Well-typed programs can’t be blamed

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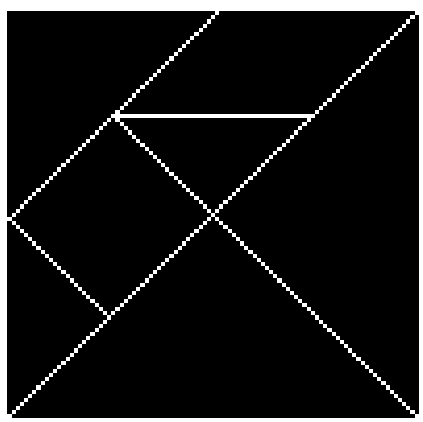
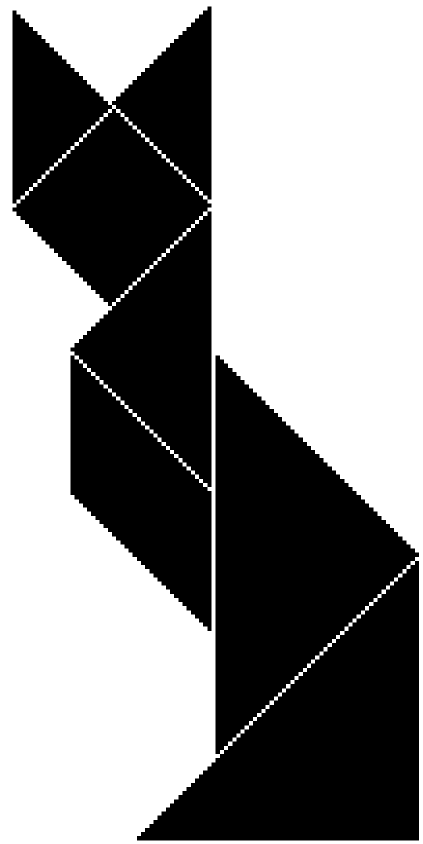



A repeated theme

Thatte (1988):
Partial types

Henglein (1994):
Dynamic typing

Findler and Felleisen (2002):
Contracts

Flanagan (2006):
Hybrid types

Siek and Taha (2006):
Gradual types

A repeated theme

Dynamics in .Net (C#, Visual Basic)

Perl 6.0

Javascript

Dart

Part I

Evolving a program

An untyped program

$$
\begin{array}{l}
\ulcorner\texttt{let} \\
\quad x = \texttt{2} \\
\quad f = \lambda y.\, y + \texttt{1} \\
\quad h = \lambda g.\, g\,(g\,x) \\
\texttt{in} \\
\quad h\,f\urcorner
\end{array}
$$

$$\longrightarrow$$

$$\ulcorner\texttt{4}\urcorner$$

A typed program

$$
\begin{aligned}
&\texttt{let} \\
&\quad x = \texttt{2} \\
&\quad f = \lambda y : \texttt{Int}.\, y + \texttt{1} \\
&\quad h = \lambda g : \texttt{Int} \to \texttt{Int}.\, g\,(g\,x) \\
&\texttt{in} \\
&\quad h\,f
\end{aligned}
$$

$$\longrightarrow$$

$$\texttt{4} : \texttt{Int}$$

A partly typed program—narrowing

$$\begin{array}{l}
\texttt{let} \\
\quad x = \texttt{2} \\
\quad f = \lceil \lambda y.\, y + \texttt{1} \rceil : \star \stackrel{p}{\Rightarrow} \texttt{Int} \to \texttt{Int} \\
\quad h = \lambda g : \texttt{Int} \to \texttt{Int}.\, g\,(g\,x) \\
\texttt{in} \\
\quad h\,f
\end{array}$$

$$\longrightarrow$$

$$\texttt{4} : \texttt{Int}$$

A partly typed program—narrowing

$$
\begin{array}{l}
\texttt{let} \\
\quad x = \texttt{2} \\
\quad f = \lceil \lambda y.\, \texttt{false} \rceil : \star \stackrel{p}{\Longrightarrow} \texttt{Int} \to \texttt{Int} \\
\quad h = \lambda g : \texttt{Int} \to \texttt{Int}.\, g\,(g\,x) \\
\texttt{in} \\
\quad h\ f
\end{array}
$$

