Answering Graph Pattern Queries Using Views

Wenfei Fan [1,2] Xin Wang [3] Yinghui Wu [4]
[1]University of Edinburgh [2]RCBD and SKLSDE Lab, Beihang University
[3]Southwest Jiaotong University [4]UC Santa Barbara
{wenfei@inf, x.wang-36@sms}.ed.ac.uk, yinghui@cs.ucsb.edu

***Abstract*—Answering queries using views has proven an effective technique for querying relational and semistructured data. This paper investigates this issue for graph pattern queries based on (bounded) simulation, which have been increasingly used in, *e.g.*, social network analysis. We propose a notion of *pattern containment* to characterize graph pattern matching using graph pattern views. We show that a graph pattern query can be answered using a set of views *if and only if* the query is contained in the views. Based on this characterization we develop efficient algorithms to answer graph pattern queries. In addition, we identify three problems associated with graph pattern containment. We show that these problems range from quadratic-time to NP-complete, and provide efficient algorithms for containment checking (approximation when the problem is intractable). Using real-life data and synthetic data, we experimentally verify that these methods are able to efficiently answer graph pattern queries on large social graphs, by using views.**

I. Introduction

Answering queries using views has been extensively studied for relational queries [19], [20], [25], XML [22], [36], [37] and semistructured data [11], [32], [38]. Given a query Q and a set $\mathcal{V} = \{\mathsf{V}_1, \ldots, \mathsf{V}_n\}$ of views, the idea is to find another query A such that A is equivalent to Q, and A only refers to views in $\mathcal{V}$ [19]. This yields an effective technique for evaluating Q: if such a query A exists, then given a database D, one can compute the answer $Q(D)$ to Q in D by using A, which uses only the data in the materialized views $\mathsf{V}_i(D)$, *without accessing* D. This is particular effective when D is "big" and/or distributed. Indeed, views have been advocated for *scale independence*, to query big data independent of the size of the underlying data [8]. They are also useful in data integration [25], data warehousing, semantic caching [13], and access control [14].

The need for studying this problem is even more evident for answering graph pattern queries (*a.k.a.* graph pattern matching) [16], [21]. Graph pattern queries have been increasingly used in social network analysis [10], [16], among other things. Real-life social graphs are typically large, and are often distributed. For example, Facebook currently has more than 1 billion users with 140 billion links [3], and the data is geo-distributed to various data centers [18]. One of the major challenges for social network analysis is how to cope with the sheer size of real-life social data when evaluating graph pattern queries. Graph pattern matching using views provides an effective method to query such data.

Example 1: A fraction of a recommendation network is depicted as a graph G in Fig. 1 (a), where each node denotes a person with name and job title (*e.g.*, project manager (PM), database administrator (DBA), programmer (PRG), business analyst (BA) and software tester (ST)); and each edge indicates collaboration, *e.g.*, (Bob, Dan) indicates that Dan worked well with Bob on a project led by Bob.

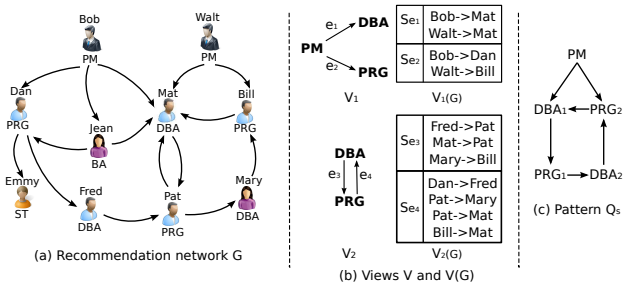


Fig. 1: Data graph, views and pattern queries

To build a team, a human resource manager issues a pattern query [23]. The query, expressed as Q_s in Fig. 1 (c), is to find a group of PM, DBA and PRG. It requires that (1) DBA_1 and PRG_2 worked well under the project manager PM; (2) each PRG (resp. DBA) had been supervised by a DBA (resp. PRG), represented as a collaboration cycle [23] in Q_s. For pattern matching based on *graph simulation* [16], [34], the answer $\mathsf{Q}_\mathsf{s}(G)$ to Q_s in G can be denoted as a set of pairs (e, S_e) such that for each pattern edge e in Q_s, S_e is a set of edges (a match set) for e in G. For example, pattern edge $(\mathsf{PM}, \mathsf{PRG}_2)$ has a match set $S_e = \{(\mathsf{Bob}, \mathsf{Dan}), (\mathsf{Walt}, \mathsf{Bill})\}$, in which each edge matches the node labels and satisfies the connectivity constraint of the pattern edge $(\mathsf{PM}, \mathsf{PRG}_2)$.

It is known that it takes $O(|\mathsf{Q}_\mathsf{s}|^2 + |\mathsf{Q}_\mathsf{s}||G| + |G|^2)$ time to compute $\mathsf{Q}_\mathsf{s}(G)$ [16], [21], where $|G|$ (resp. $|\mathsf{Q}_\mathsf{s}|$) is the size of G (resp. Q_s). This is a daunting cost when G is big. For example, to identify the match set of each pattern edge $(\mathsf{DBA}_i, \mathsf{PRG}_i)$ (for $i \in [1, 2]$), each pair of (DBA, PRG) in G has to be checked, and moreover, a number of *join* operations have to be performed to eliminate invalid matches.

One can do better by leveraging a set of *views*. Suppose that a set of views $\mathcal{V} = \{\mathsf{V}_1, \mathsf{V}_2\}$ is defined, materialized and cached ($\mathcal{V}(G) = \{\mathsf{V}_1(G), \mathsf{V}_2(G)\}$), as shown in Fig. 1 (b). Then as will be shown later, to compute $\mathsf{Q}_\mathsf{s}(G)$, (1) we only need to visit views in $\mathcal{V}(G)$, *without* accessing the original big graph G; and (2) $\mathsf{Q}_\mathsf{s}(G)$ can be efficiently computed by "merging" views in $\mathcal{V}(G)$. Indeed, $\mathcal{V}(G)$ already contains partial answers to Q_s in G: for each query edge e in Q_s, the matches of e (*e.g.*, $(\mathsf{DBA}_1, \mathsf{PRG}_1)$) are contained either in $\mathsf{V}_1(G)$ or $\mathsf{V}_2(G)$ (*e.g.*, the matches of e_3 in V_2). These partial answers can be used to construct $\mathsf{Q}_\mathsf{s}(G)$. As a result, the cost of computing $\mathsf{Q}_\mathsf{s}(G)$ is quadratic in $|\mathsf{Q}_\mathsf{s}|$ and $|\mathcal{V}(G)|$, where $\mathcal{V}(G)$ is typically *much smaller than* G. □

This example suggests that we conduct graph pattern matching by capitalizing on available views. To do this, several

questions have to be settled. (1) How to decide whether a pattern query Q_s can be answered by a set $\mathcal{V}$ of views? (2) If so, how to efficiently compute $Q_s(G)$ from $\mathcal{V}(G)$? (3) Which views in $\mathcal{V}$ should we choose to answer Q_s?

Contributions. This paper investigates these questions for answering *graph pattern queries* using *graph pattern views*. We focus on pattern matching defined in terms of *graph simulation* [21] and *bounded simulation* [16], which are particularly useful in detecting *social communities and positions* [10].

(1) To characterize when graph pattern queries can be answered using views based on graph simulation, we propose a notion of *pattern containment* (Section III). It extends the traditional notion of query containment [6] to deal with *a set of views*. Given a pattern query Q_s and a set $\mathcal{V} = \{V_1, \ldots, V_n\}$ of view definitions, we show that Q_s can be answered using $\mathcal{V}$ *if and only if* Q_s is contained in $\mathcal{V}$.

We also provide an evaluation algorithm for answering graph pattern queries using views (Section III). Given Q_s and a set $\mathcal{V}(G)$ of views on a graph G, the algorithm computes $Q_s(G)$ in $O(|Q_s||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time, *without accessing* G at all when Q_s is contained in $\mathcal{V}$. It is far less costly than $O(|Q_s|^2+|Q_s||G|+|G|^2)$ for evaluating Q_s directly on G [16], [21], since G is typically *much larger* than $\mathcal{V}(G)$ in practice.

(2) To decide which views in $\mathcal{V}$ to use when answering Q_s, we identify three fundamental problems for pattern containment (Section IV). Given Q_s and $\mathcal{V}$, (i) the *containment problem* is to decide whether Q_s is contained in $\mathcal{V}$; (ii) the *minimal containment problem* is to identify a subset of $\mathcal{V}$ that *minimally contains* Q_s, and (iii) the *minimum containment problem* is to find a minimum subset of $\mathcal{V}$ that contains Q_s.

We establish the complexity of these problems. We show that the first two problems are in quadratic-time, whereas the last one is NP-complete and approximation-hard. These results are not only useful in answering pattern queries using views, but are also interesting for *query minimization*. Indeed, when $\mathcal{V}$ contains a single view, the containment problem becomes the classical query containment problem [6].

These results are a nice surprise. Note that even for relational conjunctive queries, the problem of query containment is NP-complete [6]; for XPath fragments, it is EXPTIME-complete or even undecidable [30]. In contrast, the (minimal) containment problem for graph pattern queries is in low PTIME, although graph pattern matching via (bounded) simulation may be "recursively defined" (for cyclic patterns).

(3) We develop efficient algorithms for checking (minimal, minimum) pattern containment (Section V). For containment and minimal containment checking, we provide quadratic-time algorithms in the sizes of *query* Q_s and *view definitions* $\mathcal{V}$, which are *much smaller* than graph G in practice. For minimum containment, we provide an efficient approximation algorithm with performance guarantees.

(4) We show that all these results carry over to bounded simulation [16] (Section VI). More specifically, the notion of pattern containment, the algorithm for answering pattern queries using views, the three containment problems, and the (approximation) algorithms for checking (minimal, minimum) pattern containment can be extended to bounded simulation, with the same or comparable complexity.

(5) Using real-life data (Amazon, YouTube and Citation) and synthetic data, we experimentally verify the effectiveness and efficiency of our view-based matching method (Section VII). We find that this method reduces 94% of the time used by prior methods for bounded pattern queries on large datasets on average [16]. Moreover, our matching algorithm scales well with both the data size and pattern size; and our algorithms for (minimal, minimum) pattern containment checking take less than 0.5 second on complex (cyclic) patterns. Furthermore, we find that our optimization methods by identifying minimal (minimum) containment effectively reduce redundant views and improve the performance by 46% on average.

This work is a first step toward understanding graph pattern matching using views, from theory to practical methods. We contend that the method is effective: one may pick and cache previous query results, and efficiently answer pattern queries using these views *without* accessing the large social graphs. Better still, incremental methods are already in place to efficiently maintain cached pattern views (*e.g.,* [15]). The view-based method can be readily *combined* with existing distributed, compression and incremental techniques for graphs, and yield a promising approach to querying "big" social data.

