

A practical theory of Language Integrated Query

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What is the difference between theory and
practice?

In theory there is no difference.

But in practice there is.

A tale of two languages

Links

Cooper, Lindley, Wadler, Yallop
(Edinburgh)

LINQ for C#, VB, F#

Hejlsberg, Meijer, Syme
(Microsoft Redmond & Cambridge)

Goals

Series of examples

Join queries

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

Dynamic generation of queries

Nested intermediate data

Type safety

Avoid Scylla and Charybdis

Each host query generates one SQL query

Scylla: failure to generate a query

Charybdis: multiple queries, avalanche



SHARKS; Dogs of Scylla.

BRITANNIA between SCYLLA & CHARYBDIS.

or — *The Vessel of the Constitution. steered clear of the Rock of Democracy, and the Whirlpool of Arbitrary Power.*

Pubd April 6th 1795, by H. Humphreys No 10 Strand Street.

J: G: del: et fecit: scilicet: publicus:



Limitations

Restrictions on the theory:

We consider only comprehensions, unions, and existence tests.

Future work to extend to *grouping*, *sorting*, and *aggregation*.

Notational convention:

We treat *bags* (multisets) as lists.

Part I

A first example

A database

people

name	age
"Alex"	60
"Bert"	56
"Cora"	33
"Drew"	31
"Edna"	21
"Fred"	60

couples

her	him
"Alex"	"Bert"
"Cora"	"Drew"
"Edna"	"Fred"

A query in SQL

select w.name **as** name, w.age – m.age **as** diff

from couples **as** c,

people **as** w,

people **as** m

where c.her = w.name **and** c.him = m.name **and** w.age > m.age

name	diff
“Alex”	4
“Cora”	2

A database as data

{people =

[{name = "Alex" ; age = 60};

{name = "Bert" ; age = 56};

{name = "Cora" ; age = 33};

{name = "Drew"; age = 31};

{name = "Edna"; age = 21};

{name = "Fred" ; age = 60}] ;

couples =

[{her = "Alex" ; him = "Bert" } ;

{her = "Cora" ; him = "Drew"} ;

{her = "Edna" ; him = "Fred" }] }

Importing the database (naive)

```
type DB =  
  {people :  
    {name : string; age : int} list;  
  couples :  
    {her : string; him : string} list}  
let db' : DB = database("People")
```

A query as a comprehension (naive)

```
let differences' : {name : string; diff : int} list =  
  for c in db'.couples do  
  for w in db'.people do  
  for m in db'.people do  
  if c.her = w.name && c.him = m.name && w.age > m.age then  
  yield {name : w.name; diff : w.age – m.age}
```

differences'

```
[ {name = "Alex"; diff = 4}  
  {name = "Cora"; diff = 2} ]
```


Importing the database (quoted)

```
type DB =  
  {people :  
    {name : string; age : int} list;  
  couples :  
    {her : string; him : string} list}  
let db : Expr< DB > = <@ database("People") @>
```

A query as a comprehension (quoted)

```
let differences : Expr< {name : string; diff : int} list > =  
  <@ for c in (%db).couples do  
    for w in (%db).people do  
      for m in (%db).people do  
        if c.her = w.name && c.him = m.name && w.age > m.age then  
          yield {name : w.name; diff : w.age – m.age} @>
```

run(differences)

```
[ {name = "Alex"; diff = 4}  
  {name = "Cora"; diff = 2} ]
```

Running a query

1. compute quoted expression
2. simplify quoted expression
3. translate query to SQL
4. execute SQL
5. translate answer to host language

Scylla and Charybdis:

Each **run** generates one query if

- A. answer type is flat (bag of record of scalars)
- B. only permitted operations (e.g., no recursion)
- C. only refers to one database

Scala (naive)

```
val differences:
  List[{ val name: String; val diff: Int }] =
  for {
    c <- db.couples
    w <- db.people
    m <- db.people
    if c.her == w.name && c.him == m.name && w.age > m.age
  } yield new Record {
    val name = w.name
    val diff = w.age - m.age
  }
```

Scala (quoted)

```
val differences:
  Rep[List[{ val name: String; val diff: Int }]] =
  for {
    c <- db.couples
    w <- db.people
    m <- db.people
    if c.her == w.name && c.him == m.name && w.age > m.age
  } yield new Record {
    val name = w.name
    val diff = w.age - m.age
  }
```