$$\longrightarrow$$

$$\texttt{blame}\ p$$

Positive (covariant): blame the term contained in the cast

Another partly typed program—widening

$$
\begin{array}{l}
\texttt{let} \\
\quad x = \lceil \texttt{2} \rceil \\
\quad f = (\lambda y : \texttt{Int}.\, y + \texttt{1}) : \texttt{Int} \to \texttt{Int} \stackrel{p}{\Longrightarrow} \star \\
\quad h = \lceil \lambda g.\, g\,(g\,x) \rceil \\
\texttt{in} \\
\quad \lceil h\,f \rceil
\end{array}
$$

$$\longrightarrow$$

$$\lceil 4 \rceil$$

Another partly typed program—widening

$$
\begin{array}{l}
\texttt{let} \\
\quad x = \lceil \texttt{true} \rceil \\
\quad f = (\lambda y : \texttt{Int}.\, y + 1) : \texttt{Int} \to \texttt{Int} \stackrel{p}{\Rightarrow} \star \\
\quad h = \lceil \lambda g.\, g\,(g\,x) \rceil \\
\texttt{in} \\
\quad \lceil h\ f \rceil
\end{array}
$$

$$\longrightarrow$$

$$\texttt{blame}\ \bar{p}$$

Negative (contravariant): blame the context containg the cast

Part II

Untyped and supertyped

Untyped = Uni-typed

$$
\begin{array}{lcll}
\lceil x \rceil & = & x & \\
\lceil c \rceil & = & c : A \overset{p}{\Rightarrow} \star & \text{if } \mathbf{ty}(c) = A \\
\lceil op(\vec{M}) \rceil & = & op(\lceil \vec{M} \rceil : \vec{\star} \overset{\vec{p}}{\Rightarrow} \vec{A}) : B \overset{p}{\Rightarrow} \star & \text{if } \mathbf{ty}(op) = \vec{A} \to B \\
\lceil \lambda x.\, N \rceil & = & (\lambda x : \star.\, \lceil N \rceil) : \star \to \star \Rightarrow \star & \\
\lceil L\ M \rceil & = & (\lceil L \rceil : \star \overset{p}{\Rightarrow} \star \to \star)\ \lceil M \rceil &
\end{array}
$$

(slogan due to Bob Harper)

Contracts

$$\texttt{Nat} = \{x : \texttt{Int} \mid x \geq \texttt{0}\}$$

$$\texttt{let}$$

$$x = \texttt{2} : \texttt{Int} \overset{p}{\Longrightarrow} \texttt{Nat}$$

$$f = (\lambda y : \texttt{Int}.\, y + \texttt{1}) : \texttt{Int} \rightarrow \texttt{Int} \overset{q}{\Longrightarrow} \texttt{Nat} \rightarrow \texttt{Nat}$$

$$h = \lambda g : \texttt{Nat} \rightarrow \texttt{Nat}.\, g\,(g\,x)$$

$$\texttt{in}$$

$$h\ f$$

$$\longrightarrow$$

$$\texttt{4} : \texttt{Nat}$$

Part III

The Blame Game

Reductions (simplified)

$$
\begin{array}{rcl}
v : \iota \Rightarrow \star \overset{p}{\Rightarrow} \iota & \longrightarrow & v \\
v : \iota \Rightarrow \star \overset{p}{\Rightarrow} \iota' & \longrightarrow & \texttt{blame}\ p \quad \textbf{if}\ \iota \neq \iota' \\
(v : A \to B \overset{p}{\Rightarrow} A' \to B')\ w & \longrightarrow & (v\ (w : A' \overset{\bar{p}}{\Rightarrow} A) : B \overset{p}{\Rightarrow} B')
\end{array}
$$

