The Essence of
Language Integrated Query

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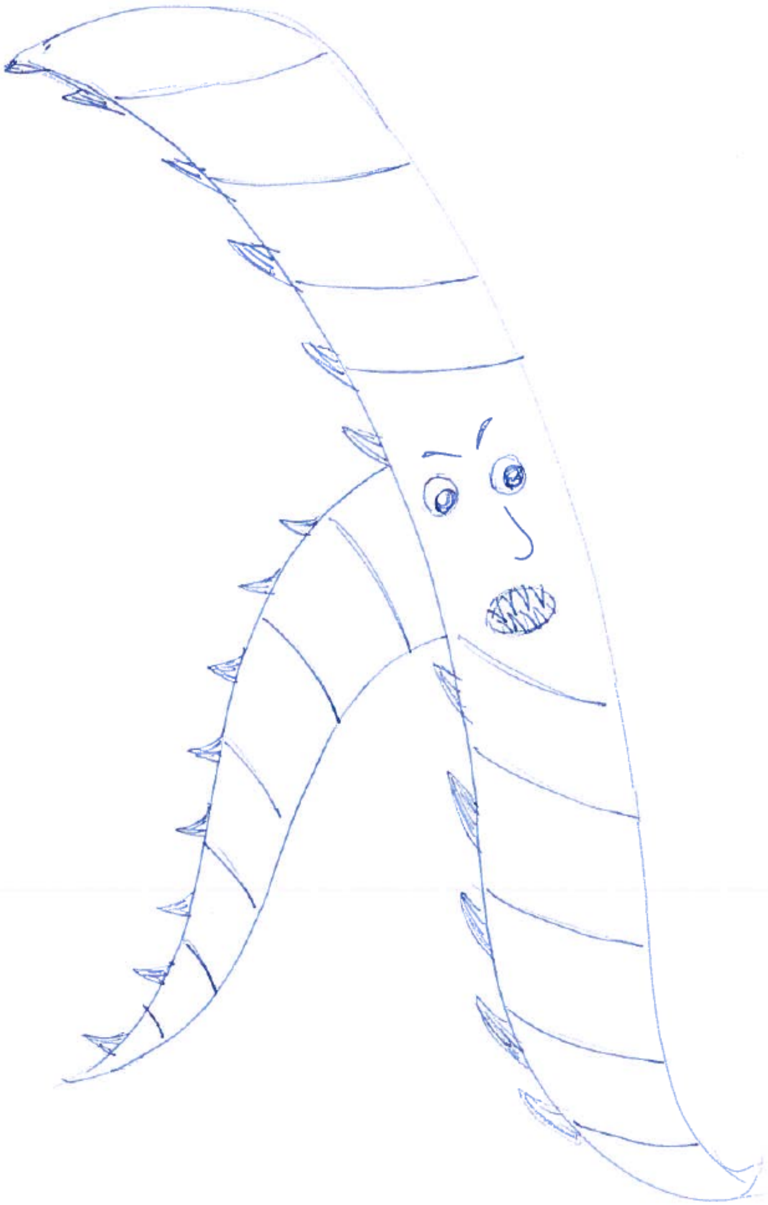
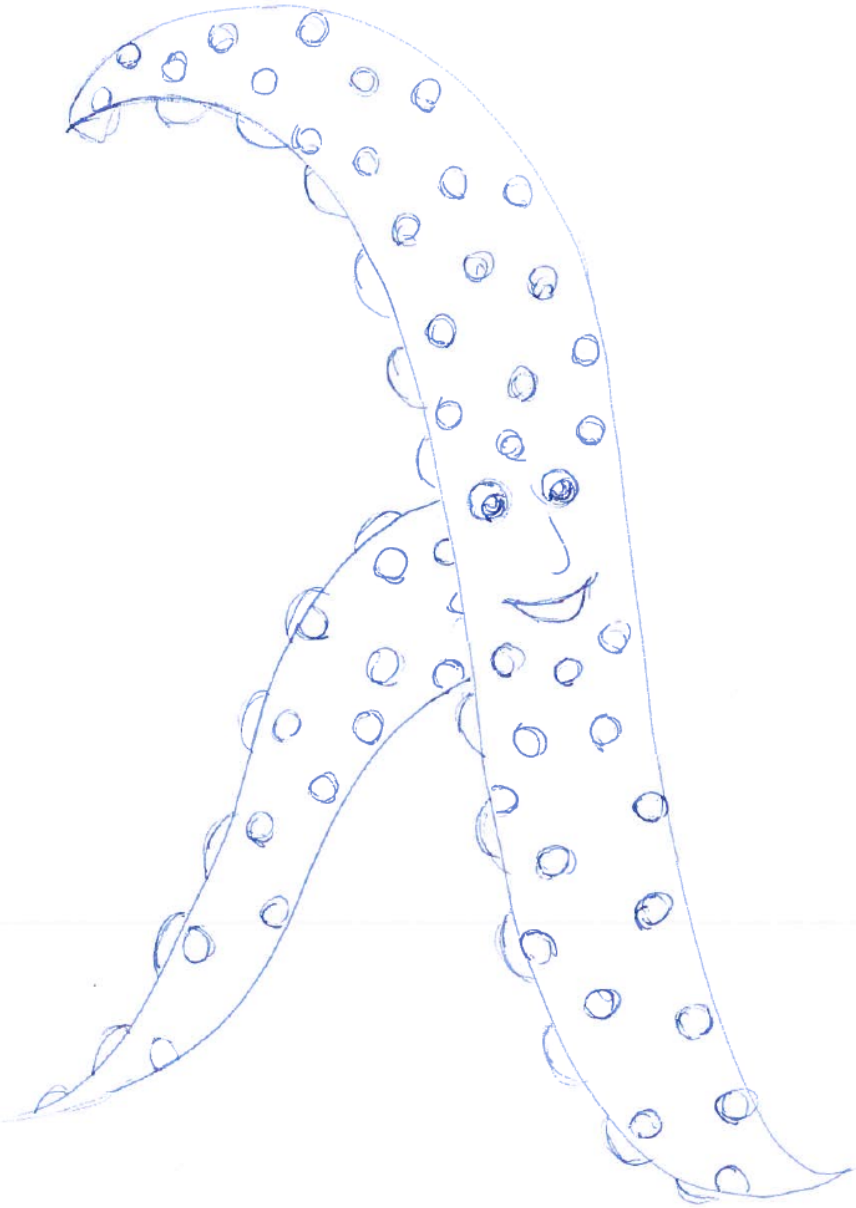




