

Well-typed programs can't be blamed

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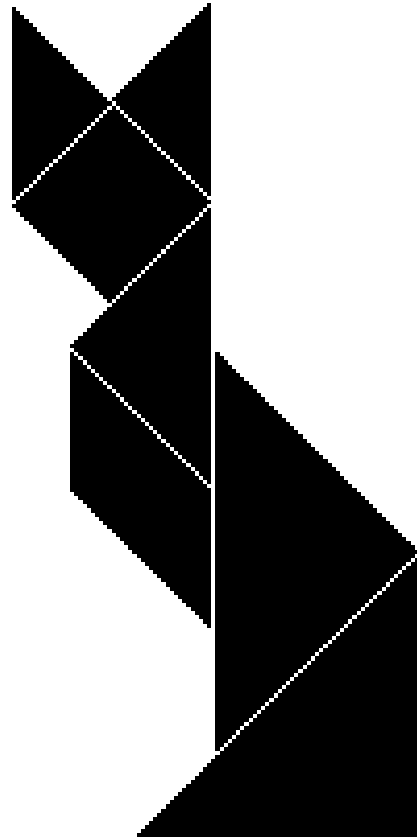
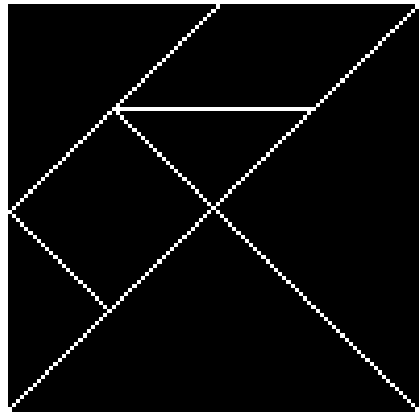
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A repeated theme

Thatte (1988):

Partial types

Henglein (1994):

Dynamic typing

Findler and Felleisen (2002):

Contracts

Flanagan (2006):

Hybrid types

Siek and Taha (2006):

Gradual types

A repeated theme

Dynamics in .Net (C#, Visual Basic)

Perl 6.0

Javascript

Dart

Part I

Evolving a program

An untyped program

```
[let  
   $x = 2$   
   $f = \lambda y. y + 1$   
   $h = \lambda g. g (g x)$   
in  
   $h f$ ]
```

→

```
[4]
```

A typed program

let

$x = 2$

$f = \lambda y : \text{Int}. y + 1$

$h = \lambda g : \text{Int} \rightarrow \text{Int}. g (g x)$

in

$h f$

→

$4 : \text{Int}$

A partly typed program—narrowing

let

$x = 2$

$f = [\lambda y. y + 1] : \star \xRightarrow{p} \text{Int} \rightarrow \text{Int}$

$h = \lambda g : \text{Int} \rightarrow \text{Int}. g (g x)$

in

$h f$

→

$4 : \text{Int}$

A partly typed program—narrowing

```
let
  x = 2
  f = [λy. false] : ★  $\stackrel{p}{\Rightarrow}$  Int → Int
  h = λg : Int → Int. g (g x)
in
  h f
→
  blame p
```

Positive (covariant): blame the term contained in the cast

Another partly typed program—widening

let

$x = [2]$

$f = (\lambda y : \text{Int}. y + 1) : \text{Int} \rightarrow \text{Int} \xRightarrow{p} \star$

$h = [\lambda g. g (g x)]$

in

$[h f]$

→

$[4]$

Another partly typed program—widening

let

$x = [\text{true}]$

$f = (\lambda y : \text{Int}. y + 1) : \text{Int} \rightarrow \text{Int} \xRightarrow{p} \star$

$h = [\lambda g. g (g x)]$

in

$[h f]$

→

blame \bar{p}

Negative (contravariant): blame the context containing the cast

Part II

Untyped and supertyped

Untyped = Uni-typed

$$[x] = x$$

$$[c] = c : A \xrightarrow{p} \star \quad \text{if } \text{ty}(c) = A$$

$$[op(\vec{M})] = op([M] : \star \xrightarrow{\vec{p}} \vec{A}) : B \xrightarrow{p} \star \quad \text{if } \text{ty}(op) = \vec{A} \rightarrow B$$

$$[\lambda x. N] = (\lambda x : \star. [N]) : \star \rightarrow \star \Rightarrow \star$$

$$[L M] = ([L] : \star \xrightarrow{p} \star \rightarrow \star) [M]$$

(slogan due to Bob Harper)