Part II

Abstraction, composition, dynamic generation

Abstracting over values

```
type Names = {name : string} list
```

```
let range : Expr< (int, int) → Names > =
```

```
<@ fun(a, b) → for w in (%db).people do
```

```
    if a ≤ w.age && w.age < b then
```

```
        yield {name : w.name} @>
```

```
run(<@ (%range)(30, 40) @>)
```

```
[{name = "Cora"}; {name = "Drew"}]
```

Abstracting over a predicate

let satisfies : Expr< (int → bool) → Names > =

<@ **fun**(p) → **for** w **in** (%db).people **do**

if p(w.age) **then**

yield {name : w.name} @>

run(<@ (%satisfies)(**fun**(x) → 30 ≤ x && x < 40) @>)

[{name = "Cora"}; {name = "Drew"}]

run(<@ (%satisfies)(**fun**(x) → x mod 2 = 0) @>)

[{name = "Alex"}; {name = "Bert"}; {name = "Fred"}]

Composing queries

```
let getAge : Expr< string → int list > =
```

```
<@ fun(s) → for u in (%db).people do  
    if u.name = s then  
    yield u.age @>
```

```
let compose : Expr< (string, string) → Names > =
```

```
<@ fun(s, t) → for a in (%getAge)(s) do  
    for b in (%getAge)(t) do  
    (%range)(a, b) @>
```

```
run(<@ (%compose)("Edna", "Bert") @>)
```

```
[ {name = "Cora"}; {name = "Drew"}; {name = "Edna"} ]
```

Dynamically generated queries (1)

type Predicate =

| Above **of** int

| Below **of** int

| And **of** Predicate × Predicate

| Or **of** Predicate × Predicate

| Not **of** Predicate

let t₀ : Predicate = And(Above(30), Below(40))

let t₁ : Predicate = Not(Or(Below(30), Above(40)))

Dynamically generated queries (2)

let rec P(*t* : Predicate) : Expr<int → bool> =

match *t* **with**

| Above(*a*) → <@ **fun**(*x*) → (%**lift**(*a*)) ≤ *x* @>

| Below(*a*) → <@ **fun**(*x*) → *x* < (%**lift**(*a*)) @>

| And(*t*, *u*) → <@ **fun**(*x*) → (%P(*t*))(*x*) && (%P(*u*))(*x*) @>

| Or(*t*, *u*) → <@ **fun**(*x*) → (%P(*t*))(*x*) || (%P(*u*))(*x*) @>

| Not(*t*) → <@ **fun**(*x*) → **not**((%P(*t*))(*x*)) @>

Dynamically generated queries (3)

$P(t_0)$

$\langle @ \text{fun}(x) \rightarrow (\text{fun}(x) \rightarrow 30 \leq x)(x) \ \&\& \ (\text{fun}(x) \rightarrow x < 40)(x) \ @ \rangle$

$\langle @ \text{fun}(x) \rightarrow 30 \leq x \ \&\& \ x < 40 \ @ \rangle$

run($\langle @ (\%satisfies)(\%P(t_0)) \ @ \rangle$)

[{name = "Cora"}; {name = "Drew"}]

run($\langle @ (\%satisfies)(\%P(t_1)) \ @ \rangle$)

[{name = "Cora"}; {name = "Drew"}]

Part III

Nesting

Flat data

```
{departments =
```

```
  [ {dpt = "Product"};
```

```
    {dpt = "Quality"};
```

```
    {dpt = "Research"};
```

```
    {dpt = "Sales"} ];
```

```
employees =
```

```
  [ {dpt = "Product"; emp = "Alex"};
```

```
    {dpt = "Product"; emp = "Bert"};
```

```
    {dpt = "Research"; emp = "Cora"};
```

```
    {dpt = "Research"; emp = "Drew"};
```

```
    {dpt = "Research"; emp = "Edna"};
```

```
    {dpt = "Sales"; emp = "Fred"} ];
```

