Coherence Generalises Duality

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Multiparty Session Types as Coherence Proofs

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Abstract

We propose a Curry-Howard correspondence between a language for programming multiparty sessions and a generalisation of Classical Linear Logic (CLL). In this framework, propositions correspond to the local behaviour of a participant in a multiparty session type, proofs to processes, and proof normalisation to executing communications. Our key contribution is generalising duality, from CLL, to a new notion of n-ary compatibility, called *coherence*. Building on coherence as a principle of compositionality, we generalise the cut rule of CLL to a new rule for composing many processes communicating in a multiparty session. We prove the soundness of our model by showing the admissibility of our new rule, which entails deadlock-freedom via our correspondence.

1998 ACM Subject Classification F1.1 Models of Computation

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Coherence Generalises Duality: a logical explanation of multiparty session types

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Abstract

Wadler introduced Classical Processes (CP), a calculus based on a propositions-as-types correspondence between propositions of classical linear logic and session types. Carbone *et al.* introduced Multiparty Classical Processes, a calculus that generalises CP to multiparty session types, by replacing the duality of classical linear logic (relating two types) with a more general notion of coherence (relating an arbitrary number of types). This paper introduces variants of CP and MCP, plus a new intermediate calculus of Globally-governed Classical Processes (GCP). We show a tight relation between these three calculi, giving semantics-preserving translations from GCP to CP and from MCP to GCP. The translation from GCP to CP interprets a coherence proof as an arbiter process that mediates communications in a session, while MCP adds annotations that permit processes to communicate directly without centralised control.

1998 ACM Subject Classification F1.2 Modes of Computation: Parallelism and concurrency; F4.1 Mathematical logic: Proof theory

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Multiparty Session Types
Within A Canonical Binary Theory, and Beyond

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Abstract. A widespread approach to software service analysis uses *session types*. Very different type theories for *binary* and *multiparty* protocols have been developed; establishing precise connections between them remains an open problem. We present the first formal relation between two existing theories of binary and multiparty session types: a binary system rooted in linear logic, and a multiparty system based on automata theory. Our results enable the analysis of multiparty protocols using a (much simpler) type theory for binary protocols, ensuring protocol fidelity and deadlock-freedom. As an application, we offer the first theory of multiparty session types with *behavioral genericity*. This theory is natural and powerful; its analysis techniques reuse results for binary session types.

ABCD, 2013

Multiparty Session Types. Before the work of Honda et al. (2008), research had focussed on binary (two-party) sessions. However, there are cases where binary endpoint session types are not powerful enough for describing and validating interactions which involve more than two parties.

We shall illustrate the key idea of multiparty session types with a simple ring protocol, where participant Buyer1 sends a message of type `bool` to Buyer2, then Buyer2 forwards the message of type `bool` to Seller. After receiving the message, Seller sends an acknowledgement of type `int` to Buyer1. To develop the code for this protocol, we start by specifying the global type as

$$\mathrm{Buyer1} \rightarrow \mathrm{Buyer2}{:}\langle \mathtt{bool} \rangle ; \mathrm{Buyer2} \rightarrow \mathrm{Seller}{:}\langle \mathtt{bool} \rangle ; \mathrm{Seller} \rightarrow \mathrm{Buyer1}{:}\langle \mathtt{bool} \rangle ; \mathsf{end} \tag{3}$$

where $\rightarrow$ signifies the flow of communication. With agreement on the global type in (3) as a specification for participants Seller, Buyer1 and Buyer2, each program can be implemented separately. Then for type-checking, the global type in (3) is *projected* into the three endpoint session types:

Buyer1's endpoint type:	$\mathrm{Buyer2}!\langle \mathtt{bool} \rangle ; \mathrm{Seller}?\langle \mathtt{int} \rangle ; \mathsf{end}$
Buyer2's endpoint type:	$\mathrm{Buyer1}?\langle \mathtt{bool} \rangle ; \mathrm{Seller}!\langle \mathtt{bool} \rangle ; \mathsf{end}$
Seller's endpoint type:	$\mathrm{Buyer2}?\langle \mathtt{bool} \rangle ; \mathrm{Buyer1}!\langle \mathtt{int} \rangle ; \mathsf{end}$

where $\mathrm{Buyer2}!\langle \mathtt{bool} \rangle$ means (output to Buyer2 with `bool`-type), and $\mathrm{Seller}?\langle \mathtt{int} \rangle$ means (input from Seller with `int`-type). Then each process is type-checked againt its own endpoint type. When the three processes are executed, their interactions automatically follow the stipulated scenario.

If we use three separate binary session types (one between Buyer1 and Buyer2, one between Buyer2 and Seller and one between Buyer1 and Seller) without using the multiparty session type, then we lose essential sequencing information in this interaction scenario and we can no longer gurantee deadlock-freedom among these three parties. Since the three separate binary sessions can be interleaved freely, Buyer1 who conforms to $\mathrm{Seller}?\langle \mathtt{int} \rangle ; \mathrm{Buyer2}!\langle \mathtt{bool} \rangle ; \mathsf{end}$ becomes typable. This causes the situation that each of the three parties blocks indefinitely while waiting for a message to be delivered. For validating this ring topology as a whole, therefore, the structure should be represented as a *single multiparty session*. The multiparty session type discipline can validate whether a program is typable or not, given global types (as a shared agreement) and an individual program (as its end-point realiser). The resulting type discipline guarantees all the original key properties of binary session types, such as deadlock-freedom, race-freedom and session fidelity in a general n-party session, underpinning its practical use.

Part I

CP:

Classical Processes

Types

$A, B, C ::=$

	output types		*input types*
X	type variable	$X^\perp$	dual of type variable
$A \otimes B$	output A then behave as B	$A \parr B$	input A then behave as B
$A \oplus B$	select from A or B	$A \with B$	offer choice of A or B
$?A$	client request	$!A$	server accept
1	unit for $\otimes$	$\perp$	unit for $\parr$
0	unit for $\oplus$	$\top$	unit for $\with$

Duals

$$(X)^\perp = X^\perp \qquad (X^\perp)^\perp = X$$

$$(A \otimes B)^\perp = A^\perp \parr B^\perp \qquad (A \parr B)^\perp = A^\perp \otimes B^\perp$$

$$(A \oplus B)^\perp = A^\perp \,\&\, B^\perp \qquad (A \,\&\, B)^\perp = A^\perp \oplus B^\perp$$

$$(!A)^\perp = ?A^\perp \qquad (?A)^\perp = !A^\perp$$

$$1^\perp = \perp \qquad \perp^\perp = 1$$

$$0^\perp = \top \qquad \top^\perp = 0$$

CP: Old and new

Old

$$\frac{}{x{\leftrightarrow}y \vdash x : A^{\perp}, y : A}\ \textsc{Axiom} \qquad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^{\perp}}{\boldsymbol{\nu} x{:}A.\,(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

New

$$\frac{}{x \to y^{A} \vdash x : A^{\perp}, y : A}\ \textsc{Axiom} \qquad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, y : A^{\perp}}{(\boldsymbol{\nu} x^{A} y)\,(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

CP: Processes

$$\frac{}{x \to y^{A} \vdash x : A^{\perp}, y : A}\ \textsc{Axiom} \qquad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, y : A^{\perp}}{(\boldsymbol{\nu} x^{A} y)\,(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

$$\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ \otimes \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A \parr B}\ \parr$$

$$\frac{P \vdash \Gamma, x : A}{x[\mathsf{inl}].P \vdash \Gamma, x : A \oplus B}\ \oplus_1 \qquad \frac{P \vdash \Gamma, x : B}{x[\mathsf{inr}].P \vdash \Gamma, x : A \oplus B}\ \oplus_2$$