Contracts

$$\text{Nat} = \{x : \text{Int} \mid x \geq 0\}$$

let

$$x = 2 : \text{Int} \xRightarrow{p} \text{Nat}$$

$$f = (\lambda y : \text{Int}. y + 1) : \text{Int} \rightarrow \text{Int} \xRightarrow{q} \text{Nat} \rightarrow \text{Nat}$$

$$h = \lambda g : \text{Nat} \rightarrow \text{Nat}. g (g x)$$

in

$$h f$$

→

$$4 : \text{Nat}$$

Part III

The Blame Game

Reductions (simplified)

$$v : \iota \Rightarrow \star \xRightarrow{p} \iota \longrightarrow v$$

$$v : \iota \Rightarrow \star \xRightarrow{p} \iota' \longrightarrow \text{blame } p \text{ if } \iota \neq \iota'$$

$$(v : A \rightarrow B \xRightarrow{p} A' \rightarrow B') w \longrightarrow (v (w : A' \xRightarrow{\bar{p}} A) : B \xRightarrow{p} B')$$

Blame

$[2] : \star \xRightarrow{p} \text{Int}$

=

$2 : \text{Int} \Rightarrow \star \xRightarrow{p} \text{Int}$

→

2

$[\text{true}] : \star \xRightarrow{p} \text{Int}$

=

$\text{true} : \text{Bool} \Rightarrow \star \xRightarrow{p} \text{Int}$

→

blame p

The Blame Game—widening

$((\lambda y : \text{Int}. y + 1) : \text{Int} \rightarrow \text{Int} \xRightarrow{p} \star \rightarrow \star) [2]$

→

$(\lambda y : \text{Int}. y + 1) ([2] : \star \xRightarrow{\bar{p}} \text{Int}) : \text{Int} \xRightarrow{p} \star$

→

[3]

The Blame Game—widening

$((\lambda y : \text{Int}. y + 1) : \text{Int} \rightarrow \text{Int} \xrightarrow{p} \star \rightarrow \star) [\text{true}]$

→

$(\lambda y : \text{Int}. y + 1) ([\text{true}] : \star \xrightarrow{\bar{p}} \text{Int}) : \text{Int} \xrightarrow{p} \star$

→

$\text{blame } \bar{p}$

Widening can give rise to negative blame, but never positive blame

The Blame Game—narrowing

$$((\lambda y : \star. [y + 1]) : \star \rightarrow \star \xRightarrow{p} \text{Int} \rightarrow \text{Int}) 2$$

→

$$(\lambda y : \star. [y + 1]) (2 : \text{Int} \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} \text{Int}$$

→

3

The Blame Game—narrowing

$((\lambda y : \star. [\text{false}]) : \star \rightarrow \star \xrightarrow{p} \text{Int} \rightarrow \text{Int}) 2$

→

$(\lambda y : \star. [\text{false}]) (2 : \text{Int} \xrightarrow{\bar{p}} \star) : \star \xrightarrow{p} \text{Int}$

→

`blame p`

Narrowing can give rise to positive blame, but never negative blame

Part IV

Blame calculus in detail

Notation

It took us four years to find the right notation!

$$\langle A \Rightarrow B \rangle^p s$$

$$\langle B \Leftarrow A \rangle^p s$$

$$s : A \xRightarrow{p} B$$

We want composition to be easy to read:

$$\langle B \Rightarrow C \rangle^q \langle A \Rightarrow B \rangle^p s$$

$$\langle C \Leftarrow B \rangle^q \langle B \Leftarrow A \rangle^p s$$

$$s : A \xRightarrow{p} B : B \xRightarrow{q} C$$

And there is a convenient abbreviation:

$$s : A \xRightarrow{p} B \xRightarrow{q} C$$

Syntax

Blame labels p, q

Base types ι

Types $A, B, C ::= \iota \mid A \rightarrow B \mid \star$

Ground types $G, H ::= \iota \mid \star \rightarrow \star$

Terms $s, t ::= c \mid op(\vec{t}) \mid x \mid \lambda x:A. t \mid t s \mid$
 $s : A \xRightarrow{p} B \mid s : G \Rightarrow \star \mid \text{blame } p$

