The RPC Calculus

Symmetrical RPC in an Asymmetrical World

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Dream of unified web programming

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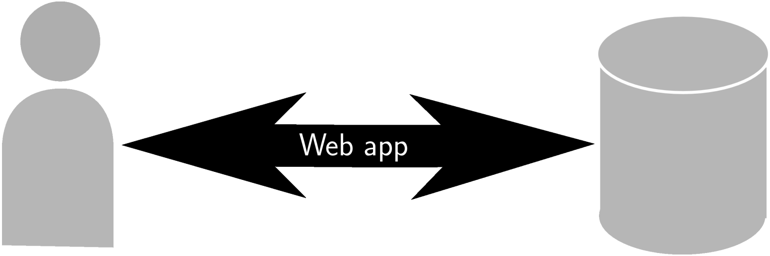

What we want

Reality of web programming

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It's a bit more fiddly.

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Here’s why:

Traditional program

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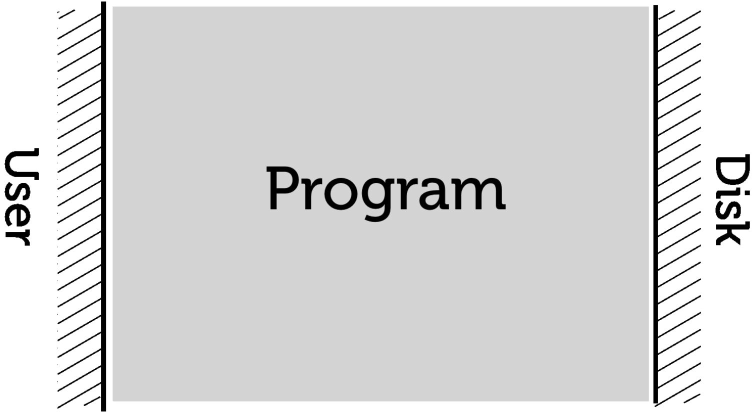
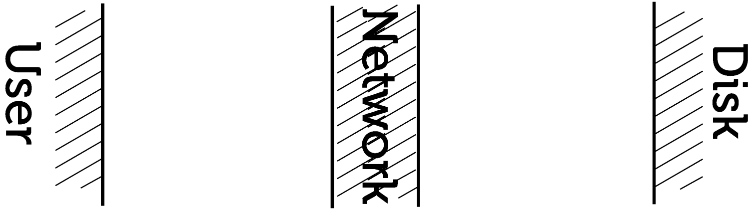


Traditional web program



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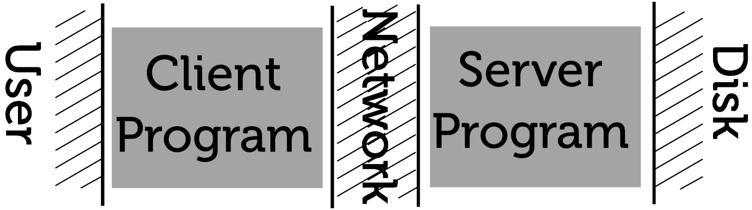


Unified program

Unified program

Simplifies the work of web programming

Unified program

Simplifies the work of web programming

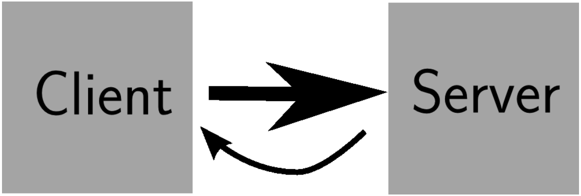
The Links language:
http://groups.inf.ed.ac.uk/links

Dream of a unified language

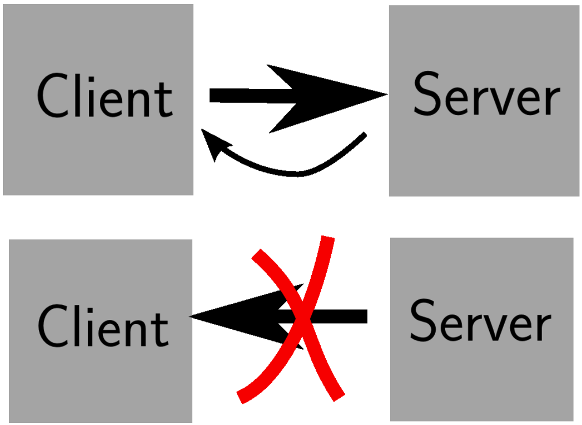
Waking up:

- Desire to control location explicitly, with a light touch;
- Need control for performance and security reasons;
- Tricky because of asymmetrical client/server relationship.