Database programming languages

Kleisli

Buneman, Libkin, Suciu, Tannen, Wong (Penn)

Ferry

Grust, Mayr, Rittinger, Schreiber (Tübingen)

Links

Cooper, Lindley, Wadler, Yallop (Edinburgh)

SML#

Ohori, Ueno (Tohoku)

Ur/Web

Chlipala (Harvard/MIT)

LINQ for C#, VB, F#

Helsbjorg, Meijer, Syme (Microsoft Redmond & Cambridge)

Our goals:

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

Dynamic generation of queries

Type-safety

Goldilocks:

Exactly one query per *run*

Not too few (failure)

Not too many (avalanche)

Our restrictions:

We consider only *select-from-where* queries,
with *exists* and *union*.

We equate *bags* and *lists*.

Future work to extend to *group-by* and *sort-by*.

Part I

A first example

A database

people

name	age
“Alex”	60
“Bert”	56
“Cora”	33
“Drew”	31
“Edna”	21
“Fred”	60

couples

her	him
“Alex”	“Bert”
“Cora”	“Drew”
“Edna”	“Fred”

A query in SQL

```
select w.name as name, w.age − m.age as diff
from couples as c,
       people as w,
       people as m
where c.her = w.name and c.him = m.name and
        w.age > m.age
```

name	diff
“Alex”	4
“Cora”	2

A database as data

$$
\begin{array}{l}
\{\text{people} = \\
\quad [\{\text{name} = \text{“Alex”}\ ;\ \text{age} = 60\}; \\
\quad\ \ \{\text{name} = \text{“Bert”}\ ;\ \text{age} = 56\}; \\
\quad\ \ \{\text{name} = \text{“Cora”};\ \text{age} = 33\}; \\
\quad\ \ \{\text{name} = \text{“Drew”};\ \text{age} = 31\}; \\
\quad\ \ \{\text{name} = \text{“Edna”};\ \text{age} = 21\}; \\
\quad\ \ \{\text{name} = \text{“Fred”}\ ;\ \text{age} = 60\}]; \\
\ \text{couples} = \\
\quad [\{\text{her} = \text{“Alex”}\ ;\ \text{him} = \text{“Bert”}\ \}; \\
\quad\ \ \{\text{her} = \text{“Cora”};\ \text{him} = \text{“Drew”}\}; \\
\quad\ \ \{\text{her} = \text{“Edna”};\ \text{him} = \text{“Fred”}\ \}]\}
\end{array}
$$