$$\frac{P \vdash \Gamma, x : A \quad Q \vdash \Gamma, x : B}{x.\mathsf{case}(P, Q) \vdash \Gamma, x : A \,\&\, B}\ \&$$

$$\frac{P \vdash \Gamma, y : A}{?x[y].P \vdash \Gamma, x : ?A}\ ? \qquad \frac{P \vdash\ ?\Gamma, y : A}{!x(y).P \vdash\ ?\Gamma, x : !A}\ !$$

$$\frac{P \vdash \Gamma}{P \vdash \Gamma, x : ?A}\ \textsc{Weaken} \qquad \frac{P \vdash \Gamma, y : ?A, z : ?A}{P\{x/y, x/z\} \vdash \Gamma, x : ?A}\ \textsc{Contract}$$

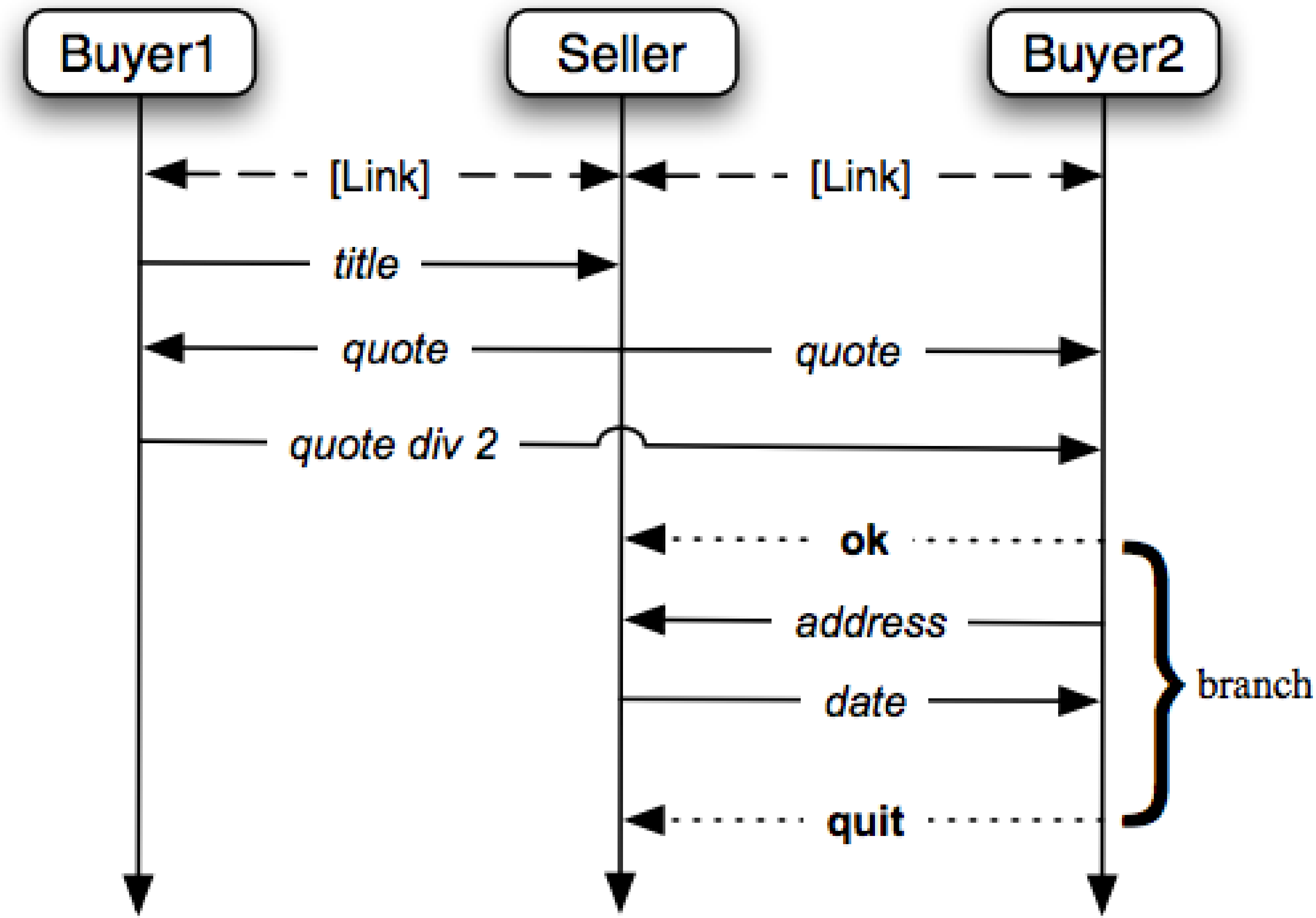
$$\frac{}{x[\,] \vdash x : 1}\ 1 \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \perp}\ \perp \qquad \text{(no rule for 0)} \qquad \frac{}{x.\mathsf{case}() \vdash \Gamma, x : \top}\ \top$$

