

ABCD

A Basis for Concurrency and Distribution

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CoCo, Loch Lomond, 8 May 2014



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Session Types

$$S = \&\langle \text{auth1: ?}[\text{str}, \text{str}] . \oplus$$
$$\langle \text{accept: !}[\text{str}] . \&\langle \text{auth2: ?}[\text{int}] . \oplus$$
$$\langle \text{accept: !}[\text{str}] . \text{end} \rangle,$$
$$\langle \text{reject: !}[\text{str}] . \text{end} \rangle \rangle \rangle,$$
$$\langle \text{reject: !}[\text{str}] . \text{end} \rangle \rangle$$

Linear Logic: CP

$$\begin{array}{c}
 \frac{}{w \leftrightarrow x \vdash w : A^\perp, x : A} \text{Ax} \quad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^\perp}{\nu x : A. (P \mid Q) \vdash \Gamma, \Delta} \text{Cut} \\
 \\
 \frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x[y]. (P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B} \otimes \quad \frac{R \vdash \Theta, y : A, x : B}{x(y). R \vdash \Theta, x : A \wp B} \wp \\
 \\
 \frac{P \vdash \Gamma, x : A}{x[\text{inl}]. P \vdash \Gamma, x : A \oplus B} \oplus_1 \quad \frac{P \vdash \Gamma, x : B}{x[\text{inr}]. P \vdash \Gamma, x : A \oplus B} \oplus_2 \quad \frac{Q \vdash \Delta, x : A \quad R \vdash \Delta, x : B}{x.\text{case}(Q, R) \vdash \Delta, x : A \& B} \& \\
 \\
 \frac{P \vdash ?\Gamma, y : A}{!x(y). P \vdash ?\Gamma, x : !A} ! \quad \frac{Q \vdash \Delta, y : A}{?x[y]. Q \vdash \Delta, x : ?A} ? \quad \frac{Q \vdash \Delta}{Q \vdash \Delta, x : ?A} \text{Weaken} \quad \frac{Q \vdash \Delta, x' : ?A, x'' : ?A}{Q\{x/x', x/x''\} \vdash \Delta, x : ?A} \text{Contract} \\
 \\
 \frac{P \vdash \Gamma, x : B\{A/X\}}{x[A]. P \vdash \Gamma, x : \exists X. B} \exists \quad \frac{Q \vdash \Delta, x : B}{x(X). Q \vdash \Delta, x : \forall X. B} \forall \quad (X \notin \text{fv}(\Delta)) \\
 \\
 \frac{}{x[.]. 0 \vdash x : 1} 1 \quad \frac{P \vdash \Gamma}{x(). P \vdash \Gamma, x : \perp} \perp \quad (\text{no rule for } 0) \quad \frac{}{x.\text{case}() \vdash \Gamma, x : \top} \top
 \end{array}$$

Figure 1. CP, classical linear logic as a session-typed process calculus

Linear Logic: GV

$$\begin{array}{c}
 \frac{}{x : T \vdash x : T} \text{Id} \quad \frac{}{\vdash \text{unit} : \text{Unit}} \text{Unit} \quad \frac{\Phi \vdash N : U \quad \text{un}(T)}{\Phi, x : T \vdash N : U} \text{Weaken} \quad \frac{\Phi, x' : T, x'' : T \vdash N : U \quad \text{un}(T)}{\Phi, x : T \vdash N\{x/x', x/x''\} : U} \text{Contract} \\
 \\
 \frac{\Phi, x : T \vdash N : U}{\Phi \vdash \lambda x. N : T \multimap U} \multimap\text{-I} \quad \frac{\Phi \vdash L : T \multimap U \quad \Psi \vdash M : T}{\Phi, \Psi \vdash LM : U} \multimap\text{-E} \quad \frac{\Phi \vdash L : T \multimap U \quad \text{un}(\Phi)}{\Phi \vdash L : T \rightarrow U} \rightarrow\text{-I} \quad \frac{\Phi \vdash L : T \rightarrow U}{\Phi \vdash L : T \multimap U} \rightarrow\text{-E} \\
 \\
 \frac{\Phi \vdash M : T \quad \Psi \vdash N : U}{\Phi, \Psi \vdash (M, N) : T \otimes U} \otimes\text{-I} \quad \frac{\Phi \vdash M : T \otimes U \quad \Psi, x : T, y : U \vdash N : V}{\Phi, \Psi \vdash \text{let } (x, y) = M \text{ in } N : V} \otimes\text{-E} \\
 \\
 \frac{\Phi \vdash M : T \quad \Psi \vdash N : !T.S}{\Phi, \Psi \vdash \text{send } MN : S} \text{Send} \quad \frac{\Phi \vdash M : ?T.S}{\Phi \vdash \text{receive } M : T \otimes S} \text{Receive} \\
 \\
 \frac{\Phi \vdash M : \oplus\{l_i : S_i\}_{i \in I}}{\Phi \vdash \text{select } l_j M : S_j} \text{Select} \quad \frac{\Phi \vdash M : \&\{l_i : S_i\}_{i \in I} \quad (\Psi, x : S_i \vdash N_i : T)_{i \in I}}{\Phi, \Psi \vdash \text{case } M \text{ of } \{l_i : x.N_i\}_{i \in I} : T} \text{Case} \\
 \\
 \frac{\Phi, x : S \vdash M : \text{end}_! \quad \Psi, x : \bar{S} \vdash N : T}{\Phi, \Psi \vdash \text{with } x \text{ connect } M \text{ to } N : T} \text{Connect} \quad \frac{\Phi \vdash M : T \otimes \text{end}_?}{\Phi \vdash \text{terminate } M : T} \text{Terminate}
 \end{array}$$

Figure 5. GV, a session-typed functional language