The proofs of the results of this work can be found in [4].

Related Work. There are two view-based approaches for query processing: query rewriting and query answering [20], [25]. Given a query Q and a set $\mathcal{V}$ of views, (1) query rewriting is to reformulate Q into an equivalent query Q' in a fixed language (*i.e.,* for all D, $Q(D) = Q'(D)$), such that Q' refers only to $\mathcal{V}$; and (2) query answering is to compute $Q(D)$ by evaluating an equivalent query A of Q, while A refers only to $\mathcal{V}$ and its extensions $\mathcal{V}(D)$. While the former requires that Q' is in a fixed language, the latter imposes no constraint on A. We study answering graph pattern queries using pattern views.

We next review previous work on these issues for relational databases, XML data and general graphs.

Relational data. Query answering using views has been extensively studied for relational data (see [6], [20], [25] for surveys). It is known that for conjunctive queries, query answering and rewriting using views are already intractable [20], [25]. For the containment problem, the homomorphism theorem shows that one conjunctive query is contained in another if and only if there exists a homomorphism between the tableaux representing the queries, and it is NP-complete to determine the existence of such a homomorphism [6]. Moreover, the containment problem for conjunctive queries is NP-complete, and is undecidable for relational algebra [6].

XML queries. There has been a host of work on processing XML queries using views [29], [30], [33]. In [29], the containment of simple XPath queries is shown coNP-complete. When disjunction, DTDs and variables are taken into account, the problem ranges from coNP-complete to EXPTIME-complete to undecidable for various XPath classes [30]. In [7], pattern containment and query rewriting of XML are studied under constraints expressed as a structural summary. For tree pattern queries (a fragment of XPath), [22], [36] study maximally contained rewriting instead of equivalent rewriting.

Semistructured data and RDF. There has also been work on view-based query processing for semistructured data and RDF, which are also modeled as graphs.

(1) *Semistructure data.* Views defined in Lorel are studied in, *e.g.,* [38], which are quite different from graph patterns considered here. View-based query rewriting for regular path queries (RPQs) is shown PSPACE-complete in [11], and an EXPTIME rewriting algorithm is given in [32]. The containment problem is shown undecidable for RPQs in the presence of path constraints [17] and for extended conjunctive RPQs [9].

(2) *RDF.* An EXPTIME query rewriting algorithm is given in [24] for SPARQL. It is shown in [12] that query containment is in EXPTIME for PSPARQL, which supports regular expressions. There has also been work on evaluating SPARQL queries on RDF based on cached query results [13].

Our work differs from the prior work in the following. (1) We study query answering using views for graph pattern queries via (bounded) simulation, which are quite different from previous settings, from complexity bounds to processing techniques. (2) We show that the containment problem for the pattern queries is in PTIME, in contrast to its intractable counterparts for *e.g.,* XPath, regular path queries and SPARQL. (3) We study a more general form of query containment between a query $\mathsf{Q_s}$ and *a set of* queries, to identify an equivalent query for $\mathsf{Q_s}$ that is not necessarily a pattern query. (4) The high complexity of previous methods for query answering using views hinders their applications in the real world. In contrast, our algorithms have performance guarantees and yield a practical method for querying real-life social networks.

We focus on (bounded) simulation in this work as it is widely used in social data analysis [10], [16]. Nonetheless, the techniques can be extended to revisions of simulation such as dual and strong simulation [28] (see Section VIII).

II. Graphs, Patterns and Views

We first review pattern queries and graph simulation. We then state the problem of pattern matching using views.

A. Data Graphs and Graph Pattern Queries

Data graphs. A *data graph* is a directed graph $G = (V, E, L)$, where (1) V is a finite set of nodes; (2) $E \subseteq V \times V$, in which (v, v') denotes an edge from node v to v'; and (3) L is a function such that for each node v in V, $L(v)$ is a set of labels from an alphabet Σ. Intuitively, L specifies the attributes of a node, *e.g.,* name, keywords, social roles [23].

Pattern queries [16]. A graph pattern query, denoted as $\mathsf{Q_s}$, is a directed graph $\mathsf{Q_s} = (V_p, E_p, f_v)$, where (1) V_p and E_p are the set of *pattern nodes* and the set of *pattern edges*, respectively; and (2) f_v is a function defined on V_p such that for each node $u \in V_p$, $f_v(u)$ is a label in Σ. We remark that f_v can be readily extended to specify search conditions in terms of Boolean predicates [16] (see Fig. 7 for examples).

Graph pattern matching via simulation. We say that a data graph $G = (V, E, L)$ *matches* a query $\mathsf{Q_s} = (V_p, E_p, f_v)$ *via simulation*, denoted by $\mathsf{Q_s} \trianglelefteq_{\mathsf{sim}} G$, if there exists a binary relation $S \subseteq V_p \times V$, refereed to as a *match* in G for $\mathsf{Q_s}$, such that

- for each node $u \in V_p$, there exists a node $v \in V$ such that $(u, v) \in S$, referred to as a *match* of u; and
- for each pair $(u, v) \in S$, $f_v(u) \in L(v)$; for each pattern edge $e = (u, u')$ in E_p, there exists an edge (v, v') in E, referred to as a *match* of e in S, such that $(u', v') \in S$.

When $\mathsf{Q_s} \trianglelefteq_{\mathsf{sim}} G$, it is known that there exists a *unique maximum* match S_o in G for $\mathsf{Q_s}$ [21]. We derive $\{(e, S_e) \mid e \in E_p\}$ from S_o, where S_e is the set of all matches of e in S_o, called *the match set* of e. Here S_e is *nonempty* for all $e \in E_p$.

We define the *result* of $\mathsf{Q_s}$ in G, denoted as $\mathsf{Q_s}(G)$, to be the unique maximum set $\{(e, S_e) \mid e \in E_p\}$ if $\mathsf{Q_s} \trianglelefteq_{\mathsf{sim}} G$, and let $\mathsf{Q_s}(G) = \emptyset$ otherwise. We denote the size of query $\mathsf{Q_s}$ by $|\mathsf{Q_s}|$, and the size of result $\mathsf{Q_s}(G)$ by $|\mathsf{Q_s}(G)|$ (see Table I).

Example 2: Consider the pattern query $\mathsf{Q_s}$ shown in Fig. 1 (c), where each pattern node carries a search condition (job title), and each pattern edge indicates collaboration relationship between two people. When $\mathsf{Q_s}$ is posed on the network G of Fig. 1 (a), the result $\mathsf{Q_s}(G)$ is shown in the table below:

Edge	Matches
(PM, DBA_1)	{(Bob, Mat), (Walt, Mat)}
(PM, PRG_2)	{(Bob, Dan), (Walt, Bill)}
(DBA_1, PRG_1) (DBA_2, PRG_2)	{(Fred, Pat), (Mat,Pat), (Mary, Bill)}
(PRG_1, DBA_2) (PRG_2, DBA_1)	{(Dan, Fred), (Pat, Mary), (Pat, Mat), (Bill, Mat)}

Here (1) both Bob and Walt are matches of pattern node PM as they satisfy the search condition of PM; similarly, Fred, Mat, Mary match DBA, and Dan, Pat, Bill match PRG; (2) query edge (PM, DBA_1) has two matches in G; and (3) query edges (DBA_1, PRG_1) and (DBA_2, PRG_2) (resp. (PRG_1, DBA_2) and (PRG_2, DBA_1)) have the same matches. □

B. Graph Pattern Matching Using Views

We next formulate the problem of graph pattern matching using views. We study *views* V defined as a graph pattern query, and refer to the query result $\mathsf{V}(G)$ in a data graph G as the *view extension* for V in G or simply as a *view* [19].

Given a pattern query $\mathsf{Q_s}$ and a *set* $\mathcal{V} = \{\mathsf{V}_1, \ldots, \mathsf{V}_n\}$ of view definitions, *graph pattern matching using views* is to find another query A such that (1) A is equivalent to $\mathsf{Q_s}$, *i.e.,* $A(G) = \mathsf{Q_s}(G)$ for *all* data graphs G; and (2) A only refers to views $\mathsf{V}_i \in \mathcal{V}$ and their extensions $\mathcal{V}(G) = \{\mathsf{V}_1(G), \ldots, \mathsf{V}_n(G)\}$ in G, without accessing G. If such a query A exists, we say that $\mathsf{Q_s}$ *can be answered using* $\mathcal{V}$.

In contrast to query rewriting using views [19] but along the same lines as query answering using views [25], here A is *not* required to be a pattern query. For example, Fig. 1 (b) depicts a view definition set $\mathcal{V} = \{\mathsf{V}_1, \mathsf{V}_2\}$ and their extensions $\mathcal{V}(G) = \{\mathsf{V}_1(G), \mathsf{V}_2(G)\}$. To answer the query $\mathsf{Q_s}$ (Fig. 1 (c)), we want to find a query A that computes $\mathsf{Q_s}(G)$ by using only $\mathcal{V}$ and $\mathcal{V}(G)$, where A is not necessarily a graph pattern.

For a set $\mathcal{V}$ of view definitions, we define the size $|\mathcal{V}|$ of $\mathcal{V}$ to be the total size of V_i's in $\mathcal{V}$, and the cardinality $\mathsf{card}(\mathcal{V})$ of $\mathcal{V}$ to be the number of view definitions in $\mathcal{V}$.

The notations of the paper are summarized in Table I.

symbols	notations
$Q_s = (V_p, E_p, f_v)$	graph pattern query
$Q_s(G)$	query result of Q_s in G
$\mathcal{V} = (V_1, \ldots, V_n)$	a set of view definitions V_i
$\mathcal{V}(G) = (V_1(G), \ldots, V_n(G))$	a set of view extensions $V_i(G)$
$Q_s \trianglelefteq_{sim} G$ (resp. $Q_b \trianglelefteq^B_{sim} G$)	simulation (resp. bounded simulation)
$Q_s \sqsubseteq \mathcal{V}$ (resp. $Q_b \sqsubseteq \mathcal{V}$)	Q_s (resp. Q_b) is contained in $\mathcal{V}$
$M_V^{Q_s}$ (resp. $M_V^{Q_b}$)	view match from a view V to Q_s (resp. Q_b)
$\lvert Q_s\rvert$ (resp. $\lvert Q_b\rvert$ and $\lvert V\rvert$)	size (total number of nodes and edges) of Q_s (resp. Q_b and view definition V)
$\lvert Q_s(G)\rvert$ (resp. $\lvert Q_b(G)\rvert$)	total number of edges in sets S_e for all edges e in Q_s (resp. Q_b)
$\lvert\mathcal{V}\rvert$	total size of view definitions in $\mathcal{V}$
$\mathsf{card}(\mathcal{V})$	the number of view definitions in $\mathcal{V}$

TABLE I: A summary of notations

Remark. (1) We assume *w.l.o.g.* that graph patterns are connected, since isolated pattern nodes can be easily handled using the same matching semantic. (2) Our techniques can be readily extended to graphs and queries with edge labels. Indeed, an edge-labeled graph can be transformed to a node-labeled graph: for each edge e, add a "dummy" node carrying the edge label of e, along with two unlabeled edges.