Duplicating a bit

$$P_x \stackrel{\text{def}}{=} x[y].(y[\mathsf{inl}].y[] \mid x[\mathsf{inl}].x[]) \qquad Q_x \stackrel{\text{def}}{=} x[y].(y[\mathsf{inr}].y[] \mid x[\mathsf{inr}].x[])$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{y[] \vdash y : 1}\,1}{y[\mathsf{inl}].y[] \vdash y : 1 \oplus 1}\,\oplus_1 \quad \dfrac{\dfrac{}{x[] \vdash x : 1}\,1}{x[\mathsf{inl}].x[] \vdash x : 1 \oplus 1}\,\oplus_1}{P_x \vdash x : (1 \oplus 1) \otimes (1 \oplus 1)}\,\otimes}{w().P_x \vdash w : \bot, x : (1 \oplus 1) \otimes (1 \oplus 1)}\,\bot \quad \dfrac{\dfrac{\dfrac{\dfrac{}{y[] \vdash y : 1}\,1}{y[\mathsf{inr}].y[] \vdash y : 1 \oplus 1}\,\oplus_2 \quad \dfrac{\dfrac{}{x[] \vdash x : 1}\,1}{x[\mathsf{inr}].x[] \vdash x : 1 \oplus 1}\,\oplus_2}{Q_x \vdash x : (1 \oplus 1) \otimes (1 \oplus 1)}\,\otimes}{w().Q_x \vdash w : \bot, x : (1 \oplus 1) \otimes (1 \oplus 1)}\,\bot}{\dfrac{w.\mathsf{case}(w().P_x, w().Q_x) \vdash w : \bot \,\&\, \bot, x : (1 \oplus 1) \otimes (1 \oplus 1)}{x(w).w.\mathsf{case}(w().P_x, w().Q_x) \vdash x : (\bot \,\&\, \bot) \parr ((1 \oplus 1) \otimes (1 \oplus 1))}\,\parr}\,\&$$

Two-buyer protocol



Two-buyer protocol

$$b_1(b_1').s[s'].(b_1' \to s'^{\mathbf{name}} \mid s(s').b_1[b_1'].(s' \to b_1'^{\mathbf{cost}} \mid$$
$$s(s').b_2[b_2'].(s' \to b_2'^{\mathbf{cost}} \mid b_1(b_1').b_2[b_2'].(b_1' \to b_2'^{\mathbf{cost}} \mid$$
$$b_2.\mathsf{case}(s[\mathsf{inl}].b_2(b_2').s[s'].(b_2' \to s'^{\mathbf{addr}} \mid b_1().b_2().s[]), s[\mathsf{inr}].b_1().b_2().s[])))))$$

$$\vdash$$

$$b_1 : \mathbf{name}^\perp \parr (\mathbf{cost} \otimes (\mathbf{cost}^\perp \parr \perp)),$$

$$b_2 : \mathbf{cost} \otimes (\mathbf{cost} \otimes ((\mathbf{addr}^\perp \parr \perp) \& \perp)),$$

$$s : \mathbf{name} \otimes (\mathbf{cost}^\perp \parr (\mathbf{cost}^\perp \parr ((\mathbf{addr} \otimes 1) \oplus 1)))$$