Blame

$$\lceil \mathtt{2} \rceil : \star \overset{p}{\Longrightarrow} \mathtt{Int}$$

$$=$$

$$\mathtt{2} : \mathtt{Int} \Rightarrow \star \overset{p}{\Longrightarrow} \mathtt{Int}$$

$$\longrightarrow$$

$$\mathtt{2}$$

$$\lceil \mathtt{true} \rceil : \star \overset{p}{\Longrightarrow} \mathtt{Int}$$

$$=$$

$$\mathtt{true} : \mathtt{Bool} \Rightarrow \star \overset{p}{\Longrightarrow} \mathtt{Int}$$

$$\longrightarrow$$

$$\mathtt{blame}\ p$$

The Blame Game—widening

$$((\lambda y : \texttt{Int}.\, y + 1) : \texttt{Int} \to \texttt{Int} \overset{p}{\Longrightarrow} \star \to \star)\; \lceil \texttt{2} \rceil$$

$$\longrightarrow$$

$$(\lambda y : \texttt{Int}.\, y + 1)\; (\lceil \texttt{2} \rceil : \star \overset{\bar{p}}{\Longrightarrow} \texttt{Int}) : \texttt{Int} \overset{p}{\Longrightarrow} \star$$

$$\longrightarrow$$

$$\lceil \texttt{3} \rceil$$

The Blame Game—widening

$$((\lambda y : \texttt{Int}.\, y + 1) : \texttt{Int} \to \texttt{Int} \stackrel{p}{\Rightarrow} \star \to \star)\ \lceil \texttt{true} \rceil$$

$$\longrightarrow$$

$$(\lambda y : \texttt{Int}.\, y + 1)\ (\lceil \texttt{true} \rceil : \star \stackrel{\overline{p}}{\Rightarrow} \texttt{Int}) : \texttt{Int} \stackrel{p}{\Rightarrow} \star$$

$$\longrightarrow$$

$$\texttt{blame}\ \overline{p}$$

Widening can give rise to negative blame, but never positive blame

The Blame Game—narrowing

$$((\lambda y : \star.\, \lceil y + \mathtt{1} \rceil) : \star \to \star \stackrel{p}{\Longrightarrow} \mathtt{Int} \to \mathtt{Int})\ \mathtt{2}$$

$$\longrightarrow$$

$$(\lambda y : \star.\, \lceil y + \mathtt{1} \rceil)\ (\mathtt{2} : \mathtt{Int} \stackrel{\bar{p}}{\Longrightarrow} \star) : \star \stackrel{p}{\Longrightarrow} \mathtt{Int}$$

$$\longrightarrow$$

$$\mathtt{3}$$

The Blame Game—narrowing

$$((\lambda y : \star.\, \lceil \texttt{false} \rceil) : \star \to \star \stackrel{p}{\Longrightarrow} \texttt{Int} \to \texttt{Int})\; \texttt{2}$$

$$\longrightarrow$$

$$(\lambda y : \star.\, \lceil \texttt{false} \rceil)\; (\texttt{2} : \texttt{Int} \stackrel{\bar{p}}{\Longrightarrow} \star) : \star \stackrel{p}{\Longrightarrow} \texttt{Int}$$

$$\longrightarrow$$

$$\texttt{blame}\; p$$

Narrowing can give rise to positive blame, but never negative blame

Part IV

Blame calculus in detail

Notation

It took us four years to find the right notation!

$$\langle A \Rightarrow B \rangle^p \; s$$

$$\langle B \Leftarrow A \rangle^p \; s$$

$$s : A \stackrel{p}{\Rightarrow} B$$

We want composition to be easy to read:

$$\langle B \Rightarrow C \rangle^q \; \langle A \Rightarrow B \rangle^p \; s$$

$$\langle C \Leftarrow B \rangle^q \; \langle B \Leftarrow A \rangle^p \; s$$

$$s : A \stackrel{p}{\Rightarrow} B : B \stackrel{q}{\Rightarrow} C$$

And there is a convenient abbreviation:

$$s : A \stackrel{p}{\Rightarrow} B \stackrel{q}{\Rightarrow} C$$

Syntax

Blame labels	p, q		
Base types	ι		
Types	A, B, C	$::=$	$\iota \mid A \to B \mid \star$
Ground types	G, H	$::=$	$\iota \mid \star \to \star$
Terms	s, t	$::=$	$c \mid op(\vec{t}) \mid x \mid \lambda x{:}A.\, t \mid t\ s \mid$
			$s : A \stackrel{p}{\Rightarrow} B \mid s : G \Rightarrow \star \mid \texttt{blame}\ p$
Environments	Γ	$::=$	$\cdot \mid \Gamma, x : A$
Values	v, w	$::=$	$c \mid \lambda x{:}A.\, t \mid v : G \Rightarrow \star$
Contexts	E	$::=$	$[\cdot] \mid op(\vec{v}, E, \vec{t}) \mid E\ s \mid v\ E \mid$
			$E : A \stackrel{p}{\Rightarrow} B \mid E : G \Rightarrow \star$