III. Pattern Containment: A Characterization

In this section we propose a characterization of graph pattern matching using views, *i.e.,* a *sufficient and necessary* condition for deciding whether a pattern query can be answered by using *a set of views*. We also provide a quadratic-time algorithm for answering pattern queries using views.

Pattern containment. We introduce a notion of pattern containment, by extending the traditional notion of query containment to *a set* of views. Consider a pattern query $Q_s = (V_p, E_p, f_v)$ and a set $\mathcal{V} = \{V_1, \ldots, V_n\}$ of view definitions, where $V_i = (V_i, E_i, f_i)$. We say that Q_s is *contained* in $\mathcal{V}$, denoted by $Q_s \sqsubseteq \mathcal{V}$, if there exists a mapping λ from E_p to powerset $\mathcal{P}(\bigcup_{i\in[1,n]} E_i)$, such that for all data graphs G, the match set $S_e \subseteq \bigcup_{e'\in\lambda(e)} S_{e'}$ for all edges $e \in E_p$.

Example 3: Recall G, $\mathcal{V}$ and Q_s in Fig. 1. Then $Q_s \sqsubseteq \mathcal{V}$. Indeed, there exists a mapping λ from E_p of Q_s to sets of edges in $\mathcal{V}$, which maps edges $(\mathsf{PM}, \mathsf{DBA}_1)$, $(\mathsf{PM}, \mathsf{PRG}_2)$ of Q_s to their counterparts in V_1; both $(\mathsf{DBA}_1, \mathsf{PRG}_1)$, $(\mathsf{DBA}_2, \mathsf{PRG}_2)$ of Q_s to e_3, and $(\mathsf{PRG}_1, \mathsf{DBA}_2)$, $(\mathsf{PRG}_2, \mathsf{DBA}_1)$ to e_4 in V_2. One may verify that for any graph G and any edge e of Q_s, its matches in G are contained in the union of the match sets of the edges in $\lambda(e)$, *e.g.,* the match set of pattern edge $(\mathsf{DBA}_1, \mathsf{PRG}_1)$ in G is $\{(\mathsf{Fred}, \mathsf{Pat}), (\mathsf{Mat}, \mathsf{Pat}), (\mathsf{Mary}, \mathsf{Bill})\}$, which is contained in the match set of e_3 of V_2 in G. □

Pattern containment and query answering. The main result of this section is as follows: (1) pattern containment indeed characterizes pattern matching using views; and (2) when $Q_s \sqsubseteq \mathcal{V}$, for all graphs G, $Q_s(G)$ can be efficiently computed by using views $\mathcal{V}(G)$ only, *independent of* $|G|$. In Sections IV and V we will show how to decide whether $Q_s \sqsubseteq \mathcal{V}$ by inspecting Q_s and $\mathcal{V}$ only, also *independent of* $|G|$.

Theorem 1: *(1) A pattern query Q_s can be answered using $\mathcal{V}$* if and only if $Q_s \sqsubseteq \mathcal{V}$. *(2) For any graph G, $Q_s(G)$ can be computed in $O(|Q_s||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time if $Q_s \sqsubseteq \mathcal{V}$.* □

Proof sketch: Below we outline the proof (see [4] for details).

Input: A pattern query Q_s, a set of view definitions $\mathcal{V}$ and their extensions $\mathcal{V}(G)$, a mapping λ.
Output: The query result M as $Q_s(G)$.

1. **for** each edge e in Q_s **do** $S_e := \emptyset$;
2. $M := \{(e, S_e) \mid e \in Q_s\}$;
3. **for each** $e \in Q_s$ **do**
4. **for each** $e' \in \lambda(e)$ **do** $S_e := S_e \cup S_{e'}$;
5. **while** there is change in S_{e_p} for an edge $e_p = (u, u'')$ in Q_s **do**
6. **for each** $e = (u', u)$ in Q_s and $e' = (v', v) \in S_e$ **do**
7. **if** there is $e_1 = (u', u_1)$ but no $e'_1 = (v', v_1)$ in S_{e_1} **then**
8. $S_e := S_e \setminus \{e'\}$;
9. **if** there is $e_2 = (u, u_2)$ but no $e'_2 = (v, v_2)$ in S_{e_2} **then**
10. $S_e := S_e \setminus \{e'\}$;
11. **if** $S_e = \emptyset$ **then return** $\emptyset$;
12. **return** $M = \{(e, S_e) \mid e \in Q_s\}$, which is $Q_s(G)$;

Fig. 2: Algorithm MatchJoin

(I) We prove the **Only If** condition in Theorem 1(1) by contradiction. Assume that Q_s can be answered using $\mathcal{V}$ and $Q_s \not\sqsubseteq \mathcal{V}$. By $Q_s \not\sqsubseteq \mathcal{V}$, there exists at least an edge e of Q_s that cannot be mapped to any edge in the match sets from $\mathcal{V}$. We show that for such an edge e, one may always construct a data graph G that matches Q_s, but no match in G can be identified by using only $\mathcal{V}$. This contradicts the assumption that Q_s can be answered using $\mathcal{V}$ by definition (Section II-B).

(II) We show the **If** condition of Theorem 1(1) by a constructive proof: we next present an algorithm to evaluate Q_s using $\mathcal{V}(G)$, if $Q_s \sqsubseteq \mathcal{V}$. We verify Theorem 1(2) by showing that the algorithm is in $O(|Q_s||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time. □

Algorithm. The algorithm, denoted as MatchJoin, is shown in Fig. 2. It takes as input (1) a pattern query Q_s and a set of view definitions $\mathcal{V} = \{V_i \mid i \in [1, n]\}$, (2) a mapping λ for $Q_s \sqsubseteq \mathcal{V}$ (we defer the computation of λ to Section V); and (3) view extensions $\mathcal{V}(G) = \{V_i(G) \mid i \in [1, n]\}$. In a nutshell, it computes $Q_s(G)$ by "merging" (joining) views $V_i(G)$ as guided by λ. The merge process iteratively identifies and removes those edges that are not matches of Q_s, until *a fixpoint* is reached and $Q_s(G)$ is correctly computed.

More specifically, MatchJoin works as follows. It first initializes M with empty match sets S_e for each pattern edge e (lines 1-2). MatchJoin sets S_e as $\bigcup_{e'\in\lambda(e)} S_{e'}$, where $S_{e'}$ is extracted from $\mathcal{V}(G)$ (lines 3-4), following the definition of $\lambda(e)$ (Section II). It then performs a fixpoint computation to remove all invalid matches from S_e (lines 5-10). Specifically, for a pattern edge $e_p = (u, u'')$ in Q_s with changed match set S_e, it checks whether each match e' of a pattern edge $e = (u', u)$ still remains to be a match of e (lines 7-10), by the definition of simulation (Section II-A). If e' is no longer a match, it is removed from S_e (lines 8,10). In the process, if S_e becomes empty for some edge e, MatchJoin returns $\emptyset$ since Q_s has no match in G. Otherwise, the process (lines 5-11) proceeds until M=$Q_s(G)$ is computed and returned (line 12).

Example 4: Given Q_s, $\mathcal{V}$, $\mathcal{V}(G)$ of Fig. 1, and the mapping λ of Example 3, MatchJoin evaluates Q_s using $\mathcal{V}$ and $\mathcal{V}(G)$. For each edge e of Q_s, its match set S_e is exactly $\bigcup_{e'\in\lambda(e)} S_{e'}$, which yields the same $Q_s(G)$ as given in Example 2.

Consider another example shown in Fig. 3. One can verify $Q_s \sqsubseteq \mathcal{V}$ by a mapping λ that maps $(\mathsf{AI}, \mathsf{Bio})$, $(\mathsf{PM}, \mathsf{AI})$ to e_1, e_2

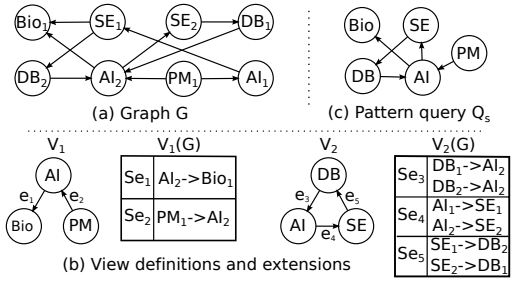


Fig. 3: Answering pattern queries using views

in V_1, respectively; and $(\mathsf{DB}, \mathsf{AI})$, $(\mathsf{AI}, \mathsf{SE})$, $(\mathsf{SE}, \mathsf{DB})$ to e_3, e_4, e_5 in V_2, respectively. MatchJoin then merges view matches guided by λ. It next removes $(\mathsf{AI}_1, \mathsf{SE}_1)$ from $S_{(\mathsf{AI},\mathsf{SE})}$, which is not a valid match for $(\mathsf{AI}, \mathsf{SE})$ in $\mathsf{Q_s}$. This further leads to the removal of $(\mathsf{SE}_1, \mathsf{DB}_2)$ from $S_{(\mathsf{SE},\mathsf{DB})}$, $(\mathsf{DB}_2, \mathsf{AI}_2)$ from $S_{(\mathsf{DB},\mathsf{AI})}$, and yields $\mathsf{Q_s}(G)$ shown in the table below.

Edge	Matches	Edge	Matches
$(\mathsf{PM}, \mathsf{AI})$	$(\mathsf{PM}_1, \mathsf{AI}_2)$	$(\mathsf{AI}, \mathsf{Bio})$	$(\mathsf{AI}_2, \mathsf{Bio}_1)$
$(\mathsf{DB}, \mathsf{AI})$	$(\mathsf{DB}_1, \mathsf{AI}_2)$	$(\mathsf{AI}, \mathsf{SE})$	$(\mathsf{AI}_2, \mathsf{SE}_2)$
$(\mathsf{SE}, \mathsf{DB})$	$(\mathsf{SE}_2, \mathsf{DB}_1)$		

□

Correctness & Complexity. Denote the match set in G as S_e^* for each edge e in $\mathsf{Q_s}$. One may verify that MatchJoin preserves two invariants: (1) at any time, for each edge e of $\mathsf{Q_s}$, $S_e^* \subseteq S_e$; and (2) $S_e = S_e^*$ when MatchJoin terminates. Indeed, S_e is initialized with $\bigcup_{e' \in \lambda(e)} S_{e'}$, hence $S_e^* \subseteq S_e$ due to $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. During the **while** loop (lines 5-10), MatchJoin repeatedly refines S_e by removing invalid matches only (lines 8,10) until S_e can no longer be refined. Thus, $S_e = S_e^*$ when the algorithm terminates. From these the correctness of MatchJoin follows.