Importing the database (naive)

$$
\begin{array}{l}
\textbf{type}\ \mathsf{DB} = \\
\quad \{\mathsf{people} : \\
\qquad \{\mathsf{name} : \textbf{string};\ \mathsf{age} : \textbf{int}\}\ \textbf{list}; \\
\quad\ \mathsf{couples} : \\
\qquad \{\mathsf{her} : \textbf{string};\ \mathsf{him} : \textbf{string}\}\ \textbf{list}\} \\
\textbf{let}\ \mathsf{db}' : \mathsf{DB} = \textbf{database}(\text{“People”})
\end{array}
$$

A query as a comprehension (naive)

```
let differences′ : {name : string; diff : int} list =
  for c in db′.couples do
  for w in db′.people do
  for m in db′.people do
  if c.her = w.name && c.him = m.name && w.age > m.age then
  yield {name : w.name; diff : w.age − m.age}
```

```
differences’

[{name = “Alex” ; diff = 4}
 {name = “Cora”; diff = 2}]
```

Importing the database (quoted)

```
type DB =
  {people :
      {name : string; age : int} list;
   couples :
      {her : string; him : string} list}
let db : Expr< DB > = <@ database(“People”) @>
```

A query as a comprehension (quoted)

```
let differences : Expr< {name : string; diff : int} list > =
   <@ for c in (%db).couples do
      for w in (%db).people do
      for m in (%db).people do
      if c.her = w.name && c.him = m.name && w.age > m.age then
      yield {name : w.name; diff : w.age − m.age} @>
```

```
run(differences)

[{name = “Alex” ; diff = 4}
 {name = “Cora”; diff = 2}]
```

Execute **run** as follows:

1. compute quoted expression
2. simplify quoted expression
3. translate query to SQL
4. execute SQL
5. translate answer to host language

Each **run** generates one query if:

A. answer type is flat (bag-of-record-of-scalars)

B. only permitted operations (e.g., no recursion)

C. consistent use of **database** (all same)

Part II

Abstraction, composition, dynamic generation

Abstracting over values

```
type Names = {name : string} list
let range : Expr< (int, int) → Names > =
  <@ fun(a, b) → for w in (%db).people do
                   if a ≤ w.age  &&  w.age < b then
                   yield {name : w.name} @>


           run(<@ (%range)(30, 40) @>)
      [ {name = “Cora”}; {name = “Drew”} ]
```

Abstracting over a predicate

```
let satisfies : Expr< (int → bool) → Names > =
  <@ fun(p) → for w in (%db).people do
              if p(w.age) then
              yield {name : w.name} @>
```

```
run(<@ (%satisfies)(fun(x) → 30 ≤ x  &&  x < 40) @>)
```

```
[ {name = “Cora”};  {name = “Drew”} ]
```

```
run(<@ (%satisfies)(fun(x) → x mod 2 = 0) @>)
```

```
[ {name = “Alex”};  {name = “Bert”};  {name = “Fred”} ]
```

Composing queries

```
let ageFromName : Expr< string → int list > =
  <@ fun(s) → for u in (%db).people do
              if u.name = s then
              yield u.age @>
let rangeFromNames : Expr< (string, string) → Names > =
  <@ fun(s, t) → for a in (%ageFromName)(s) do
                 for b in (%ageFromName)(t) do
                 (%range)(a, b) @>
```

```
run(<@ (%nameRange)(“Edna”, “Bert”) @>)
[ {name = “Cora”}; {name = “Drew”}; {name = “Edna”} ]
```

Dynamically generated queries (1)

type Predicate =
  | Above **of int**
  | Below **of int**
  | And **of** Predicate × Predicate
  | Or **of** Predicate × Predicate
  | Not **of** Predicate
let t_0 : Predicate = And(Above(30), Below(40))
let t_1 : Predicate = Not(Or(Below(30), Above(40)))