Blame calculus: Compatibility

$$A \prec \star \qquad \star \prec B$$

$$\iota \prec \iota$$

$$\frac{A' \prec A \qquad B \prec B'}{A \to B \prec A' \to B'}$$

Types

$$\frac{\mathrm{ty}(c) = \iota}{\Gamma \vdash c : \iota} \qquad \frac{\Gamma \vdash \vec{t} : \vec{A} \qquad \mathrm{ty}(op) = \vec{A} \to B}{\Gamma \vdash op(\vec{t}) : B} \qquad \frac{x : A \in \Gamma}{\Gamma \vdash x : A}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x{:}A.\, t : A \to B} \qquad \frac{\Gamma \vdash t : A \to B \qquad \Gamma \vdash s : A}{\Gamma \vdash t\, s : B}$$

$$\frac{\Gamma \vdash s : A \qquad A \prec B}{\Gamma \vdash (s : A \stackrel{p}{\Rightarrow} B) : B} \qquad \frac{\Gamma \vdash s : G}{\Gamma \vdash (s : G \Rightarrow \star) : \star}$$

$$\Gamma \vdash \texttt{blame}\, p : A$$

Beta, Delta, contextual closure

$$
\begin{array}{rcl}
(\lambda x{:}A.\, t)\, v & \longrightarrow & t[x{:=}v] \\
\boldsymbol{op}(\vec{v}\,) & \longrightarrow & \delta(\boldsymbol{op}, \vec{v}\,)
\end{array}
$$

$$
\frac{s \longrightarrow t}{E[s] \longrightarrow E[t]} \qquad \frac{E \neq [\cdot]}{E[\texttt{blame}\ p] \longrightarrow \texttt{blame}\ p}
$$

Wrap, Id, Ground, Collapse, Conflict

$$
\begin{array}{rcll}
v : A \to B \stackrel{p}{\Rightarrow} A' \to B' & \longrightarrow & \lambda x' {:} A'.\, (v\ (x' : A' \stackrel{\overline{p}}{\Rightarrow} A) : B \stackrel{p}{\Rightarrow} B') & \\
v : \iota \stackrel{p}{\Rightarrow} \iota & \longrightarrow & v & \\
v : A \stackrel{p}{\Rightarrow} \star & \longrightarrow & v : A \stackrel{p}{\Rightarrow} G \Rightarrow \star & \text{if } A \prec G \text{ and } A \neq \star \\
v : G \Rightarrow \star \stackrel{p}{\Rightarrow} A & \longrightarrow & v : G \stackrel{p}{\Rightarrow} A & \text{if } G \prec A \\
v : G \Rightarrow \star \stackrel{p}{\Rightarrow} A & \longrightarrow & \mathtt{blame}\ p & \text{if } G \not\prec A
\end{array}
$$

Part V

Subtyping

$$<: \qquad <:^{+} \qquad <:^{-} \qquad <:_{n}$$

Subtype

$$\frac{}{\star <: \star}$$

$$\frac{}{\iota <: \iota} \qquad \frac{A <: G}{A <: \star}$$

$$\frac{A' <: A \qquad B <: B'}{A \to B <: A' \to B'}$$

Example:

$$\frac{\dfrac{\dfrac{}{\texttt{Int} <: \texttt{Int}}}{\texttt{Int} <: \star} \qquad \dfrac{\dfrac{}{\texttt{Int} <: \texttt{Int}}}{\texttt{Int} <: \star}}{\star \to \texttt{Int} <: \texttt{Int} \to \star}$$

Positive subtype—widening

$$\frac{}{A <:^{+} \star}$$

$$\frac{}{\iota <: \iota}$$

$$\frac{A' <:^{-} A \qquad B <:^{+} B'}{A \to B <:^{+} A' \to B'}$$

Example:

$$\frac{\dfrac{}{\star <:^{-} \mathtt{Int}} \qquad \dfrac{}{\mathtt{Int} <:^{+} \star}}{\mathtt{Int} \to \mathtt{Int} <: \star \to \star}$$