For the complexity, it takes $O(|\mathsf{Q_s}|)$ time to initialize M (lines 1-2), and $O(|\mathsf{Q_s}||\mathcal{V}|)$ time to initialize S_e (lines 3-4). MatchJoin then removes invalid matches by using a dynamically maintained index, which maps pattern edges to their possible matches (see [4] for details). The **while** loop (lines 5-11) is bounded by $O(|\mathcal{V}(G)|^2)$ time. Putting these together, MatchJoin is in $O(|\mathsf{Q_s}||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time.

The analysis above completes the proof of Theorem 1.

Remark. (1) It takes $O(|\mathsf{Q_s}|^2 + |\mathsf{Q_s}||G| + |G|^2)$ time to evaluate $\mathsf{Q_s}(G)$ directly on G [16]. In contrast, MatchJoin is in $O(|\mathsf{Q_s}||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time, *without* accessing G. As will be seen in Section VII, $\mathcal{V}(G)$ is much smaller than G, and MatchJoin is more efficient than the algorithm of [16]. Indeed, for *Youtube* graph in our experiments, only 3 to 6 views are used to answer $\mathsf{Q_s}$, and the overall size of $\mathcal{V}(G)$ is no more than 4% of the size of the *Youtube* graph.

Optimization. MatchJoin may visit each S_e multiple times. To reduce unnecessary visits, below we introduce an optimization strategy for MatchJoin. The strategy evaluates $\mathsf{Q_s}$ by using *ranks* in $\mathsf{Q_s}$ as follows. Given a pattern $\mathsf{Q_s}$, the strongly connected component graph $\mathsf{G_{SCC}}$ of $\mathsf{Q_s}$ is obtained by collapsing each strongly connected component SCC of $\mathsf{Q_s}$ into a single node $s(u)$. The rank $r(u)$ of each node u in $\mathsf{Q_s}$ is computed as follows: (a) $r(u) = 0$ if $s(u)$ is a leaf in $\mathsf{G_{SCC}}$, where u is in the SCC $s(u)$; and (b) $r(u) = \max\{(1 + r(u')) \mid (s(u), s(u')) \in E_{\mathsf{SCC}}\}$ otherwise. Here E_{SCC} is the edge set of the $\mathsf{G_{SCC}}$ of $\mathsf{Q_s}$. The rank $r(e)$ of an edge $e = (u', u)$ in $\mathsf{Q_s}$ is set to be $r(u)$.

Bottom-up strategy. We revise MatchJoin by processing edges e in $\mathsf{Q_s}$ following an ascending order of their ranks (lines 5-11). One may verify that this "bottom-up" strategy guarantees the following for the number of visits.

Lemma 2: *For all edges $e = (u', u)$, where u' and u do not reach any non-singleton* SCC *in* $\mathsf{Q_s}$, MatchJoin *visits its match set S_e at most once, using the bottom-up strategy.* □

In particular, when $\mathsf{Q_s}$ is a DAG pattern (*i.e.*, acyclic), MatchJoin visits each match set at most once, and the total visits are bounded by the number of the edges in $\mathsf{Q_s}$. As will be verified in Section VII, the optimization strategy improves the performance by at least 46% over (possibly cyclic) patterns, and is even more effective over denser data graphs.

IV. Pattern Containment Problems

We have seen that given a pattern query $\mathsf{Q_s}$ and a set $\mathcal{V}$ of views, we can efficiently answer $\mathsf{Q_s}$ by using the views when $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. In the next two sections, we study how to determine whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. Our main conclusion is that there are efficient algorithms for these, with their costs as a function of $|\mathsf{Q_s}|$ and $|\mathcal{V}|$, which are typically small in practice, and are *independent of* data graphs and materialized views.

In this section we study three problems in connection with pattern containment, and establish their complexity. In the next section, we will develop effective algorithms for checking $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ and computing mapping λ from $\mathsf{Q_s}$ to $\mathcal{V}$.

Pattern containment problem. The *pattern containment problem* is to determine, given a pattern query $\mathsf{Q_s}$ and a set $\mathcal{V}$ of view definitions, whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. The need for studying this problem is evident: Theorem 1 tells us that $\mathsf{Q_s}$ can be answered by using views of $\mathcal{V}$ if and only if $\mathsf{Q_s} \sqsubseteq \mathcal{V}$.

The result below tells us that $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ can be efficiently decided, in quadratic-time in $|\mathsf{Q_s}|$ and $|\mathcal{V}|$. We will prove the result in Section V, by providing such an algorithm.

Theorem 3: *Given a pattern query $\mathsf{Q_s}$ and a set $\mathcal{V}$ of view definitions, it is in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2 + |\mathcal{V}|^2 + |\mathsf{Q_s}||\mathcal{V}|)$ time to decide whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ and if so, to compute a mapping λ from $\mathsf{Q_s}$ to $\mathcal{V}$, where $|\mathcal{V}|$ is the size of view definitions.* □

A special case of pattern containment is the classical *query containment* problem [6]. Given two pattern queries $\mathsf{Q_{s1}}$ and $\mathsf{Q_{s2}}$, the latter is to decide whether $\mathsf{Q_{s1}} \sqsubseteq \mathsf{Q_{s2}}$, *i.e.*, whether for all graphs G, $\mathsf{Q_{s1}}(G)$ is contained in $\mathsf{Q_{s2}}(G)$. Indeed, when $\mathcal{V}$ contains only a single view definition $\mathsf{Q_{s2}}$, pattern containment becomes query containment. From this and Theorem 3 the result below immediately follows.

Corollary 4: *The query containment problem for graph pattern queries is in quadratic time.* □

Like for relational queries (see, *e.g.*, [6]), the query containment analysis is important in minimizing and optimizing

pattern queries. Corollary 4 shows that the analysis can be efficiently conducted for graph patterns, as opposed to the intractability of its counterpart for relational conjunctive queries.

Minimal containment problem. As shown in Section III, the complexity of pattern matching using views is dominated by $|\mathcal{V}(G)|$. This suggests that we reduce the number of views used for answering $\mathsf{Q_s}$. Indeed, the less views are used, the smaller $|\mathcal{V}(G)|$ is. This gives rise to *the minimal containment problem*. Given $\mathsf{Q_s}$ and $\mathcal{V}$, it is to find a minimal subset $\mathcal{V}'$ of $\mathcal{V}$ that contains $\mathsf{Q_s}$. That is, (1) $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$, and (2) for any proper subset $\mathcal{V}''$ of $\mathcal{V}'$, $\mathsf{Q_s} \not\sqsubseteq \mathcal{V}''$.

The good news is that the minimal containment problem does not make our lives harder. We will prove the next result in Section V by developing a quadratic-time algorithm.

Theorem 5: *Given $\mathsf{Q_s}$ and $\mathcal{V}$, it is in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2+|\mathcal{V}|^2+|\mathsf{Q_s}||\mathcal{V}|)$ time to find a minimal subset $\mathcal{V}'$ of $\mathcal{V}$ containing $\mathsf{Q_s}$ and a mapping λ from $\mathsf{Q_s}$ to $\mathcal{V}'$ if $\mathsf{Q_s} \sqsubseteq \mathcal{V}$.* □

Minimum containment problem. One may also want to find a *minimum* subset $\mathcal{V}'$ of $\mathcal{V}$ that contains $\mathsf{Q_s}$. The *minimum containment* problem, denoted by MMCP, is to find a subset $\mathcal{V}'$ of $\mathcal{V}$ such that (1) $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$, and (2) for any subset $\mathcal{V}''$ of $\mathcal{V}$, if $\mathsf{Q_s} \sqsubseteq \mathcal{V}''$, then $\mathsf{card}(\mathcal{V}') \leq \mathsf{card}(\mathcal{V}'')$.

As will be seen shortly (Examples 6 and 7) and verified by our experimental study, MMCP analysis often finds smaller $\mathcal{V}'$ than that found by minimal containment checking.

MMCP is, however, nontrivial: its decision problem is NP-complete and it is APX-hard. Here APX is the class of problems that allow PTIME algorithms with approximation ratio bounded by a constant (see [35] for APX). Nonetheless, we show that MMCP is approximable within $O(\log|E_p|)$ in low polynomial time, where $|E_p|$ is the number of edges of $\mathsf{Q_s}$. That is, there exists an efficient algorithm that identifies a subset $\mathcal{V}'$ of $\mathcal{V}$ with *performance guarantees* whenever $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ such that $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$ and $|\mathsf{card}(\mathcal{V}')| \leq \log(|E_p|)|\mathsf{card}(\mathcal{V}_{\mathsf{OPT}})|$, where $\mathcal{V}_{\mathsf{OPT}}$ is a minimum subset of $\mathcal{V}$ that contains $\mathsf{Q_s}$.

Theorem 6: *The minimum containment problem is (1) NP-complete (its decision problem) and APX-hard, but (2) it is approximable within $O(\log|E_p|)$ in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2+|\mathcal{V}|^2+|\mathsf{Q_s}||\mathcal{V}|+(|\mathsf{Q_s}|\cdot\mathsf{card}(\mathcal{V}))^{3/2})$ time.* □

Proof sketch: (I) The decision problem of MMCP is to decide whether there exists a subset $\mathcal{V}'$ of $\mathcal{V}$ such that $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$ and $\mathsf{card}(\mathcal{V}') \leq k$, where k is an integer bound. It is in NP since there exists an NP algorithm that first guesses $\mathcal{V}'$ and then checks whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$ and $\mathsf{card}(\mathcal{V}') \leq k$ in PTIME. The lower bound is verified by reduction from the NP-complete *set cover* problem (cf. [31]). Given a set X, a collection $\mathcal{U}$ of its subsets and an integer B, the latter is to decide whether there is a B-element subset of $\mathcal{U}$ that covers X. We show that there exists a set cover of size k if and only if there exists a k-element subset $\mathcal{V}'$ of $\mathcal{V}$ that contains $\mathsf{Q_s}$.

(II) The APX-hardness of MMCP is verified by approximation preserving reduction [35] from the minimum set cover problem, which is APX-hard (cf. [35]; see [4] for details). □

We defer the proof of Theorem 6(2) to Section V, where an approximation algorithm is provided.

V. Determining Pattern Containment

We next prove Theorems 3, 5 and 6(2) by providing effective (approximation) algorithms for checking pattern containment, minimal containment and minimum containment, in Sections V-A, V-B and V-C, respectively.