Dynamically generated queries (2)

```
let rec P(t : Predicate) : Expr< int → bool > =
  match t with
  | Above(a)→ <@ fun(x) → (%lift(a)) ≤ x @>
  | Below(a)→ <@ fun(x) → x < (%lift(a)) @>
  | And(t, u) → <@ fun(x) → (%P(t))(x)  &&  (%P(u))(x) @>
  | Or(t, u)    → <@ fun(x) → (%P(t))(x)  ||  (%P(u))(x) @>
  | Not(t)      → <@ fun(x) → not((%P(t))(x)) @>
```

Dynamically generated queries (3)

$\mathsf{P}(\mathsf{t}_0)$

`<@` **fun**(x) → (**fun**(x) → 30 ≤ x)(x) && (**fun**(x) → x < 40)(x) `@>`

`<@` **fun**(x) → 30 ≤ x && x < 40 `@>`

run(`<@` (%satisfies)(%P(t_0)) `@>`)

[{name = "Cora"}; {name = "Drew"}]

run(`<@` (%satisfies)(%P(t_1)) `@>`)

[{name = "Cora"}; {name = "Drew"}]

Part III

Nesting

Flat data

$$
\begin{array}{l}
\{\text{departments} = \\
\qquad [\,\{\text{dpt} = \text{“Product”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Quality”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Research”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Sales”}\}\,]\,; \\
\;\text{employees} = \\
\qquad [\,\{\text{dpt} = \text{“Product”}; \text{emp} = \text{“Alex”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Product”}; \text{emp} = \text{“Bert”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Research”}; \text{emp} = \text{“Cora”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Research”}; \text{emp} = \text{“Drew”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Research”}; \text{emp} = \text{“Edna”}\}; \\
\qquad\;\; \{\text{dpt} = \text{“Sales”}; \text{emp} = \text{“Fred”}\}\,]\,;
\end{array}
$$

Flat data (continued)

$$
\begin{array}{l}
\text{tasks} = \\
\quad [\,\{\text{emp} = \text{“Alex”}; \text{tsk} = \text{“build”}\}; \\
\quad\;\; \{\text{emp} = \text{“Bert”}; \text{tsk} = \text{“build”}\}; \\
\quad\;\; \{\text{emp} = \text{“Cora”}; \text{tsk} = \text{“abstract”}\}; \\
\quad\;\; \{\text{emp} = \text{“Cora”}; \text{tsk} = \text{“build”}\}; \\
\quad\;\; \{\text{emp} = \text{“Cora”}; \text{tsk} = \text{“design”}\}; \\
\quad\;\; \{\text{emp} = \text{“Drew”}; \text{tsk} = \text{“abstract”}\}; \\
\quad\;\; \{\text{emp} = \text{“Drew”}; \text{tsk} = \text{“design”}\}; \\
\quad\;\; \{\text{emp} = \text{“Edna”}; \text{tsk} = \text{“abstract”}\}; \\
\quad\;\; \{\text{emp} = \text{“Edna”}; \text{tsk} = \text{“call”}\}; \\
\quad\;\; \{\text{emp} = \text{“Edna”}; \text{tsk} = \text{“design”}\}; \\
\quad\;\; \{\text{emp} = \text{“Fred”}; \text{tsk} = \text{“call”}\}\,]\,\}
\end{array}
$$

Importing the database

```
type Org = {departments : {dpt : string} list;
            employees :   {dpt : string; emp : string} list;
            tasks :       {emp : string; tsk : string} list }
let org : Expr< Org > = <@ database("Org") @>
```

Departments where every employee can do a given task

```
let expertise′ : Expr< string → {dpt : string} list > =
  <@ fun(u) → for d in (%org).departments do
              if not(exists(
                 for e in (%org).employees do
                 if d.dpt = e.dpt  &&  not(exists(
                    for t in (%org).tasks do
                    if e.emp = t.emp  &&  t.tsk = u then yield { })
                 )) then yield { })
              )) then yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise’)(“abstract”) @>)
[ {dpt = “Quality”}; {dpt = “Research”} ]
```

Nested data

```
[{dpt = “Product”; employees =
   [{emp = “Alex”; tasks = [“build”]}
    {emp = “Bert”; tasks = [“build”]}]};
 {dpt = “Quality”; employees = []};
 {dpt = “Research”; employees =
   [{emp = “Cora”; tasks = [“abstract”; “build”; “design”]};
    {emp = “Drew”; tasks = [“abstract”; “design”]};
    {emp = “Edna”; tasks = [“abstract”; “call”; “design”]}]};
 {dpt = “Sales”; employees =
   [{emp = “Fred”; tasks = [“call”]}]}]
```

