Q) \in \mathcal{R}$ iff

1. whenever $P \xrightarrow{\tau} P'$ then there exists $Q' \in \mathcal{P}_{Loc}$ such that $Q \Longrightarrow Q'$ and $(P', Q') \in \mathcal{R}$, and
2. for all $a \in \mathcal{L}, u \in Loc$, whenever $P \overset{a}{\underset{u}{\longrightarrow}} P'$ then there exists $Q' \in \mathcal{P}_{Loc}$ such that $Q \overset{a}{\underset{u}{\Longrightarrow}} Q'$ and $(P', Q') \in \mathcal{R}$.

Location equivalence $\approx_l$ is defined to be the largest location bisimulation

Example

$$a.nil \mid b.nil \not\approx_l a.b.nil + b.a.nil$$

Consider the following transitions for $l, m \in Loc$

$$(a.nil \mid b.nil) \overset{a}{\underset{l}{\Longrightarrow}} (l :: nil \mid b.nil) \overset{b}{\underset{m}{\Longrightarrow}} (l :: nil \mid m :: nil)$$

whereas

$$a.b.nil + b.a.nil \overset{a}{\underset{l}{\Longrightarrow}} l :: b.nil \overset{b}{\underset{lm}{\Longrightarrow}} l :: m :: nil$$

Other non-interleaving equivalences

- local/global cause equivalence (Kiehn) $P \xrightarrow[A,B,l]{a} P'$
- causal bisimilarity (Darondeau & Degano) $P \stackrel{\langle a,B\rangle}{\longrightarrow} P'$
- distributed bisimulation equivalence (Castellani & Hennessy)

  $$P \stackrel{a}{\longrightarrow} \langle P', P'' \rangle$$

- refine equivalence/ST-equivalence (Hennessy)

  $$a.P \stackrel{s(a_i)}{\longrightarrow} f(a_i).P \quad \text{and} \quad f(a_i).P \stackrel{f(a_i)}{\longrightarrow} P$$

- read/write equivalence (Priami & Yankelvich)

  $$(a.c.b \mid d.\overline{c}.e)\backslash c \not\approx_{rw} (a.\overline{c}.b \mid d.c.e)\backslash c$$

Comparison

- Why?
  - to determine the relationship between different equivalences
  - to determine which equivalence to use in a given situation
- How?
  - in terms of CCS processes
  - in terms of labelled transition systems

Comparison in terms of CCS processes

$\approx$	observation equivalence
$\approx_d$	distributed bisimulation equivalence
$\approx_l$	location equivalence
$\approx_{ll}$	loose location equivalence
$\approx_l^s$	static location equivalence
$\approx_{dg}$	distributed grapes equivalence
$\approx_c$	causal bisimilarity
$\approx_{lc}$	local cause equivalence
$\approx_{gc}$	global cause equivalence
$\approx_{lg}$	local/global cause equivalence
$\approx_{rw}$	read/write equivalence
$\approx_{ST}$	ST-equivalence

Comparison in terms of labelled transition system

- Underlying process domain
- Language-independent approach
- Commonalities

Extended single-action labelled transition system (esaLTS)

$$\mathcal{L} = (\mathcal{S}, \mathcal{A}, \mathcal{D}, \mathcal{U})$$

- $\mathcal{S}$, set of states
- $\mathcal{A}$, set of (atomic) actions
- $\mathcal{D}$, data structure
- $\mathcal{U} \subseteq (\mathcal{S} \times \mathcal{A} \times \mathcal{D} \times \mathcal{S}) \cup (\mathcal{S} \times \{\tau\} \times \mathcal{S})$
- Write $s \xrightarrow[d]{a} s'$ for $(s, a, d, s') \in \mathcal{U}$ and $s \xrightarrow{\tau} s'$ for $(s, \tau, s') \in \mathcal{U}$
- Define $\overset{a}{\underset{d}{\Longrightarrow}} = \Longrightarrow \xrightarrow[d]{a} \Longrightarrow$ and $\overset{\tau}{\Longrightarrow} = \Longrightarrow \xrightarrow{\tau} \Longrightarrow$

esaLTS bisimulation

A *(weak) esaLTS bisimulation* is a symmetric binary relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ such that $(s, t) \in \mathcal{R}$ if

1. whenever $s \xrightarrow{\tau} s'$, then there exists $t' \in \mathcal{S}$ such that $t \Longrightarrow t'$ and $(s', t') \in \mathcal{R}$, and
2. for all $a \in \mathcal{A}, d \in \mathcal{D}$, whenever $s \xrightarrow[d]{a} s'$, then there exists $t' \in \mathcal{S}$ such that $t \overset{a}{\underset{d}{\Longrightarrow}} t'$ and $(s', t') \in \mathcal{R}$,

Two states, s_1 and s_2 are *(esaLTS-)bisimilar* ($s_1 \approx_{\mathcal{D}} s_2$) if there exists a bisimulation $\mathcal{R}$ such that $(s_1, s_2) \in \mathcal{R}$,

esaLTS homomorphism

$(h_\sigma, h_\delta, h_\upsilon) : (\mathcal{S}_1, \mathcal{A}, \mathcal{D}_1, \mathcal{U}_1) \rightarrow (\mathcal{S}_2, \mathcal{A}, \mathcal{D}_2, \mathcal{U}_2)$ with