Roadblock: Asymmetrical client/server relationship



Roadblock: Asymmetrical client/server relationship



Stateless server

Web applications should not store control state at the server.

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Server should encode all state and give it to client.

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Server should encode all state and give it to client.

For this talk, *state* = call stack.

Example program

```
fun checkPassword(name, password) {
    # load this user's row from database & check password
    var u = lookupUser(name);
    u.password == password
}

fun validate() {
    var auth = checkPassword(fieldValue("name"),
                             fieldValue("password")));
    if (auth)
        displaySecretDocument();
    else
        displayErrorMessage();
}
```

Example located program

```
fun checkPassword(name, password) server {
  # load this user's row from database & check password
  var u = lookupUser(name);
  u.password == password
}

fun validate() client {
  var auth = checkPassword(fieldValue("name"),
                           fieldValue("password"));
  if (auth)
    displaySecretDocument();
  else
    displayErrorMessage();
}
```

Example located program: server push

```
fun findFlights(flightQuery) server {
  # Query each vendor for its own matching flights
  for (vendor ← airlines()) {
    var flights = queryVendor(vendor, flightQuery);
    # Send this vendor's flights to the browser
    displayFlights(flights);
  }
}

fun displayFlights(flights) client {
  # Add each flight to the page
  for (flight ← flights)
    addToPage(flight);
}
```

Example: higher-order functions

How should this code behave?

$$\begin{array}{l}\textbf{fun } \mathit{usernameMap}(f)\ \textbf{server}\ \{\\ \quad \textbf{var } \mathit{users} = \mathit{getUsersFromDatabase}();\\ \quad \textbf{for}\ (u \leftarrow \mathit{users})\,[f(u.\mathrm{name})]\\ \}\end{array}$$

$$\begin{array}{l}\textbf{fun } \mathit{userNameFirstThree}()\ \textbf{client}\ \{\\ \quad \mathit{usersMap}(\mathit{fun}(\mathit{name})\{\mathit{take}(3, \mathit{name})\});\\ \}\end{array}$$

Example: higher-order functions

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\begin{array}{l}
\textbf{fun}\ \mathit{usernameMap}(f)\ \textbf{server}\ \{ \\
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\} \\
\\
\textbf{fun}\ \mathit{userNameFirstThree}()\ \textbf{client}\ \{ \\
\quad \mathit{usersMap}(\mathit{fun}(\mathit{name})\{\mathit{take}(3, \mathit{name})\}); \\
\}
\end{array}
$$

▷ Functions in *lexical* client-context execute on client.

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- How to compile this language for the asymmetrical client-server model,

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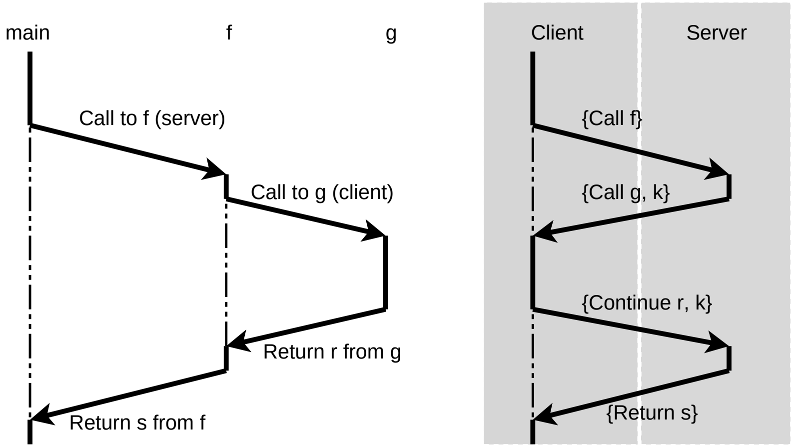
- How to compile this language for the asymmetrical client-server model,
- How the compilation factors into standard techniques,
- How these these techniques can be presented formally and concisely.

