Abstract

A fundamental concern of information integration in an XML context is the ability to embed one or more source documents in a target document so that (a) the target document conforms to a target schema and (b) the information in the source document(s) is preserved. In this paper, information preservation for XML is formally studied, and the results of this study guide the definition of a novel notion of schema embedding between two XML DTD schemas as graphs. Schema embedding generalizes the conventional notion of graph similarity by allowing an edge in a source DTD schema to be mapped to a path in the target DTD. Instance-level embeddings can be defined from the schema embedding in a straightforward manner, such that conformance to a target schema and information preservation are guaranteed. We show that it is NP-complete to find an embedding between two DTD schemas. We also provide efficient heuristic algorithms to find candidate embeddings, along with experimental results to evaluate and compare the algorithms. These yield the first systematic and effective approach to finding information preserving XML mappings.

Example 1.1: Consider two source DTDs $S_0$, $S_1$ and a target DTD $S$ represented as graphs in Fig. 1 (we omit the $\text{str}$ child under $\text{cno}$, $\text{credit}$, $\text{title}$, $\text{year}$, $\text{term}$, $\text{instructor}$, $\text{gpa}$ in Fig. 1(c)). A document of $S_0$ contains information of classes currently being taught at a school, and a document of $S_1$ consists of student data of the school. The user wants to map the document of $S_0$ and the document of $S_1$ to a single instance of $S$, which is to collect data about courses and students of the school in the last five years. Here we use edges of different types to represent different constructs of a DTD, namely, solid edges for a concatenation type (a unique occurrence of each child), dashed edges for disjunction (one or only one child), and star edges (edge labeled ‘*’) for Kleene star (zero or more children).

In this example, invertibility asks for the ability to reconstruct the original class and student documents from an integrated school document, while query preservation requires the ability to answer XML queries posed on class and student documents using the school document. Two natural questions are: (a) can one determine whether an XML mapping is information preserving? (b) is there an efficient method to find information-preserving XML mappings?

While type safety and information preservation are clearly desirable, an additional feature is the ability to map documents of DTDs that have different structures. A given source DTD may differ in structure from a desired target DTD. This is typical in data integration, where the target DTD needs to accommodate data from multiple sources and thus cannot be similar to any of the sources; see, e.g., the class, student DTDs and the school DTD in Fig. 1.

Background. While information preservation has been studied for traditional database transformations [3, 16, 28, 29], to our knowledge, no previous work has considered it for XML mappings. In fact, a variety of tools and models have been proposed for finding XML mappings at schema- or instance-level [13, 23, 25, 26, 27, 30]; however, none has addressed invertibility and query preservation for XML. Most tools either focus on highly similar structures, or adopt a strict graph similarity model like bisimulation (see, e.g., [1]) to match structures, which is incapable of mapping DTDs with different structures such as those shown in Fig. 1, and can ensure neither invertibility nor query preservation w.r.t. XML query languages. Another issue is that it is unclear that mappings found by some of these tools guarantee type safety when it comes to complex XML DTDs.

Contribution. To this end we study information preserving XML mappings, and make the following contributions.
no sensible mapping from them to
beddings. We show that it is NP-complete to nd an em-
from writing and type-checking a complex mapping query.
functions
cursive. Thus algorithms for nding embeddings are nec-
the notions of invertibility and query preservation [3, 16,
Moreover, we show that the
bedding between two
propagation introduced in [24]), and ensure type safety.
XSLT
mappings (Section 2). While the two no-
tions coincide for relational mappings w.r.t. relational cal-
is general different for
XML
of schema embeddings. A
mapping dened in a simple fragment of
XML
an
XPath queries proposed and
studied in [24], denoted by \( \mathcal{X_R} \) and de ned as follows:
\[
p := \epsilon | A | p/\text{text}() | p/p | p/p \cup p | p^* | p[q],
q := p | p/\text{text}() = \text{c'} | \text{position()} = k
\]
where \( \epsilon \) is the empty path (self), \( A \) is a label (element type),
\( \cup \) is the union operator, \( \text{/} \) is the child-axis, and \( * \) is the
Kleene star; \( p \) is an \( \mathcal{X_R} \) expressions, \( k \) is a natural number,
\( c \) is a string constant, and \( \neg, \land, \lor \) are the Boolean negation,
conjunction and disjunction operators, respectively.

An XPath fragment of \( \mathcal{X_R} \), denoted by \( \chi \), is de ned by \( \chi \), is de ned by replacing \( p^* \) with \( p/p \) in the definition above, where \( / \) is the
descendant-or-self axis.

A (regular) XPath query \( p \) is evaluated at a context node \( v \) in an XML tree \( T \), and its result is the set of nodes (ids) of \( T \) reachable via \( p \) from \( v \), denoted by \( v[p] \).

2 DTDs, XPath, Information Preservation

In this section we review DTDs and (regular) XPath, and revisit information preservation [16, 29] for XML.

2.1 XPath and Regular XPath

We consider a class of regular XPath queries proposed and
studied in [24], denoted by \( \mathcal{X_R} \) and de ned as follows:
\[
p := \epsilon | A | p/\text{text}() | p/p | p/p \cup p | p^* | p[q],
q := p | p/\text{text}() = \text{c'} | \text{position()} = k
\]
where \( \epsilon \) is the empty path (self), \( A \) is a label (element type),
\( \cup \) is the union operator, \( \text{/} \) is the child-axis, and \( * \) is the
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2.2 DTDs

We consider DTDs of the form \((\text{Ele}, P, r)\), where \( \text{Ele} \) is a
finite set of element types; \( r \) is a distinguished type in \( \text{Ele} \),
called the root type; \( P \) defines the element types: for each
\( A \) in \( \text{Ele} \), \( P(A) \) is a regular expression of the form:
\[ \alpha ::= \text{str} \mid \varepsilon \mid B_1, \ldots, B_n \mid B_1 + \ldots + B_n \mid B^* \]

where \text{str} denotes PCDATA, \varepsilon is the empty word, \(B\) is a type in \textit{Ele} (referred to as a \textit{child} of \(A\)), and ‘+’, ‘\ldots\’, and ‘\*’ denote \textit{disjunction} (with \(n > 1\)), \textit{concatenation} and the \textit{Kleene star}, respectively. We refer to \(A \rightarrow P(A)\) as the \textit{production} of \(A\). Note that this form of a tree does not lose generality since any DTDs \(S\) can be converted to \(S'\) of this form (in linear time) by introducing new element types, and (regular) XPath queries on \(S\) can be rewritten into equivalent (regular) XPath queries on \(S'\) in PTIME [6].

\textbf{Schema Graphs.} We represent a DTD \(S\) as a labeled graph \(G_S\), referred to as the \textit{graph} of \(S\). For each element type \(A\) in \(S\), there is a unique node labeled \(A\) in \(G_S\), referred to as the \textit{A node}. From the \textit{A node} there are edges to nodes representing child types in \(P(A)\), determined by the production \(A \rightarrow P(A)\) of \(A\). There are three different types of edges indicating different constructs. Specifically, if \(P(A) = B_1, \ldots, B_n\) then there is a \textit{solid edge} from the \textit{A node} to each \(B_i\) node; it is labeled with a position \(k\) if \(B_i\) is the \(k\)-th occurrence of a type \(B_i\) in \(P(A)\) (the label can be omitted if \(B_i\)'s are distinct). If \(P(A) = B_1 + \ldots + B_n\), then there is a \textit{dashed edge} from the \textit{A node} to each \(B_i\) node (w.l.o.g. assume that \(B_i\)'s are distinct in disjunction). If \(P(A) = B^*\), then there is a \textit{solid edge} with a \textit{\*} label from the \textit{A node} to the \textit{B node}. Note that a DTD is recursive if its graph is cyclic. When it is clear from the context, we shall use the DTD and its graph interchangeably, both referred to as \(S\); similarly, for an element type and an 

For example, Fig. 1 shows graphs representing three DTDs, where Figs. 1(a) and 1(c) depict recursive DTDs.

An XML \textit{instance} of a DTD \(S\) is a node-labeled tree that conforms to \(S\). We use \(I(S)\) to denote the set of all instances of \(S\). A DTD \(S\) is \textit{consistent} if it has no useless element types, i.e., each type of \(S\) has an instance. In the sequel we consider consistent DTDs only.

\subsection*{2.3 Invertibility and Query Preservation}

For XML DTDs \(S_1, S_2\), a mapping \(\sigma_d : I(S_1) \rightarrow I(S_2)\) is \textit{invertible} if there exists an inverse \(\sigma_d^{-1}\) of \(\sigma_d\) such that for any XML instance \(T \in I(S_1)\), \(\sigma_d^{-1}(\sigma_d(T)) = T\), where the notation \(f(T)\) denotes the result of applying a function (or mapping, query) \(f\) to \(T\). In other words, the composition \(\sigma_d^{-1} \circ \sigma_d\) is equivalent to the identity mapping \(id\), which maps an XML document to itself.

For an XML query language \(\mathcal{L}\), a mapping \(\sigma_d\) is \textit{query preserving w.r.t.} \(\mathcal{L}\) if there exists a computable function \(F : \mathcal{L} \rightarrow \mathcal{L}\) such that for any XML query \(Q \in \mathcal{L}\) and any \(T \in I(S_1)\), \(Q(T) = F(Q)(\sigma_d(T))\), i.e., \(Q = F(Q) \circ \sigma_d\).

In a nutshell, invertibility is the ability that the original source XML document can be recovered from the target document; query preservation w.r.t. \(\mathcal{L}\) indicates whether or not all queries of \(\mathcal{L}\) on any source \(T\) of \(S_1\) can be effectively answered over \(\sigma_d(T)\), i.e., the mapping \(\sigma_d\) does not lose information of \(T\) when \(\mathcal{L}\) queries are concerned.

The notions of invertibility and query preservation are inspired by (calculus) \textit{dominance} and \textit{query dominance} that were proposed in [16] for relational mappings and later studied in [3, 28, 29]. In contrast to query dominance, query preservation is defined w.r.t. a given XML query language that does not necessarily support query composition.

Invertibility is defined for XML mappings and it only requires \(\sigma_d^{-1}\) to be a partial function defined on \(\sigma_d(I(S_1))\).

We say that a mapping \(\sigma_d : I(S_1) \rightarrow I(S_2)\) is \textit{information preserving w.r.t.} \(\mathcal{L}\) if it is both invertible and query preserving w.r.t. \(\mathcal{L}\).

\section*{3 Information Preservation}

In this section we establish basic results for separation and equivalence of the invertibility and query preservation of XML mappings, as well as complexity of determining whether a given XML mapping is information preserving.

\textbf{Invertibility and Query Preservation.} It was shown [16] that calculus dominance and query dominance are equivalent for relational mappings. In contrast, invertibility and query preservation do not necessarily coincide for XML mappings and query languages. Recall the class \(\mathcal{X}\) of XPath queries defined in the last section.

\textbf{Theorem 3.1:} There exists an invertible XML mapping that is not query preserving w.r.t. \(\mathcal{X}\); and there exists an XML mapping that is not invertible but is query-preserving w.r.t. the class \(\mathcal{X}\) of XML queries without position() qualifier.

We identify sufficient conditions for the two to coincide:

\textbf{Theorem 3.2:} Let \(\mathcal{L}\) be any XML query language and \(\sigma_d\) be a mapping from \(I(S_1) \rightarrow I(S_2)\).

- If the identity mapping \(id\) is definable in \(\mathcal{L}\) and \(\sigma_d\) is query preserving w.r.t. \(\mathcal{L}\), then \(\sigma_d\) is invertible.
- Suppose \(\mathcal{L}\) is composable, i.e., for any \(Q_1, Q_2\) in \(\mathcal{L}\), \(Q_2 \circ Q_1\) is in \(\mathcal{L}\); if \(\sigma_d\) is invertible and \(\sigma_d^{-1}\) is expressible in \(\mathcal{L}\), then \(\sigma_d\) is query preserving w.r.t. \(\mathcal{L}\).

Recall the class \(X_{ft}\) of regular XPath queries defined in Section 2. Although the identity mapping \(id\) is not definable in \(X_{ft}\), we show below that query preservation w.r.t. \(X_{ft}\) is a stronger property than invertibility.

\textbf{Theorem 3.3:} If an XML mapping \(\sigma_d\) is query preserving w.r.t. \(X_{ft}\), then \(\sigma_d\) is invertible. Conversely, there exists a mapping \(\sigma_d\) that is invertible but is not query preserving w.r.t. \(X_{ft}\).

\textbf{Complexity.} It is common to find XML mappings defined in XQuery or XSLT. A natural yet important question is to decide whether or not an XML mapping is invertible or query preserving w.r.t. a query language \(\mathcal{L}\). Unfortunately, this is impossible for XML mappings defined in any language that subsumes first-order logic (FO, or relational algebra), e.g., XQuery, XSLT, even when \(\mathcal{L}\) consists of projection queries only. Thus it is beyond reach to answer these questions for XQuery or XSLT mappings.