Environments $\Gamma ::= \cdot \mid \Gamma, x : A$

Values $v, w ::= c \mid \lambda x:A. t \mid v : G \Rightarrow \star$

Contexts $E ::= [\cdot] \mid op(\vec{v}, E, \vec{t}) \mid E s \mid v E \mid$
 $E : A \xRightarrow{p} B \mid E : G \Rightarrow \star$

Blame calculus: Compatibility

$$A \prec \star \quad \star \prec B$$

$$\iota \prec \iota$$

$$\frac{A' \prec A \quad B \prec B'}{A \rightarrow B \prec A' \rightarrow B'}$$

Types

$$\frac{\text{ty}(c) = \iota}{\Gamma \vdash c : \iota} \quad \frac{\Gamma \vdash \vec{t} : \vec{A} \quad \text{ty}(op) = \vec{A} \rightarrow B}{\Gamma \vdash op(\vec{t}) : B} \quad \frac{x : A \in \Gamma}{\Gamma \vdash x : A}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x:A. t : A \rightarrow B} \quad \frac{\Gamma \vdash t : A \rightarrow B \quad \Gamma \vdash s : A}{\Gamma \vdash t s : B}$$

$$\frac{\Gamma \vdash s : A \quad A \prec B}{\Gamma \vdash (s : A \xRightarrow{p} B) : B} \quad \frac{\Gamma \vdash s : G}{\Gamma \vdash (s : G \Rightarrow \star) : \star}$$

$$\Gamma \vdash \text{blame } p : A$$

Beta, Delta, contextual closure

$$\begin{aligned}(\lambda x:A. t) v &\longrightarrow t[x:=v] \\ op(\vec{v}) &\longrightarrow \delta(op, \vec{v})\end{aligned}$$

$$\frac{s \longrightarrow t}{E[s] \longrightarrow E[t]} \qquad \frac{E \neq [\cdot]}{E[\text{blame } p] \longrightarrow \text{blame } p}$$

Wrap, Id, Ground, Collapse, Conflict

$$\begin{aligned}v : A \rightarrow B \xRightarrow{p} A' \rightarrow B' &\longrightarrow \lambda x' : A'. (v (x' : A' \xRightarrow{\bar{p}} A) : B \xRightarrow{p} B') \\v : \iota \xRightarrow{p} \iota &\longrightarrow v \\v : A \xRightarrow{p} \star &\longrightarrow v : A \xRightarrow{p} G \Rightarrow \star \quad \text{if } A \prec G \text{ and } A \neq \star \\v : G \Rightarrow \star \xRightarrow{p} A &\longrightarrow v : G \xRightarrow{p} A \quad \text{if } G \prec A \\v : G \Rightarrow \star \xRightarrow{p} A &\longrightarrow \text{blame } p \quad \text{if } G \not\prec A\end{aligned}$$

Part V

Subtyping

$\langle :$

$\langle :^+$

$\langle :^-$

$\langle :n$

Subtype

$$\frac{}{\star <: \star}$$

$$\frac{}{\iota <: \iota} \quad \frac{A <: G}{A <: \star}$$

$$\frac{A' <: A \quad B <: B'}{A \rightarrow B <: A' \rightarrow B'}$$

Example:

$$\frac{\frac{}{\text{Int} <: \text{Int}}}{\text{Int} <: \star} \quad \frac{\frac{}{\text{Int} <: \text{Int}}}{\text{Int} <: \star}}{\star \rightarrow \text{Int} <: \text{Int} \rightarrow \star}$$