A. Pattern Containment

We start with a proof of Theorem 3, *i.e.*, whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ can be decided in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2+|\mathcal{V}|^2+|\mathsf{Q_s}||\mathcal{V}|)$ time. To do this, we first propose a *sufficient and necessary* condition to characterize pattern containment. We then develop a quadratic-time algorithm based on the characterization.

Sufficient and necessary condition. To characterize pattern containment, we introduce a notion of *view matches*.

Consider a pattern query $\mathsf{Q_s}$ and a set $\mathcal{V}$ of view definitions. For each $\mathsf{V} \in \mathcal{V}$, let $\mathsf{V}(\mathsf{Q_s}) = \{(e_\mathsf{V}, S_{e_\mathsf{V}}) \mid e_\mathsf{V} \in \mathsf{V}\}$, by treating $\mathsf{Q_s}$ as a *data graph*. Obviously, if $\mathsf{V} \trianglelefteq_{\mathsf{sim}} \mathsf{Q_s}$, then S_{e_V} is the nonempty match set of e_V for each edge e_V of V (see Section II-A). We define the *view match* from V to $\mathsf{Q_s}$, denoted by $M^{\mathsf{Q_s}}_{\mathsf{V}}$, to be the union of S_{e_V} for all e_V in V.

The result below shows that view matches yield a characterization of pattern containment.

Proposition 7: *For view definitions $\mathcal{V}$ and pattern $\mathsf{Q_s}$ with edge set E_p, $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ if and only if $E_p = \bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$.* □

Proof sketch: We outline the proof below (see [4] for details). (1) Assume $E_p = \bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$. We construct a mapping λ from E_p to the edges of the views in $\mathcal{V}$, as a "reversed" view match relation. The mapping λ ensures that for any data graph G, if e_o in G is a match of e in $\mathsf{Q_s}$ ($e_o \in S_e$), there must exist an edge $e_i \in \lambda(e)$ such that $e_o \in S_{e_i}$. Thus, $\mathsf{Q_s} \sqsubseteq \mathcal{V}$.

(2) Assume by contradiction $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ but $E_p \neq \bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$. Then by $E_p \neq \bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$, there exists an edge e in E_p but not in $\bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$. Since $\mathsf{Q_s} \sqsubseteq \mathcal{V}$, if an edge e_o in G matches e in $\mathsf{Q_s}$, then e_o is in S_{e_i} of $\mathsf{V}(G)$ for some $\mathsf{V} \in \mathcal{V}$. These together lead to the contradiction, since if such an e exists, we can expand mapping $\lambda(e)$ by including e_i of V; thus e is "covered" by $M^{\mathsf{Q_s}}_{\mathsf{V}}$. Therefore, if $\mathsf{Q_s} \sqsubseteq \mathcal{V}$, E_p=$\bigcup_{\mathsf{V}\in\mathcal{V}} M^{\mathsf{Q_s}}_{\mathsf{V}}$. □

Algorithm. Following Proposition 7, we present an algorithm, denoted as contain (not shown) to check whether $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. Given a pattern query $\mathsf{Q_s}$ and a set $\mathcal{V}$ of view definitions, it returns a boolean value ans that is true if and only if $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. The algorithm first initializes an empty edge set E to record view matches from $\mathcal{V}$ to $\mathsf{Q_s}$. It then checks the condition of Proposition 7 as follows. (1) Compute view match $M^{\mathsf{Q_s}}_{\mathsf{V}}$ for each V in $\mathcal{V}$, by invoking the simulation evaluation algorithm in [16]. (2) Extend E with $M^{\mathsf{Q_s}}_{\mathsf{V}}$ by union, since $M^{\mathsf{Q_s}}_{\mathsf{V}}$ is a *subset* of E_p. After all view matches are merged, contain then checks whether $\mathsf{E} = E_p$. It returns true if so, and false otherwise.

Example 5: Recall the pattern query $\mathsf{Q_s}$ and views $\mathcal{V} = \{\mathsf{V}_1, \mathsf{V}_2\}$ given in Fig. 1. As remarked earlier, $\mathsf{Q_s} \sqsubseteq \mathcal{V}$. Indeed, one can verify that $\bigcup_{i\in[1,2]} M^{\mathsf{Q_s}}_{\mathsf{V}_i} = E_p$.

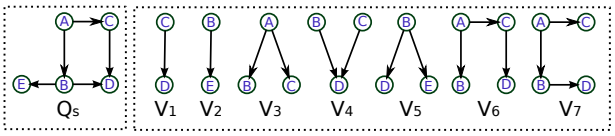


Fig. 4: Containment for pattern queries

V_i	$M_{V_i}^{Q_s}$	V_i	$M_{V_i}^{Q_s}$
V_1	$\{(C, D)\}$	V_2	$\{(B, E)\}$
V_3	$\{(A, B), (A, C)\}$	V_4	$\{(B, D), (C, D)\}$
V_5	$\{(B, D), (B, E)\}$	V_6	$\{(A, B), (A, C), (C, D)\}$
V_7	$\{(A, B), (A, C), (B, D)\}$		

Consider another pattern query Q_s and a set of view definitions $\mathcal{V} = \{V_i \mid i \in [1, 7]\}$ given in Fig. 4. The view matches $M_{V_i}^{Q_s}$ of V_i for $i \in [1, 7]$ are shown in the table above. Given Q_s and $\mathcal{V}$, contain returns true since $\bigcup_{V_i \in \mathcal{V}} M_{V_i}^{Q_s}$ is the set of edges of Q_s. One can verify that $Q_s \sqsubseteq \mathcal{V}$. □

Correctness & Complexity. The correctness of algorithm contain follows from Proposition 7. For each $V \in \mathcal{V}$, it takes $O(|Q_s||V|+|Q_s|^2+|V|^2)$ time to compute $M_V^{Q_s}$ [16], and O(1) time for set union. The **for** loop (lines 2-3) has $\mathsf{card}(\mathcal{V})$ iterations, and it takes $O(\mathsf{card}(\mathcal{V})|Q_s|^2 + |\mathcal{V}|^2 + |Q_s||\mathcal{V}|)$ time in total, since both $\mathsf{card}(\mathcal{V}) \cdot |V|$ and $|V|$ are bounded by $|\mathcal{V}|$.

From these and Proposition 7, Theorem 3 follows.

Remarks. (1) Algorithm contain can be easily adapted to return a mapping λ that specifies pattern containment (Section III), to serve as input for algorithm MatchJoin. This can be done by following the construction given in the proof of Proposition 7. (2) In contrast to regular path queries and relational queries, pattern containment checking is in PTIME.

B. Minimal Containment Problem

We now prove Theorem 5 by presenting an algorithm that, given Q_s and $\mathcal{V}$, finds a minimal subset $\mathcal{V}'$ of $\mathcal{V}$ containing Q_s in $O(\mathsf{card}(\mathcal{V})|Q_s|^2 + |\mathcal{V}|^2 + |Q_s||\mathcal{V}|)$ time if $Q_s \sqsubseteq \mathcal{V}$.

Algorithm. The algorithm, denoted as minimal, is shown in Fig. 5. Given a query Q_s and a set $\mathcal{V}$ of view definitions, it returns either a nonempty subset $\mathcal{V}'$ of $\mathcal{V}$ that minimally contains Q_s, or $\emptyset$ to indicate that $Q_s \not\sqsubseteq \mathcal{V}$.

The algorithm initializes (1) an empty set $\mathcal{V}'$ for selected views, (2) an empty set S for view matches of $\mathcal{V}'$, and (3) an empty set E for edges in view matches. It also maintains an index M that maps each edge e in Q_s to a set of views (line 1). Similar to contain, minimal first computes $M_{V_i}^{Q_s}$ for all $V_i \in \mathcal{V}$ (lines 2-7). However, instead of simply merging the view matches as in contain, it extends S with a new view match $M_{V_i}^{Q_s}$ only if $M_{V_i}^{Q_s}$ contains a new edge not in E, and updates M accordingly (lines 4-7). The **for** loop stops as soon as $E = E_p$ (line 7), as Q_s is already contained in $\mathcal{V}'$. If $E \neq E_p$ after the loop, it returns $\emptyset$ (line 8), since Q_s is not contained is $\mathcal{V}$ (Proposition 7). The algorithm then eliminates redundant views $V_j \in \mathcal{V}'$ (lines 9-11), by checking whether the removal of V_j causes $M(e) = \emptyset$ for some $e \in M_{V_j}^{Q_s}$ (line 10). If no such e exists, it removes V_j from $\mathcal{V}'$ (line 11). After all view matches are checked, minimal returns $\mathcal{V}'$ (line 12).

Input: A pattern query Q_s, and a set of view definitions $\mathcal{V}$.
Output: A subset $\mathcal{V}'$ of $\mathcal{V}$ that minimally contains Q_s.
1. set $\mathcal{V}' := \emptyset$; $S := \emptyset$; $E := \emptyset$; map $M := \emptyset$;
2. **for each** view definition $V_i \in \mathcal{V}$ **do**
3. compute $M_{V_i}^{Q_s}$;
4. **if** $M_{V_i}^{Q_s} \setminus E \neq \emptyset$ **then**
5. $\mathcal{V}' := \mathcal{V}' \cup \{V_i\}$; $S := S \cup \{M_{V_i}^{Q_s}\}$; $E := E \cup M_{V_i}^{Q_s}$;
6. **for each** $e \in M_{V_i}^{Q_s}$ **do** $M(e) := M(e) \cup \{V_i\}$;
7. **if** $E = E_p$ **then break** ;
8. **if** $E \neq E_p$ **then return** $\emptyset$;
9. **for each** $M_{V_j}^{Q_s} \in S$ **do**
10. **if** there is no $e \in M_{V_j}^{Q_s}$ such that $M(e) \setminus \{V_j\} = \emptyset$ **then**
11. $\mathcal{V}' := \mathcal{V}' \setminus \{V_j\}$; update M;
12. **return** $\mathcal{V}'$;

Fig. 5: Algorithm minimal

Example 6: Consider Q_s and $\mathcal{V}$ given in Fig. 4. After $M_{V_i}^{Q_s}$ ($i \in [1, 4]$) are computed, algorithm minimal finds that E already equals E_p, and breaks the loop, where M is initialized to be $\{((A, B) : \{V_3\}), ((A, C) : \{V_3\}), ((B, D) : \{V_4\}), ((C, D) : \{V_1, V_4\}), ((B, E) : \{V_2\})\}$. As the removal of V_1 does not make any $M(e)$ empty, minimal removes V_1 and returns $\mathcal{V}' = \{V_2, V_3, V_4\}$ as a minimal subset of $\mathcal{V}$. □

Correctness & Complexity. To see the correctness of minimal, observe the following: (1) $Q_s \sqsubseteq \mathcal{V}'$ if $\mathcal{V}' \neq \emptyset$; indeed, $\mathcal{V}'$ is returned *only if* the union of the view matches in S equals E_p, *i.e.*, $Q_s \sqsubseteq \mathcal{V}'$ by Proposition 7; and (2) $Q_s \not\sqsubseteq \mathcal{V}''$ for any $\mathcal{V}'' \subset \mathcal{V}'$. To see this, note that by the strategy of minimal for reducing redundant views in $\mathcal{V}'$ (lines 9-11), for *any* $\mathcal{V}'' \subset \mathcal{V}'$, $\bigcup_{V \in \mathcal{V}''} M_V^{Q_s}$ is not equal to E_p, the edge set of Q_s. Hence again by Proposition 7, $Q_s \not\sqsubseteq \mathcal{V}''$.