\textbf{Theorem 3.4:} It is undecidable to determine, given an XML mapping \(\sigma_d\) defined in any language subsuming \(FO\),

\begin{itemize}
  \item whether or not \(\sigma_d\) is invertible;
\end{itemize}
2. whether or not \(\sigma_A\) is query preserving w.r.t. projection queries.
\(\Box\)

4 Schema Embeddings for XML

The negative results in Section 3 tell us that it is already hard to determine whether or not an XML mapping is information preserving, not to mention finding one. This motivates us to look for a class of XML mappings that are guaranteed to be information preserving.

We approach this problem by specifying XML mappings at the schema level embeddings, and providing an automated derivation of instance-level mappings from these embeddings. Our notion of schema embeddings is novel, and extends the conventional notion of graph similarity by allowing edges in a source DTD schema to be mapped to a path in a target DTD with a “larger information capacity.” For example, a STAR edge can only be mapped to a path with at least one STAR edge.

In this section we define XML schema embeddings, present an algorithm for deriving an instance-level mapping from a schema embedding, and verify that the resulting mappings ensure information preservation.

4.1 Schema Level Embeddings

Consider a source XML DTD schema \(S_1 = (E_1, P_1, r_1)\) and a target DTD \(S_2 = (E_2, P_2, r_2)\). In a nutshell, a schema embedding \(\sigma\) is a pair of functions \((\lambda, \text{path})\) that maps each \(A\) type in \(E_1\) to a \(\lambda(A)\) type in \(E_2\), and each edge \((A, B)\) in \(S_1\) to a unique \(\text{path}(A, B)\) from \(\lambda(A)\) to \(\lambda(B)\) in \(S_2\), such that the \(S_2\) paths mapped from sibling edges in \(S_1\) are sufficiently distinct to allow information to be preserved. To define \(\lambda\) and \(\text{path}\) we first introduce a few notations.

\(\lambda\)\(\text{Path} s.\) An \(\lambda\)\(\text{Path}\) over a DTD \(S = (E, P, r)\) is an \(\lambda\)\(\text{Path}\) query of the form \(\rho = \eta_1 / \ldots / \eta_k\), where \(k \geq 1\), \(\eta_i\) is of the form \(A[g]\), and \(g\) is either true or a position() qualifier, such that \(\rho\) is a path in \(S\) and it carries all the position labels on the path. An \(\lambda\)\(\text{Path}\) query is called an AND path (resp. OR path, and STAR path) if it is nonempty and consists of only solid or star edges (resp. of solid edges and at least one dashed edge, and of solid edges and at least one edge labeled *). Referring to Fig. 1(c), for example, basic/class/semester/title is an AND path as well as a STAR path, and mandatory/regular is an OR path.

\(\text{Name Similarity.}\) A similarity matrix for \(S_1\) and \(S_2\) is an \([|E_1| \times |E_2|]\) matrix \(\text{att}\) of numbers in the range \([0, 1]\). For any \(A \in E_1\) and \(B \in E_2\), \(\text{att}(A, B)\) indicates the suitability of mapping \(A\) to \(B\), as determined by human domain experts or computed by an existing algorithm, e.g., [13, 22].

\(\text{Type Mapping.}\) A type mapping \(\lambda\) from \(S_1\) to \(S_2\) is a (total) function from \(E_1\) to \(E_2\); in particular, it maps the root of \(S_1\) to the root of \(S_2\), i.e., \(\lambda(r_1) = r_2\). A type mapping \(\lambda\) is valid w.r.t. \(\text{similarity matrix att}\) if for any \(A \in E_1\), \(\text{att}(A, \lambda(A)) > 0\).

\(\text{Path Mapping.}\) A path mapping from \(S_1\) to \(S_2\), denoted by \(\sigma : S_1 \rightarrow S_2\), is a pair \((\lambda, \text{path})\), where \(\lambda\) is a type mapping and \(\text{path}\) is a function that maps each edge \((A, B)\) in \(S_1\) to an \(\lambda(A)\) path \(\text{path}(A, B)\) that is from \(\lambda(A)\) to \(\lambda(B)\) in \(S_2\). For a particular element type \(A\) in \(E_1\), we say that \(\sigma\) is valid for \(A\) if the following conditions hold, based on the production \(A \rightarrow P_1(A)\) in \(S_1\):

- if \(P_1(A) = B_1, \ldots, B_k\), then for each \(i\), \(\text{path}(A, B_i)\) is an AND path from \(\lambda(A)\) to \(\lambda(B_i)\) that is not a prefix of \(\text{path}(A, B_j)\) for any \(j \neq i\);
- if \(P_1(A) = B_1 + \ldots + B_k\), then for each \(i\), \(\text{path}(A, B_i)\) is an OR path from \(\lambda(A)\) to \(\lambda(B_i)\) that is not a prefix of \(\text{path}(A, B_j)\) for any \(j \neq i\);
- if \(P_1(A) = B^*\), then \(\text{path}(A, B_i)\) is a STAR path;
- if \(P_1(A) = \text{str}\), then \(\text{path}(A, \text{str})\) is an AND path ending with text().

The validity requires a path type condition and a prefix-free condition, which, as will be seen shortly, are important for deriving the instance-level mapping from \(\sigma\).

Example 4.1: Consider pairs of source (on the left) and target (on the right) DTDs depicted in Fig. 2, for which a type mapping \(\lambda\) is defined as \(\lambda(X) = X'\) for \(X\) in \(\{A, B, C\}\), except in Fig. 2(c) where both \(\lambda(C) = B'\) and \(\lambda(B) = B''\). Observe the following. For Fig. 2(a), there is no valid path mapping from the source DTD to the target; intuitively, \(B\) and \(C\) must coexist in a source document while only one of \(B'\) and \(C'\) exists in the target. Similarly for Fig. 2(b), where the source cannot be mapped to the target since there is no valid \(\lambda\)\(\text{Path}\) mapping from the source DTD to the target; intuitively, \(B\) and \(C\) must coexist in a source document while only one of \(B'\) and \(C'\) exists in the target. Finally, we define XML schema embeddings as follows.

**Schema Embedding.** A schema embedding from \(S_1\) to \(S_2\) valid w.r.t. a similarity matrix \(\text{att}\) is a path mapping \(\sigma = (\lambda, \text{path})\) from \(S_1\) to \(S_2\) such that \(\lambda\) is valid w.r.t. \(\text{att}\), and \(\sigma\) is valid for every element \(A\) in \(E_1\).

Example 4.2: Assume a similarity matrix \(\text{att}\) such that \(\text{att}(A, A') = 1\) for all \(A\) in the DTD \(S_0\) of Fig. 1(a) and \(A'\) in \(S\) of Fig. 1(c). The source DTD \(S_0\) can be embedded in the target \(S\) via \(\sigma = (\lambda_1, \text{path}_1)\) defined as follows:

\[\lambda_1(\text{db}) = \text{school}, \quad \lambda_1(\text{class}) = \text{course}, \quad \lambda_1(\text{type}) = \text{category}, \quad \lambda_1(\text{A}) = A / ^* A: \text{cn}, \text{title}, \text{regular, project, prereq, str} *\]  

\[\text{path}_1(\text{db}, \text{class}) = \text{courses/current/course} \quad \text{path}_1(\text{class, cn}) = \text{basic/cno} \quad \text{path}_1(\text{class, title}) = \text{basic/class/semester/title}\]
Note that \( \text{path}_{1}(A, B) \) is a path in \( S \) denoting how to reach \( \lambda_{1}(B) \) from \( \lambda_{1}(A) \), i.e., the path is relative to \( \lambda_{1}(A) \). For example, \( \text{path}_{1}(\text{type}, \text{project}) \) indicates how to reach \( B_1 \) from a context node in \( S \), where \( B_1 \) is mapped from \( \text{type} \) in \( S_0 \) by \( \lambda_{1} \). Also observe that the similarity matrix \( \text{att} \) imposes no restrictions: any name in the source can be mapped to any name in the target; thus the embedding here is decided solely on the \( \text{DTD} \) structures.

In contrast, one cannot map \( S_0 \) to \( S \) by graph similarity, which requires that node \( A \) in the source is mapped (similar) to \( B \) in the target only if all children of \( A \) are mapped (similar) to children of \( B \). In other words, graph similarity maps an edge in the source to an edge in the target. □

**Embedding Quality.** There are many possible metrics. In this paper we consider only a simple one: the quality of a schema embedding \( \sigma = (\lambda, \text{path}) \) w.r.t. \( \text{att} \), which is the sum of \( \text{att}(A, \lambda(A)) \) for \( A \in E_1 \), and we say that \( \sigma \) is invalid if \( \lambda \) is invalid w.r.t. \( \text{att} \). We refer to this metric as \( \text{qual}(\sigma, \text{att}) \).

### 4.2 Instance Level Mapping

For a valid schema embedding \( \sigma = (\lambda, \text{path}) \) from \( S_1 \) to \( S_2 \), we give its semantics by defining an instance-level mapping \( \sigma_d : \mathcal{I}(S_1) \rightarrow \mathcal{I}(S_2) \), referred to as the XML mapping of \( \sigma \).

We define \( \sigma_d \) by presenting an algorithm that, given an instance \( T_1 \) of \( S_1 \), computes an instance \( T_2 = \sigma_d(T_1) \) of \( S_2 \). In a nutshell, \( \sigma_d \) constructs \( T_2 \) top-down starting from the root \( r_2 \) of \( T_2 \), mapped from the root \( r_1 \) of \( T_1 \) (recall \( \lambda(r_1) = r_2 \)). Inductively, for each \( \lambda(A) \) element \( u \) in \( T_2 \) that is mapped from an \( A \) element \( v \) in \( T_1 \), \( \sigma_d \) generates a distinct \( \lambda(B) \) node \( u' \) in \( T_2 \) for each distinct \( B \) child \( v' \) of \( v \) in \( T_1 \), such that \( u' \) is reached from \( u \) by \( \text{path}(A, B) \) in \( T_2 \), i.e., \( u' \) is uniquely identified by the \( \lambda_R \) path from \( u \). More specifically, the construction is based on the production \( A \rightarrow P_1(A) \) in \( S_1 \).

1. \( P_1(A) = B_1, \ldots, B_n \). For each child \( v_i \) of \( A \), \( \sigma_d \) creates a node \( u_i \) bearing the same id as \( v_i \). These nodes are added to \( T_2 \) as follows. For each \( i \in [1, n] \), \( u_i \) is added to \( T_2 \) by creating \( \text{path}(A, B_i) \) emanating from \( u \) to \( u_i \), such that the path shares any prefix already in \( T_2 \) which were created for, e.g., \( \text{path}(A, B_j) \) for \( j < i \).

2. \( P_1(A) = B_1 + \ldots + B_n \). Here \( v \) in \( T_1 \) must have a unique child \( v_i \). For \( v_i \), \( \sigma_d \) creates a node \( u_i \) bearing the same id as \( v_i \), and adds \( u_i \) to \( T_2 \) via \( \text{path}(A, B_i) \) as above.

3. \( P_1(A) = B^* \). By the definition of valid path function, \( \text{path}(A, B) \) is of the form \( \text{path}(A, A_1)/B_1/\text{path}(B_1, B) \), where \( A_1 \) is the first type defined in terms of Kleene star in \( P_2 \), i.e., \( P_2(A_1) = B^*_1 \). Let \( v_1, \ldots, v_k \) be all the children of \( v \). Then \( \sigma_d \) creates \( u_1, \ldots, u_k \) bearing the same id’s as \( v_1, \ldots, v_k \), and adds these nodes to \( T_2 \) as follows. It first generates a single \( \text{path}(A_i, A_j) \) from \( u \) to an \( A' \) node \( u' \) if it does not already exist in \( T_2 \), and for each \( i \in [1, k] \), it creates a distinct \( i \)-th \( B_i \) child if it is not already in \( T_2 \). From the \( i \)-th \( B_i \) node it generates \( \text{path}(B_i, B) \) leading to \( u_i \), in the same way as in (1) above. Note that the order of the children of \( v \) is preserved by \( \sigma_d \).

4. \( P_1(A) = \text{str} \). Same as (1) except the last node of \( \text{path}(A, \text{str}) \) in \( T_2 \) is a text node holding the same value as the text node in \( T_1 \).

We repeat the process until all nodes in \( T_1 \) are mapped to nodes in \( T_2 \). We finally complete \( \sigma_d(T) \) by adding necessary default elements such that \( \sigma_d(T) \) conforms to \( S_2 \). Recall from Section 2 that we can assume w.l.o.g. consistent \( \text{DTD} \). Thus for each element type \( A \) in \( S_2 \), we can pick a fixed instance \( I_A \) of \( A \) and use it as \( A \)’s default element.