Positive subtype—widening

$$\overline{A <:^+ \star}$$

$$\overline{l <: l}$$

$$\frac{A' <:^- A \quad B <:^+ B'}{A \rightarrow B <:^+ A' \rightarrow B'}$$

Example:

$$\frac{\overline{\star <:^- \text{Int}} \quad \overline{\text{Int} <:^+ \star}}{\text{Int} \rightarrow \text{Int} <: \star \rightarrow \star}$$

Negative subtype—narrowing

$$\overline{\star <:^{-} A}$$

$$\frac{}{\overline{\iota <: \iota}} \quad \frac{A <:^{-} G}{\overline{A <:^{-} \star}}$$

$$\frac{A' <:^{+} A \quad B <:^{-} B'}{\overline{A \rightarrow B <:^{-} A' \rightarrow B'}}$$

Example:

$$\frac{\overline{\text{Int} <:^{+} \star} \quad \overline{\star <:^{-} \text{Int}}}{\overline{\star \rightarrow \star <:^{-} \text{Int} \rightarrow \text{Int}}}$$

Naive subtype

$$\frac{}{A <{:}_n \star}$$

$$\frac{}{l <{:}_n l}$$

$$\frac{A <{:}_n A' \quad B <{:}_n B'}{A \rightarrow B <{:}_n A' \rightarrow B'}$$

Example:

$$\frac{\frac{}{\text{Int} <{:}_n \star} \quad \frac{}{\text{Int} <{:}_n \star}}{\text{Int} \rightarrow \text{Int} <{:} \star \rightarrow \star}}$$

Part VI

The Blame Theorem

Safety

$$\frac{}{x \text{ sf } p} \quad \frac{t \text{ sf } p}{\lambda x. t \text{ sf } p} \quad \frac{s \text{ sf } p \quad t \text{ sf } p}{s t \text{ sf } p}$$

$$\frac{s \text{ sf } p \quad A <:^+ B}{s : A \xRightarrow{p} B \text{ sf } p}$$

$$\frac{s \text{ sf } p \quad A <:^- B}{s : A \xRightarrow{\bar{p}} B \text{ sf } p}$$

$$\frac{s \text{ sf } p \quad p \neq q \quad \bar{p} \neq q}{s : A \xRightarrow{q} B \text{ sf } p}$$

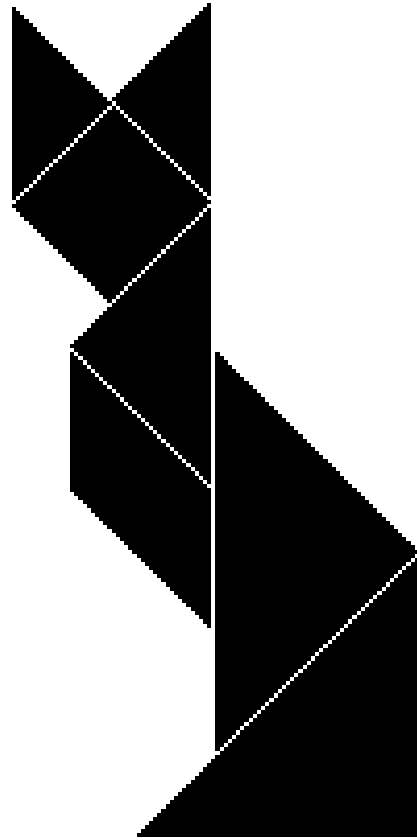
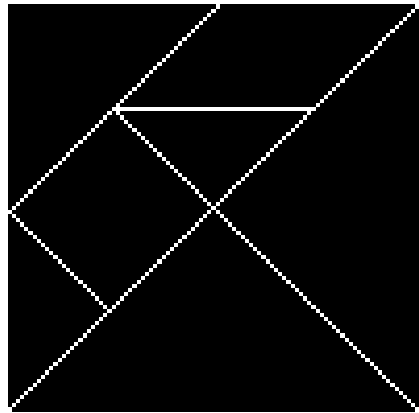
The Blame Theorem

Preservation

If s sf p and $s \longrightarrow t$ then t sf p .

Progress

If s sf p then $s \not\rightarrow$ blame p .