Part II

GCP:

Globally-governed Classical Processes

GCP: Coherence

$$\frac{}{x^A \to y \vDash x : A, y : A^\perp}\ \textsc{Axiom}$$

$$\frac{G \vDash (x_i : A_i)_i, y : C \quad H \vDash \Gamma, (x_i : B_i)_i, y : D}{\tilde{x} \to y(G).H \vDash \Gamma, (x_i : A_i \otimes B_i)_i, y : C \parr D}\ \otimes\parr$$

$$\frac{G \vDash \Gamma, x : A, (y_i : C_i)_i \quad H \vDash \Gamma, x : B, (y_i : D_i)_i}{x \to \tilde{y}.\mathsf{case}(G, H) \vDash \Gamma, x : A \oplus B, (y_i : C_i \mathbin{\&} D_i)_i}\ \oplus\&$$

$$\frac{G \vDash x : A, (y_i : B_i)_i}{!x \to \tilde{y}(G) \vDash x : ?A, (y_i : !B_i)_i}\ ?!$$

$$\frac{}{\tilde{x} \to y \vDash (x_i : 1)_i, y : \perp}\ 1\perp \qquad \frac{}{x \to \tilde{y}.\mathsf{case}() \vDash \Gamma, x : 0, (y_i : \top)_i}\ 0\top$$

GCP: Processes

$$\frac{(P_i \vdash \Gamma_i, x_i : A_i)_i \quad G \vDash (x_i : A_i)_i}{(\boldsymbol{\nu} \tilde{x}^{\tilde{A}} : G)\,(\tilde{P}) \vdash \tilde{\Gamma}} \text{ CCUT}$$

From GCP to CP

Global cut as binary cut

$$[\![(\boldsymbol{\nu}\tilde{x}^{\tilde{A}} : G)\,(\tilde{P})]\!] \stackrel{\text{def}}{=} (\boldsymbol{\nu}x_1{}^{A_1}y_1)\,([\![P_1]\!] \mid \cdots(\boldsymbol{\nu}x_n{}^{A_n}y_n)\,([\![P_n]\!] \mid [\![G]\!]\{\tilde{y}/\tilde{x}\})\cdots)$$

($\tilde{y}$ fresh)

Global types as processes

$$\begin{aligned}
[\![x^A \to y]\!] &\stackrel{\text{def}}{=} x \to y^A \\
[\![\tilde{x} \to y(G).H]\!] &\stackrel{\text{def}}{=} x_1(u_1).\cdots x_n(u_n).y[v].([\![G]\!]\{\tilde{u}/\tilde{x}, v/y\} \mid [\![H]\!]) \\
[\![x \to \tilde{y}.\mathsf{case}(G,H)]\!] &\stackrel{\text{def}}{=} x.\mathsf{case}(y_1[\mathsf{inl}].\cdots y_n[\mathsf{inl}].[\![G]\!], y_1[\mathsf{inr}].\cdots y_n[\mathsf{inr}].[\![H]\!]) \\
[\![!x \to \tilde{y}(G)]\!] &\stackrel{\text{def}}{=} !x(u).?y_1[v_1].\cdots ?y_n[v_n].[\![G]\!]\{u/x, \tilde{v}/\tilde{y}\} \\
[\![\tilde{x} \to y]\!] &\stackrel{\text{def}}{=} x_1().\cdots x_n().y[] \\
[\![x \to \tilde{y}.\mathsf{case}()]\!] &\stackrel{\text{def}}{=} x.\mathsf{case}()
\end{aligned}$$

($\tilde{u}, v, u, \tilde{v}$ fresh)