**Example 4.3:** Consider the XML mapping \( \sigma_d \) of the embedding defined in Example 4.2. Given an instance \( T_1 \) of \( S_1 \) of Fig. 1(a), \( \sigma_d \) generates a tree \( T_2 \) of \( S \) of Fig. 1(c) as follows: \( \sigma_d \) first creates the root \( \text{school} \) of \( T_2 \), bearing the node id of the root \( \text{db} \) of \( T_1 \). Then, \( \sigma_d \) creates a single \( \text{courses} \) child \( x \) of \( \text{school} \), a single \( \text{current} \) child \( y \) of \( x \), and for each \( \text{class} \) child \( c \) of \( \text{db} \), \( \sigma_d \) creates a distinct course \( z \) of \( y \) bearing the id of \( c \), such that the course children of \( y \) are in the same order as the class children of \( db \). It then maps the \( \text{cno}, \text{title}, \text{type} \) children of \( c \) to \( \text{cno}, \text{title}, \text{category} \) descendants of \( z \) in \( T_2 \), based on \( \text{path}_1 \). In particular, to map \( \text{title} \) in \( S_1 \), it creates a single class child \( x_c \) of the basic element, a single \( \text{semester} \) child \( x_{c_s} \) under \( x_c \) (although class is defined with a Kleene star), and then a \( \text{title} \) child under \( x_{c_s} \). For the category element \( w \) mapped from the type \( c \) of \( c_d \), \( \sigma_d \) creates a distinct path advanced/project under \( w \) if \( t \) has a project child, or a mandatory/regulat project otherwise, but not both. The process proceeds until all nodes in \( T_1 \) are mapped to \( T_2 \). Finally, default elements of \( \text{history}, \text{credit}, \text{year}, \text{term}, \text{instructor} \) and \( \text{gpa} \) are added to \( T_2 \) such that \( T_2 \) conforms to \( S \). At the last stage, no children of disjunctive types \( \text{category}, \text{mandatory} \) or \( \text{advanced} \) are added, and no children are created under \( \text{history} \). That is, default elements are added only when necessary. □

Note that \( \sigma_d(T) \) does not have to be a new document of \( S \) constructed starting from scratch. Under certain conditions a mild variation of \( \sigma_d \) allows us to expand an existing instance \( T_2 \) of \( S \) to accommodate nodes of \( T_1 \), by modifying default elements in \( T_2 \) and introducing new elements.

We next show that \( \sigma_d \) is well defined. That is, given any \( T_1 \) in \( \mathcal{I}(S_1) \), \( \sigma_d(T) \) is an XML tree that conforms to \( S_2 \). This is nontrivial due to the interaction between different paths defined for disjunction types in the schema mapping \( \sigma \), among other things. Consider, for example, \( \text{path}(\text{type}, \text{project}) \) in Example 4.2. The path requires the existence of a regular child under a mandatory element \( m \), which is in turn a child under a category element \( c \) in an instance of \( S \). Thus it rules out the possibility of adding an advanced child under \( c \) or a lab child under \( m \), perhaps requested by a conflicting path in \( \sigma \). However, Theorem 4.1 below shows that the prefix-free condition in the definition of valid path functions ensures that conflicting paths do not exist, and
thus prevents violation caused by conflicting paths.

Theorem 4.1 also shows that $\sigma_d$ is injective: it maps distinct nodes in $T_1$ to distinct nodes in $\sigma_d(T_1)$, a property necessary for information preservation. Indeed, $\sigma$ determines an injective path-mapping function $\delta$ such that, for each $\mathcal{X}$ path $\rho = A_1[i_1]/\ldots/A_k[i_k]$ in $S_1$ from $r_1$, $\delta(\rho)$ is path$(r_1, A_1[i_1]/\ldots/path(A_k-1, A_k[i_k])$, an $\mathcal{X}$ path in $S_2$ from $r_2$, by substituting path$(A_1, A_i+1)$ for each $A_{i+1}$ in $\rho$. Since each node in $T_1$ is uniquely determined by an $\mathcal{X}$ path from the root, it follows that $\sigma_d$ is injective.

**Theorem 4.1:** The XML mapping $\sigma_d$ of a valid schema embedding $\sigma : S_1 \rightarrow S_2$ is well defined and injective. □

### 4.3 Properties of Schema Embeddings

We have shown that the XML mapping $\sigma_d$ of a valid schema embedding $\sigma$ is guaranteed to type check. We next show that $\sigma_d$ and $\sigma$ also have all the other desired properties.

**Information Preservation.** In contrast to Theorem 3.4, information preservation is guaranteed by schema embeddings. Recall regular XPath $\mathcal{X}$ from Section 2.

**Theorem 4.2:** The XML mapping $\sigma_d$ of a valid schema embedding $\sigma : S_1 \rightarrow S_2$ is invertible and is query preserving w.r.t. $\mathcal{X}$. More precisely, (a) there exists an inverse $\sigma_d^{-1}$ of $\sigma_d$, that, given any $\sigma_d(T)$, recovers $T$ in $O(|\sigma_d(T)|^2)$ time; and (b) there is a query translation function $F$ that given any $\mathcal{X}$ query $Q$ over $S_1$, computes an $\mathcal{X}$ query $F(Q)$ equivalent w.r.t. $\sigma_d$ over $S_2$ in $O(|Q||\sigma_d|(|S_1|)$ time.

**Example 4.4:** The $\mathcal{X}$ query $Q$ below, over $S_0$ of Fig. 1(a), is to find all the courses that are (direct or indirect) prerequisites of CS331. It is translated to an $\mathcal{X}$ query $Q'$ over $S$ of Fig. 1(c), which is equivalent w.r.t. the mapping $\sigma_d$ given in Example 4.3, i.e. $Q(T) = Q'(\sigma_d(T))$ for any $T \in \mathcal{T}(S_0)$, when evaluated on $T$ with the root as the context node.

\[
Q : \text{class}[cno/text()=\text{CS331}]/(type/regular/prereq/class)*. \\
Q' : \text{courses}/\text{current}/\text{course}[cno/text()=\text{CS331}]/(category/mandatory/regular/required/prereq/course)*.
\]

In contrast, the notion of graph similarity ensures neither invertibility nor query preservation w.r.t. $\mathcal{X}$. As a simple example, the source and target schemas in Fig. 2(a) are bisimilar (if the conventional definition of graph similarity is not extended to accommodate the cardinality constraints of different DTD constructs). However, there exists no instance-level mapping from the source to the target, not to mention inverse mappings and query translation.

**Multiple sources.** In contrast to graph similarity, it is possible to embed multiple source DTD schemas to a single target DTD, as illustrated by the example below. This property is particularly useful in data integration.

**Example 4.5:** The embedding $\sigma_2 = (\lambda_2, \text{path}_2)$ below maps $S_1$ of Fig. 1(b) to the target DTD $S$ of Fig. 1(c).

\[
\begin{align*}
\lambda_2(\text{db}) &= \text{school} \\
\lambda_2(A) &= A \quad /\ast A: \text{student, ssn, name, taking, cno} \ast/ \\
\text{path}_2(\text{db, student}) &= \text{students/student} \\
\text{path}_2(\text{student, B}) &= B \quad /\ast B: \text{ssn, name, taking} \ast/ \\
\text{path}_2(\text{taking, cno}) &= \text{cno} \\
\text{path}_2(\text{C, str}) &= \text{text}() \quad /\ast C: \text{ssn, name, cno} \ast/
\end{align*}
\]

Taken together with $\sigma_1$ of Example 4.2, this allows us to integrate a course document of $S_0$ and a student document of $S_1$ into a single school instance of the target DTD $S$. □

In general, given multiple source DTDs $S_1, \ldots, S_n$ and a single target DTD $S$, one can define schema embeddings $\sigma_i : S_i \rightarrow S$ to simultaneously map $S_i$ to $S$. Their XML mappings $\sigma_1, \ldots, \sigma_n$ are invertible and query preserving w.r.t. $\mathcal{X}$ as long as $\delta_i, \delta_j$ are pairwise disjoint, where $\delta_i$ is the path mapping function derived from $\sigma_i$ to map $\mathcal{X}$ paths from root in $S_i$ to $\mathcal{X}$ paths from root in $S$. The instance-level XML mapping $\sigma_d$ is a composition of individual $\sigma_1, \ldots, \sigma_n$, such that $\sigma_d$ increments the document constructed by $\sigma_j$’s for $j < i$ by modifying default elements or introducing new elements, as described earlier.

**Small model property.** The result below gives us an upper bound on the length $|\text{path}(A, B)|$, and allows us to reduce the search space when defining or finding an embedding.

**Theorem 4.3:** If there exists a valid schema embedding $\sigma : S_1 \rightarrow S_2$, then there exists one such that for any edge $(A, B)$ in $S_1$, $|\text{path}(A, B)| \leq (k + 1)|E_2|$, where $S_2 = (E_2, P_2, r_2)$, and $k$ is the size of the production $P_2(A)$. □

## 5 Computing Schema Embeddings

In this section we address the computation of XML schema embeddings as defined by the following problem, stated in terms of two XML DTD schemas $S_1 = (E_1, P_1, r_1)$ and $S_2 = (E_2, P_2, r_2)$, and a similarity matrix $\text{att}$:

<table>
<thead>
<tr>
<th>PROBLEM: Schema-Embedding</th>
</tr>
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<tbody>
<tr>
<td>INPUT: Two DTDs $S_1$ and $S_2$ and matrix att.</td>
</tr>
<tr>
<td>OUTPUT: A schema embedding $\sigma : S_1 \rightarrow S_2$ valid w.r.t. att if one exists.</td>
</tr>
</tbody>
</table>

In practice, a reasonable goal is to find an embedding $\sigma : S_1 \rightarrow S_2$ with as high a value for $\text{qual}(\sigma, \text{att})$ as possible. The ability to efficiently find good solutions to this problem will lead to an automated tool that, given two DTD schemas, compute candidate embeddings to recommend to users.

However desirable, this problem is NP-complete. Its intractability is rather robust: it remains NP-hard for nonrecursive DTDs even when they are defined in terms of concatenation types only.

**Theorem 5.1:** The Schema-Embedding problem is NP-complete. It remains NP-hard for nonrecursive DTDs. □

In light of the intractable results we develop two efficient yet accurate heuristic algorithms for computing schema embedding candidates in the rest of the section.

**Notations.** Recall that a schema embedding is a path mapping $\sigma$ that is valid for each element type $A$ in $S_1$. Since the validity conditions for $A$ involve only $A$’s immediate children, it is useful to talk about mappings local to $A$. A local
5.1 Finding Valid Local Mappings

developed heuristic tool [9] is available.

We start by giving an algorithm to find a local embedding defined to be

where

and similarly for

Consider two partial mappings, \( \sigma_0 = (\lambda_0, \text{path}_0) \) and \( \sigma_1 = (\lambda_1, \text{path}_1) \). We say that \( \lambda_0 \) and \( \lambda_1 \) conflict on \( A \) if both \( \lambda_0(A) \) and \( \lambda_1(A) \) are defined, but \( \lambda_0(A) \neq \lambda_1(A) \), and similarly for \( \text{path}_0 \) and \( \text{path}_1 \). We say \( \sigma_0 \) and \( \sigma_1 \) are consistent if they do not conflict, either on \( A \) or path. The union of consistent partial mappings, denoted by \( \sigma_0 \cup \sigma_1 \), is a partial embedding defined to be

\[
\lambda_1(A) \uplus \lambda_2(A) = \begin{cases} 
\lambda_1(A) & \text{if } \lambda_2(A) \text{ is } \perp \text{ (undefined)} \\
\lambda_2(A) & \text{if } \lambda_1(A) \text{ is } \perp \\
\lambda_1(A) & \text{otherwise}
\end{cases}
\]

similarly for \( \text{path}_1(A,B) \uplus \text{path}_2(A,B) \).

Outline. In the rest of the section we first present a technique for finding local embeddings, already a nontrivial yet interesting problem. Making use of this algorithm, we then provide three heuristics for finding embedding candidates. The first two are based on randomized programming and the last is by reduction from our problem to the Max-Weight-Independent-Set problem for which a well-developed heuristic tool [9] is available.

5.1 Finding Valid Local Mappings

We start by giving an algorithm to find a local embedding defined to be

when the partial type mapping \( \lambda_0 \) is fixed, as this is a key building block of our schema-embedding algorithms. We then extend the algorithm to handle the general case when \( \lambda_0 \) is not given. To simplify the presentation we focus on nonrecursive DTDs, i.e., DTDs with a directed acyclic graph (DAG) structure, but we show that our technique also works on recursive (cyclic) DTDs.