Flat data (continued)

tasks =

```
[ {emp = "Alex"; tsk = "build"};  
  {emp = "Bert"; tsk = "build"};  
  {emp = "Cora"; tsk = "abstract"};  
  {emp = "Cora"; tsk = "build"};  
  {emp = "Cora"; tsk = "design"};  
  {emp = "Drew"; tsk = "abstract"};  
  {emp = "Drew"; tsk = "design"};  
  {emp = "Edna"; tsk = "abstract"};  
  {emp = "Edna"; tsk = "call"};  
  {emp = "Edna"; tsk = "design"};  
  {emp = "Fred"; tsk = "call"} ] }
```

Importing the database

```
type Org = {departments : {dpt : string} list;  
            employees :  {dpt : string; emp : string} list;  
            tasks :      {emp : string; tsk : string} list }  
let org : Expr< Org > = <@ database("Org") @>
```

Departments where every employee can do a given task

```
let expertise' : Expr< string → {dpt : string} list > =
```

```
<@ fun(u) → for d in (%org).departments do
```

```
  if not(exists(
```

```
    for e in (%org).employees do
```

```
    if d.dpt = e.dpt && not(exists(
```

```
      for t in (%org).tasks do
```

```
        if e.emp = t.emp && t.tsk = u then yield { })
```

```
    )) then yield { })
```

```
  )) then yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise')("abstract") @>)
```

```
[ {dpt = "Quality"}; {dpt = "Research"} ]
```

Nested data

```
[ {dpt = "Product"; employees =  
  [ {emp = "Alex"; tasks = [ "build" ] }  
    {emp = "Bert"; tasks = [ "build" ] } ] ];  
{dpt = "Quality"; employees = [ ] };  
{dpt = "Research"; employees =  
  [ {emp = "Cora"; tasks = [ "abstract"; "build"; "design" ] } ;  
    {emp = "Drew"; tasks = [ "abstract"; "design" ] } ;  
    {emp = "Edna"; tasks = [ "abstract"; "call"; "design" ] } ] ];  
{dpt = "Sales"; employees =  
  [ {emp = "Fred"; tasks = [ "call" ] } ] ] ]
```


Higher-order queries

let any : Expr< (*A list*, *A* → **bool**) → **bool** > =

<@ **fun**(*xs*, *p*) →
 exists(**for** *x* **in** *xs* **do**
 if *p*(*x*) **then**
 yield { }) @>

let all : Expr< (*A list*, *A* → **bool**) → **bool** > =

<@ **fun**(*xs*, *p*) →
 not((%**any**)(*xs*, **fun**(*x*) → **not**(*p*(*x*)))) @>

let contains : Expr< (*A list*, *A*) → **bool** > =

<@ **fun**(*xs*, *u*) →
 (%**any**)(*xs*, **fun**(*x*) → *x* = *u*) @>

Departments where every employee can do a given task

```
let expertise : Expr< string → {dpt : string} list > =  
  <@ fun(u) → for d in (%nestedOrg)  
    if (%all)(d.employees,  
      fun(e) → (%contains)(e.tasks, u) then  
    yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise)("abstract") @>)
```

```
[{dpt = "Quality"}; {dpt = "Research"}]
```

Part IV

Quotations vs. functions

Abstracting over values

```
let range : Expr< (int, int) → Names > =  
  <@ fun(a, b) → for w in (%db).people do  
    if a ≤ w.age && w.age < b then  
      yield {name : w.name} @>  
run(<@ (%range)(30, 40) @>)
```

vs.

```
let range'(a : Expr< int >, b : Expr< int >) : Names =  
  <@ for w in (%db).people do  
    if (%a) ≤ w.age && w.age < (%b) then  
      yield {name : w.name} @>  
run(range'(<@ 30 @>, <@ 40 @>))
```

Composing queries

let compose : Expr< (**string**, **string**) → Names > =
 <@ **fun**(s, t) → **for a in** (%getAge)(s) **do**
 for b in (%getAge)(t) **do**
 (%range)(a, b) @>

vs.

let compose' : Expr< (**string**, **string**) → Names > =
 <@ **fun**(s, t) → **for a in** (%getAge)(s) **do**
 for b in (%getAge)(t) **do**
 (%range'(<@ a @>, <@ b @>)) @>

Prefer
closed quotations
to
open quotations.

Prefer
quotations of functions
to
functions of quotations.