It takes minimal $O(\mathsf{card}(\mathcal{V})|Q_s|^2 + |\mathcal{V}|^2 + |Q_s||\mathcal{V}|)$ time to find all the view matches of $\mathcal{V}$ (line 3). Its nested loop for M (line 6) takes $O(\mathsf{card}(\mathcal{V}) \cdot |Q_s|)$ time. The redundant elimination is processed in $O(\mathsf{card}(\mathcal{V}) \cdot |Q_s|)$ time (lines 9-11). Thus minimal is in $O(\mathsf{card}(\mathcal{V})|Q_s|^2 + |\mathcal{V}|^2 + |Q_s||\mathcal{V}|)$ time.

From the algorithm and its analyses Theorem 5 follows. Again algorithm minimal can be readily extended to return a mapping λ that specifies containment of Q_s in $\mathcal{V}'$.

C. Minimum Containment Problem

We next prove Theorem 6 (2), *i.e.*, MMCP is approximable within $O(\log |E_p|)$ in $O(\mathsf{card}(\mathcal{V})|Q_s|^2+|\mathcal{V}|^2+|Q_s||\mathcal{V}|+(|Q_s| \cdot \mathsf{card}(\mathcal{V}))^{3/2})$ time. We give such an algorithm for MMCP, following the greedy strategy of the approximation of [35] for the set cover problem. The algorithm of [35] achieves an approximation ratio $O(\log n)$, for an n-element set.

Algorithm. The algorithm is denoted as minimum (not shown). Given a pattern Q_s and a set $\mathcal{V}$ of view definitions, minimum identifies a subset $\mathcal{V}'$ of $\mathcal{V}$ such that (1) $Q_s \sqsubseteq \mathcal{V}'$ if $Q_s \sqsubseteq \mathcal{V}$ and (2) $\mathsf{card}(\mathcal{V}') \leq \log(|E_p|) \cdot \mathsf{card}(\mathcal{V}_{\mathsf{OPT}})$, where $\mathcal{V}_{\mathsf{OPT}}$ is a minimum subset of $\mathcal{V}$ that contains Q_s. In other words, minimum approximates MMCP with approximation ratio $O(\log |E_p|)$. Note that $|E_p|$ is typically small.

Algorithm minimum iteratively finds the "top" view whose view match can cover most edges in Q_s that are not covered.

To do this, we define a metric $\alpha(\mathsf{V})$ for a view V, where

$$\alpha(\mathsf{V}) = \frac{|M_{\mathsf{V}}^{\mathsf{Q_s}} \setminus \mathsf{E_c}|}{|E_p|}.$$

Here $\mathsf{E_c}$ is the set of edges in E_p that have been covered by selected view matches, and $\alpha(V)$ indicates the amount of *uncovered* edges that $M_{\mathsf{V}}^{\mathsf{Q_s}}$ covers. We select V with the largest α in each iteration, and maintain α accordingly.

Similar to minimal, algorithm minimum computes the view match $M_{\mathsf{V}_i}^{\mathsf{Q_s}}$ for each $\mathsf{V}_i \in \mathcal{V}$, and collects them in a set S. It then does the following. (1) It selects view V_i with the largest α, and removes $M_{\mathsf{V}_i}^{\mathsf{Q_s}}$ from S. (2) It merges $\mathsf{E_c}$ with $M_{\mathsf{V}_i}^{\mathsf{Q_s}}$ if $M_{\mathsf{V}_i}^{\mathsf{Q_s}}$ contains some edges that are not in $\mathsf{E_c}$, and extends $\mathcal{V}'$ with V_i. During the loop, if $\mathsf{E_c}$ equals E_p, the set $\mathcal{V}'$ is returned. Otherwise, minimum returns $\emptyset$, indicating that $\mathsf{Q_s} \not\sqsubseteq \mathcal{V}$.

Example 7: Given $\mathsf{Q_s}$ and $\mathcal{V} = \{\mathsf{V}_1, \ldots, \mathsf{V}_7\}$ of Fig. 4, minimum selects views based on their α values. More specifically, in the loop it first chooses V_6, since its view match $M_{\mathsf{V}_6}^{\mathsf{Q_s}}$ = $\{(A, B), (A, C), (C, D)\}$ makes $\alpha(\mathsf{V}_6)$ = 0.6, the largest one. Then V_6 is followed by V_5, as $\alpha(\mathsf{V}_5)$ = 0.4 is the largest one in that iteration. After V_5 and V_6 are selected, algorithm minimum finds that $\mathsf{E_c} = E_p$, and thus $\mathcal{V}' = \{\mathsf{V}_5, \mathsf{V}_6\}$ is returned as a minimum subset that contains $\mathsf{Q_s}$. □

Correctness & Complexity. Observe that minimum finds a nonempty $\mathcal{V}'$ such that $\mathsf{Q_s} \sqsubseteq \mathcal{V}'$ if and only if $\mathsf{Q_s} \sqsubseteq \mathcal{V}$ (Proposition 7). The approximation ratio of minimum can be verified by an approximation-preserving reduction from MMCP to the *set cover problem* [31], by treating each $M_{\mathsf{V}_i}^{\mathsf{Q_s}}$ in S as a subset of E_p. Algorithm minimum extends the algorithm of [35] (with approximation ratio $\log(n)$ for n-element set) to query containment, and preserves approximation ratio $\log|E_p|$.

For the complexity, minimum computes view matches in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2 + |\mathcal{V}|^2 + |\mathsf{Q_s}||\mathcal{V}|)$ time (lines 1-3). The **while** loop is executed $O((|\mathsf{Q_s}| \cdot \mathsf{card}(\mathcal{V}))^{1/2})$ times. Each iteration takes $O(|\mathsf{Q_s}| \cdot \mathsf{card}(\mathcal{V}))$ time to find a view with the largest α. Thus, minimum is in $O(\mathsf{card}(\mathcal{V})|\mathsf{Q_s}|^2 + |\mathcal{V}|^2 + |\mathsf{Q_s}||\mathcal{V}| + (|\mathsf{Q_s}| \cdot \mathsf{card}(\mathcal{V}))^{3/2})$ time, where $|\mathsf{Q_s}|$ and $\mathsf{card}(\mathcal{V})$ are often smaller than $|\mathcal{V}|$. This completes the proof of Theorem 6 (2).

VI. Bounded Pattern Matching Using Views

In this section, we show that the results of the previous sections carry over to *bounded pattern* queries, which extend patterns with distance constraints on pattern edges, and have been verified effective in social network analysis [16].

Bounded pattern queries [16]. A bounded pattern query, denoted as $\mathsf{Q_b}$, is a directed graph (V_p, E_p, f_v, f_e), where (1) V_p, E_p and f_v are the same as in a pattern $\mathsf{Q_s}$ (Section II), and (2) f_e is a function defined on E_p such that for all (u, u') in E_p, $f_e(u, u')$ is either a positive integer k or a symbol $*$.

A data graph $G = (V, E, L)$ *matches* $\mathsf{Q_b}$ *via bounded simulation*, denoted by $\mathsf{Q_b} \trianglelefteq^{\mathsf{B}}_{\mathsf{sim}} G$ (Table I), if there exists a binary relation $S \subseteq V_p \times V$ such that (1) for each node $u \in V_p$, there exists a *match* $v \in V$ such that $(u, v) \in S$, and (2) for each pair $(u, v) \in S$, $f_v(u) \in L(v)$, and for each pattern edge $e = (u, u')$ in E_p, there exists a nonempty *path* from v to v' in G, with its length bounded by k if $f_e(u, u') = k$. When $f_e(u, u') = *$, there is no constraint on the path length.

Intuitively, $\mathsf{Q_b}$ extends pattern queries by mapping an edge (u, u') in E_p to a nonempty path from v to v' in data graph G, such that v can reach v' within $f_e(u, u')$ hops.

It is known that when $\mathsf{Q_b} \trianglelefteq^{\mathsf{B}}_{\mathsf{sim}} G$, there exists a *unique maximum* match S_o in G for $\mathsf{Q_b}$ [16]. Along the same lines as Section II, we define the query result $\mathsf{Q_b}(G)$ to be the *maximum* set $\{(e, S_e) \mid e \in E_p\}$ derived from S_o, where S_e is a set of node pairs for $e = (u, u')$ such that (1) v (resp. v') is a match of u (resp. u'), and (2) the *distance* d from v to v' satisfies the bound specified in $f_e(e)$, *i.e.*, $d \leq k = f_e(e)$.

Example 8: Consider $\mathsf{Q_b} = (V_p, E_p, f_v, f_e)$, a bounded pattern in which (1) V_p, E_p and f_v are the same as in $\mathsf{Q_s}$ of Fig 3; and (2) $f_e(\mathsf{AI}, \mathsf{Bio})$ = 2, and $f_e(e)$ = 1 for all the other edges e. The result $\mathsf{Q_b}(G)$ in graph G of Fig. 3 (a) is:

Edge	Matches	Edge	Matches
$(\mathsf{PM}, \mathsf{AI})$	$(\mathsf{PM}_1, \mathsf{AI}_1), (\mathsf{PM}_1, \mathsf{AI}_2)$	$(\mathsf{AI}, \mathsf{Bio})$	$(\mathsf{AI}_1, \mathsf{Bio}_1), (\mathsf{AI}_2, \mathsf{Bio}_1)$
$(\mathsf{DB}, \mathsf{AI})$	$(\mathsf{DB}_1, \mathsf{AI}_2), (\mathsf{DB}_2, \mathsf{AI}_2)$	$(\mathsf{AI}, \mathsf{SE})$	$(\mathsf{AI}_1, \mathsf{SE}_1), (\mathsf{AI}_2, \mathsf{SE}_2)$
$(\mathsf{SE}, \mathsf{DB})$	$(\mathsf{SE}_1, \mathsf{DB}_2), (\mathsf{SE}_2, \mathsf{DB}_1)$		

Note that the pattern edge $(\mathsf{AI}, \mathsf{Bio})$ has a match $(\mathsf{AI}_1, \mathsf{Bio}_1)$, which denotes a path $\big((\mathsf{AI}_1, \mathsf{SE}_1), (\mathsf{SE}_1, \mathsf{Bio}_1)\big)$ of length 2. □

Observe that pattern queries (Section II) are a special case of bounded patterns when $f_e(e) = 1$ for all edges e. While bounded patterns are more expressive, they do not incur extra complexity when it comes to query answering using views (Section VI-A) and their containment analysis (Section VI-B).