Finding Valid Paths. Let \( A \in E_1 \) be a source element type with production \( A \rightarrow P_1(A) \), in which the element types appearing in \( P_1(A) \) are \( B_1, \ldots, B_k \). Assume that the type mapping \( \lambda_0 \) is already given as a partial function from \( E_1 \) to \( E_2 \) that is defined on \( B_1, \ldots, B_k \) and \( A \). The Valid-Paths problem is to find paths \( \text{path}_0(A, B_1), \ldots, \text{path}_0(A, B_k) \) such that \( \lambda_0, \text{path}_0 \) is a valid local mapping for \( A \).

The validity conditions stated for embeddings in Section 4.1 require that (a) target paths for each edge are of the appropriate type (AND, OR, or STAR path), and (b) that the target path for an edge is not a prefix of a sibling’s target path. We abstract the second condition as a directed-graph problem: Given a directed graph \( G = (V, E) \), a source vertex \( s \) and a bag of target vertices \( L_{tar} = \{t_1, \ldots, t_k\} \), find paths \( p_1, \ldots, p_k \) such that no path is equal to or is a prefix of another. That is, for all \( i \neq j \), \( p_i \neq p_j \) for any \( p_j \) including the empty path. In contrast to most sub-problems of Schema-Embedding, this can be solved in polynomial time. We introduce our solution by giving an algorithm that works only on a DAG and discuss extending it to handle cycles below.

We present our algorithm, findPathsDAG, in Fig. 3, for finding prefix-free paths in a DAG. The algorithm depends on the recursive procedure traverse, shown in Fig. 4. The intuition of this algorithm is to modify a simple (but exponential) algorithm to recursively enumerate all paths in a DAG in such a way that prefix-free paths are found, but excessive running time is avoided. In a nutshell, traverse conducts a depth-first-search on the input graph \( G \), enumerating paths from the source node \( s \) to target nodes in \( L_{tar} \), and identifies prefix-free ones. It uses a (global) boolean array \( \text{marked}(n) \) to keep track of whether the subgraph rooted at a node \( n \) has been searched and yielded no matches for nodes in \( L_{tar} \), and if so, it does not re-enter the subgraph. A (local) variable \( \text{ret} \) is used to indicate whether the search of a subgraph finds any matches to nodes in \( L_{tar} \).

To see that traverse is correct, consider removing line 5 in which the algorithm returns early, and line 11 in which nodes are marked to avoid revisiting them. It is clear that the resulting algorithm considers every possible path leading to nodes in \( L_{tar} \), and assigns one path to each \( n \in L_{tar} \), but it does not avoid assigning one node the prefix of another path. However, the prefix-free condition is assured by the return at line 5 without affecting correctness, since a suffix of the path assigned to \( n \) could only be generated by continuing the recursion from this node. Thus it remains to argue that the algorithm is still correct if line 11 is in place.
The intuition of line 11 is simple: if no new target nodes were found in the subtree of a node when it was explored by the recursive calls of lines 7-10, then the current node will not be on any path to any other remaining in $L_{tar}$.

**Example 5.1:** Consider the schema embedding problem shown in Figure 1. Say that `att` (regular, seminar), and `att` (project, advanced) in $S_0$ are 0.75. This means that the bag of possible target matchings for source tags `{regular, project}` in $S_0$ can be `{seminar, advanced}` from $S$. We then invoke `traverse` with $S$, category, $\rho$ (which is empty), and $L_{tar}$ as `{seminar, advanced}`. The first call to `traverse` would result in all edges from `category` to be recursed. Say, our algorithm first picks the edge to `advanced`. Line 2 of `traverse` would check `advanced` to be in $L_{tar}$ and add the path to `advanced` into $\mathcal{P}$. It would then return back from the recursion and try the other edges from `category` in lines 7 though 10. This would result in a prefix-free path candidate/seminar which would then be added to $\mathcal{P}$.

To analyze the performance of `findPathsDAG`, consider `traverse` as a sequence of forward and backward traversals of edges in the graph. A forward traversal occurs at line 9 and a backward traversal at lines 1, 5 and 12. Clearly, the number of forward traversals and backward traversals in a run are the same. Further, observe that one returns from an un-marked node at line 5 only on the path `back` from some node newly removed from $L_{tar}$. Thus, there can be at most $|L_{tar}| \cdot |V|$ such backward steps, and at most $|E|$ other backward steps (which mark the child of the edge traversed). Since $G$ is a DAG, the algorithm is in $O(|L_{tar}| \cdot |V|)$ time.

To use `findPathsDAG` in our algorithms for schema embedding, we must further ensure that the paths returned match the types needed for $n \in L_{tar}$. That is easy to accomplish, as the type of a path can be maintained incrementally as it is lengthened and shortened (by storing counts of nodes of each type), and be checked at line 2.

**Schema Embeddings with a Given $\lambda$.** This algorithm can be used to directly find a schema embedding $\sigma = (\lambda, \text{path})$ from $S_1$ to $S_2$ when the type mapping $\lambda$ is a given total function from $E_1$ to $E_2$. As remarked earlier, the validity conditions for any $n \in E_1$ involve only $A$'s children; thus to find path we only need to find valid paths for each $n \in E_1$ and take the union of these valid local embeddings. This yields an $O(|S_1| \cdot |S_2|)$ algorithm to find embeddings in this special setting, which is not so uncommon since one may know in advance which target type a source type should map to, based on, e.g., machine-learning techniques [13].

**Handling Multiple Targets.** However, to find valid local mappings when $\lambda$ is not given, we must consider that there are multiple possible target nodes for each source node. The general Local-Embedding problem is to find a local embedding $(\lambda_0, \text{path}_0)$ when $\lambda_0$ may not be fixed. This problem is no longer tractable as shown below.

**Theorem 5.2:** The Local-Embedding problem is NP-hard for nonrecursive DTDs.

One heuristic approach to finding local embeddings is to extend `findPathsDAG` as follows. We compute the set of all pairings of source nodes $A$ and possible matches for $A$ from $\text{att}$ and pass it as $L_{tar}$. We also modify line 3 of `traverse` to (a) pick an arbitrary pair with the current node as the target from $L_{tar}$ at line 2 and (b) remove all pairs associated with source node $A$ from $L_{tar}$ at line 3. While this may work, it is essentially a greedy algorithm and may not find a solution if one exists. To compensate for this, we actually use a randomized variant `findPathsRand` (not shown) which (a) picks a random source node associated with $n$ at line 2 of `traverse`, and (b) tries outgoing edges from $n$ at line 7 in random order. The ability of `findPathsRand` to find embeddings varies with the size of $L_{tar}$, and will be investigated in Section 6.

**Handling Cycles.** Of course, schemas are frequently cyclic (recursive), and the algorithms as presented so far only handle DAGs. In fact, handling cycles generally is somewhat more complicated, but not hard – it is easy to see that an arbitrary number of paths can be generated by repeated loops around some cycle on the path to a target, and careful use of these paths can guarantee the prefix-free property (Figure 2(e) gives such an example, in which the cycle is unfolded once to get a prefix-free path, in contrast to Fig. 2(d)). While we present this full algorithm in [7], the complication is not warranted here since long cyclic paths are almost certainly semantically uninteresting. In practice, we have extended `findPathsDAG` once again to allow limited exploration of cycles limited by (a) no more than $k$ trips through visited nodes and (b) no more than $l$ total path length. A bound on $k$ and $l$ is given in Theorem 4.3 and usually $k$ and $l$ are set to small numbers.

**5.2 Three Methods for Finding Schema Embeddings**

We next introduce our three heuristic embedding-search algorithms, namely, **QualityOrdered**, **RandomOrdered** and **RandomMaxInd**.

**Finding Solutions with Ordered Algorithms.** Our first two heuristics are based on a common subroutine `Ordered`, shown in Fig. 5. A key data structure is a table, $C$, where $C(A)$ is a set of known local embeddings for a source node $A$. The initialization of this table is discussed later. Given $C$ and an ordered set $O$ of source schema element types, `Ordered` tries to assemble a consistent solution $\sigma$ by considering each $A$ in $O$ order (line 2), and trying to find a local embedding $\sigma_A$ in $C(A)$ which can be merged with the existing $\sigma$ without a conflict (lines 3-8).

Our first algorithm based on `Ordered`, `QualityOrdered`, is shown in Figure 6. In this algorithm, $C(A)$ is initialized with a single randomly chosen local embedding for each source node $A$. The ordering $O$ is sorted by the quality of the local embedding.

In our second algorithm `RandomOrdered` (not shown), $C$ is the complete set of local embeddings discovered so far for each source node (lines 4 and 5 in Figure 6), while $O$ is a random ordering of source nodes (line 6 in Figure 6).

**A Reduction Approach.** We now discuss our third heuristic, `RandomMaxInd`. To understand this heuristic, consider
the following problem defined on the table \( C \) of local mappings defined above:

**PROBLEM:** Assemble-Embedding  
**INPUT:** Two DTDs \( S_1 \) and \( S_2 \), a similarity matrix \( \text{att} \), and a table \( C \).  
**OUTPUT:** A schema embedding \( \sigma : S_1 \rightarrow S_2 \), valid w.r.t. \( \text{att} \), formed as the union of a subset of embeddings in \( C \) if one exists.

Composing a schema embedding from local embeddings in \( C \) is nontrivial:

**Theorem 5.3:** The Assemble-Embedding problem is NP-hard for nonrecursive DTDs. \[ \square \]

To cope with this, the RandomMaxInd heuristic takes the approach of reducing the Assemble-Embeddings problem to the problem of finding high-weight independent sets in a graph. It uses an existing heuristic solution \([9]\) to produce partial or complete solutions to this problem, which can be used to create partial or complete embeddings.

Before describing our reduction, we review the definition of Max-Weight-Independent-Set. That problem is defined on an undirected graph \( G = (V, E) \) (not to be confused with a schema graph) with node weights \( w[v], v \in V \). The goal is to find a subset \( V' \) of \( V \) such that for \( v_i \) and \( v_j \) in \( V' \), there is no edge from \( v_i \) to \( v_j \). In other words, \((v_i, v_j) \notin E\) and the weight of \( V' \), defined as \( \sum_{v \in V'} w[v] \), is maximized.

Given an instance of the Assemble-Embedding problem, it is straightforward to construct an instance of Max-Weight-Independent-Set. First, for each local mapping \( \sigma_a \in C(A) \) for any \( A \in E_1 \), we construct a vertex \( v_{\sigma_a} \) in \( V \). Second, for each pair \( \sigma_a, \sigma_b \) of such mappings, we construct an edge between \( v_{\sigma_a} \) and \( v_{\sigma_b} \) if \( \sigma_a \) and \( \sigma_b \) conflict. The weight of \( v_{\sigma_a} \) is given as \( \text{qual}(\sigma_a, \text{att}) \).

To complete the algorithm on the resulting graph, we use an existing heuristic tool for Max-Weight-Independent-Set, which returns a subset \( V' \) of \( V \). Finally, we construct an embedding \( \sigma \) by adding local embedding \( \sigma_a \) to \( \sigma \) for each \( v_{\sigma_a} \in V' \). The quality of \( \sigma \) is warranted by the heuristic tool used, and its correctness is verified below.

**Theorem 5.4:** If \( |V'| = |E_1| \), \( \sigma \) constructed as above is a schema embedding from \( S_1 \) to \( S_2 \). \[ \square \]
each leaf in the new subtree is visited, and with probability .5, an edge is added to an existing leaf outside the newly-added subtree. (This leaf may later have a subtree added under it.) The intuition for this last step is that confusion between different parts of the tree is more likely to arise if the same “attributes” (leaf nodes) appear in multiple places.

**Generating the att.** The similarity array, att, is initialized by computing pairwise string-edit distances between source and target tags (string edit distance with unit cost is also known as Damerau-Levenshtein distance). Furthermore, if a minimum threshold, sel, of similarity is not met by a pair, the similarity of that pair is set to 0, and as a result the tags cannot be matched. Note that the “similar names” introduced above range in similarity from .5 for short strings to over .8 for longer strings, and will be counted as potential matches in many experiments. There are also similar names in the schemas themselves, caused by the conversion of the schema to our graph format.

Clearly, sel, referred to as the *selectivity* of att, is an important parameter, as it directly determines the size of the candidate pool of target tags matching each source tag. Larger selectivities make the problem easier, and for our experimental data if sel is 1.0 (exact matches only), then finding a schema embedding reduces to finding valid prefix-free paths for each local embedding in the source schema.