Part V

From XPath to SQL

Part VI

Idealised LINQ

Terms

VAR

$$\frac{}{\Gamma, x : A \vdash x : A}$$

FUN

$$\frac{\Gamma, x : A \vdash N : B}{\Gamma \vdash \mathbf{fun}(x) \rightarrow N : A \rightarrow B}$$

APP

$$\frac{\Gamma \vdash L : A \rightarrow B \quad \Gamma \vdash M : A}{\Gamma \vdash L M : B}$$

SINGLETON

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash \mathbf{yield} M : A \mathbf{list}}$$

FOR

$$\frac{\Gamma \vdash M : A \mathbf{list} \quad \Gamma, x : A \vdash N : B \mathbf{list}}{\Gamma \vdash \mathbf{for} x \mathbf{in} M \mathbf{do} N : B \mathbf{list}}$$

REC

$$\frac{\Gamma, f : A \rightarrow B, x : A \vdash N : B}{\Gamma \vdash \mathbf{rec} f(x) \rightarrow N : A \rightarrow B}$$

Quoted terms

VARQ

$$\frac{}{\Gamma; \Delta, x : A \vdash x : A}$$

FUNQ

$$\frac{\Gamma; \Delta, x : A \vdash N : B}{\Gamma; \Delta \vdash \mathbf{fun}(x) \rightarrow N : A \rightarrow B}$$

APPQ

$$\frac{\Gamma; \Delta \vdash L : A \rightarrow B \quad \Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash L M : B}$$

SINGLETONQ

$$\frac{\Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash \mathbf{yield} M : A \mathbf{list}}$$

FORQ

$$\frac{\Gamma; \Delta \vdash M : A \mathbf{list} \quad \Gamma; \Delta, x : A \vdash N : B \mathbf{list}}{\Gamma; \Delta \vdash \mathbf{for} x \mathbf{in} M \mathbf{do} N : B \mathbf{list}}$$

DATABASE

$$\frac{\Sigma(\mathbf{db}) = \{\overline{\ell : T}\}}{\Gamma; \Delta \vdash \mathbf{database}(\mathbf{db}) : \{\overline{\ell : T}\}}$$

Quotation and anti-quotation

QUOTE

$$\frac{\Gamma; \cdot \vdash M : A}{\Gamma \vdash \langle @ M @ \rangle : \text{Expr} \langle A \rangle}$$

ANTIQUOTE

$$\frac{\Gamma \vdash M : \text{Expr} \langle A \rangle}{\Gamma; \Delta \vdash (\%M) : A}$$

RUN

$$\frac{\Gamma \vdash M : \text{Expr} \langle T \rangle}{\Gamma \vdash \mathbf{run}(M) : T}$$

LIFT

$$\frac{\Gamma \vdash M : O}{\Gamma \vdash \mathbf{lift}(M) : \text{Expr} \langle O \rangle}$$

Normalisation: symbolic evaluation

(fun(x) $\rightarrow N$) $M \rightsquigarrow N[x := M]$

$\{\overline{\ell = M}\}.l_i \rightsquigarrow M_i$

for x **in** (yield M) **do** $N \rightsquigarrow N[x := M]$

for y **in** (for x **in** L **do** M) **do** $N \rightsquigarrow$ **for** x **in** L **do** (for y **in** M **do** N)

for x **in** (if L **then** M) **do** $N \rightsquigarrow$ **if** L **then** (for x **in** M **do** N)

for x **in** [] **do** $N \rightsquigarrow$ []

for x **in** (L @ M) **do** $N \rightsquigarrow$ (for x **in** L **do** N) @ (for x **in** M **do** N)

if **true** **then** $M \rightsquigarrow M$

if **false** **then** $M \rightsquigarrow$ []

Normalisation: *ad hoc* rewriting

for x **in** L **do** $(M @ N)$ \hookrightarrow (**for** x **in** L **do** M) @ (**for** x **in** L **do** N)

for x **in** L **do** $[\]$ \hookrightarrow $[\]$

if L **then** $(M @ N)$ \hookrightarrow (**if** L **then** M) @ (**if** L **then** N)

if L **then** $[\]$ \hookrightarrow $[\]$

if L **then** (**for** x **in** M **do** N) \hookrightarrow **for** x **in** M **do** (**if** L **then** N)

if L **then** (**if** M **then** N) \hookrightarrow **if** $(L \ \&\& \ M)$ **then** N

Properties of reduction

On well-typed terms, the relations \rightsquigarrow and \hookrightarrow

- *preserve* typing,
- are *strongly normalising*, and
- are *confluent*.