How it's done



Source language:
call/return style

Implementation:
request/response style

Getting technical

Source language: the located lambda calculus

Source language: the located lambda calculus

$$\begin{aligned} L, M, N &::= LM \mid \lambda^{a} x.N \mid \lambda x.N \mid x \mid c \\ a, b &::= \mathbf{c} \mid \mathbf{s} \end{aligned}$$

Source language: the located lambda calculus

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We eliminate un-located forms $\lambda x.N$ by explicitly copying the location of their lexical context.

So $\lambda^{\mathbf{c}} x.L(\lambda y.N)$ becomes $\lambda^{\mathbf{c}} x.L(\lambda^{\mathbf{c}} y.N)$

Source language: the located lambda calculus

$$
\begin{array}{rcl}
L, M, N & ::= & L M \mid \lambda^{a} x. N \mid x \mid c \\
a, b & ::= & \mathbf{c} \mid \mathbf{s}
\end{array}
$$

Semantics

Read $M \Downarrow_a V$ as "M evaluates, starting at a, to V."

$$V \Downarrow_a V \qquad \text{(Value)}$$

$$\frac{L \Downarrow_a \lambda^b x.N \qquad M \Downarrow_a W \qquad N\{W/x\} \Downarrow_b V}{LM \Downarrow_a V} \qquad \text{(Beta)}$$

$$\frac{L \Downarrow_a c \qquad M \Downarrow_a W \qquad \delta_a(c, W) \Downarrow_a V}{LM \Downarrow_a V} \qquad \text{(Delta)}$$

Translation to a client-server system

Three techniques:

- CPS translation:
  reifies the control state
- Defunctionalization:
  turns higher-order functions into data (serializable)
- Trampolining:
  inverts control, so state resides at client.

CPS translation (due to Fischer, 1972, via Sabry and Wadler, 1997)

Source:

$$
\begin{array}{rcl}
L, M, N & ::= & LM \mid V \\
V & ::= & \lambda x.N \mid x
\end{array}
$$

CPS translation:

$$
\begin{array}{rcl}
(LM)^\dagger K & = & L^\dagger(\lambda f.M^\dagger(\lambda x.fxK)) \\
V^\dagger K & = & KV^\circ
\end{array}
$$

$$
\begin{array}{rcl}
(\lambda x.N)^\circ & = & \lambda x.\lambda k.N^\dagger k \\
x^\circ & = & x
\end{array}
$$

Defunctionalization

Defunctionalization target

$$
\begin{array}{rcl}
\mathcal{D} & ::= & \textbf{letrec}\ D\ \textbf{and}\ \cdots\ \textbf{and}\ D \\
D & ::= & f(\vec{x}) = \textbf{case}\ x\ \textbf{of}\ \mathcal{A} \\
\mathcal{A} & ::= & \text{a set of } A \text{ items} \\
A & ::= & F(\vec{c}) \Rightarrow M \\
M & ::= & f(\vec{M}) \mid F(\vec{M}) \mid x \mid c
\end{array}
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\end{array}
$$

$$
\begin{array}{rl}
\text{function names} & f, g \\
\text{constructor names} & F, G
\end{array}
$$

Defunctionalization (orig. Reynolds, 1972)

$$
\begin{array}{rcl}
[\![M]\!]^{\text{top}} & = & \textbf{letrec}\ \mathit{apply}(\mathit{fun}, \mathit{arg}) = \textbf{case}\ \mathit{fun}\ \textbf{of}\ [\![M]\!]^{\text{fun}^*} \\
 & & \quad \textbf{in}\ [\![M]\!] \\
[\![\lambda x.N]\!]^{\text{fun}} & = & \ulcorner \lambda x.N \urcorner(\vec{y}) \Rightarrow [\![N]\!]\{\mathit{arg}/x\} \\
 & & \hfill \text{where } \vec{y} = \mathrm{FV}(\lambda x.N) \\
 & & \\
[\![LM]\!] & = & \mathit{apply}([\![L]\!], [\![M]\!]) \\
[\![V]\!] & = & V^{\circ} \\
 & & \\
(\lambda x.N)^{\circ} & = & \ulcorner \lambda x.N \urcorner(\vec{y}) \qquad \text{where } \vec{y} = \mathrm{FV}(\lambda x.N) \\
x^{\circ} & = & x
\end{array}
$$

The operation $\ulcorner M \urcorner$ gives an opaque identifier for the term M.