A second important parameter is the *accuracy* of att. This matters greatly for heuristic algorithms, since the valid embedding in our generated data always has the highest average quality. Accuracy is implemented with a parameter \(c\), which varies between 0 and 1. Each entry \(m\) in att is replaced by \(cm + (1-c) \cdot \text{rnd}\), where \(\text{rnd}\) is a random number from 0 to 1. A low accuracy tends to mislead heuristics that rely heavily on att. Combining a low accuracy with a very low selectivity makes the problem very difficult to solve.

**Experimental Setting.** Experiments are conducted by copying the source schema, adding some amount of noise based on the parameter noise, and adjusting the att according to sel and c. Then the three algorithms given in Section 5 (RandomOrdered, QualityOrdered and RandomMaxInd) are used to try to find embeddings. For the ordered algorithms, the set \(C\) is initialized by finding 3 random mappings for each \(A_i\) , and discarding the two with the lowest qual ratings. When not otherwise stated, experiments are run with \(\text{sel} = 0.6, c = 0.75\) (accuracy) and noise = 0.25. Since all algorithms (and the noise introduction) have a random component, they are repeated with 40 different random seeds, and an average is used.

The software is written in Java, except for the external heuristic for maximum independent sets [8], which is an optimized C program. Experiments are run on a variety of machines with Pentium III processors running at either 933MHz or 1.0GHz, with 256MB of RAM.

**Accuracy Results.** Figure 7 shows how the three algorithms perform while varying accuracy, with noise = 0.25. The y axis shows the percentages of runs for which a successful embedding is found. For this noise amount, the target schema is approximately three times as large as the source schema. This graph shows that QualityOrdered is extremely sensitive to the quality of the att values. It uses att extensively in its search pattern, and thus cannot find solutions unless att is accurate.

Figure 7 also shows that RandomOrdered finds correct solutions more frequently than RandomMaxInd. While RandomOrdered takes into account att when it is seeking its solution set, it tries to find alternative solutions based on the conflicts it detects, independent of the att values. RandomMaxInd seeks alternative solutions for nodes based solely on their weights, as defined by att. It does not use conflicts to guide its search.

**Varying Target Schema Size.** We also consider target schemas with different numbers of erroneous nodes and edges introduced. These results are shown in Fig. 8. Because this graph shows results when accuracy is 0.75, QualityOrdered does not do well, as expected. RandomOrdered and RandomMaxInd both find the correct solution the majority of the time, decreasing somewhat as noise increases. The running times are shown in Fig. 9.

**Different Source Schemas.** We also run tests with different source schemas. We vary noise over five different source schemas, using RandomOrdered and accuracy = 0.75. Figure 10 shows the running times for the various source schemas. For all runs across the different schemas, a solution was found more than 90% of the time (not shown).

**Varying Selectivity.** We also run experiments with different values of selectivity. Both RandomOrdered and RandomMaxInd find solutions less frequently as selectivity decreases (not shown). QualityOrdered is relatively indifferent to the selectivity level, finding approximately the same number of solutions at sel = 0.3 as at sel = 0.7. The running time increases dramatically, however, once sel falls below 0.4. The results are shown in Figure 11.

**Discussion.** Our experimental results show that, when a feasible matching exists, it is likely to be almost completely found for schema sizes of up to a few hundred
nodes. While this does not demonstrate that similar results can be obtained with differing target schemas and the use of real-world tools to produce att, it is certainly promising. Further, we found that the randomized algorithm RandomOrdered performs better than RandomMaxInd, and that QualityOrdered only does well with a highly accurate att. Based on these results, we plan to integrate RandomOrdered and RandomMaxInd, since the external independent set heuristic is very fast in practice. Finally, we note that QualityOrdered may be important in practice, where the att values may in fact be reliable.

7 Related Work

A wide variety of techniques have been developed to solve different forms of schema matching for relational, ER and object-oriented models (e.g., [18, 22, 32, 12, 20]; see [33] for a recent survey). While these are not focused on XML DTD schema matching, some techniques, such as linguistic analyses and machine learning, are useful for finding name/label similarity, which our algorithms take as input.

Closer to XML schema matching are [13, 23, 25, 26, 27, 30]. LSD [13] proposes powerful machine-learning techniques that make use of instance-level information to determine XML DTD tag mapping. Systems of [23, 25, 26] target a wide class of schemas and can be tailored to a variety of data models. The similarity flooding algorithm of [25] provides a novel schema matching tool based on graph-similarity. Cupid [23] is a generic schema matching tool that encompasses a variety of techniques such as linguistic analyses, referential constraints and context dependencies. Rondo [26] proposes a powerful set of model mapping operators. For structure-level schema matching, these systems adopt graph similarity to map a single source schema to a target. TransScm [30] considers instance-level mappings based on schema matching, and uses a semi-automated mechanism to match highly similar schemas. Clio [27] also focuses on deriving instance translation from schema mappings. Both TransScm and Clio adopt a similarity heuristic, and decouple the process of getting an instance mapping from schema matching. To our knowledge, no previous work has addressed information preservation for XML DTD schemas. Our notion of schema embedding extends graph similarity by allowing a source DTD schema to be mapped to a structurally different target DTD. It is capable of mapping multiple source DTDs to a single target schema, which is typical in data integration. Furthermore, an instance mapping can be automatically derived from a schema embedding and it guarantees both invertibility and query preserving w.r.t. regular XPath queries.

Information preservation has been studied for nested relational and complex data models (e.g., [3, 16, 28, 29]). [16] proposed several notions of dominance and studied their relationships, which were revisited in [28]. The focus of [3, 29] has mainly been on the information capacity of type constructs and structural transformation rules. Our study of information preservation is inspired by the prior work: our notions of invertibility and query preservation are mild extensions of calculus dominance and query dominance [16]. We revise these notions and study their basic properties for XML DTD schemas and XML queries, and our focus is to develop the notion of DTD schema embedding that preserves information by ensuring both effective invertible mapping and efficient XML query translation.

Query preservation is related to query rewriting using views, which has been extensively studied for conjunctive and datalog queries for relational databases and regular path queries on semistructured data (e.g., [2, 10, 11, 21]; see [15, 19] for surveys). View-based query rewriting mainly studies whether a given query on the source can be answered using materialized data from a set of views (lossless), by translating the query to an equivalent query in a particular language on the views. In contrast, query preservation deals with the issue whether all queries in an (infinite) query language on an XML source can be rewritten to equivalent queries over XML target (view). Moreover, the focus of this work is to generate XML mappings (views) that automatically preserves all the queries in an XML query language, rather than to determine the losslessness of views. It is worth remarking that Theorem 3.2 establishes a connection between invertibility and query rewriting. For example, if the query language \( \mathcal{L} \) includes the identity query \( id \), then a view \( \sigma_d \) is invertible and \( \sigma_d^{-1} \) is in \( \mathcal{L} \) iff \( id \) has a rewriting in \( \mathcal{L} \) using \( \sigma_d \).

Also related to schema matching is the problem of type checking one schema against another. For XML schemas, the problem of determining the existence of a subsumption map has been studied (e.g., [17]), but data restructuring and query translation are not addressed in that context.
8 Conclusions

We have studied information-preservation criteria for XML mappings by revising the notions of invertibility and query preservation [3, 16, 28, 29], and by establishing separation, equivalence and complexity results. In light of the difficulties of determining information preservation, we have introduced a novel notion of schema embedding for XML DTD schemas, from which an instance-level mapping is automatically derived and is guaranteed to be information preserving, type checking, and able to accommodate multiple source schemas. While we show that finding a schema embedding is NP-complete, we have provided heuristic algorithms to compute embedding candidates, which are efficient and accurate as shown by our experimental results. These provide a practical approach to specifying and computing information-preserving XML mappings.

This is a first step toward developing a useful tool for lossless XML data migration and integration. For future work, we plan to extend the notion of schema embedding to (a) accommodate more general XML schemas with constraints and inheritance, (b) allow one source type to map to different target types in different contexts, (c) allow certain queries in XQuery in the path function, and (d) preserve XQuery fragments as query languages.

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References


Appendix

Proof of Theorem 3.1

(1) We first show that invertibility does not entail query preservation w.r.t. \( \mathcal{X} \). Consider a source DTD \( S_1 \) and a target DTD \( S_2 \):

\[
S_1 = \{(r, A, B, C), P_1, r\}, \text{ where } P_1 \text{ is:} \\
r \rightarrow A, \quad A \rightarrow B, C, \quad B \rightarrow A + \varepsilon, \quad C \rightarrow \varepsilon.
\]

\[
S_2 = \{(r, A), P_2, r\}, \text{ where } P_2 \text{ is:} \\
r \rightarrow A, \quad A \rightarrow A + \varepsilon.
\]

The mapping \( \sigma_d : \mathcal{I}(S_1) \rightarrow \mathcal{I}(S_2) \) is defined such that for any \( T \in \mathcal{I}(S_1) \), the root \( r_1 \) of \( T \) is mapped to the root \( r_2 \) of \( \sigma_d(T) \), the \( A \) child of \( r_1 \) is mapped to the \( A \) child of \( r_2 \); and inductively, if an \( A \) element \( v \) in \( T \) is mapped to an \( A \) element \( v' \) in \( \sigma_d(T) \), then the \( B \), \( C \) children of \( v \) are mapped to the child and the grandchild of \( v' \), respectively, and the \( A \) child of the \( B \) node is mapped to the great-grandchild of \( v' \). Formally, this can be expressed by the function \( \text{path} \) from the edges of \( T \) to (relative) paths of \( \sigma_d(T) \) as follows:

\[
\text{path}(r, A) = A \\
\text{path}(A, C) = A/A \\
\text{path}(A, B) = A/A \\
\text{path}(B, C) = A/A
\]

Obviously \( \sigma_d \) is invertible: one can restore the original \( T \) from \( \sigma_d(T) \) inductively top-down from the root \( r_1 \) of \( T \).

Now consider an \( \mathcal{X} \) query \( Q = /B \). An equivalent translation of \( Q \) over \( \sigma_d(T) \) is to find all the elements in the \( A \)-chain of \( \sigma_d(T) \) that are reachable from \( r_2 \) via \( A^{2k+1} \). It is easy to prove that \( A^{2k+1} \) is not expressible in \( \mathcal{X} \) by contradiction. Thus \( \sigma_d \) is not query preserving w.r.t. \( \mathcal{X} \).

(2) We next show that query preservation w.r.t. \( \mathcal{X} \) does not entail invertibility. Consider a source DTD \( S_1 \):

\[
S_1 = \{(r, A), P_1, r\}, \text{ where } P_1 \text{ is:} \\
r \rightarrow A^*, \quad A \rightarrow \text{str}
\]

and the target DTD \( S_2 \) is identical to \( S_1 \).

The mapping \( \sigma_d : \mathcal{I}(S_1) \rightarrow \mathcal{I}(S_2) \) is defined such that for any \( T \in \mathcal{I}(S_1) \), the root \( r_1 \) of \( T \) is mapped to the root \( r_2 \) of \( \sigma_d(T) \), the \( A \) children of \( r_1 \) are mapped to \( A \) children of \( r_2 \) such that there is a bijection from the \( A \) children of \( r_1 \) to the \( A \) children of \( r_2 \); however, the \( A \)-children of \( r_2 \) are ordered based on their string values (str).

One can pose the following forms of \( \mathcal{X} \) queries over \( T \in \mathcal{I}(S_1) \): \( \varepsilon, A, A[q] \), where \( q \) is a boolean formula defined in terms of atomic formulas of the form \( \text{text}() = 'c' \). Since the identity mapping from \( \mathcal{X} \) to \( \mathcal{X} \) yields equivalent queries over \( \sigma_d(T) \) for these queries, the mapping \( \sigma_d \) is query preserving w.r.t. \( \mathcal{X} \). However, \( \sigma_d \) is not invertible: one cannot recover the original order of the \( A \) elements of \( r_1 \).

\[\square\]

Proof of Theorem 3.2

(1) Suppose that \( \sigma_d \) is query preserving w.r.t. \( \mathcal{L} \). Then there exists a computable function \( F : \mathcal{L} \rightarrow \mathcal{L} \) such that for any \( Q \in \mathcal{L} \) and any \( T \in \mathcal{I}(S_1) \), \( Q(T) = F(Q)(\sigma_d(T)) \). Since \( \sigma_d \) is invertible, we have \( T = \sigma_d^{-1}(T) = F(id) \circ \sigma_d \circ F^{-1}(id) \) for any \( T \in \mathcal{I}(S_1) \). Thus \( \sigma_d^{-1} = F(id) \circ \sigma_d \circ F^{-1}(id) \). Thus \( F \) is an effective query translation function for \( \mathcal{L} \).

\[\square\]

Proof of Theorem 3.3

Suppose that \( \sigma_d : S_1 \rightarrow S_2 \) is query preserving w.r.t. \( \mathcal{X}_R \). We show that \( \sigma_d \) is invertible by providing an algorithm for computing \( \sigma_d^{-1} \). Given \( \sigma_d(T) \), the algorithm recovers \( T \). It first creates the root \( r_1 \) of \( T \), which is identical to the root \( r_2 \) of \( \sigma_d(T) \). It then recursively expands \( T \) top-down, until \( T \) cannot be expanded further. More specifically, for each node \( v \) created for \( T \), it recovers the children of \( v \) based on its type \( A \), the production \( A \rightarrow \alpha \) of \( A \) in \( S_1 \), and the query translation function \( F : \mathcal{X}_R \rightarrow \mathcal{X}_R \), as follows. Note that there is a unique \( \mathcal{X}_R \) path \( \rho \) from \( r_1 \) to \( v \).