Terms in normal form under \rightsquigarrow satisfy the *subformula property*: with the exception of predicates (such as $<$ or **exists**), the type of a subterm must be a subformula of either the type of a free variable or of the type of the term.

Rewriting in Scala

```
/*
  for x in [] do N --> []
  for x in (yield Q) do R --> R[x:=Q]
  for x in (P @ Q) do R -->
    (for x in P do R) @ (for x in Q do R)
  for x in (if P then Q) do R -->
    if P then (for x in Q do R)
  for y in (for x in P do Q) do R -->
    for x in P do (for y in Q do R)
*/
def dbfor[A,B](l: Exp[List[A]], f: Exp[A] => Exp[List[B]])
  = l match {
  case Empty()      => List()
  case Yield(a)     => f(a)
  case Concat(a,b)  => a.flatMap(f) ++ b.flatMap(f)
  case IfThen(c,a)  =>
    if (c) for (x <- a; y <- f(x)) yield y else List()
  case For(l2,f2)   =>
    for (x <- l2; y <- f2(x); z <- f(y)) yield z
    ...
}
```


Example (1): query

```
run(<@ (%compose)("Edna", "Bert") @>)
```

Example (2): after splicing

```
(fun(s, t) →  
  for a in (fun(s) →  
    for u in database("People").people do  
    if u.name = s then yield u.age)(s) do  
  for b in (fun(s) →  
    for u in database("People").people do  
    if u.name = s then yield u.age)(t) do  
  (fun(a, b) →  
    for w in database("People").people do  
    if a ≤ w.age && w.age < b then  
    yield {name : w.name})(a, b))  
("Edna", "Bert")
```

Example (3): beta reduction \rightsquigarrow

```
for a in (for u in database("People").people do  
    if u.name = "Edna" then yield u.age) do  
for b in (for u in database("People").people do  
    if u.name = "Bert" then yield u.age) do  
for w in database("People").people do  
if a ≤ w.age && w.age < b then  
yield {name : w.name}
```

Example (4): other rewriting \rightsquigarrow

```
for u in database("People").people do  
if u.name = "Edna" then  
for v in database("People").people do  
if v.name = "Bert" then  
for w in database("People").people do  
if u.age  $\leq$  w.age && w.age < v.age then  
yield {name : w.name}
```

Example (5): *ad hoc* reductions \hookrightarrow

```
for u in database("People").people do  
for v in database("People").people do  
for w in database("People").people do  
if u.name = "Edna" && v.name = "Bert" &&  
    u.age  $\leq$  w.age && w.age < v.age then  
yield {name : w.name}
```

Example (6): SQL

```
select w.name as name
from people as u,
      people as v,
      people as w
where u.name = "Edna" and v.name = "Bert" and
      u.age ≤ w.age and w.age < v.age
```

Part VII

Results

Example	F# 2.0	F# 3.0	us	(norm)
differences	17.6	20.6	18.1	0.5
range	×	5.6	2.9	0.3
satisfies	2.6	×	2.9	0.3
satisfies	4.4	×	4.6	0.3
compose	×	×	4.0	0.8
P(t ₀)	2.8	×	3.3	0.3
P(t ₁)	2.7	×	3.0	0.3
expertise'	7.2	9.2	8.0	0.6
expertise	×	66.7 ^{av}	8.3	0.9
xp ₀	×	8.3	7.9	1.9
xp ₁	×	14.7	13.4	1.1
xp ₂	×	17.9	20.7	2.2
xp ₃	×	3744.9	3768.6	4.4

^{av} marks query avalanche. All times in milliseconds.