Negative subtype—narrowing

$$\frac{}{\star <:^{-} A}$$

$$\frac{}{\iota <: \iota} \qquad \frac{A <:^{-} G}{A <:^{-} \star}$$

$$\frac{A' <:^{+} A \qquad B <:^{-} B'}{A \to B <:^{-} A' \to B'}$$

Example:

$$\frac{\dfrac{}{\texttt{Int} <:^{+} \star} \qquad \dfrac{}{\star <:^{-} \texttt{Int}}}{\star \to \star <:^{-} \texttt{Int} \to \texttt{Int}}$$

Naive subtype

$$\frac{}{A <:_n \star}$$

$$\frac{}{\iota <:_n \iota}$$

$$\frac{A <:_n A' \qquad B <:_n B'}{A \to B <:_n A' \to B'}$$

Example:

$$\frac{\dfrac{}{\texttt{Int} <:_n \star} \qquad \dfrac{}{\texttt{Int} <:_n \star}}{\texttt{Int} \to \texttt{Int} <: \star \to \star}$$

Part VI

The Blame Theorem

Safety

$$\frac{}{x \ \mathrm{sf}\ p} \qquad \frac{t \ \mathrm{sf}\ p}{\lambda x.\, t \ \mathrm{sf}\ p} \qquad \frac{s \ \mathrm{sf}\ p \qquad t \ \mathrm{sf}\ p}{s\ t \ \mathrm{sf}\ p}$$

$$\frac{s \ \mathrm{sf}\ p \qquad A <:^{+} B}{s : A \stackrel{p}{\Rightarrow} B \ \mathrm{sf}\ p}$$

$$\frac{s \ \mathrm{sf}\ p \qquad A <:^{-} B}{s : A \stackrel{\bar{p}}{\Rightarrow} B \ \mathrm{sf}\ p}$$

$$\frac{s \ \mathrm{sf}\ p \qquad p \neq q \qquad \bar{p} \neq q}{s : A \stackrel{q}{\Rightarrow} B \ \mathrm{sf}\ p}$$