(1) \( \alpha = A_1, \ldots, A_n \). For each \( A_i \), define an \( \mathcal{X}_R \) query \( Q_i = F(\rho/A_i) \) and evaluate \( Q_i(\sigma_d(T)) \) at the context node \( v \) in \( \sigma_d(T) \), where \( k \) is the \( k \)-th \( A_i \) element in \( \alpha \) if it has multiple \( A_i \) elements. Since \( \sigma_d(T) \) is mapped from an XML tree \( T \), \( Q_i \) always returns a single node \( v_i \). Treat \( v_1, \ldots, v_n \) as the children of \( v \), and for each \( i \in [1, n] \), proceed to expand the subtree at \( v_i \) in the same way.

(2) \( \alpha = A_1 + \ldots + A_n \). For each \( A_i \), let \( Q_i = F(\rho/A_i) \) and evaluate \( Q_i(\sigma_d(T)) \) at the context node \( v \) in \( \sigma_d(T) \). Since \( \sigma_d(T) \) is mapped from an XML tree \( T \), there exists one and only one \( v_k \) such that \( Q_k(\sigma_d(T)) \) returns a single node \( v_i \) (and the others return empty). Treat \( v_k \) as the only child of \( v \) and proceed to expand the subtree at \( v_i \) in the same way.

(3) \( \alpha = B^* \). For each natural number \( k \), evaluate \( Q_k(\sigma_d(T)) \) at the context node \( v \) in \( \sigma_d(T) \), where \( Q_k = F(\rho/B[position] = k) \), until it reaches a \( k \)-th \( B \) such that \( Q_k(\sigma_d(T)) = \emptyset \). Since \( \sigma_d(T) \) is mapped from an XML tree \( T \), there exists one and only one \( v_k \) returned by \( Q_k(\sigma_d(T)) \) for each \( k < k_0 \), and for any \( k \geq k_0 \), \( Q_k(\sigma_d(T)) = \emptyset \). Treat \( v_k \) as the \( k \)-th child of \( v \) for all \( k < k_0 \), and proceed to expand the subtree at each \( v_k \) in the same way.

(4) \( \alpha = \text{str} \). Find the string value via \( \rho/\text{text}() \).

(5) \( \alpha = \varepsilon \). Nothing needs be done here.

This process terminates since \( \sigma_d(T) \) is generated from a finite \( T \). One can verify that \( T = \sigma_d^{-1}(\sigma_d(T)) \), i.e., the algorithm above indeed computes \( \sigma_d^{-1} \). Thus \( \sigma_d^{-1} \) is computable and \( \sigma_d \) is invertible.
To show that invertibility does not necessarily lead to query preservation w.r.t. $X_R$, recall the DTDS $S_1$ and $S_2$ defined in the proof of Theorem 3.1. Consider a mapping $\sigma_2 : \mathcal{I}(S_1) \rightarrow \mathcal{I}(S_2)$ such that for any $T \in \mathcal{I}(S_1)$, the root $r_1$ of $T$ is mapped to the root $r_2$ of $\sigma_2(T)$, the $A$ child of $r_1$ is mapped to the $A$ child of $r_2$; and inductively, if an $A$ element $v$ in $T$ is mapped to an $A$ element $v'$ in $\sigma_2(T)$, then the $B$, $C$ children of $v$ are also mapped to $v'$, and the $A$ child of the $B$ node is mapped to the $A$ child of $v'$, such that the number of $A$ nodes in $T$ is the same as that in $\sigma_2(T)$. Obviously, $\sigma_2$ is invertible: given any $\sigma_2(T)$ one can recover $T$ such that the number of $A$ nodes in $T$ is the same as that in $\sigma_2(T)$, and each $A$ node in $T$ has a $B$ child followed by a $C$ child. However, one cannot translate a query $A/B$ over $S_1$ to an equivalent one over $S_2$. □

Proof of Theorem 3.4

(1) We prove the undecidability by reduction from the equivalence problem for relational-algebra (RA) queries; that is the problem to decide, given two RA queries $Q_1, Q_2 : R_1 \rightarrow R_2$, whether or not $Q_1 \equiv Q_2$, i.e., whether or not for any relational database $I_1$ of $R_1$, $Q_1(I) = Q_2(I)$. The equivalence problem is undecidable (cf. [4]).

It suffices to show that the invertibility problem is undecidable for relational mappings defined in RA. Since relational data can be coded in XML and RA queries can be expressed in $FO$ over XML trees, the undecidability carries over to XML mappings defined in $FO$.

Given two RA queries $Q_1, Q_2 : R_1 \rightarrow R_2$, we define a RA mapping $V : R_1 \times R_2 \rightarrow R_1 \times R_2$, as follows:

$$V = \pi_{R_1} \times (\pi_{R_2} \cup \Delta(Q_1, Q_2))$$

$$\Delta(Q_1, Q_2) = Q_1 \setminus Q_2 \cup Q_2 \setminus Q_2$$

Observe that $Q_1 \equiv Q_2$ iff $\Delta(Q_1, Q_2) = \emptyset$, i.e., the query always returns an empty set.

We show that $V$ is invertible iff $Q_1 \equiv Q_2$. If $Q_1 \equiv Q_2$, then $\Delta(Q_1, Q_2) = \emptyset$. Then $V$ is the identity query and is certainly invertible. Conversely, if $Q_1 \not\equiv Q_2$, then there exists an instance $I$ of $R_1$ such that $\Delta(Q_1, Q_2)(I)$ is nonempty. Consider two distinct instances of $R_1 \times R_2$: $I_1 = (I, \Delta(Q_1, Q_2)(I))$ and $I_2 = (I, \emptyset)$. Since $V(I_1) = V(I_2) = (I, \Delta(Q_1, Q_2)(I))$, $V$ is not injective and thus is not invertible (there exists no inverse function for $V$).

(2) The proof is similar to (1), by reduction from the equivalence problem for RA queries.

Given two RA queries $Q_1, Q_2 : R_1 \rightarrow R_2$, we use the same RA mapping $V$ given above to show that $V$ is query preserving w.r.t. a fixed query iff $Q_1 \equiv Q_2$. Consider a fixed query $Q = \pi_{R_1}$. First, suppose that $Q_1 \equiv Q_2$. Then one can define $F$ such that $F(Q) = Q$. This shows that $V$ is query preserving w.r.t. $Q$. Conversely, suppose that $Q_1 \not\equiv Q_2$. Suppose, by contradiction, that there is a computable query translation function $F$ such that $Q' = F(Q)$. Recall $I_1$, $I_2$ given above. Obviously, $Q(I_1) \not\equiv Q(I_2)$, while $Q'(V(I_1)) = Q'(V(I_2))$. Thus either $Q(I_1) \not\equiv Q'(V(I_1))$ or $Q(I_2) \not\equiv Q'(V(I_2))$; that is, $F$ does not translate $Q$ to an equivalent query over the target, which contradicts the assumption above. Thus $V$ is not query preserving w.r.t. $Q$. □

Proof of Theorem 4.1

The proof consists of three parts. We first show that $\delta$ maps distinct $X_R$ paths from $r_1$ to distinct $X_R$ paths in $S_2$ from $r_2$. Then, using this we show that $\sigma_d$ is injective. Based on this we finally show that $\sigma_d$ is well defined.

(1) We first define a function $\delta$ that maps $X_R$ paths from the root $r_1$ in $S_1$ to $X_R$ paths from the root $r_2$ in $S_2$. Given an $X_R$ path $p = A_1[q_1] / \ldots / A_k[q_k]$ in $S_1$ from $r_1$, $\delta(p)$ is defined to be $\text{path}(r_1, A_1)[q_1] / \ldots / \text{path}(A_{k-1}, A_k)[q_k]$, an $X_R$ path in $S_2$ from $r_2$, by substituting $\text{path}(A_i, A_{i+1})$ for each $A_{i+1}$ in $\rho$. We show that $\delta$ maps distinct $X_R$ paths in $S_1$ from $r_1$ to distinct $X_R$ paths in $S_2$ from $r_2$. Let $p_1, p_2$ be distinct $X_R$ paths from $r_1$ in $S_1$. Consider the following two cases. First, $p_1$ is a prefix of $p_2$. That is, $p_2 = p_1 / \rho$ where $\rho$ is nonempty since $p_1$ and $p_2$ are distinct. Then $\rho$ is mapped to a nonempty $X_R$ path in $S_2$ by the definition of $\sigma$, and thus $\delta(p_1) \neq \delta(p_2)$; similarly if $p_2$ is a prefix of $p_1$. Second, neither $p_1$ is a prefix of $p_2$ nor $p_2$ is a prefix of $p_1$. Then there exist $\rho, \rho_1, A[q], B_1[q_1]$ and $B_2[q_2]$ such that $\rho_1 = \rho / A[q]/B_1[q_1]/\rho', \rho_2 = \rho / A[q]/B_2[q_2]/\rho'$, and $B_1[q_1], B_2[q_2]$ are the first labels that differ in $p_1$ and $p_2$. Then $B_1, B_2$ are child types of $A$. $A$ is either a concatenation type or a disjunction type, and moreover, either $B_1, B_2$ are distinct labels, or $q_1, q_2$ indicate different positions of the same label. By the definition of schema embedding, neither $\text{path}(A, B_1)$ is a prefix of $\text{path}(A, B_2)$ nor the other way around. That is, $\text{path}(A, B_1) = q/\eta_1/\rho_1$ and $\text{path}(A, B_2) = q/\eta_2/\rho_2$ such that $\eta_1$ and $\eta_2$ are distinct. Thus $\delta(p_1) \neq \delta(p_2)$.

(2) From (1) and the definition of $\sigma_d$ it follows that $\sigma_d$ is injective. Indeed, any node in an XML tree is uniquely determined by an $X_R$ path from the root. Thus by the definition of $\sigma_d$, any node $v$ in $T \in \mathcal{I}(S_1)$ is mapped to a distinct node in $\sigma_d(T)$. More specifically, this obviously holds if the parent of $v$ is of a concatenation or disjunction type or str; and moreover, if the type parent of $v$ is defined with a Kleene star, the children of $v$ are mapped to distinct nodes preserving the original order, by the definition of $\sigma_d$.

(3) We next show that $\sigma_d$ is well defined, i.e., for any $T \in \mathcal{I}(S_1)$, $\sigma_d(T)$ conforms to $S_2$. One possible violation of $S_2$ may occur when there exists an $A$-node $u$ in $\sigma_d(T)$ such that $A$ is a disjunction type $A \\rightarrow B_1 + \ldots + B_k$, and $\sigma_d(T)$ forces the presences of both $B_i$ and $B_j$ children of $u$. Then there must be two nodes $v_1, v_2$ in $T$ identified by $X_R$ paths $\rho_1, \rho_2$ from root over $S_1$ such that $\rho_1 = \rho / A[q]/\rho'_1$ and $\rho_2 = \rho / A[q]/\rho'_2$ such that $v_1, v_2$ have the lowest common ancestor $v$ that is identified by $\rho$ and mapped to either $u$ by $\sigma_d$ or an ancestor $u'$ of $u$. If $v$ is mapped to $u$ then $v$ must have a disjunction type by the definition of schema embedding, and thus $v_1, v_2$ cannot coexist, which contra-
dicts the assumption. If $v$ is mapped to $v'$, since $v_1, v_2$ both exist and $v$ is the lowest common ancestor of $v_1$ and $v_2$, $v'$ must have a concatenation type $A' \rightarrow B'_1, \ldots, B'_n$ such that $v_1, v_2$ are descendants of the $B'_i, B'_j$ children of $v'$; indeed, $A'$ cannot be a Kleene star type since otherwise by the definition of $\sigma_d, v_1, v_2$ cannot be mapped to nodes that have a common ancestor $u$; and $A'$ cannot be a disjunction type since $v$ is the lowest common ancestor of $v_1, v_2$. Since both $\text{path}(A', B'_i)$ and $\text{path}(A', B'_j)$ end up to be suffixes of $\rho[A]$ it contradicts the definition of schema embedding since either $\text{path}(A', B'_i)$ is a prefix of $\text{path}(A', B'_j)$ or the other way around.