Nested data from flat data

```
type NestedOrg = [{dpt : string; employees :
                                    [{emp : string; tasks : [string]}]}]
let nestedOrg : Expr< NestedOrg > =
  <@ for d in (%org).departments do
     yield {dpt = d.dpt; employees =
                for e in (%org).employees do
                if d.dpt = e.dpt then
                yield {emp = e.emp; tasks =
                           for t in (%org).tasks do
                           if e.emp = t.emp then
                           yield t.tsk}}} @>
```

Higher-order queries

```
let any : Expr< (A list, A → bool) → bool > =
   <@ fun(xs, p) →
         exists(for x in xs do
                  if p(x) then
                  yield { }) @>
let all : Expr< (A list, A → bool) → bool > =
   <@ fun(xs, p) →
         not((%any)(xs, fun(x) → not(p(x)))) @>
let contains : Expr< (A list, A) → bool > =
   <@ fun(xs, u) →
         (%any)(xs, fun(x) → x = u) @>
```

Departments where every employee can do a given task

```
let expertise : Expr< string → {dpt : string} list > =
  <@ fun(u) → for d in (%nestedOrg)
              if (%all)(d.employees,
                   fun(e) → (%contains)(e.tasks, u) then
              yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise)("abstract") @>)
```

```
[{dpt = "Quality"}; {dpt = "Research"}]
```

Part IV

Quotations vs. functions

Abstracting over values

```
let range : Expr< (int, int) → Names > =
  <@ fun(a, b) → for w in (%db).people do
                   if a ≤ w.age  &&  w.age < b then
                   yield {name : w.name} @>
run(<@ (%range)(30, 40) @>)
```

vs.

```
let range′(a : Expr< int >, b : Expr< int >) : Names =
  <@ for w in (%db).people do
       if (%a) ≤ w.age  &&  w.age < (%b) then
       yield {name : w.name} @>
run(range′(<@ 30 @>, <@ 40 @>))
```

Composing queries

```
let rangeFromNames : Expr< (string, string) → Names > =
  <@ fun(s, t) → for a in (%ageFromName)(s) do
                 for b in (%ageFromName)(t) do
                 (%range)(a, b) @>
```

vs.

```
let rangeFromNames′ : Expr< (string, string) → Names > =
  <@ fun(s, t) → for a in (%ageFromName)(s) do
                 for b in (%ageFromName)(t) do
                 (%range′(<@ a @>, <@ b @>)) @>
```

Prefer
closed quotations
to
open quotations.

Prefer
quotations of functions
to
functions of quotations.

Part V

From XPath to SQL

Part VI

Idealised LINQ

Terms

VAR

$$\frac{}{\Gamma, x : A \vdash x : A}$$

FUN

$$\frac{\Gamma, x : A \vdash N : B}{\Gamma \vdash \textbf{fun}(x) \to N : A \to B}$$

APP

$$\frac{\Gamma \vdash L : A \to B \qquad \Gamma \vdash M : A}{\Gamma \vdash L\ M : B}$$

SINGLETON

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash \textbf{yield}\ M : A\ \textbf{list}}$$

FOR

$$\frac{\Gamma \vdash M : A\ \textbf{list} \qquad \Gamma, x : A \vdash N : B\ \textbf{list}}{\Gamma \vdash \textbf{for}\ x\ \textbf{in}\ M\ \textbf{do}\ N : B\ \textbf{list}}$$

REC

$$\frac{\Gamma, f : A \to B, x : A \vdash N : B}{\Gamma \vdash \textbf{rec}\ f(x) \to N : A \to B}$$

Quoted terms

VARQ

$$\frac{}{\Gamma; \Delta, x : A \vdash x : A}$$

FUNQ

$$\frac{\Gamma; \Delta, x : A \vdash N : B}{\Gamma; \Delta \vdash \textsf{\textbf{fun}}(x) \to N : A \to B}$$

APPQ

$$\frac{\Gamma; \Delta \vdash L : A \to B \qquad \Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash L\ M : B}$$