The First Tangram Theorem

$A <: B$ if and only if $A <:^+ B$ and $A <:^- B$

The First Blame Corollary

Let t be a term where $s : A \xRightarrow{p} B$ is the only subterm with label p . If $A <: B$ then $t \not\rightarrow \text{blame } p$ and $t \not\rightarrow \text{blame } \bar{p}$.

The Second Tangram Theorem

$A <:_n B$ if and only if $A <:^+ B$ and $B <:^- A$

The Second Blame Corollary

Let t be a term where $s : A \xRightarrow{p} B$ is the only subterm with label p . If $A <:_n B$ then $t \not\rightarrow \text{blame } p$.

Let t be a term where $s : A \xRightarrow{p} B$ is the only subterm with label p . If $B <:_n A$ then $t \not\rightarrow \text{blame } \bar{p}$.

Part VII

Adding polymorphism

Explicit binding

$$\frac{\Gamma, X:=A \vdash t : B \quad X \notin \text{ftv}(B)}{\Gamma \vdash \nu X:=A. t : B}$$

$$\frac{\Gamma \vdash t : B \quad (X:=A) \in \Gamma}{\Gamma \vdash t : B[X:=A]}$$

$$\frac{\Gamma \vdash t : B[X:=A] \quad (X:=A) \in \Gamma}{\Gamma \vdash t : B}$$

$$(\Lambda X. v) A \longrightarrow \nu X:=A. v$$

Global store vs. Local bindings

George Neis, Derek Dreyer, and Andreas Rossberg. Non-parametric parametricity. ICFP 2009, Edinburgh.

Compatibility and reductions

$$X \prec X \quad \frac{A \prec B}{A \prec \forall X. B} \quad X \notin \text{ftv}(A) \quad \frac{A[X:=\star] \prec B}{\forall X. A \prec B}$$

$$v : A \xRightarrow{p} (\forall X. B) \longrightarrow \Lambda X. v : A \xRightarrow{p} B \quad \text{if } X \notin \text{ftv}A$$

$$v : (\forall X. A) \xRightarrow{p} B \longrightarrow v \star : A[X:=\star] \xRightarrow{p} B$$

$$v : X \xRightarrow{p} \star \xRightarrow{q} X \longrightarrow v$$

$$v : X \xRightarrow{p} \star \xRightarrow{q} Y \longrightarrow \text{blame } q \quad \text{if } X \neq Y$$

Instantiate

$$v : \forall X. A \xRightarrow{p} B \longrightarrow v \star : A[X := \star] \xRightarrow{p} B$$

$$K = \lambda x : X. \lambda y : X. x$$

$$\begin{aligned} & (((\lambda X. K) : \forall X. X \rightarrow X \rightarrow X \xRightarrow{p} \star \rightarrow \star \rightarrow \star) \ 42 \ 7) \\ \longrightarrow & (((\lambda X. K) \star : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \star \rightarrow \star \rightarrow \star) \ 42 \ 7) \\ \longrightarrow & \nu X := \star. (K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \star \rightarrow \star \rightarrow \star) \ 42 \ 7 \\ \longrightarrow & \nu X := \star. (K (42 : \star \xRightarrow{\bar{p}} \star) (7 : \star \xRightarrow{\bar{p}} \star)) : \star \xRightarrow{p} \star \\ \longrightarrow & \nu X := \star. (42 : \star \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} \star \\ \longrightarrow & \nu X := \star. 42 \\ \longrightarrow & 42 \end{aligned}$$

★ is a Jack-of-all-Trades

$$v : \forall X. A \xRightarrow{p} B \longrightarrow v \star : A[X := \star] \xRightarrow{p} B$$

$$K = \lambda x : X. \lambda y : X. x$$

$$\begin{aligned} & ((\Lambda X. K) : \forall X. X \rightarrow X \rightarrow X \xRightarrow{p} \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}) \ 42 \ 7 \\ \longrightarrow & ((\Lambda X. K) \star : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}) \ 42 \ 7 \\ \longrightarrow & \nu X := \star. (K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}) \ 42 \ 7 \\ \longrightarrow & \nu X := \star. (K (42 : \text{Int} \xRightarrow{\bar{p}} \star) (7 : \text{Int} \xRightarrow{\bar{p}} \star)) : \star \xRightarrow{p} \text{Int} \\ \longrightarrow & \nu X := \star. (42 : \text{Int} \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} \text{Int} \\ \longrightarrow & \nu X := \star. 42 \\ \longrightarrow & 42 \end{aligned}$$