Similarly, it can be verified that violations cannot be caused by AND and STAR paths either.

**Proof of Theorem 4.2**

(1) We first show that $\sigma_d$ is invertible and query preserving w.r.t. $X_R$. It suffices to define a query translation function $F : X_R \rightarrow X_R$. For if it holds, then $\sigma_d$ is query preserving w.r.t. $X_R$ and in addition, by Theorem 3.3 it is also invertible. The translation function $F$ extends the mapping $\delta$ on $X_R$ paths given earlier, by substituting $\text{path}(A, B)$ for each edge $(A, B)$ in a given $X_R$ query. More specifically, given an $X_R$ query $Q$ over $S_1$, $F(Q)$ is computed by using functions $f$ and $\text{reach}$ defined below. For each element type $A$ in $E_1$ and each sub-query $Q_1$ of $Q$,

- the local translation $f(Q_1, A)$ of $Q_1$ at $A$ is an $X_R$ query over $S_2$ such that for any instance $T$ of $S_1$ and any $A$ element $a$ in $T$, the result of evaluating $Q_1$ at $a$ in $T$ is the same as the result of evaluating $f(Q_1, A)$ at $a'$ on $\sigma_d(T)$, where $a'$ is mapped from $a$ by $\sigma_d$;

- $\text{reach}(Q_1, A)$ is the set of element types in $S_1$ that are reached via $Q_1$ when evaluated at an $A$ element in an instance $T$ of $S_1$.

More specifically, $f(Q_1, A)$ and $\text{reach}(Q_1, A)$ are defined based on the structure of query $Q_1$ as follows.

(a) If $Q_1$ is $\varepsilon$, then

\[ \text{reach}(Q_1, A) = \{ A \}, \]

\[ f(Q_1) = \varepsilon. \]

(b) If $Q_1$ is $B$, then

\[ \text{reach}(Q_1, A) = \{ B \}, \]

\[ f(Q_1, A) = \text{path}(A, B). \]

(c) If $Q_1$ is $p/text()$, then

\[ \text{reach}(Q_1, A) = \emptyset, \]

\[ f(Q_1, A) = f(p, A)/(\bigcup_{B \in \text{reach}(p, A)} \text{path}(B, \text{str})). \]

(d) If $Q_1$ is $p_1 \lor p_2$, then

\[ \text{reach}(Q_1, A) = \bigcup_{B \in \text{reach}(p_1, A)} \text{reach}(p_2, B), \]

\[ f(Q_1, A) = f(p_1, A)/(\bigcup_{B \in \text{reach}(p_1, A)} f(p_2, B)). \]

(e) If $Q_1$ is $p_1 \lor p_2$, then

\[ \text{reach}(Q_1, A) = \text{reach}(p_1, A) \cup \text{reach}(p_2, A), \]

\[ f(Q_1, A) = f(p_1, A) \cup f(p_2, A). \]

(f) If $Q_1$ is $p^*$, then

\[ \text{reach}(Q_1, A) = \{ A \} \cup \text{reach}(p, A), \]

\[ f(Q_1, A) = f(p, A)^* \]

(g) If $Q_1$ is $p[q]$, then

\[ \text{reach}(Q_1, A) = \text{reach}(p, A), \]

\[ f(Q_1, A) = f(p, A)[\bigcup_{B \in \text{reach}(p, A)} f(q, B)]. \]

(h) If $Q_1$ is $[p]$, then

\[ \text{reach}(Q_1, A) = \text{reach}(p, A), \]

\[ f(Q_1, A) = f(p, A). \]

(i) If $Q_1$ is $[p/text() = e]$, then

\[ \text{reach}(Q_1, A) = \emptyset, \]

\[ f(Q_1, A) = [\text{position()} = k]. \]

(j) If $Q_1$ is $[\text{position()} = k]$, then

\[ \text{reach}(Q_1, A) = \emptyset, \]

\[ f(Q_1, A) = [\neg f(q, A)]. \]

(k) If $Q_1$ is $\neg q$, then

\[ \text{reach}(Q_1, A) = \emptyset, \]

\[ f(Q_1, A) = [f(q_1, A) \land f(q_2, A)]. \]

(l) If $Q_1$ is $q_1 \land q_2$, then

\[ \text{reach}(Q_1, A) = \emptyset, \]

\[ f(Q_1, A) = [f(q_1, A) \lor f(q_2, A)]. \]

Given these, we define $F(Q)$ to be $f(Q, r_1)$. One can easily verify that $F$ is indeed a query translation function such that for any instance $T$ of $S_1$, $Q(T) = F(Q)(\sigma_d(T))$, again by induction on the structure of $Q$.

(2) For the complexity of the query translation function $F$, note that $|\text{reach}(Q, A)|$ is bounded by $|S_1|$ and thus $f(Q, A)$ is bounded by $O(|Q| |\text{path}| |S_1|)$. The computation of $f(Q, A)$ and $\text{reach}(Q, A)$ can be conducted by dynamic programming, and it takes at most $O(|Q| |\text{path}| |S_1|)$ time to compute $F(Q)$.

The inverse function $\sigma_d^{-1}$ is defined along the same lines as the function in the proof of Theorem 3.3. Given any $\sigma_d(T)$ in $T(S_2)$, it takes at most $O(|\sigma_d(T)|^2)$ time to compute the source instance $T$. 

\[ \square \]
Proof of Theorem 4.3

Suppose that there exists a valid embedding $\sigma : S_1 \rightarrow S_2$, where $S_1 = (E_1, P_1, r_1)$ and $S_2 = (E_2, P_2, r_2)$, and $\sigma = (\lambda, \text{path})$. Consider an arbitrary edge $(A, B)$ in $S_1$.

(1) $A$ is a concatenation type. Then $\text{path}(A, B)$ is an AND $X_R$ path that can be simplified to one that contains at most $k$ cycles, where $k$ cycles may be necessary to ensure that $\text{path}(A, B)$ is not a prefix of any $\text{path}(A, B')$ for distinct subelement types $B, B'$ of $A$. Any other cycles can be removed, and all of the $k$ cycles can be made simple cycles (i.e., a cycle that does not contain repeated labels), while the modified $\sigma$ remains well defined. Thus $|\text{path}(A, B)|$ is bounded by $k |E_2|$. 

(2) $A$ is a disjunction type. Then $\text{path}(A, B)$ is a disjunction $X_R$ path that can be simplified to one that contains at most $k + 1$ simple cycles: $k$ cycles to ensure that $\text{path}(A, B)$ is not a prefix of any $\text{path}(A, B')$, where $B'$ is another subelement type of $A$, and an additional cycle to include a dashed edge. After the simplification the modified $\sigma$ remains well defined. Thus $|\text{path}(A, B)| \leq (k + 1) |E_2|$. 

(3) $A$ is defined to be a Kleene closure $A \rightarrow B^*$. Then $\text{path}(A, B)$ is a star $X_R$ path, which can be simplified such that $\text{path}(A, B)$ contains at most one simple cycle (to include a star edge). Thus $\text{path}(A, B) \leq 2 |E_2|$. 

(4) $A$ is defined to be $A \rightarrow \text{str}$. As in (1), $\text{path}(A, B)$ is no longer than $|E_2|$. 

Proof of Theorem 5.1

We show that the schema embedding problem is NP-complete. A NP algorithm is as follows: guess a mapping, and then check whether it is an embedding; the latter can be done in PTIME. The NP-hardness is verified by reduction from 3SAT, which is NP-complete (cf. [14]). It suffices to show that the problem is NP-hard for nonrecursive DTDs, by reduction from 3SAT. An instance of 3SAT is a well-formed Boolean formula $\phi = C_1 \land \cdots \land C_n$ of which we want to decide satisfiability.

Given an instance $\phi$ of 3SAT, we define two nonrecursive DTDs $S_1, S_2$ such that if $\phi$ is satisfiable iff there is a valid schema embedding from $S_1$ to $S_2$. We define a similarity matrix $\mathbf{att}$ such that for all element types $A$ in $S_1$ and $B$ in $S_2$, $\mathbf{att}(A, B) = 1$, i.e., there is no restriction on the mapping. Assume that all the propositional variables in $\phi$ are $x_1, \ldots, x_m$. We define $S_1, S_2$ as follows.

$S_1 = (E_1, P_1, r_1)$, where

$E_1 = \{r_1, Z, W\} \cup \{C_i \mid i \in [1, n]\} \cup \{Y_s \mid s \in [1, m]\}$

$P_1$ is defined as:

$r \rightarrow C_i_1, \ldots, C_i_n, Y_1, \ldots, Y_m$

$C_i \rightarrow Z, \ldots, Z, \; /\!* n + i \text{ occurrences of } Z/\!

Y_s \rightarrow W, \ldots, W, \; /\!* 2n + s \text{ occurrences of } W^*/\!

A \rightarrow \epsilon \; /\!* \text{for } A \text{ ranging over } W, Z/\!

$(a)$ Source $S_1$

$S_2 = (E_2, P_2, r_2)$, where

$E_2 = \{r_2, Z, W\} \cup \{C_i \mid i \in [1, n]\}$

$\cup \{X_s, T_s, F_s \mid s \in [1, m]\}$

$P_2$ is defined as:

$r \rightarrow C_i_1, \ldots, C_i_m, Z, W$

$C_i \rightarrow \epsilon \; /\!* \text{for } C_i \text{ ranging over } W, Z/\!

\text{The DTDs are depicted in Fig. 12(a) and 12(b), respectively. Note that both}$

$S_1, S_2$ are nonrecursive and are defined in terms of concatenation types only. Intuitively, $S_2$ encodes $\phi$, and $S_1$ is to assert the existence of a truth assignment to $x_1, \ldots, x_m$ that satisfies all the clauses in $\phi$.

In both $S_1$ and $S_2$, $C_i$ is to code clause $C_i$, which has a "signature" consisting of $n + i$ occurrences of $Z$ that is to ensure that $C_i$ in $S_1$ is mapped to $C_i$ in $S_2$. In $S_2$, $X_j$ codes the variable $x_j$ in $\phi$, which may have either a true value or false, indicated by $T_i$ and $F_i$, respectively. In DTD $S_1$, $Y_s, Y_1, \ldots, Y_m$ are to code the "negation" of a truth assignment $\mu$ to variables in $\phi$: $Y_s$ is mapped to $F_s$ if $\mu(x_s)$ is true for some $j \in [1, m]$, and $Y_s$ is mapped to $T_s$ if $\mu(x_s)$ is false. This is asserted by the number of $W$ children below $Y_s$ and $T_s, F_s$.

We next show that $S_1, S_2$ are indeed a reduction from 3SAT, i.e., there is a valid embedding from $S_1$ to $S_2$ iff $\phi$ is satisfiable. First, suppose that $\phi$ is satisfiable. Then there exists a truth assignment $\mu$ to $x_1, \ldots, x_m$ that satisfies $\phi$. We define an embedding $\sigma = (\lambda, \text{path})$ such that $\lambda(C_i) = C_i$, $\lambda(Z) = Z$, $\lambda(W) = W$, $\lambda(Y_j) = F_j$ if $\mu(x_j)$ is true, $\lambda(Y_j) = T_j$ if $\mu(x_j)$ is false; furthermore, $\text{path}(r_1, C_i)$ is a path $r_2$ from $r_2$ to $C_i$ in $S_2$ such that there exists $j \in [1, m]$ and $X_j / T_j$ is on $r_2$ if clause $C_i$ is satisfied by $\mu(x_j) = true$, and $X_j / F_j$ is on $r_1$ if clause $C_i$ is satisfied by $\mu(x_j) = false$; since $\phi$ is satisfied by $\mu$, there must exist such a
variable $x_j$ for every $C_i$. It is easy to verify that $\sigma$ is indeed an embedding from $S_1$ to $S_2$.

Conversely, suppose that there exists a valid embedding $\sigma = (\lambda, \text{path})$ from $S_1$ to $S_2$. Observe that $\sigma$ must have the following properties. (1) $\lambda(C_i)$ is either $C_i$ or $V_i$, where $V_i$ is either $T_i$ or $F_i$; and (2) $\lambda(Y_j)$ is mapped to $V_j$, where $V_j$ is either $T_j$ or $F_j$, such that $\lambda(Y_j) \neq \lambda(Y_k)$ and $\lambda(Y_j) \neq \lambda(Y_i)$ for $k \neq j, i \neq j$; and furthermore, for each $x_j$ there exists $Y_j$ such that $\lambda(Y_j) = V_j$. This is because, by the definitions of $S_1, S_2$, (1) $\lambda(C_i)$ must have $n + i$ descendants of type $Z$, like $C_i$ in $S_1$; and (2) $\lambda(Y_j)$ must have $2n + j$ descendants of type $W$, and may not be an ancestor of $\lambda(Y_j)$ or $\lambda(C_i)$, and vice versa. We define a truth assignment $\mu$ such that $\mu(x_s)$ is true if $\lambda(Y_s) = F_s$ and $\mu(x_s)$ is false if $\lambda(Y_s) = T_s$. It is easy to verify that $\mu$ satisfies $\phi$. □

**Proof of Theorem 5.2**

We show that the Local-Embedding problem is NP-hard for nonrecursive DTDs, by reduction from 3SAT. Given an instance $\phi = C_1 \land \cdots \land C_n$ of 3SAT, we define two nonrecursive DTDs $S_1, S_2$, where $S_1$ consists of a single production, and a similarity matrix att such that $\phi$ is satisfiable iff there is a valid schema embedding from $S_1$ to $S_2$ w.r.t. att. Assume that all the propositional variables in $\phi$ are $x_1, \ldots, x_m$. We define $S_1, S_2$ as follows.

