Proposal for ESSLI course:
Propositions as Types

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a. Personal information

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b. General proposal information

Title: Propositions as Types
Category: Foundational/intermediate
c. Contents information

Abstract

The principle of Propositions as Types describes a fundamental connection between logic and computation which views

\[ \text{propositions as types,} \]
\[ \text{proofs as programs, and} \]
\[ \text{normalisation of proofs as evaluation of programs.} \]

The proposed course is intended to begin at the foundations and introduce students to a few intermediate or advanced topics. It is suitable for students in both logic and computing, presuming no previous knowledge of either, though familiarity with logic and computing will be helpful.

Motivation and description

[Taken from the introduction to Wadler [2015].]

Powerful insights arise from linking two fields of study previously thought separate. Examples include Descartes’s coordinates, which links geometry to algebra, Planck’s Quantum Theory, which links particles to waves, and Shannon’s Information Theory, which links thermodynamics to communication. Such a synthesis is offered by the principle of Propositions as Types, which links logic to computation. At first sight it appears to be a simple coincidence—almost a pun—but it turns out to be remarkably robust, inspiring the design of theorem provers and programming languages, and continuing to influence the forefronts of computing.

Propositions as Types is a notion with many names and many origins. It is closely related to the BHK Interpretation, a view of logic developed by the intuitionists Brouwer, Heyting, and Kolmogorov in the 1930s. It is often referred to as the Curry-Howard Isomorphism, referring to a correspondence observed by Curry in 1958 and refined by Howard in 1969 (though not published until 1980, in a Festschrift dedicated to Curry). Others draw attention to significant contributions from de Bruijn’s Automath and Martin-Löf’s Type Theory in the 1970s. Many variant names appear in the literature, including Formulae as Types, Curry-Howard-de Bruijn Correspondence, Brouwer’s Dictum, and others.
Propositions as Types is a notion with depth. It describes a correspondence between a given logic and a given programming language, for instance, between Gentzen’s intuitionistic natural deduction and Church’s simply-typed lambda calculus. At the surface, it says that for each proposition in the logic there is a corresponding type in the programming language—and vice versa. Thus we have

\[ \text{propositions as types.} \]

But it goes deeper, in that for each proof of a given proposition, there is a program of the corresponding type—and vice versa. Thus we also have

\[ \text{proofs as programs.} \]

And it goes deeper still, in that for each way to normalise a proof there is a corresponding way to evaluate a program—and vice versa. Thus we further have

\[ \text{normalisation of proofs as evaluation of programs.} \]

Hence, we have not merely a shallow bijection between propositions and types, but a true isomorphism preserving the deep structure of proofs and programs, normalisation and evaluation.

Propositions as Types is a notion with breadth. It applies to a range of logics including propositional, predicate, second-order, intuitionistic, classical, modal, and linear. It underpins the foundations of functional programming, explaining features including functions, products, sums, parametric polymorphism, data abstraction, continuations, linear types, and session types. It has inspired theorem provers and programming languages including Agda, Automath, Coq, Epigram, F#, F*, Haskell, LF, ML, NuPRL, Scala, Singularity, and Trellys. Applications include CompCert, a certified compiler for the C programming language verified in Coq, a computer-checked proof of the four-colour theorem also verified in Coq, parts of the Ensemble distributed system verified in NuPRL, and ten thousand lines of browser plug-ins verified in F*.

Propositions as Types is a notion with mystery. Why should it be the case that intuitionistic natural deduction, as developed by Gentzen in the 1930s, and simply-typed lambda calculus, as developed by Church around the same time for an unrelated purpose, should be discovered forty years later to be essentially identical? And why should it be the case that the
same correspondence arises again and again? The logician Girard and the computer scientist Reynolds independently developed the same calculus, now dubbed Girard-Reynolds. The logician Hindley and the computer scientist Milner independently developed the same type system, now dubbed Hindley-Milner. Curry-Howard is a double-barrelled name that ensures the existence of other double-barrelled names. Those of us that design and use programming languages may often feel they are arbitrary, but Propositions as Types assures us some aspects of programming are absolute.

Outline

- Introduction to propositions as types, including the history of the subject. (Based on Wadler [2015].)
- Detailed description of the correspondence for implication/function, conjunction/product, truth/unit, disjunction/sum, and false/zero. (Based on Girard et al. [1989].)
- Application of the correspondence to second order logic and polymorphic lambda calculus. (Based on Wadler [2007].)
- Application of the correspondence to classical logic and sequent calculus. (Based on Wadler [2003].)
- Application of the correspondence to linear logic and session types. (Based on Wadler [2012, 2014].)

Although texts by myself will be main texts of the course, there is much related work cited by those texts that we will also review: Gentzen [1935], Church [1940], Girard [1972], Reynolds [1983], Griffin [1990], Parigot [1992], Curien and Herbelin [2000], Girard [1987], Caires and Pfenning [2010], to name but a few.

Expected level and prerequisites

The course is intended to be suitable for those without previous experience, and so in that sense is foundational, but will touch on further developments in the area and so in that sense is intermediate or even advanced.
References


d. Practical information

I know of no relevant immediately preceding meetings and events, or of external sources of funding.