A. Answering Bounded Pattern Queries

Given a bounded pattern query $\mathsf{Q_b}$ and a set $\mathcal{V}$ of view definitions (expressed as bounded pattern queries), the problem of answering queries using views is to compute $\mathsf{Q_b}(G)$ by only referring to $\mathcal{V}$ and their extensions $\mathcal{V}(G)$.

Pattern containment for $\mathsf{Q_b}$ is defined in the same way as for pattern queries. That is, $\mathsf{Q_b}$ is *contained in* $\mathcal{V}$, denoted as $\mathsf{Q_b} \sqsubseteq \mathcal{V}$, if there exists *a mapping* λ that maps each $e \in E_p$ to a set $\lambda(e)$ of edges in $\mathcal{V}$, such that for any data graph G, the match set $S_e \subseteq \bigcup_{e' \in \lambda(e)} S_{e'}$ for all edges e of $\mathsf{Q_b}$. Along the same lines as Theorem 1, one can readily verify that pattern containment also characterizes whether bounded pattern queries can be answered using views.

Theorem 8: *A bounded pattern query* $\mathsf{Q_b}$ *can be answered using views* $\mathcal{V}$ *if and only if* $\mathsf{Q_b}$ *is contained in* $\mathcal{V}$. □

Better still, answering bounded pattern queries using views is no harder than its counterpart for pattern queries.

Theorem 9: *Answering bounded pattern query* $\mathsf{Q_b}$ *on graph* G *using views* $\mathcal{V}$ *is in* $O(|\mathsf{Q_b}||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ *time.* □

To prove Theorem 9, we outline an algorithm for computing $\mathsf{Q_b}(G)$ by using $\mathcal{V}$ and $\mathcal{V}(G)$ when $\mathsf{Q_b} \sqsubseteq \mathcal{V}$. To cope with edge-to-path mappings, it uses an auxiliary index $I(\mathcal{V})$ such that for each match (v, v') in $\mathcal{V}(G)$ of some edge in $\mathcal{V}$, $I(\mathcal{V})$

includes a pair $\langle (v, v'), d\rangle$, where d is the distance from v to v' in G. Note that the size of $I(\mathcal{V})$ is bounded by $|\mathcal{V}(G)|$.

Algorithm. The algorithm, denoted by BMatchJoin (not shown), takes as input Q_b, $\mathcal{V}$, $\mathcal{V}(G)$, $I(\mathcal{V})$ and a mapping λ from the edges of Q_b to edge sets in $\mathcal{V}$. Similar to algorithm MatchJoin (Fig. 2), it evaluates Q_b by (1) "merging" views in $\mathcal{V}(G)$ to M according to λ, and (2) removing invalid matches. It differs from MatchJoin in the following: for an edge $e_p = (u, u'')$ of Q_b with *changed* S_{e_p}, it reduces match set S_e of a "parent" edge $e = (u', u)$ in Q_b by getting *the distance* d (by querying $I(\mathcal{V})$ in $O(1)$ time) from v' to v_1 (resp. v to v_2), checking whether $(v', v_1) \in S_{e_1}$ (resp. $(v, v_2) \in S_{e_2}$) for pattern edge $e_1 = (u', u_1)$ (resp. $e_2 = (u, u_2)$) such that distance d is no greater than $f_e(u', u_1)$ (resp. $f_e(u, u_2)$), and removing (v', v) from S_e if no (v', v_1) (resp. (v, v_2)) exists. The removal of (v', v) may introduce more invalid matches in M, which are removed repeatedly by BMatchJoin until a fixpoint is reached. Then M is returned as the answer.

The correctness of BMatchJoin follows from Theorem 8. One can verify that BMatchJoin takes $O(|Q_b||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time, the same as the complexity of MatchJoin.

Remarks. (1) Evaluating Q_b directly in a graph G takes cubic-time $O(|Q_b||G|^2)$ [16]. In contrast, it takes $O(|Q_b||\mathcal{V}(G)| + |\mathcal{V}(G)|^2)$ time using views, and $\mathcal{V}(G)$ is *much smaller* than G in practice. (2) The optimization strategy in Section III can be naturally incorporated into BMatchJoin (see details in [4]).

B. Bounded Pattern Containment

We next show that the containment analysis of bounded pattern queries is in cubic-time, up from quadratic-time.

Theorem 10: *Given a bounded pattern query Q_b and a set $\mathcal{V}$ of view definitions, (1) it is in $O(|Q_b|^2|\mathcal{V}|)$ time to decide whether $Q_b \sqsubseteq \mathcal{V}$; (2) the minimal containment problem is also in $O(|Q_b|^2|\mathcal{V}|)$ time; and (3) the minimum containment problem (denoted as* BMMCP*) is (i) NP-complete (decision version) and* APX*-hard, but (ii) approximable within $O(\log |E_p|)$ in $O(|Q_b|^2|\mathcal{V}| + (|Q_b| \cdot \mathsf{card}(\mathcal{V}))^{3/2})$ time.* □

To prove Theorem 10, we extend the notion of view matches (Section IV) to bounded pattern queries. Given a bounded pattern $Q_b = (V_p, E_p, f_v, f_e)$ and a view definition $V = (V^V, E^V, f_v^V, f_e^V)$, we define the *view match* from V to Q_b as follows. (1) We treat Q_b as a *weighted data graph* in which each edge e has a weight $f_e(e)$. The *distance from node u to u' in Q_b* is given by the minimum sum of the edge weights on shortest paths from u to u'. (2) We define $V(Q_b) = \{(e_V, S_{e_V}) \mid e_V \in V\}$ as its counterpart for Q_s, except that for each edge $e_V = (v, v')$ in V, the distance from u to u' in all pairs $(u, u') \in S_{e_V}$ is bounded by k if $f_e^V(e_V) = k$. (3) One may verify that there exists a unique, nonempty maximum set $V(Q_b)$ if $V \trianglelefteq_{sim}^B Q_b$. The *view match* $M_V^{Q_b}$ from V to Q_b is the union of S_{e_V} for all e_V in V.

Example 9: Consider Q_b and $\mathcal{V} = \{V_1, \ldots, V_7\}$ shown in Fig. 6. One may verify that $M_{V_3}^{Q_b} = \{(A, B), (B, E)\}$, where the corresponding node pairs in Q_b satisfies the length constraints imposed by V_3. As another example, it can be

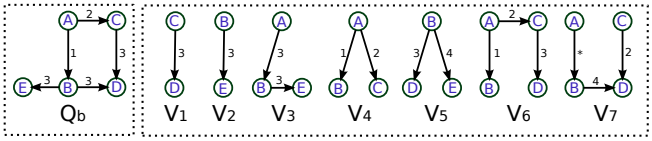


Fig. 6: Containment for bounded pattern queries

shown that the view match $M_{V_7}^{Q_b}$ from V_7 to Q_b is $\emptyset$, since the distance from C to D in Q_b is greater than 2. □

Similar to Proposition 7, the result below gives a *sufficient and necessary* condition for Q_b containment checking.

Proposition 11: *For view definitions $\mathcal{V}$ and bounded pattern query Q_b, $Q_b \sqsubseteq \mathcal{V}$ if and only if $E_p = \bigcup_{V \in \mathcal{V}} M_V^{Q_b}$.* □

Bounded pattern containment. To prove Theorem 10 (1), we give an algorithm for checking bounded pattern containment following Proposition 11, denoted by Bcontain (not shown). Bcontain is the same as contain (Section III) except that it computes view matches differently. More specifically, it extends the algorithm for evaluating bounded pattern queries [16] to weighted graphs. It can be easily verified that it is in $O(|Q_b|^2|\mathcal{V}|)$ time to find all view matches for $\mathcal{V}$. Thus Bcontain decides whether Q_b is contained in $\mathcal{V}$ in $O(|Q_b|^2|\mathcal{V}|)$ time, from which Theorem 10 (1) follows.

Minimal bounded containment. To show Theorem 10 (2), we give an algorithm for minimal containment checking, denoted by Bminimal (not shown). Similar to minimal (Fig. 5), Bminimal first computes view matches for each $V \in \mathcal{V}$, in $O(|Q_b|^2|\mathcal{V}|)$ time, and unions view matches until E equals the edge set E_p of Q_b as described above. Bminimal then follows the same strategies as minimal to eliminate redundant views V_i whose removal will not cause any $M(e) = \emptyset$ for each $e \in M_{V_i}^{Q_b}$. Thus Bminimal is in $O(|Q_b|^2|\mathcal{V}|)$ time.

Minimum bounded containment. Theorem 10 (3) (i) follows from Theorem 6(1), since MMCP is a special case of BMMCP when $f_e(e) = 1$ for all edges. To show Theorem 10 (3) (ii), we give an algorithm for minimum containment checking, denoted by Bminimum (not shown). It is similar to minimum, except that it computes view matches differently. Bminimum takes $O(|Q_b|^2|\mathcal{V}|)$ time to find all view matches of $\mathcal{V}$. Thus, it takes $O(|Q_b|^2|\mathcal{V}|+(|Q_b|\cdot\mathsf{card}(\mathcal{V}))^{3/2})$ time to return a subset of $\mathcal{V}$ no larger than $\log(|E_p|) \cdot \mathsf{card}(\mathcal{V}_{\mathsf{OPT}})$, where $\mathcal{V}_{\mathsf{OPT}}$ is a *minimum* subset of $\mathcal{V}$ that contains Q_b.

VII. Experimental Evaluation

Using real-life and synthetic data, we conducted four sets of experiments to evaluate (1) the efficiency and scalability of algorithm MatchJoin for graph pattern matching using views; (2) the effectiveness of optimization techniques for MatchJoin; (3) the efficiency and effectiveness of (minimal, minimum) containment checking algorithms; and (4) the counterparts of the algorithms in (1) and (3) for bounded pattern queries.

Experimental setting. We used the following data.

(1) Real-life graphs. We used three real-life graphs: (a) *Amazon* [1], a product co-purchasing network with 548K nodes and 1.78M edges. Each node has attributes such as title, group and sales-rank, and an edge from product x to y indicates that people who buy x also buy y. (b) *Citation* [2], with 1.4M nodes and 3M edges, in which nodes represent papers with attributes such as title, authors, year and venue, and edges denote citations. (c) *YouTube* [5], a recommendation network with 1.6M nodes and 4.5M edges. Each node is a video with attributes such as category, age and rate, and each edge from x to y indicates that y is in the related list of x.