η-expansion

$$\dfrac{}{x \to y^{A \otimes B} \vdash x : A^\perp \mathbin{⅋} B^\perp, y : A \otimes B}\ \textsc{Axiom}$$

$$\Longrightarrow$$

$$\dfrac{\dfrac{\dfrac{}{u \to v^{A} \vdash u : A^\perp, v : A}\ \textsc{Axiom} \qquad \dfrac{}{x \to y^{B} \vdash x : B^\perp, y : B}\ \textsc{Axiom}}{y[v].(u \to v^{A} \mid x \to y^{B}) \vdash u : A^\perp, x : B^\perp, y : A \otimes B}\ \otimes}{x(u).y[v].(u \to v^{A} \mid x \to y^{B}) \vdash x : A^\perp \mathbin{⅋} B^\perp, y : A \otimes B}\ ⅋$$

From duality to coherence

Duality

$$\dfrac{\dfrac{[\![G]\!] \vdash u : A, v : C \qquad [\![H]\!] \vdash x : B, y : D}{y[v].([\![G]\!] \mid [\![H]\!]) \vdash u : A, x : B, y : C \otimes D}\otimes}{x(u).y[v].([\![G]\!] \mid [\![H]\!]) \vdash x : A \mathbin{\wp} B, y : C \otimes D}\wp$$

Coherence

$$\dfrac{\dfrac{\dfrac{[\![G]\!] \vdash u_1 : A_1, u_2 : A_2, v : C \qquad [\![H]\!] \vdash x_1 : B_1, x_2 : B_2, y : D}{y[v].([\![G]\!] \mid [\![H]\!]) \vdash u_1 : A_1, x_1 : B_1, u_2 : A_2, x_2 : B_2, y : C \otimes D}\otimes}{x_2(u_2).y[v].([\![G]\!] \mid [\![H]\!]) \vdash u_1 : A_1, x_1 : B_1, x_2 : A_2 \mathbin{\wp} B_2, y : C \otimes D}\wp}{x_1(u_1).x_2(u_2).y[v].([\![G]\!] \mid [\![H]\!]) \vdash x_1 : A_1 \mathbin{\wp} B_1, x_2 : A_2 \mathbin{\wp} B_2, y : C \otimes D}\wp$$

From GCP to CP

Theorem 1 (Type preservation from GCP to CP)

1. *If* $P \vdash \Gamma$ *in GCP, then* $[\![P]\!] \vdash \Gamma$ *in CP.*

2. *If* $G \vDash \Gamma$ *in GCP, then* $[\![G]\!] \vdash \Gamma^{\perp}$ *in CP.*

Theorem 2 (Simulation of GCP in CP)

1. *If* $P \vdash \Gamma$ *and* $P \equiv Q$ *in GCP, then* $[\![P]\!] \equiv [\![Q]\!]$ *in CP.*

2. *If* $P \vdash \Gamma$ *and* $P \longrightarrow_{\eta} Q$ *in GCP, then* $[\![P]\!] \longrightarrow_{\eta} [\![Q]\!]$ *in CP.*

3. *If* $G \vDash \Gamma$ *and* $G \longrightarrow_{\eta} H$ *in GCP, then* $[\![G]\!] \longrightarrow_{\eta} [\![H]\!]$ *in CP.*

4. *If* $P \vdash \Gamma$ *and* $P \longrightarrow Q$ *in GCP, then* $[\![P]\!] \Longrightarrow^{+} [\![Q]\!]$ *in CP.*

Theorem 3 (Reflection of CP in GCP) *If* $P \vdash \Gamma$ *in GCP and* $[\![P]\!] \longrightarrow Q'$ *in CP, then there exists* Q *such that* $P \Longrightarrow Q$ *in GCP and* $Q' \Longrightarrow^{*} [\![Q]\!]$ *in CP.*

From CP to GCP

Binary cut as global cut

$$(\!|(\boldsymbol{\nu} x^A y)\,(P \mid Q)|\!) = (\boldsymbol{\nu} x, y : x^{A^\perp} \to y)\,((\!|P|\!) \mid (\!|Q|\!))$$

Theorem 4 (Type preservation from CP to GCP) *If* $P \vdash \Gamma$, *then* $(\!|P|\!) \vdash \Gamma$.