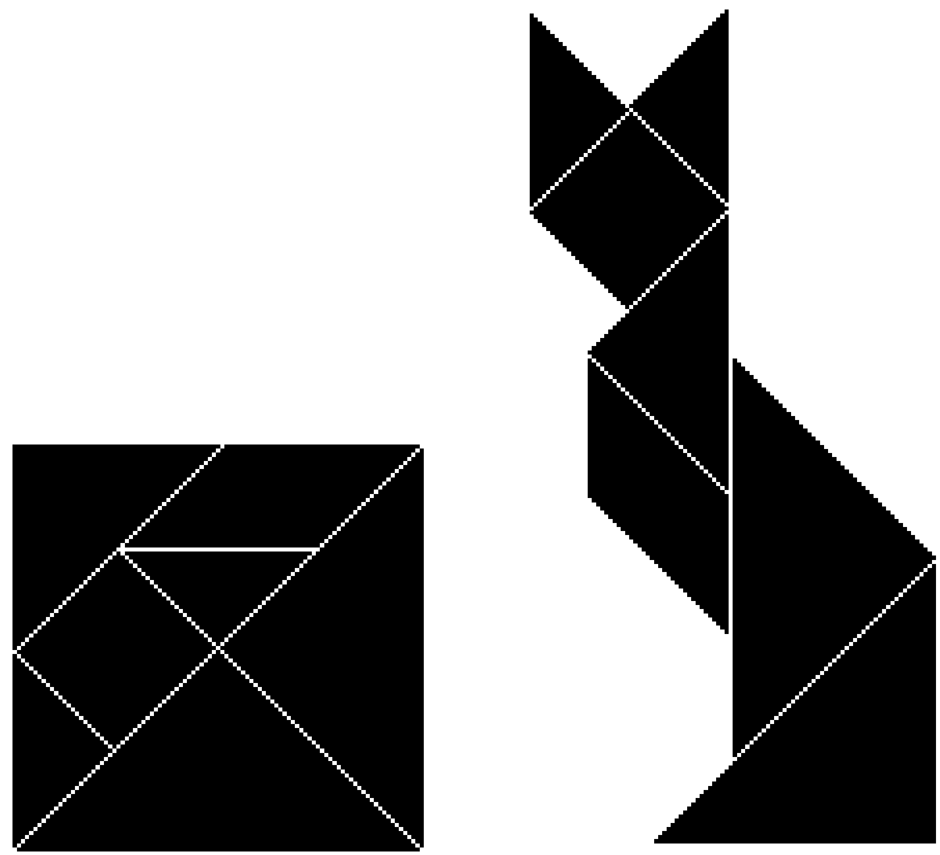
The Blame Theorem

Preservation

If s sf p and $s \longrightarrow t$ then t sf p.

Progress

If s sf p then $s \not\longrightarrow \texttt{blame}\ p$.



The First Tangram Theorem

$$A <: B \text{ if and only if } A <:^{+} B \text{ and } A <:^{-} B$$

The First Blame Corollary

Let t be a term where $s : A \stackrel{p}{\Longrightarrow} B$ is the only subterm with label p. If $A <: B$ then $t \not\longrightarrow \texttt{blame}\ p$ and $t \not\longrightarrow \texttt{blame}\ \bar{p}$.

The Second Tangram Theorem

$$A <:_n B \text{ if and only if } A <:^+ B \text{ and } B <:^- A$$

The Second Blame Corollary

Let t be a term where $s : A \stackrel{p}{\Rightarrow} B$ is the only subterm with label p. If $A <:_n B$ then $t \not\longrightarrow \texttt{blame}\ p$.

Let t be a term where $s : A \stackrel{p}{\Rightarrow} B$ is the only subterm with label p. If $B <:_n A$ then $t \not\longrightarrow \texttt{blame}\ \overline{p}$.

Part VII

Adding polymorphism

Explicit binding

$$\frac{\Gamma, X{:=}A \vdash t : B \qquad X \notin \mathrm{ftv}(B)}{\Gamma \vdash \nu X{:=}A.\, t : B}$$

$$\frac{\Gamma \vdash t : B \qquad (X{:=}A) \in \Gamma}{\Gamma \vdash t : B[X{:=}A]} \qquad\qquad \frac{\Gamma \vdash t : B[X{:=}A] \qquad (X{:=}A) \in \Gamma}{\Gamma \vdash t : B}$$

$$(\Lambda X.\, v)\; A \quad\longrightarrow\quad \nu X{:=}A.\, v$$

Global store vs. Local bindings

George Neis, Derek Dreyer, and Andreas Rossberg. Non-parametric parametricity. ICFP 2009, Edinburgh.

Compatibility and reductions

$$X \prec X \qquad \frac{A \prec B}{A \prec \forall X.\, B}\; X \notin \mathrm{ftv}(A) \qquad \frac{A[X{:=}\star] \prec B}{\forall X.\, A \prec B}$$

$$\begin{array}{rcll}
v : A \stackrel{p}{\Rightarrow} (\forall X.\, B) & \longrightarrow & \Lambda X.\, v : A \stackrel{p}{\Rightarrow} B & \text{if } X \notin \mathrm{ftv} A \\
v : (\forall X.\, A) \stackrel{p}{\Rightarrow} B & \longrightarrow & v\,\star : A[X{:=}\star] \stackrel{p}{\Rightarrow} B & \\
v : X \stackrel{p}{\Rightarrow} \star \stackrel{q}{\Rightarrow} X & \longrightarrow & v & \\
v : X \stackrel{p}{\Rightarrow} \star \stackrel{q}{\Rightarrow} Y & \longrightarrow & \mathtt{blame}\; q & \text{if } X \neq Y
\end{array}$$