$S_1 = (E_1, P_1, r_1)$, where $E_1 = \{r_1\} \cup \{C_i \mid i \in [1, n]\} \cup \{Y_s \mid s \in [1, m]\}$;  
$P_1$ is defined as: $r \rightarrow C_1, \ldots, C_n, Y_1, \ldots, Y_m$

$S_2 = (E_2, P_2, r_2)$, where $E_2 = \{r_2\} \cup \{C_i \mid i \in [1, n]\} \cup \{X_s, T_s, F_s \mid s \in [1, m]\}$;  
$P_2$ is defined as: $r \rightarrow X_1, \ldots, X_m$,  
$X_i \rightarrow T_i, F_i$,  
$T_i \rightarrow C_1, \ldots, C_n \forall$ all $C_j$ in which $x_i$ appears*/  
$F_i \rightarrow C_1, \ldots, C_n \forall$ all $C_j$ in which $\neg x_i$ appears */  
$A \rightarrow \epsilon \forall$ for $A$ ranging over $C_i$.

Similar to the proof of Theorem 5.1, we use $Y_1, \ldots, Y_m$ in $S_1$ to code the “negation” of a truth assignment $\mu$ to variables in $\phi$. In both $S_1$ and $S_2$, $C_i$ is to code clause $C_i$. Note that both $S_1, S_2$ are nonrecursive and are defined in terms of concatenation types only. Furthermore, $S_1$ consists of a single production.

The similarity matrix att is defined such that $\text{att}(C_i, C_j) = 1$, and $\text{att}(C_i, X) = 0$ for any $X \neq C_i$.  
$\text{att}(Y_j, Z_j) = 1$ if $Z_j = T_j$ or $Z_j = F_j$, and  
$\text{att}(Y_j, Z) = 0$ if $Z \neq T_j$ and $Z \neq F_j$.

That is, $C_i$ in $S_1$ can only map to $C_i$ in $S_2$, and $Y_j$ in $S_1$ can only map to either $T_j$ or $F_j$.

Along the same lines as in the proof of Theorem 5.1, one can verify that $Y_j$ in $S_1$ can only be mapped to the negation of the truth value of $X_j$, and as a result, $\phi$ is satisfiable iff there is a valid schema embedding from $S_1$ to $S_2$ w.r.t. att. □

**Algorithm findpaths**

**Input:** Directed graph $G$, source node $s$, a bag of target nodes $L_{ta} = \{t_1, \ldots, t_k\}$.

**Output:** Paths $p_1, \ldots, p_k$ satisfying the prefix-free condition, or “no” if such paths do not exist.

1. Let $G_s$ be the subgraph of $G$ reachable from $s$.  
2. if any $t_i$ is not in $G_s$, return “no”;  
3. Let $G'_s$ be the component graph of $G_s$; /* see text */  
4. Let $C$ be the subset of the components $c_i$ in $G'_s$ with a nonempty target shadow; /* see text */  
5. Let $L_{ta}'$ be the subset of $L_{ta}$ not in the shadow of any $c_i$;  
6. Add each $c_i$ with a nonempty target shadow to $L_{ta}'$;  
7. Let $G'' = G'_s$ with the shadow of each $c_i$ removed;  
8. Let $\mathcal{P} = \text{findPathsDAG}(G'', s, L_{ta}'')$;  
9. For each $P_i \in \mathcal{P}$ that ends at a new target for some $c_i$:  
10. remove $P_i$ from $\mathcal{P}$;  
11. add $\text{findPathCycle}(G, G_s, P_i, c_i, L_{ta})$ to $\mathcal{P}$;  
12. return $\mathcal{P}$;

**Figure 13:** Algorithm findpaths

**Prefix-free Paths in Cyclic Graphs**

In Section 5.1, an algorithm was given for finding paths in an acyclic graph. In this section, we describe how this algorithm can be generalized to handle cycles.

The overall algorithm has three parts: 1) break up the original graphs into a DAG of connected components, 2) solve this problem for the DAG case, 3) add back in the connected components and 4) use the cycles found in these components to create prefix-free paths to nodes in the component or reachable therefrom. We now describe these steps in more detail.

In Figure 13 the overall algorithm for finding a set of prefix-free paths from a single root $s$ to a bag of target nodes in a possibly cyclic graph is given. In step 3 the algorithm computes a “component graph” of a graph $G$ which we define as the graph produced from $G$ by replacing each cyclic connected component in $G$ with a new node, $c_i$, and creating edges to and from $c_i$ for each edge which entered or left component $i$ in the original graph. Note that the resulting graph is a directed graph containing component nodes $c_i$ as well as any node in $G$ that did not participate in a cycle.

We now introduce some notations: a node $n \in V$ is said to be in the shadow of $c_i$ if $n$ can be reached in $G$ by a node in $c_i$. The target shadow of $c_i$ is the subset of $L_{ta}$ in the shadow of $c_i$. Note that a target node may be in the shadow of more than one component node.

To continue with the discussion of findpaths, the algorithm at line 4-5 removes cyclic components and all nodes reachable from them from the component graph. For each cyclic node that contains a target node or from which a target node is reachable, a node for that component is added back to the graph as a new target node. The idea is that the computed path to this node will be used as the prefix for all paths in that component or reachable therefrom. A bag $L''$ is computed from $L_{ta}$ by removing any target
Algorithm findpathCycle \((G, G_c, \rho, c_i, L_{a})\)

Input: Directed graph \(G\), component graph \(G_c\), connected component \(c_i\), path to \(c_i\) in \(G_c\), target nodes \(T = t_1, \ldots, t_b\) in shadow of \(c_i\)

Output: Paths \(p_1, \ldots, p_\delta\) for \(t_1\) to \(t_b\)

1. While \(L_{a}\) not empty
2. Let \(Y\) be a cycle in \(L_{a}\) containing at least one node from \(L_{a} \cap c_i\) and the last node in \(\rho\);
3. \(\text{count} := 0;\)
4. For each node \(n\) in \(L_{a}\), in reverse topological order
   - If \(n\) is reachable from \(Y\)
   - Output \(p_n = \rho + \text{count trips around } Y\)
   - Path from last node in \(Y\) to \(n;\)
6. Remove \(n\) from \(L_{a}\);
7. \(\text{count}++;\)

Figure 14: Algorithm findpathCycle

nodes no longer in the graph and adding any of the just-mentioned component nodes. Finally, for each component node \(c_i\) with a non-empty target cycle, findpathCycle is called to construct paths to target nodes reachable from \(c_i\).

Procedure findpathCycle is shown in Figure 14. The intuition for this algorithm is as follows: consider a path consisting of the nodes in a cycle, say \(y_1, \ldots, y_m, y_1\), followed by three nodes, \(a, b, c\). How can we construct prefix free paths to \(a, b,\) and \(c\)? Clearly we can create an arbitrary number of paths to each node by going around \(y_1, \ldots, y_m, y_1\) some number of times. If we form paths to \(b, c,\) and \(y_1\), say \(P_b\) and \(P_c\), it is easy to see that the prefix-free property will only hold if there are more instances of the \(y\) cycle in the path to \(P_b\) than that to \(P_c\).

Proof of Theorem 5.3

We show that the Assembly-Embedding problem is \(\text{NP-hard}\) for nonrecursive \(\text{DTDs}\), by reduction from \(3\text{SAT}\). Given an instance \(\phi = C_1 \land \cdots \land C_n\) of \(3\text{SAT}\), we define two nonrecursive \(\text{DTDs}\) \(S_1, S_2\) and a table \(\text{local}\) such that \(\phi\) is satisfiable iff there exists a valid schema embedding from \(S_1\) to \(S_2\) that is formed by composing a subset of the assignments found in local. The similarity matrix \(\text{att}\) is defined such that for all element types \(A\) in \(S_1\) and \(B\) in \(S_2\), \(\text{att}(A, B) = 1\), i.e., there is no restriction on the mapping. Assume that all the propositional variables in \(\phi\) are \(x_1, \ldots, x_m\). We define \(S_1, S_2\) as follows.

\[ S_1 = (E_1, P_1, r_1), \]
\[ E_1 = \{ r_1 \} \cup \{ C_i, V_i | i \in [1, n]\} \cup \{ X_j | j \in [1, m]\}; \]
\[ P_1 \text{ is defined as:} \]
\[ r \rightarrow C_1, \ldots, C_n \]
\[ C_i \rightarrow V_i \]
\[ V_i \rightarrow X^1_i, X^2_i, X^3_i \quad /\ast \quad X^1_i, X^2_i, X^3_i \text{ are the variables in } C_i \ast/ \]

\[ S_2 = (E_2, P_2, r_2), \]
\[ E_2 = \{ r_2 \} \cup \{ C_i, V_{(i,j)} | i \in [1, n], j \in [1, 7]\} \cup \{ T_j, F_j | j \in [1, m]\}; \]
\[ P_2 \text{ is defined as:} \]
\[ r \rightarrow C_1, \ldots, C_n \]
\[ C_i \rightarrow V_{(i,1)}, \ldots, V_{(i,7)} \quad /\ast \quad \text{for } i \in [1, n]\ast/ \]
\[ V_{(i,j)} \rightarrow Y_{s_1}, Y_{s_2}, Y_{s_3} \quad /\ast \quad \text{see below}\ast/ \]

That is, each of the local embeddings in \(\text{local}(V_i)\) is a truth assignment \(\mu_i\) to those variable involved in the clause \(C_i\) such that \(\mu_i\) satisfies \(C_i\). In other words, any local embedding in \(\text{local}(V_i)\) satisfies \(C_i\). Note that a schema embedding formed composing a subset of the local has to contain one and only one local embedding from \(\text{local}(X)\) for each \(X\) in \(S_1\) such that these local embeddings do not conflict with each other. That is, each variable in the embeddings has a unique truth value: in other words, there cannot be local embeddings \(\sigma_i = (\lambda_i, \text{path}_i)\) from \(\text{local}(V_i)\) and \(\sigma_j = (\lambda_j, \text{path}_j)\) from \(\text{local}(V_j)\) such that \(\sigma_i[X_s] = T_s\) and \(\sigma_j[X_s] = F_s\) for any \(X_s\).

One can verify that \(\phi\) is satisfiable iff there exists a valid schema embedding \(\sigma : S_1 \rightarrow S_2\) formed by composing a subset of local, one from the set of local embeddings for each production in \(S_1\).

Proof of Theorem 5.4

Suppose that the algorithm for Max-Weight-Independent-Set returns a subset \(V'\) of \(V\) such that \(|V'| = |E_1|\). We show that \(\sigma\) constructed from \(V'\) is a valid embedding. To do so, it suffices to show the following: (1) for any element type \(A \in E_1\), there exists a unique \(v_{\sigma A}\) in \(V'\) such that \(\sigma_A\) is a local embedding for \(A\); and (2) for any \(v_{\sigma A}, v_{\sigma A'}\) in \(V'\), \(\sigma_A\) and \(\sigma_A'\) do not conflict with each other. For if these hold, then the union of all the local embeddings corresponding to nodes in \(V'\) is a valid embedding from \(S_1\) to \(S_2\).

To see (1), observe that in the construction of \(G\), for any pair local embeddings \(\sigma_A, \sigma_A'\) for the same element type \(A\), there is a conflict edge between \(v_{\sigma A}\) and \(v_{\sigma A'}\). Since
$V'$ is an independent set, it cannot possibly contain more than one local embedding for $A$. Then from $|V'| = |E|$ it follows that for each type $A$ in $E$, there exists exactly one local embedding $v_{\sigma_A}$ for $A$ such that $v_{\sigma_A}$ is in $V'$.

To see (2), note that $V'$ is an independent set, and thus for any nodes $v_{\sigma_A}, v_{\sigma_B}$ in $V'$, there exists no conflict edge between the two in $G$. That is, $\sigma_A$ and $\sigma_B$ are consistent with each other. \qed