Theorem 5 (Simulation of CP in GCP)

1. *If* $P \vdash \Gamma$ *and* $P \equiv Q$ *in CP, then* $(\!|P|\!) \equiv (\!|Q|\!)$ *in GCP.*
2. *If* $P \vdash \Gamma$ *and* $P \longrightarrow Q$ *in CP, then* $(\!|P|\!) \longrightarrow^+ (\!|Q|\!)$ *in GCP.*

Two-buyer protocol

$$B_1 \to S({B_1}^{\mathbf{name}} \to S).\ S \to B_1(S^{\mathbf{cost}} \to B_1).$$
$$S \to B_2(S^{\mathbf{cost}} \to B_2).\ B_1 \to B_2({B_1}^{\mathbf{cost}} \to B_2).$$
$$B_2 \to S.\mathsf{case}(B_2 \to S({B_2}^{\mathbf{addr}} \to S).(B_1, B_2) \to S,\ (B_1, B_2) \to S))$$

$\models$

$B_1 : \mathbf{name} \otimes (\mathbf{cost}^\perp \parr (\mathbf{cost} \otimes 1)),$

$B_2 : \mathbf{cost}^\perp \parr (\mathbf{cost}^\perp \parr ((\mathbf{addr} \otimes 1) \oplus 1)),$

$S : \mathbf{name}^\perp \parr (\mathbf{cost} \otimes (\mathbf{cost} \otimes ((\mathbf{addr}^\perp \parr \bot) \,\&\, \bot)))$

Part III

MCP:

Multiparty Classical Processes

Types

$A, B, C ::=$

	output types		*input types*
X	type variable	$X^{\perp}$	dual of type variable
$A \otimes^{z} B$	output A then behave as B	$A \mathbin{⅋}^{\tilde{z}} B$	input A then behave as B
$A \oplus^{\tilde{z}} B$	select from A or B	$A \mathbin{\&}^{z} B$	offer choice of A or B
$?^{\tilde{z}} A$	client request	$!^{z} A$	server accept
1^{z}	unit for $\otimes$	$\perp^{\tilde{z}}$	unit for $⅋$
$0^{\tilde{z}}$	unit for $\oplus$	$\top^{z}$	unit for $\&$

MCP: Coherence

$$\frac{|A|^{\perp} = |B|}{x^A \to y^B \vdash x : A, y : B}\ \textsc{Axiom}$$

$$\frac{G \ \vDash\ (x_i : A_i)_i,\ y : C \qquad H \ \vDash\ \Gamma,\ (x_i : B_i)_i,\ y : D}{\tilde{x} \to y(G).H \ \vDash\ \Gamma,\ (x_i : A_i \otimes^{y} B_i)_i,\ y : C \parr^{\tilde{x}} D}\ \otimes\parr$$

$$\frac{G_1 \vDash \Gamma, x : A, (y_i : C_i)_i \qquad G_2 \vDash \Gamma, x : B, (y_i : D_i)_i}{x \to \tilde{y}.\mathsf{case}(G_1, G_2) \vDash \Gamma, x : A \oplus^{\tilde{y}} B, (y_i : C_i \&^{x} D_i)_i}\ \oplus\&$$

$$\frac{G \vDash x : A, (y_i : B_i)_i}{!x \to \tilde{y}(G) \vDash x : ?^{\tilde{y}} A, (y_i : !^{x} B_i)_i}\ !?$$

$$\frac{}{\tilde{x} \to y \vDash (x_i : 1^{y})_i, y : \bot^{\tilde{x}}}\ 1\bot \qquad \frac{}{x \to \tilde{y}.\mathsf{case}() \vDash \Gamma, x : 0^{\tilde{y}}, (y_i : \top^{x})_i}\ 0\top$$