Instantiate

$$v : \forall X.\, A \overset{p}{\Rightarrow} B \;\longrightarrow\; v \star : A[X{:=}\star] \overset{p}{\Rightarrow} B$$

$$K = \lambda x : X.\, \lambda y : X.\, x$$

$$
\begin{array}{ll}
 & (((\Lambda X.\, K) : \forall X.\, X{\to}X{\to}X \overset{p}{\Rightarrow} \star{\to}\star{\to}\star)\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow & (((\Lambda X.\, K)\, \star : \star{\to}\star{\to}\star \overset{p}{\Rightarrow} \star{\to}\star{\to}\star)\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow & \nu X{:=}\star.\, (K : \star{\to}\star{\to}\star \overset{p}{\Rightarrow} \star{\to}\star{\to}\star)\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow & \nu X{:=}\star.\, (K\, (\mathtt{42} : \star \overset{\bar{p}}{\Rightarrow} \star)\, (\mathtt{7} : \star \overset{\bar{p}}{\Rightarrow} \star)) : \star \overset{p}{\Rightarrow} \star \\
\longrightarrow & \nu X{:=}\star.\, (\mathtt{42} : \star \overset{\bar{p}}{\Rightarrow} \star) : \star \overset{p}{\Rightarrow} \star \\
\longrightarrow & \nu X{:=}\star.\, \mathtt{42} \\
\longrightarrow & \mathtt{42}
\end{array}
$$

$\star$ is a Jack-of-all-Trades

$$v : \forall X.\, A \stackrel{p}{\Rightarrow} B \;\longrightarrow\; v\,\star : A[X{:=}\star] \stackrel{p}{\Rightarrow} B$$

$$K = \lambda x : X.\, \lambda y : X.\, x$$

$$\begin{aligned}
& ((\Lambda X.\, K) : \forall X.\, X{\to}X{\to}X \stackrel{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Int}{\to}\mathtt{Int})\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & ((\Lambda X.\, K)\,\star : \star{\to}\star{\to}\star \stackrel{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Int}{\to}\mathtt{Int})\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & \nu X{:=}\star.\, (K : \star{\to}\star{\to}\star \stackrel{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Int}{\to}\mathtt{Int})\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & \nu X{:=}\star.\, (K\ (\mathtt{42} : \mathtt{Int} \stackrel{\bar{p}}{\Rightarrow} \star)\ (\mathtt{7} : \mathtt{Int} \stackrel{\bar{p}}{\Rightarrow} \star)) : \star \stackrel{p}{\Rightarrow} \mathtt{Int} \\
\longrightarrow\ & \nu X{:=}\star.\, (\mathtt{42} : \mathtt{Int} \stackrel{\bar{p}}{\Rightarrow} \star) : \star \stackrel{p}{\Rightarrow} \mathtt{Int} \\
\longrightarrow\ & \nu X{:=}\star.\, \mathtt{42} \\
\longrightarrow\ & \mathtt{42}
\end{aligned}$$

… but master of none

$$v : \forall X.\, A \overset{p}{\Rightarrow} B \;\longrightarrow\; v\,\star : A[X{:=}\star] \overset{p}{\Rightarrow} B$$

$$K = \lambda x : X.\, \lambda y : X.\, x$$

$$
\begin{aligned}
& ((\Lambda X.\, K) : \forall X.\, X{\to}X{\to}X \overset{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Bool}{\to}\mathtt{Int})\; \mathtt{42}\; \mathtt{true} \\
\longrightarrow\; & ((\Lambda X.\, K)\, \star : \star{\to}\star{\to}\star \overset{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Bool}{\to}\mathtt{Int})\; \mathtt{42}\; \mathtt{true} \\
\longrightarrow\; & \nu X{:=}\star.\, (K : \star{\to}\star{\to}\star \overset{p}{\Rightarrow} \mathtt{Int}{\to}\mathtt{Bool}{\to}\mathtt{Int})\; \mathtt{42}\; \mathtt{true} \\
\longrightarrow\; & \nu X{:=}\star.\, (K\, (\mathtt{42} : \mathtt{Int} \overset{\bar{p}}{\Rightarrow} \star)\, (\mathtt{true} : \mathtt{Bool} \overset{\bar{p}}{\Rightarrow} \star)) : \star \overset{p}{\Rightarrow} \mathtt{Int} \\
\longrightarrow\; & \nu X{:=}\star.\, (\mathtt{42} : \mathtt{Int} \overset{\bar{p}}{\Rightarrow} \star) : \star \overset{p}{\Rightarrow} \mathtt{Int} \\
\longrightarrow\; & \nu X{:=}\star.\, \mathtt{42} \\
\longrightarrow\; & \mathtt{42}
\end{aligned}
$$