Q#	F# 3.0	us	(norm)
Q1	2.0	2.4	0.3
Q2	1.5	1.7	0.2
Q5	1.7	2.1	0.3
Q6	1.7	2.1	0.3
Q7	1.5	1.8	0.2
Q8	2.3	2.4	0.2
Q9	2.3	2.7	0.3
Q10	1.4	1.7	0.2
Q11	1.4	1.7	0.2
Q12	4.4	4.9	0.4
Q13	2.5	2.9	0.4
Q14	2.5	2.9	0.3

Q#	F# 3.0	us	(norm)
Q15	3.5	4.0	0.5
Q16	3.5	4.0	0.5
Q17	6.2	6.7	0.4
Q18	1.5	1.8	0.2
Q19	1.5	1.8	0.2
Q20	1.5	1.8	0.2
Q21	1.6	1.9	0.3
Q22	1.6	1.9	0.3
Q23	1.6	1.9	0.3
Q24	1.8	2.0	0.3
Q25	1.4	1.6	0.2
Q27	1.8	2.1	0.2

Q#	F# 3.0	us	(norm)
Q29	1.5	1.7	0.2
Q30	1.8	2.0	0.2
Q32	2.7	3.1	0.3
Q33	2.8	3.1	0.3
Q34	3.1	3.6	0.5
Q35	3.1	3.6	0.4
Q36	2.2	2.4	0.2
Q37	1.3	1.6	0.2
Q38	4.2	4.9	0.6
Q39	4.2	4.7	0.4
Q40	4.1	4.6	0.4
Q41	6.3	7.3	0.6

Q#	F# 3.0	us	(norm)
Q42	4.7	5.5	0.5
Q43	7.2	6.9	0.7
Q44	5.4	6.2	0.7
Q45	2.2	2.6	0.3
Q46	2.3	2.7	0.4
Q47	2.1	2.5	0.3
Q48	2.1	2.5	0.3
Q49	2.4	2.7	0.3
Q50	2.2	2.5	0.3
Q51	2.0	2.4	0.3
Q52	6.1	5.9	0.4
Q53	11.9	11.2	0.6

Q#	F# 3.0	us	(norm)
Q54	4.4	4.8	0.4
Q55	5.2	5.6	0.4
Q56	4.6	5.1	0.5
Q57	2.5	2.9	0.4
Q58	2.5	2.9	0.4
Q59	3.1	3.6	0.5
Q60	3.6	4.4	0.7

Q#	F# 3.0	us	(norm)
Q61	5.8	6.3	0.3
Q62	5.4	5.9	0.2
Q63	3.4	3.8	0.4
Q64	4.3	4.9	0.6
Q65	10.2	10.1	0.4
Q66	8.9	8.7	0.6
Q67	14.7	13.1	1.1

Comparison of F# 3.0 and us (using F# 3.0 as a back-end) on the 62 example database queries in the F# 3.0 documentation. Five query expressions (Q3, Q4, Q26, Q28, Q31) are excluded because they are executed on in-memory lists rather than generating SQL. All times in milliseconds.

Goals

Series of examples

Join queries

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

Dynamic generation of queries

Nested intermediate data

Type safety

Avoid Scylla and Charybdis

Each host query generates one SQL query

Scylla: failure to generate a query

Charybdis: multiple queries, avalanche

Theory and Practice

Host and quoted languages

Theory: different (recursion, database)

Practice: identical

Coverage

Theory: doesn't cover sorting, grouping, aggregation—work for tomorrow

Practice: covers all of LINQ—put it to work today

What is the difference between theory and
practice?

In theory there is a difference.

But in practice there isn't.



Daddy
is a
lambada
man

Appendix A7

Problems with F#

Problems with F# PowerPack

(Notes from James Cheney)

Problems fixed in F# PowerPack code:

- F# 2.0/PowerPack lacked support for singletons in nonstandard places (i.e. other than in a comprehension body).
- F# 2.0/PowerPack also lacked support for Seq.exists in certain places because it was assuming that expressions of base types (eg. booleans) did not need to be further translated.