SINGLETONQ

$$\frac{\Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash \textsf{\textbf{yield}}\ M : A\ \textsf{\textbf{list}}}$$

FORQ

$$\frac{\Gamma; \Delta \vdash M : A\ \textsf{\textbf{list}} \qquad \Gamma; \Delta, x : A \vdash N : B\ \textsf{\textbf{list}}}{\Gamma; \Delta \vdash \textsf{\textbf{for}}\ x\ \textsf{\textbf{in}}\ M\ \textsf{\textbf{do}}\ N : B\ \textsf{\textbf{list}}}$$

DATABASE

$$\frac{\Sigma(\textsf{db}) = \{\overline{\ell : T}\}}{\Gamma; \Delta \vdash \textsf{\textbf{database}}(\textsf{db}) : \{\overline{\ell : T}\}}$$

Quotation and anti-quotation

Quote

$$\frac{\Gamma; \cdot \vdash M : A}{\Gamma \vdash \texttt{<@}\ M\ \texttt{@>} : \mathsf{Expr}< A >}$$

Antiquote

$$\frac{\Gamma \vdash M : \mathsf{Expr}< A >}{\Gamma; \Delta \vdash (\texttt{\%}M) : A}$$

Run

$$\frac{\Gamma \vdash M : \mathsf{Expr}< T >}{\Gamma \vdash \textbf{run}(M) : T}$$

Lift

$$\frac{\Gamma \vdash M : O}{\Gamma \vdash \textbf{lift}(M) : \mathsf{Expr}< O >}$$

Normalisation: symbolic evaluation

$$
\begin{aligned}
(\textbf{fun}(x) \rightarrow N)\ M &\ \rightsquigarrow\ N[x := M] \\
\{\overline{\ell = M}\}.\ell_i &\ \rightsquigarrow\ M_i \\
\textbf{for}\ x\ \textbf{in}\ (\textbf{yield}\ M)\ \textbf{do}\ N &\ \rightsquigarrow\ N[x := M] \\
\textbf{for}\ y\ \textbf{in}\ (\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ M)\ \textbf{do}\ N &\ \rightsquigarrow\ \textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ (\textbf{for}\ y\ \textbf{in}\ M\ \textbf{do}\ N) \\
\textbf{for}\ x\ \textbf{in}\ (\textbf{if}\ L\ \textbf{then}\ M)\ \textbf{do}\ N &\ \rightsquigarrow\ \textbf{if}\ L\ \textbf{then}\ (\textbf{for}\ x\ \textbf{in}\ M\ \textbf{do}\ N) \\
\textbf{for}\ x\ \textbf{in}\ \texttt{[ ]}\ \textbf{do}\ N &\ \rightsquigarrow\ \texttt{[ ]} \\
\textbf{for}\ x\ \textbf{in}\ (L \mathbin{\texttt{@}} M)\ \textbf{do}\ N &\ \rightsquigarrow\ (\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ N) \mathbin{\texttt{@}} (\textbf{for}\ x\ \textbf{in}\ M\ \textbf{do}\ N) \\
\textbf{if}\ \textbf{true}\ \textbf{then}\ M &\ \rightsquigarrow\ M \\
\textbf{if}\ \textbf{false}\ \textbf{then}\ M &\ \rightsquigarrow\ \texttt{[ ]}
\end{aligned}
$$

Normalisation: *ad hoc* rewriting

$$\begin{aligned}
\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ (M \mathbin{\texttt{@}} N) &\hookrightarrow (\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ M) \mathbin{\texttt{@}} (\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ N) \\
\textbf{for}\ x\ \textbf{in}\ L\ \textbf{do}\ \texttt{[ ]} &\hookrightarrow \texttt{[ ]} \\
\textbf{if}\ L\ \textbf{then}\ (M \mathbin{\texttt{@}} N) &\hookrightarrow (\textbf{if}\ L\ \textbf{then}\ M) \mathbin{\texttt{@}} (\textbf{if}\ L\ \textbf{then}\ N) \\
\textbf{if}\ L\ \textbf{then}\ \texttt{[ ]} &\hookrightarrow \texttt{[ ]} \\
\textbf{if}\ L\ \textbf{then}\ (\textbf{for}\ x\ \textbf{in}\ M\ \textbf{do}\ N) &\hookrightarrow \textbf{for}\ x\ \textbf{in}\ M\ \textbf{do}\ (\textbf{if}\ L\ \textbf{then}\ N) \\
\textbf{if}\ L\ \textbf{then}\ (\textbf{if}\ M\ \textbf{then}\ N) &\hookrightarrow \textbf{if}\ (L\ \ \&\&\ \ M)\ \textbf{then}\ N \\
\textbf{yield}\ x &\hookrightarrow \textbf{yield}\ \{\overline{\ell = x.\ell}\} \\
\textbf{database}(\mathsf{db}).\ell &\hookrightarrow \textbf{for}\ x\ \textbf{in}\ \textbf{database}(\mathsf{db}).\ell\ \textbf{do}\ \textbf{yield}\ x
\end{aligned}$$