Trampolining (due to Ganz, Friedman and Wand)

- Continually returns control to a top-level *trampoline*;
- Works on any tail-form program,
  including CPS programs;
- Choice of the trampoline modifies the behavior.

Trampolining

$$
\begin{array}{rcl}
(LM)^{\mathrm{T}} & = & \mathit{Bounce}(\lambda z.L^{\mathrm{t}} M^{\mathrm{t}}) \\
 & & \qquad\qquad\text{(where } z \text{ is a dummy)} \\
V^{\mathrm{T}} & = & \mathit{Return}(V^{\mathrm{t}}) \\
 & & \\
(\lambda x.N)^{\mathrm{t}} & = & \lambda x.N^{\mathrm{T}} \\
x^{\mathrm{t}} & = & x
\end{array}
$$

Behavior depends on our choice of *tramp*.

Example trampolines

Trivial trampoline:

$$\begin{aligned} \mathit{tramp}(x) = \textbf{case } x \textbf{ of} \quad & \mathit{Bounce}(\mathit{thunk}) \Rightarrow \mathit{tramp}(\mathit{thunk}()) \\ \mid\; & \mathit{Return}(x) \Rightarrow x \end{aligned}$$

Example trampolines

Trivial trampoline:

$$
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\mid\ & \mathit{Return}(x) \Rightarrow x
\end{aligned}
$$

Step-counting trampoline:

$$
\begin{aligned}
\mathit{tramp}(n, x) = \textbf{case } x \textbf{ of} \quad & \\
& \mathit{Bounce}(\mathit{thunk}) \Rightarrow \mathit{print}(n); \mathit{tramp}(n + 1, \mathit{thunk}()) \\
\mid\ & \mathit{Return}(x) \Rightarrow x
\end{aligned}
$$

Our trampoline

Since the control state is reified, *tramp* can split the computation into a client- and a server-side piece.

$$
\begin{array}{rcl}
\mathit{tramp}(x) & = & \textbf{case } x \textbf{ of} \\
& & \quad \mid \mathit{Bounce}(f, x, k) \Rightarrow \\
& & \qquad \mathit{tramp}(\textbf{req } \mathit{cont}\ (k, \mathit{apply}(f, x))) \\
& & \quad \mid \mathit{Return}(x) \Rightarrow x
\end{array}
$$

(This shouldn't make sense yet; don't worry.)

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\end{array}
$$

(This shouldn't make sense yet; don't worry.)

The Big Transformation

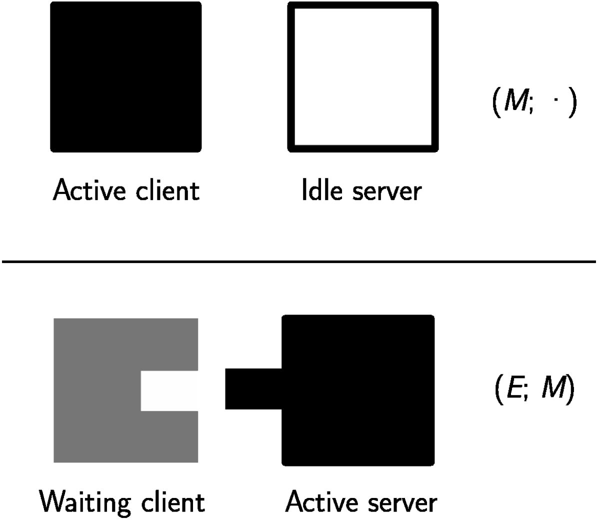
First, the target: first-order client-server calculus