F# 3.0:

- Did not exhibit the above problems
- But did exhibit translation bug where something like

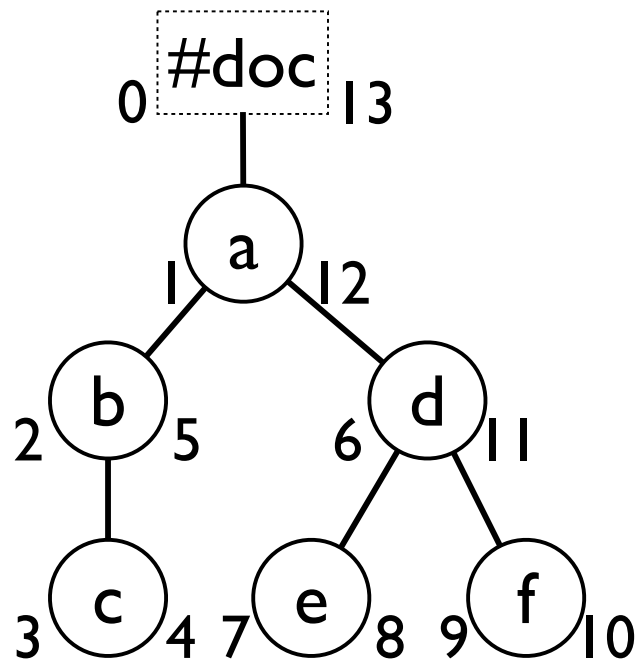
```
query if 1 = 2 then yield 3
```

leads to a run-time type error.

Appendix A7

From XPath to SQL

Representing XML



xml

id	parent	name	pre	post
0	-1	#doc	0	13
1	0	a	1	12
2	1	b	2	5
3	2	c	3	4
4	1	d	6	11
5	4	e	7	8
6	4	f	9	10

type Node =

{id : **int**, parent : **int**, name : **string**, pre : **int**, post : **int**}

Abstract syntax of XPath

type Axis =

- | Self
- | Child
- | Descendant
- | DescendantOrSelf
- | Following
- | FollowingSibling
- | Rev **of** Axis

type Path =

- | Seq **of** Path × Path
- | Axis **of** Axis
- | NameTest **of** string
- | Filter **of** Path

An evaluator for XPath: axis

let rec axis(ax : Axis) : Expr< (Node, Node) → **bool** > =

match ax **with**

| Self → <@ **fun**(s, t) → s.id = t.id @>

| Child → <@ **fun**(s, t) → s.id = t.parent @>

| Descendant → <@ **fun**(s, t) →

s.pre < t.pre && t.post < s.post @>

| DescendantOrSelf → <@ **fun**(s, t) →

s.pre ≤ t.pre && t.post ≤ s.post @>

| Following → <@ **fun**(s, t) → s.pre < t.pre @>

| FollowingSibling → <@ **fun**(s, t) →

s.post < t.pre && s.parent = t.parent @>

| Rev(axis) → <@ **fun**(s, t) → (%axis(ax))(t, s) @>

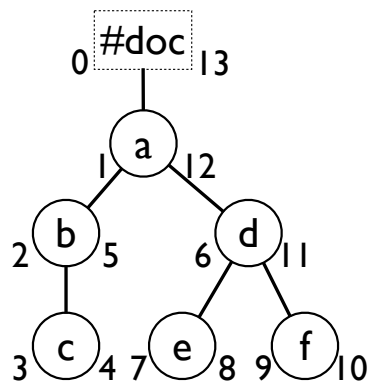
An evaluator for XPath: path

```
let rec path(p : Path) : Expr< (Node, Node) → bool > =  
match p with  
| Seq(p, q) → <@ fun(s, u) → (%any)((%db).xml,  
    fun(t) → (%path(p))(s, t) && (%path(q))(t, u)) @>  
| Axis(ax) → axis(ax)  
| NameTest(name) → <@ fun(s, t) →  
    s.id = t.id && s.name = name @>  
| Filter(p) → <@ fun(s, t) → s.id = t.id &&  
    (%any)((%db).xml, fun(u) → (%path(p))(s, u)) @>
```

An evaluator for XPath: xpath

```
let xpath(p : Path) : Expr< Node list > =  
  <@ for root in (%db).xml do  
    for s in (%db).xml do  
      if root.parent = -1 && (%path(p))(root, s) then  
        yield s @>
```

Examples



`/ * / *`

`run(xpath(Seq(Axis(Child), Axis(Child))))`

`[2; 4]`

`// * [following-sibling : : d]`

`run(xpath(Seq(Axis(Descendant),`

`Filter(Seq(Axis(FollowingSibling),`

`NameTest("d"))))))`

`[2]`