Properties of reduction

On well-typed terms, the relations $\leadsto$ and $\hookrightarrow$

- *preserve* typing,
- are *stongly normalising*, and
- are *confluent*.

Example (1): query

```
run(<@ (%nameRange)("Edna", "Bert") @>)
```

Example (2): after splicing

```
(fun(s, t) $\to$
   for a in (fun(s) $\to$
              for u in database(“People”).people do
              if u.name $=$ s then yield u.age)(s) do
   for b in (fun(s) $\to$
              for u in database(“People”).people do
              if u.name $=$ s then yield u.age)(t) do
   (fun(a, b) $\to$
      for w in database(“People”).people do
      if a $\leq$ w.age  &&  w.age $<$ b then
      yield {name : w.name})(a, b))
(“Edna”, “Bert”)
```

Example (3): beta reduction $\leadsto$

```
for a in (for u in database(“People”).people do
          if u.name = “Edna” then yield u.age) do
for b in (for u in database(“People”).people do
          if u.name = “Bert” then yield u.age) do
for w in database(“People”).people do
if a ≤ w.age  &&  w.age < b then
yield {name : w.name}
```

Example (4): other rewriting $\rightsquigarrow$

```
for u in database("People").people do
if u.name = "Edna" then
for v in database("People").people do
if v.name = "Bert" then
for w in database("People").people do
if u.age $\leq$ w.age && w.age $<$ v.age then
yield {name : w.name}
```

Example (5): *ad hoc* reductions $\hookrightarrow$

```
for u in database(“People”).people do
for v in database(“People”).people do
for w in database(“People”).people do
if u.name $=$ “Edna” && v.name $=$ “Bert” &&
  u.age $\leq$ w.age && w.age $<$ v.age then
yield $\{$name : w.name$\}$
```

Example (6): SQL

```
select w.name as name
from people as u,
     people as v,
     people as w
where u.name = “Edna” and v.name = “Bert” and
      u.age ≤ w.age and w.age < v.age
```

Part VII

Results

Example	F# 2.0	F# 3.0	Our system
differences	✔	✔	✔
range	✖	✔	✔
satisfies	✔	✖	✔
satisfies	✔	✖	✔
rangeFromNames	✖	✖	✔
P(t_0)	✔	✖	✔
P(t_1)	✔	✖	✔
expertise$'$	✔	✔	✔
expertise	✖	avalanche	✔
xp$_0$	✖	✔	✔
xp$_1$	✖	✔	✔
xp$_2$	✖	✖	✔
xp$_3$	✖	✖	✔

Our goals:

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

Dynamic generation of queries

Type-safety

Goldilocks:

Exactly one query per *run*

Not too few (failure)

Not too many (avalanche)

Deddy
as a
lambda
man

Appendix A7

Problems with F#

Problems with F# PowerPack

(Notes from James Cheney)

Problems fixed in F# PowerPack code:

- F# 2.0/PowerPack lacked support for singletons in nonstandard places (i.e. other than in a comprehension body).
- F# 2.0/PowerPack also lacked support for Seq.exists in certain places because it was assuming that expressions of base types (eg. booleans) did not need to be further translated.