The client-server calculus

Syntax

$$
\begin{array}{rrcl}
\text{configurations} & \mathcal{K} & ::= & (M;\ \cdot) \mid (E;\ M) \\
\text{terms} & L, M, N & ::= & x \mid c \mid F(\vec{M}) \mid f(\vec{M}) \mid \mathbf{req}\ f\ (\vec{M}) \\
\text{definition set} & \mathcal{D}, \mathcal{C}, \mathcal{S} & ::= & \mathbf{letrec}\ D\ \mathbf{and} \cdots \mathbf{and}\ D \\
\text{function definitions} & D & ::= & f(\vec{x}) = \mathbf{case}\ M\ \mathbf{of}\ \mathcal{A} \\
\text{alternative sets} & \mathcal{A} & & \text{a set of } A \text{ items} \\
\text{case alternatives} & A & ::= & F(\vec{x}) \Rightarrow M \\
\text{function names} & f, g & & \\
\text{constructor names} & F, G & &
\end{array}
$$

Configurations of the machine



Semantics

Client:

$$
\begin{array}{rcl}
(E[f(\vec{V})];\cdot) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E[M\{\vec{V}/\vec{x}\}];\cdot) \\
& & \quad \text{if } (f(\vec{x}) = M) \in \mathcal{C} \\
(E[\textbf{case } (F(\vec{V})) \textbf{ of } \mathcal{A}];\cdot) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E[M\{\vec{V}/\vec{x}\}];\cdot) \\
& & \quad \text{if } (F(\vec{x}) \Rightarrow M) \in \mathcal{A}
\end{array}
$$

Server:

$$
\begin{array}{rcl}
(E;\ E'[f(\vec{V})]) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E;\ E'[M\{\vec{V}/\vec{x}\}]) \\
& & \quad \text{if } (f(\vec{x}) = M) \in \mathcal{S} \\
(E;\ E'[\textbf{case } (F(\vec{V})) \textbf{ of } \mathcal{A}]) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E;\ E'[M\{\vec{V}/\vec{x}\}]) \\
& & \quad \text{if } (F(\vec{x}) \Rightarrow M) \in \mathcal{A}
\end{array}
$$

Communication:

$$
\begin{array}{rcl}
(E[\textbf{req } f\ (\vec{V})];\ \cdot) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E;\ f(\vec{V})) \\
(E;\ V) & \longrightarrow_{\mathcal{C},\mathcal{S}} & (E[V];\ \cdot)
\end{array}
$$

Now, the translation

Transformation on terms

$$
\begin{aligned}
(\lambda^a x.N)^\circ &= \ulcorner \lambda^a x.N \urcorner(\vec{y}) && \vec{y} = \mathrm{FV}(\lambda^a x.N) \\
x^\circ &= x \\
c^\circ &= c
\end{aligned}
$$

$$
\begin{aligned}
V^* &= V^\circ \\
(LM)^* &= \mathit{apply}(L^*,\ M^*)
\end{aligned}
$$

$$
\begin{aligned}
V^\dagger[\ ] &= \mathit{cont}([\ ],\ V^\circ) \\
(LM)^\dagger[\ ] &= L^\dagger(\ulcorner M \urcorner(\vec{y}, [\ ])) && \text{where } \vec{y} = \mathrm{FV}(M)
\end{aligned}
$$

Transformation to definitions (client-side)

$$
\begin{aligned}
[\![M]\!]^{\mathsf{c},\text{top}} \quad = \quad & \textbf{letrec}\, \mathit{apply}(\mathit{fun}, \mathit{arg}) = \textbf{case}\ \mathit{fun}\ \textbf{of}\ [\![M]\!]^{\mathsf{c},\text{fun}} \\
& \textbf{and}\ \mathit{tramp}(x) = \textbf{case}\ x\ \textbf{of} \\
& \qquad \mid \mathit{Call}(f, x, k) \Rightarrow \\
& \qquad\qquad \mathit{tramp}(\textbf{req}\ \mathit{cont}\ (k, \mathit{apply}(f, x))) \\
& \qquad \mid \mathit{Return}(x) \Rightarrow x
\end{aligned}
$$