... but master of none

$$v : \forall X. A \xRightarrow{p} B \longrightarrow v \star : A[X := \star] \xRightarrow{p} B$$

$$K = \lambda x : X. \lambda y : X. x$$

$$\begin{aligned} & ((\lambda X. K) : \forall X. X \rightarrow X \rightarrow X \xRightarrow{p} \text{Int} \rightarrow \text{Bool} \rightarrow \text{Int}) \ 42 \ \text{true} \\ \longrightarrow & ((\lambda X. K) \star : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \text{Int} \rightarrow \text{Bool} \rightarrow \text{Int}) \ 42 \ \text{true} \\ \longrightarrow & \nu X := \star. (K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \text{Int} \rightarrow \text{Bool} \rightarrow \text{Int}) \ 42 \ \text{true} \\ \longrightarrow & \nu X := \star. (K (42 : \text{Int} \xRightarrow{\bar{p}} \star) (\text{true} : \text{Bool} \xRightarrow{\bar{p}} \star)) : \star \xRightarrow{p} \text{Int} \\ \longrightarrow & \nu X := \star. (42 : \text{Int} \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} \text{Int} \\ \longrightarrow & \nu X := \star. 42 \\ \longrightarrow & 42 \end{aligned}$$

Generalise

$$v : A \xRightarrow{p} \forall X. B \longrightarrow \Lambda X. (v : A \xRightarrow{p} B) \quad X \text{ not free in } A$$

$$K = \lambda x : \star. \lambda y : \star. x$$

$$(K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \forall X. \forall Y. X \rightarrow Y \rightarrow X) \text{ Int Int 42 7}$$

$$\longrightarrow (\Lambda X. \Lambda Y. (K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} X \rightarrow Y \rightarrow X)) \text{ Int Int 42 7}$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (K : \star \rightarrow \star \rightarrow \star \xRightarrow{p} X \rightarrow Y \rightarrow X) \text{ 42 7}$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (K (42 : X \xRightarrow{\bar{p}} \star) (7 : Y \xRightarrow{\bar{p}} \star)) : \star \xRightarrow{p} X$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (42 : X \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} X$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. 42$$

$$\longrightarrow 42$$

Enforcing semantic parametricity

$$v : A \xRightarrow{p} \forall X. B \longrightarrow \Lambda X. (v : A \xRightarrow{p} B) \quad X \text{ not free in } A$$

$$K' = \lambda x : \star. \lambda y : \star. y$$

$$(K' : \star \rightarrow \star \rightarrow \star \xRightarrow{p} \forall X. \forall Y. X \rightarrow Y \rightarrow X) \text{ Int Int 42 7}$$

$$\longrightarrow (\Lambda X. \Lambda Y. (K' : \star \rightarrow \star \rightarrow \star \xRightarrow{p} X \rightarrow Y \rightarrow X)) \text{ Int Int 42 7}$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (K' : \star \rightarrow \star \rightarrow \star \xRightarrow{p} X \rightarrow Y \rightarrow X) \text{ 42 7}$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (K' (42 : X \xRightarrow{\bar{p}} \star) (7 : Y \xRightarrow{\bar{p}} \star)) : \star \xRightarrow{p} X$$

$$\longrightarrow \nu X := \text{Int}. \nu Y := \text{Int}. (7 : Y \xRightarrow{\bar{p}} \star) : \star \xRightarrow{p} X$$

$$\longrightarrow \text{blame } p$$

Part VIII

Conclusion

A new slogan for type safety

Milner (1978):

Well-typed programs can't go wrong.

Felleisen and Wright (1994); Harper (2002):

Well-typed programs don't get stuck.

Wadler and Findler (2008):

Well-typed programs can't be blamed.