F# 3.0:

- Did not exhibit the above problems
- But did exhibit translation bug where something like

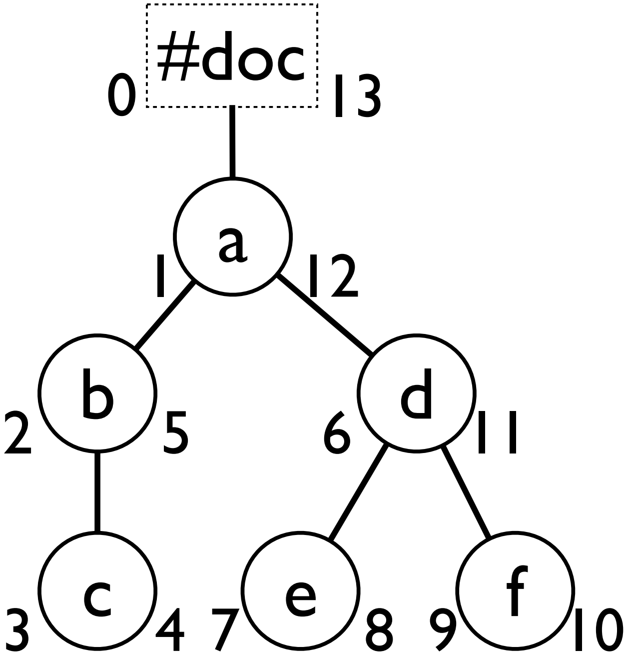
  ```
  query if 1 = 2 then yield 3
  ```

  leads to a run-time type error.

Appendix A7

From XPath to SQL

Representing XML



xml

id	parent	name	pre	post
0	-1	#doc	0	13
1	0	a	1	12
2	1	b	2	5
3	2	c	3	4
4	1	d	6	11
5	4	e	7	8
6	4	f	9	10

type Node =

$\{\mathsf{id} : \textbf{int}, \mathsf{parent} : \textbf{int}, \mathsf{name} : \textbf{string}, \mathsf{pre} : \textbf{int}, \mathsf{post} : \textbf{int}\}$

Abstract syntax of XPath

type Axis =
| Self
| Child
| Descendant
| DescendantOrSelf
| Following
| FollowingSibling
| Rev **of** Axis

type Path =
| Seq **of** Path × Path
| Axis **of** Axis
| NameTest **of string**
| Filter **of** Path

An evaluator for XPath: axis

```
let rec axis(ax : Axis) : Expr< (Node, Node) → bool > =
  match ax with
  | Self → <@ fun(s, t) → s.id = t.id @>
  | Child → <@ fun(s, t) → s.id = t.parent @>
  | Descendant → <@ fun(s, t) →
      s.pre < t.pre  &&  t.post < s.post @>
  | DescendantOrSelf → <@ fun(s, t) →
      s.pre ≤ t.pre  &&  t.post ≤ s.post @>
  | Following → <@ fun(s, t) → s.pre < t.pre @>
  | FollowingSibling → <@ fun(s, t) →
      s.post < t.pre  &&  s.parent = t.parent @>
  | Rev(axis) → <@ fun(s, t) → (%axis(ax))(t, s) @>
```

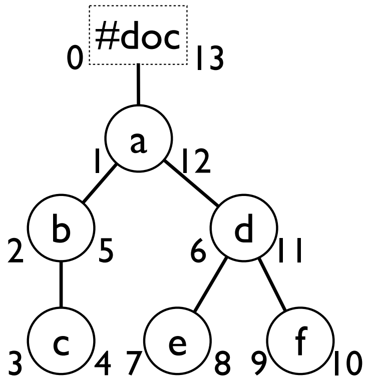
An evaluator for XPath: path

```
let rec path(p : Path) : Expr< (Node, Node) → bool > =
  match p with
  | Seq(p, q) → <@ fun(s, u) → (%any)((%db).xml,
      fun(t) → (%path(p))(s, t)  &&  (%path(q))(t, u)) @>
  | Axis(ax) → axis(ax)
  | NameTest(name) → <@ fun(s, t) →
      s.id = t.id  &&  s.name = name @>
  | Filter(p) → <@ fun(s, t) → s.id = t.id  &&
      (%any)((%db).xml, fun(u) → (%path(p))(s, u)) @>
```

An evaluator for XPath: xpath

```
let xpath(p : Path) : Expr< Node list > =
  <@ for root in (%db).xml do
     for s in (%db).xml do
     if root.parent = −1 && (%path(p))(root, s) then
     yield s @>
```

Examples



$/*/*$

run(xpath(Seq(Axis(Child), Axis(Child))))

$[2;\ 4]$

$//*$[following-sibling::d]

run(xpath(Seq(Axis(Descendant),
Filter(Seq(Axis(FollowingSibling),
NameTest(“d”))))))

$[2]$