$$
\begin{aligned}
[\![\lambda^{\mathsf{c}} x.N]\!]^{\mathsf{c},\text{fun}} \quad = \quad & \ulcorner \lambda^{\mathsf{c}} x.N \urcorner(\vec{y}) \Rightarrow N^{*}\{\mathit{arg}/x\} \\
& \qquad\qquad\qquad\qquad \text{where } \vec{y} = \mathrm{FV}(\lambda x.N) \\
[\![\lambda^{\mathsf{s}} x.N]\!]^{\mathsf{c},\text{fun}} \quad = \quad & \\
\ulcorner \lambda^{\mathsf{s}} x.N \urcorner(\vec{y}) \Rightarrow \ & \mathit{tramp}(\textbf{req}\ \mathit{apply}\ (\ulcorner \lambda^{\mathsf{s}} x.N \urcorner(\vec{y}), \mathit{arg}, \mathit{Fin}())) \\
& \qquad\qquad\qquad\qquad \text{where } \vec{y} = \mathrm{FV}(\lambda x.N)
\end{aligned}
$$

Transformation to definitions (server-side)

$$
\begin{aligned}
[\![M]\!]^{\mathbf{s},\mathsf{top}} \;=\; & \mathbf{letrec}\; apply(fun,\, arg,\, k) = \mathbf{case}\; fun\; \mathbf{of}\; [\![M]\!]^{\mathbf{s},\mathsf{fun}} \\
& \mathbf{and}\; cont(k,\, arg) = \mathbf{case}\; k\; \mathbf{of} \\
& \qquad\qquad [\![M]\!]^{\mathbf{s},\mathsf{cont}} \\
& \qquad\quad \mid App(fun,\, k) \Rightarrow apply(fun,\, arg,\, k) \\
& \qquad\quad \mid Fin() \Rightarrow Return(arg)
\end{aligned}
$$

$$
\begin{aligned}
[\![\lambda^{\mathbf{s}} x.N]\!]^{\mathbf{s},\mathsf{fun}} &= \ulcorner \lambda^{\mathbf{s}} x.N \urcorner(\vec{y}) \Rightarrow (N^{\dagger} k)\{arg/x\} \\
&\qquad\qquad\qquad \text{where } \vec{y} = \mathrm{FV}(\lambda x.N) \\
[\![\lambda^{\mathbf{c}} x.N]\!]^{\mathbf{s},\mathsf{fun}} &= \ulcorner \lambda^{\mathbf{c}} x.N \urcorner(\vec{y}) \Rightarrow Call(\ulcorner \lambda^{\mathbf{c}} x.N \urcorner(\vec{y}),\, arg,\, k) \\
&\qquad\qquad\qquad \text{where } \vec{y} = \mathrm{FV}(\lambda x.N) \\
[\![LM]\!]^{\mathbf{s},\mathsf{cont}} &= \ulcorner M \urcorner(\vec{y}, k) \Rightarrow M^{\dagger}(App(arg, k)) \\
&\qquad\qquad\qquad \text{where } \vec{y} = \mathrm{FV}(M)
\end{aligned}
$$

Correctness: Bisimulation

$$
\begin{tikzcd}
M \arrow[r, "\Downarrow"] \arrow[d, dash] & V \arrow[d, dash, dashed] \\
M' \arrow[r, dashed, "\longrightarrow\!\!\!\!\rightarrow"] & V'
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
M \arrow[r, dashed, "\Downarrow"] \arrow[d, dash] & V \arrow[d, dash, dashed] \\
M' \arrow[r, "\longrightarrow\!\!\!\!\rightarrow"] & V'
\end{tikzcd}
$$

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Summary

We can

- Enrich a functional programming language with *location annotations*,
- which designate execution location of their contents lexically,
- and whose semantics are straightforward,
- and we can execute these programs on an asymmetrical client-server system with a "stateless server"
- using a combination of classic transformations.

Summary

We can

- Enrich a functional programming language with *location annotations*,
- which designate execution location of their contents lexically,
- and whose semantics are straightforward,
- and we can execute these programs on an asymmetrical client-server system with a "stateless server"
- using a combination of classic transformations.

Thank you