$h_\sigma : \mathcal{S}_1 \rightarrow \mathcal{S}_2, \quad h_\delta : \mathcal{D}_1 \rightarrow \mathcal{D}_2$ and

$h_\upsilon : \mathcal{U}_1 \rightarrow \mathcal{U}_2$ such that

$h_\upsilon(s \xrightarrow{\tau} s') = h_\sigma(s) \xrightarrow{\tau} h_\sigma(s')$ if $h_\sigma(s) \neq h_\sigma(s')$ and

$h_\upsilon(s \xrightarrow[d]{a} s') = h_\sigma(s) \xrightarrow[h_\delta(d)]{a} h_\sigma(s')$ such that

1. for each $t \xrightarrow[d_2]{a} t' \in h_\upsilon(\mathcal{U}_1)$ and each s such that $h_\sigma(s) = t$, there exists $s \overset{a}{\underset{d_1}{\Longrightarrow}} s' \in \mathcal{U}_1$ such that $h_\sigma(s') = t'$ and $h_\delta(d_1) = d_2$.
2. for each $t \xrightarrow{\tau} t' \in h_\upsilon(\mathcal{U}_1)$ and each s such that $h_\sigma(s) = t$, there exists $s \overset{\tau}{\Longrightarrow} s' \in \mathcal{U}_1$ such that $h_\sigma(s') = t'$ and $h_\delta(d_1) = d_2$.

Given

- $(\mathcal{S}_i, \mathcal{A}, \mathcal{D}_i, \mathcal{U}_i)$ for $i = 1, 2$
- esaLTS homomorphism $(h_\sigma, h_\delta, h_\upsilon) : (\mathcal{S}_1, \mathcal{A}, \mathcal{D}_1, \mathcal{U}_1) \rightarrow (\mathcal{S}_2, \mathcal{A}, \mathcal{D}_2, \mathcal{U}_2)$
- $s_1 \approx_{\mathcal{D}_1} s_2$

then

- $h_\sigma(s) \approx_{\mathcal{D}_2} h_\sigma(s')$ in $(h_\sigma(\mathcal{S}_1), \mathcal{A}, h_\delta(\mathcal{D}_1), h_\upsilon(\mathcal{U}_1))$

Sequential esaLTS

$\mathcal{L}_{\mathcal{D}} = (\mathcal{P}_{\mathcal{D}}, \mathcal{A}, \mathcal{D}, \mathcal{U})$

- Syntax $P ::= \tau P \mid \langle a, d\rangle P \mid \sum_{i\in I} P_i$
- Operational semantics

P1 $\langle a, d\rangle P \xrightarrow[d]{a} P$
P2 $\tau P \xrightarrow{\tau} P$
P3 $P_1 \xrightarrow[d]{a} P'$ implies $P_1 + P_2 \xrightarrow[d]{a} P'$ and $P_2 + P_1 \xrightarrow[d]{a} P'$

Standard function for h_δ

$h_\delta : \mathcal{D}_1 \to \mathcal{D}_2$, surjective

Define $H_{h_\delta} = (H_\sigma, h_\delta, H_\upsilon)$ where

$$H_\upsilon(P \xrightarrow[d]{a} P') = H_\sigma(P) \xrightarrow[h_\delta(d)]{a} H_\sigma(P')$$

$$H_\upsilon(P \xrightarrow{\tau} P') = H_\sigma(P) \xrightarrow{\tau} H_\sigma(P)$$

such that $\mathcal{U}_2 = H_\upsilon(\mathcal{U}_1)$ and

$$H_\sigma(\langle a, d\rangle P) = \langle a, h_\delta(d)\rangle H_\sigma(P')$$

$$H_\sigma(\tau P) = \tau H_\sigma(P)$$

$$H_\sigma(\sum_{i\in I} P_i) = \sum_{i\in I} H_\sigma(P_i).$$

Given

- $\mathcal{L}_{\mathcal{D}_i} = (\mathcal{P}_{\mathcal{D}_i}, \mathcal{A}, \mathcal{D}_i, \mathcal{U}_i)$ for $i = 1, 2$
- $h_\delta : \mathcal{D}_1 \to \mathcal{D}_2$, surjective

then

- H_{h_δ} is an esaLTS homomorphism
- $P_1 \approx_{\mathcal{D}_1} P_2$ implies $H_{h_\delta}(P_1) \approx_{\mathcal{D}_2} H_{h_\delta}(P_2)$ implies

Questions

- Which $\mathcal{D}$'s are interesting?
- Which h_δ's are interesting?
- How does this relate to the operational semantics of a specific process algebra?
- Does this explain the known relationships between equivalences on CCS?

Conclusions and further work

- Two approaches to comparison
  - in terms of CCS processes
  - in terms of labelled transition systems
- Quantification over elements of $\mathcal{D}$
- Rule formats