ABCD

A Basis for Concurrency and Distribution

Philip Wadler

CoCo, Loch Lomond, 8 May 2014


ABCD
any body can dance

Team ABCD

- Edinburgh
  - Philip Wadler
  - Sam Lindley
  - Garrett Morris
  - Shayan Najd
- Glasgow
  - Simon Gay
  - Dimitrios Kouzapas
  - Ornela Dardha
- Imperial
  - Nabuko Yoshida
  - Laura Bocchi
  - Luca Fossati
  - Raymond Hu
  - Julien Lange
  - Rumyana Neykova
  - Nicholas Ng
  - Weizhen Yang

Session Types

$$
\begin{aligned}
S = \&\langle &\mathsf{auth1}\colon\ ?[\mathsf{str}, \mathsf{str}]\ .\ \oplus \\
&\langle \mathsf{accept}\colon\ ![\mathsf{str}]\ .\ \&\langle \mathsf{auth2}\colon\ ?[\mathsf{int}]\ .\ \oplus \\
&\qquad \langle \mathsf{accept}\colon\ ![\mathsf{str}]\ .\ \mathsf{end}\rangle, \\
&\qquad \langle \mathsf{reject}\colon\ ![\mathsf{str}]\ .\ \mathsf{end}\rangle\rangle\rangle, \\
&\langle \mathsf{reject}\colon\ ![\mathsf{str}]\ .\ \mathsf{end}\rangle\rangle
\end{aligned}
$$

Linear Logic: CP

$$\frac{}{w{\leftrightarrow}x \vdash w : A^{\perp}, x : A}\ \mathsf{Ax} \qquad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^{\perp}}{\nu x : A.(P \mid Q) \vdash \Gamma, \Delta}\ \mathsf{Cut}$$

$$\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ \otimes \qquad \frac{R \vdash \Theta, y : A, x : B}{x(y).R \vdash \Theta, x : A \parr B}\ \parr$$

$$\frac{P \vdash \Gamma, x : A}{x[\mathsf{inl}].P \vdash \Gamma, x : A \oplus B}\ \oplus_1 \qquad \frac{P \vdash \Gamma, x : B}{x[\mathsf{inr}].P \vdash \Gamma, x : A \oplus B}\ \oplus_2 \qquad \frac{Q \vdash \Delta, x : A \quad R \vdash \Delta, x : B}{x.\mathsf{case}(Q, R) \vdash \Delta, x : A \,\&\, B}\ \&$$

$$\frac{P \vdash ?\Gamma, y : A}{!x(y).P \vdash ?\Gamma, x : !A}\ ! \qquad \frac{Q \vdash \Delta, y : A}{?x[y].Q \vdash \Delta, x : ?A}\ ? \qquad \frac{Q \vdash \Delta}{Q \vdash \Delta, x : ?A}\ \mathsf{Weaken} \qquad \frac{Q \vdash \Delta, x' : ?A, x'' : ?A}{Q\{x/x', x/x''\} \vdash \Delta, x : ?A}\ \mathsf{Contract}$$

$$\frac{P \vdash \Gamma, x : B\{A/X\}}{x[A].P \vdash \Gamma, x : \exists X.B}\ \exists \qquad \frac{Q \vdash \Delta, x : B}{x(X).Q \vdash \Delta, x : \forall X.B}\ \forall\ (X \notin \mathsf{fv}(\Delta))$$

$$\frac{}{x[\,].0 \vdash x : 1}\ 1 \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \bot}\ \bot \qquad \text{(no rule for 0)} \qquad \frac{}{x.\mathsf{case}() \vdash \Gamma, x : \top}\ \top$$

Figure 1. CP, classical linear logic as a session-typed process calculus

Linear Logic: GV

$$\frac{}{x : T \vdash x : T}\ \text{Id} \qquad \frac{}{\vdash \mathsf{unit} : \mathsf{Unit}}\ \text{Unit} \qquad \frac{\Phi \vdash N : U \quad \mathsf{un}(T)}{\Phi,\ x : T \vdash N : U}\ \text{Weaken} \qquad \frac{\Phi,\ x' : T,\ x'' : T \vdash N : U \quad \mathsf{un}(T)}{\Phi,\ x : T \vdash N\{x/x', x/x''\} : U}\ \text{Contract}$$

$$\frac{\Phi,\ x : T \vdash N : U}{\Phi \vdash \lambda x.N : T \multimap U}\ {\multimap}\text{-I} \qquad \frac{\Phi \vdash L : T \multimap U \quad \Psi \vdash M : T}{\Phi,\ \Psi \vdash L\ M : U}\ {\multimap}\text{-E} \qquad \frac{\Phi \vdash L : T \multimap U \quad \mathsf{un}(\Phi)}{\Phi \vdash L : T \to U}\ {\to}\text{-I} \qquad \frac{\Phi \vdash L : T \to U}{\Phi \vdash L : T \multimap U}\ {\to}\text{-E}$$

$$\frac{\Phi \vdash M : T \quad \Psi \vdash N : U}{\Phi,\ \Psi \vdash (M, N) : T \otimes U}\ \otimes\text{-I} \qquad \frac{\Phi \vdash M : T \otimes U \quad \Psi,\ x : T,\ y : U \vdash N : V}{\Phi,\ \Psi \vdash \mathsf{let}\ (x, y) = M\ \mathsf{in}\ N : V}\ \otimes\text{-E}$$

$$\frac{\Phi \vdash M : T \quad \Psi \vdash N : !T.S}{\Phi,\ \Psi \vdash \mathsf{send}\ M\ N : S}\ \text{Send} \qquad \frac{\Phi \vdash M : ?T.S}{\Phi \vdash \mathsf{receive}\ M : T \otimes S}\ \text{Receive}$$

$$\frac{\Phi \vdash M : \oplus\{l_i : S_i\}_{i \in I}}{\Phi \vdash \mathsf{select}\ l_j\ M : S_j}\ \text{Select} \qquad \frac{\Phi \vdash M : \&\{l_i : S_i\}_{i \in I} \quad (\Psi,\ x : S_i \vdash N_i : T)_{i \in I}}{\Phi,\ \Psi \vdash \mathsf{case}\ M\ \mathsf{of}\ \{l_i : x.N_i\}_{i \in I} : T}\ \text{Case}$$

$$\frac{\Phi,\ x : S \vdash M : \mathsf{end}_! \quad \Psi,\ x : \overline{S} \vdash N : T}{\Phi,\ \Psi \vdash \mathsf{with}\ x\ \mathsf{connect}\ M\ \mathsf{to}\ N : T}\ \text{Connect} \qquad \frac{\Phi \vdash M : T \otimes \mathsf{end}_?}{\Phi \vdash \mathsf{terminate}\ M : T}\ \text{Terminate}$$

Figure 5. GV, a session-typed functional language