MCP: Processes

$$\frac{|A|^\perp = |B|}{x^A \to y^B \vdash x : A, y : B}\ \textsc{Axiom} \qquad \frac{(P_i \vdash \Gamma_i, x_i : A_i)_i \quad G \vDash (\, x_i : A_i \,)_i}{(\boldsymbol{\nu} \tilde{x}^{\tilde{A}} : G)\,(\tilde{P}) \vdash \tilde{\Gamma}}\ \textsc{CCut}$$

$$\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x^z[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes^z B}\ \otimes \qquad \frac{P \vdash \Gamma, y : A, x : B}{x^{\tilde{z}}(y).P \vdash \Gamma, x : A \parr^{\tilde{z}} B}\ \parr$$

$$\frac{P \vdash \Gamma, x : A}{x^{\tilde{z}}[\mathsf{inl}].P \vdash \Gamma, u : A \oplus^{\tilde{z}} B}\ \oplus_1 \qquad \frac{P \vdash \Gamma, x : B}{x^{\tilde{z}}[\mathsf{inr}].P \vdash \Gamma, x : A \oplus^{\tilde{z}} B}\ \oplus_2$$

$$\frac{P \vdash \Gamma, x : A \quad Q \vdash \Gamma, x : B}{x^z.\mathsf{case}(P, Q) \vdash \Gamma, x : A \&^z B}\ \&$$

$$\frac{P \vdash\ ?\Gamma, y : A}{!x^z(y).P \vdash\ ?\Gamma, x : !^z A}\ ! \qquad \frac{P \vdash \Gamma, y : A}{?x^{\tilde{z}}[y].P \vdash \Gamma, x : ?^{\tilde{z}} A}\ ?$$

$$\frac{P \vdash \Gamma}{P \vdash \Gamma, x : ?^{\tilde{z}} A}\ \textsc{Weaken} \qquad \frac{P \vdash \Gamma, y : ?^{\tilde{w}} A, z : ?^{\tilde{w}} A}{P[x/y, x/z] \vdash \Gamma, x : ?^{\tilde{w}} A}\ \textsc{Contract}$$

$$\frac{}{x^z[\,] \vdash x : 1^z}\ 1 \qquad \frac{P \vdash \Gamma}{x^{\tilde{z}}().P \vdash \Gamma, x : \bot^{\tilde{z}}}\ \bot \qquad \text{no rule for } 0 \qquad \frac{}{x^z.\mathsf{case}() \vdash \Gamma, x : \top^z}\ \top$$

Two-buyer protocol

$$B_1 \to S({B_1}^{\mathbf{name}} \to S).\ S \to B_1(S^{\mathbf{cost}} \to B_1).$$
$$S \to B_2(S^{\mathbf{cost}} \to B_2).\ B_1 \to B_2({B_1}^{\mathbf{cost}} \to B_2).$$
$$B_2 \to S.\mathsf{case}(B_2 \to S({B_2}^{\mathbf{addr}} \to S).(B_1, B_2) \to S,\ (B_1, B_2) \to S))$$

$\models$

$$B_1 : \mathbf{name} \otimes^S (\mathbf{cost}^\perp \parr^S (\mathbf{cost} \otimes^{B_2} 1^S)),$$
$$B_2 : \mathbf{cost}^\perp \parr^S (\mathbf{cost}^\perp \parr^{B_1} ((\mathbf{addr} \otimes^S 1^S) \oplus^S 1^S),$$
$$S : \mathbf{name}^\perp \parr^{B_1} (\mathbf{cost} \otimes^{B_1} (\mathbf{cost} \otimes^{B_2} ((\mathbf{addr}^\perp \parr^{B_2} \perp^{B_1, B_2}) \,\&^{B_2} \perp^{B_1, B_2})))$$

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