Generalise

$$v : A \stackrel{p}{\Rightarrow} \forall X.\, B \;\longrightarrow\; \Lambda X.\, (v : A \stackrel{p}{\Rightarrow} B) \qquad X \text{ not free in } A$$

$$K = \lambda x : \star.\, \lambda y : \star.\, x$$

$$\begin{array}{l}
\quad (K : \star\!\to\star\to\!\star \stackrel{p}{\Rightarrow} \forall X.\forall Y.\, X\!\to\! Y\!\to\! X)\ \texttt{Int}\ \texttt{Int}\ \texttt{42}\ \texttt{7} \\
\longrightarrow (\Lambda X.\, \Lambda Y.\, (K : \star\!\to\star\to\!\star \stackrel{p}{\Rightarrow} X\!\to\! Y\!\to\! X))\ \texttt{Int}\ \texttt{Int}\ \texttt{42}\ \texttt{7} \\
\longrightarrow \nu X{:=}\texttt{Int}.\, \nu Y{:=}\texttt{Int}.\, (K : \star\!\to\star\to\!\star \stackrel{p}{\Rightarrow} X\!\to\! Y\!\to\! X)\ \texttt{42}\ \texttt{7} \\
\longrightarrow \nu X{:=}\texttt{Int}.\, \nu Y{:=}\texttt{Int}.\, (K\ (\texttt{42} : X \stackrel{\bar{p}}{\Rightarrow} \star)\ (\texttt{7} : Y \stackrel{\bar{p}}{\Rightarrow} \star)) : \star \stackrel{p}{\Rightarrow} X \\
\longrightarrow \nu X{:=}\texttt{Int}.\, \nu Y{:=}\texttt{Int}.\, (\texttt{42} : X \stackrel{\bar{p}}{\Rightarrow} \star) : \star \stackrel{p}{\Rightarrow} X \\
\longrightarrow \nu X{:=}\texttt{Int}.\, \nu Y{:=}\texttt{Int}.\, \texttt{42} \\
\longrightarrow \texttt{42}
\end{array}$$

Enforcing semantic parametricity

$$v : A \overset{p}{\Rightarrow} \forall X.\, B \longrightarrow \Lambda X.\, (v : A \overset{p}{\Rightarrow} B) \qquad X \text{ not free in } A$$

$$K' = \lambda x : \star.\, \lambda y : \star.\, y$$

$$\begin{aligned}
& (K' : \star \to \star \to \star \overset{p}{\Rightarrow} \forall X.\, \forall Y.\, X \to Y \to X)\ \mathtt{Int}\ \mathtt{Int}\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & (\Lambda X.\, \Lambda Y.\, (K' : \star \to \star \to \star \overset{p}{\Rightarrow} X \to Y \to X))\ \mathtt{Int}\ \mathtt{Int}\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & \nu X{:=}\mathtt{Int}.\, \nu Y{:=}\mathtt{Int}.\, (K' : \star \to \star \to \star \overset{p}{\Rightarrow} X \to Y \to X)\ \mathtt{42}\ \mathtt{7} \\
\longrightarrow\ & \nu X{:=}\mathtt{Int}.\, \nu Y{:=}\mathtt{Int}.\, (K'\ (\mathtt{42} : X \overset{\bar{p}}{\Rightarrow} \star)\ (\mathtt{7} : Y \overset{\bar{p}}{\Rightarrow} \star)) : \star \overset{p}{\Rightarrow} X \\
\longrightarrow\ & \nu X{:=}\mathtt{Int}.\, \nu Y{:=}\mathtt{Int}.\, (\mathtt{7} : Y \overset{\bar{p}}{\Rightarrow} \star) : \star \overset{p}{\Rightarrow} X \\
\longrightarrow\ & \mathtt{blame}\ p
\end{aligned}$$

Part VIII

Conclusion

A new slogan for type safety

Milner (1978):

Well-typed programs can't go wrong.

Felleisen and Wright (1994); Harper (2002):

Well-typed programs don't get stuck.

Wadler and Findler (2008):

Well-typed programs can't be blamed.