(2) Synthetic data. We designed a generator to produce random graphs, controlled by the number $|V|$ of nodes and the number $|E|$ of edges, with node labels from an alphabet Σ.

(3) Pattern and view generator. We implemented a generator for bounded pattern queries controlled by four parameters: the number $|V_p|$ of pattern nodes, the number $|E_p|$ of pattern edges $|E_p|$, label f_v from Σ, and an upper bound k for $f_e(e)$ (Section VI), which draws an edge bound randomly from $[1, k]$. When $k = 1$ for all edges, bounded patterns are pattern queries. We use $(|V_p|, |E_p|)$ (resp. $(|V_p|, |E_p|, k)$) to present the size of a (resp. bounded) pattern query.

We generated a set $\mathcal{V}$ of 12 view definitions for *each* real-life dataset. (a) For *Amazon*, we generated 12 frequent patterns following [27], where each of the view extensions contains in average $5K$ nodes and edges. The views take 14.4% of the physical memory of the entire Amazon dataset. (b) For *Citation*, we designed 12 views to search for papers and authors in computer science. The view extensions account for 12% of the Citation graph. (c) We generated 12 views for *Youtube*, shown in Fig. 7, where each node specifies videos with Boolean search conditions specified by *e.g.*, age (A), length (L), category (C), rate (R) and visits (V). Each view extension has about 700 nodes and edges, and put together they take 4% of the memory for Youtube.

For synthetic graphs, we randomly constructed a set $\mathcal{V}$ of 22 views with node labels drawn from a set Σ of 10 labels. We cached their view extensions (query results), which take in total 26% of the memory for the data graphs.

(4) Implementation. We implemented the following algorithms, all in Java: (1) contain, minimum and minimal for checking pattern containment; (2) Bcontain, Bminimum and Bminimal for bounded pattern containment; (3) Match, $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{mnl}}$, where Match is the matching algorithm without using views [16], [21]; and $\mathsf{MatchJoin}_{\mathsf{min}}$ (resp. $\mathsf{MatchJoin}_{\mathsf{mnl}}$) revises MatchJoin by using a minimum (resp. minimal) set of views; (4) BMatch, $\mathsf{BMatchJoin}_{\mathsf{min}}$ and $\mathsf{BMatchJoin}_{\mathsf{mnl}}$, where BMatch evaluates bounded pattern queries without using views [16], and $\mathsf{BMatchJoin}_{\mathsf{min}}$ and $\mathsf{BMatchJoin}_{\mathsf{mnl}}$ are the counterparts of $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{mnl}}$ for bounded pattern queries, respectively; and (5) a version of MatchJoin (resp. BMatchJoin) without using the ranking optimization (Section III), denoted by $\mathsf{MatchJoin}_{\mathsf{nopt}}$ (resp. $\mathsf{BMatchJoin}_{\mathsf{nopt}}$).

All the experiments were run on a machine powered by an Intel Core(TM)2 Duo 3.00GHz CPU with 4GB of memory, using scientific Linux. Each experiment was run 5 times and the average is reported here.

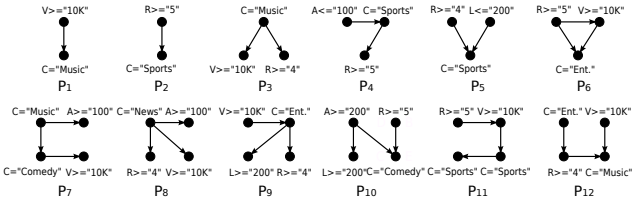


Fig. 7: Youtube views

Experimental results. We next present our findings.

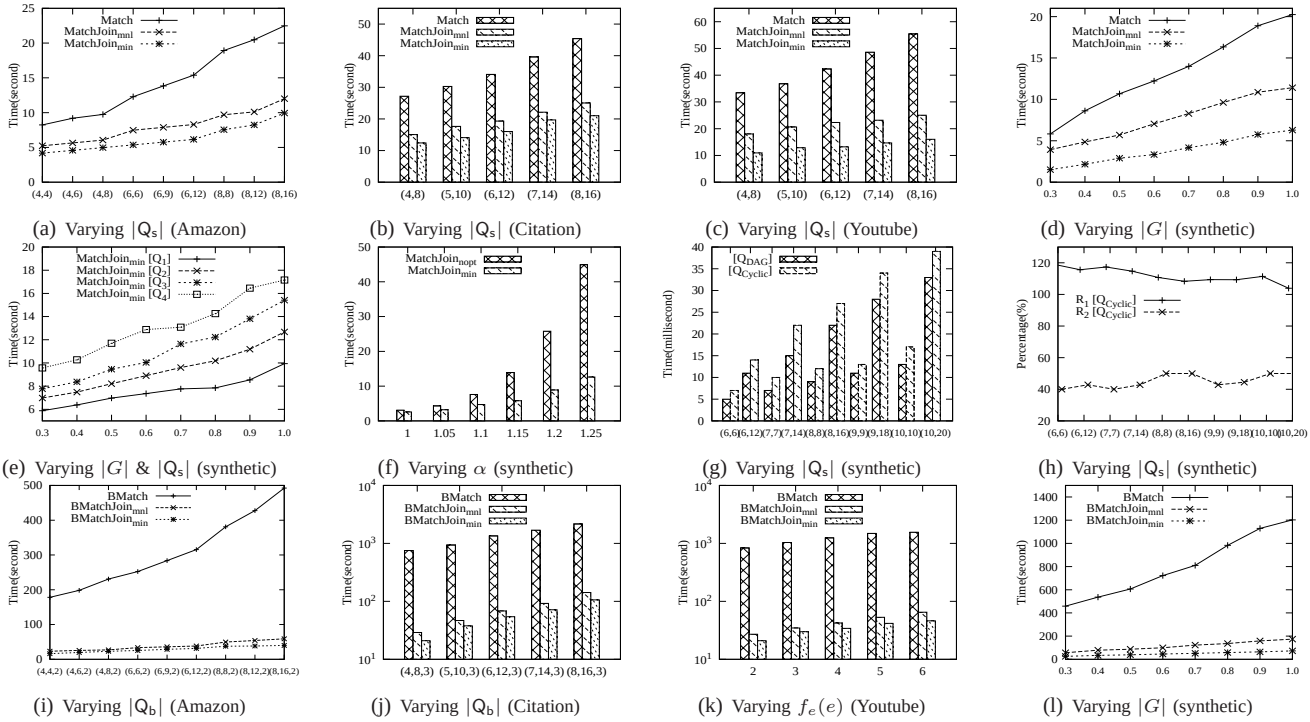
Exp-1: Query answering using views. We first evaluated the performance of graph pattern matching using views, *i.e.*, algorithms $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{mnl}}$, compared to Match [16], [21]. Using real-life data, we studied the efficiency of $\mathsf{MatchJoin}_{\mathsf{min}}$, $\mathsf{MatchJoin}_{\mathsf{mnl}}$ and MatchJoin, by varying the size of the queries. We also evaluated the scalability of these three algorithms with large synthetic datasets.

Efficiency. Figures 8(a), 8(b) and 8(c) show the results on *Amazon*, *Citation* and *YouTube*, respectively. The x-axis represents pattern size $(|V_p|, |E_p|)$. The results tell us the following. (1) $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{mnl}}$ substantially outperform Match, taking only 45% and 57% of its running time on average over all real-life datasets. (2) All three algorithms spend more time on larger patterns. Nonetheless, $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{mnl}}$ are less sensitive than Match, since they reuse previous computation cached in the views.

Scalability. Using large synthetic graphs, we evaluated the scalability of $\mathsf{MatchJoin}_{\mathsf{min}}$, $\mathsf{MatchJoin}_{\mathsf{mnl}}$ and Match. Fixing pattern size with $|V_p| = 4$ and $|E_p| = 6$, we varied the node number $|V|$ of data graphs from $0.3M$ to $1M$, in $0.1M$ increments, and set $|E| = 2|V|$. As shown in Fig. 8(d), (1) $\mathsf{MatchJoin}_{\mathsf{min}}$ scales best with $|G|$, consistent with the complexity analysis of MatchJoin; and (2) $\mathsf{MatchJoin}_{\mathsf{min}}$ accounts for about 49% of the time of $\mathsf{MatchJoin}_{\mathsf{mnl}}$. This verifies that evaluating pattern queries by using less view extensions *significantly reduces* computational time, which is consistent with the observation of Figures 8(a), 8(b) and 8(c).

To further evaluate the impact of pattern sizes on the performance of $\mathsf{MatchJoin}_{\mathsf{min}}$, we generated four sets of patterns with $(|V_p|,|E_p|)$ ranging from (4,8) to (7,14), kept $|E_p| = 2|V_p|$, and varied $|G|$ as in Fig. 8(d). The results are reported in Fig. 8(e), which tell us the following. (1) $\mathsf{MatchJoin}_{\mathsf{min}}$ scales well with $|\mathsf{Q_s}|$, which is consistent with Fig. 8(d). (2) The larger $\mathsf{Q_s}$ is, the more costly $\mathsf{MatchJoin}_{\mathsf{min}}$ is. For larger $\mathsf{Q_s}$, more views may be needed to "cover" $\mathsf{Q_s}$; and $\mathsf{MatchJoin}_{\mathsf{min}}$ takes longer time, using the selected views.

Exp-2: Optimization techniques. We also evaluated the effectiveness of the optimization strategy given in Section III, by comparing the performance of $\mathsf{MatchJoin}_{\mathsf{min}}$ and $\mathsf{MatchJoin}_{\mathsf{nopt}}$ using patterns of size $(4, 6)$ and same set of views. The synthetic graphs are generated following the densification law [26]: $|E| = |V|^{\alpha}$. Fixing $|V| = 200K$, we varied α from 1 to 1.25 in 0.05 increments. As shown in Fig. 8(f), $\mathsf{MatchJoin}_{\mathsf{min}}$ is more efficient than $\mathsf{MatchJoin}_{\mathsf{nopt}}$ over all the datasets. Indeed, the running time of $\mathsf{MatchJoin}_{\mathsf{min}}$ is on average 54% of that of $\mathsf{MatchJoin}_{\mathsf{nopt}}$. The improvement becomes more evident when α increases. This is because



(a) Varying $|\mathsf{Q_s}|$ (Amazon) (b) Varying $|\mathsf{Q_s}|$ (Citation) (c) Varying $|\mathsf{Q_s}|$ (Youtube) (d) Varying $|G|$ (synthetic)

(e) Varying $|G|$ & $|\mathsf{Q_s}|$ (synthetic) (f) Varying α (synthetic) (g) Varying $|\mathsf{Q_s}|$ (synthetic) (h) Varying $|\mathsf{Q_s}|$ (synthetic)

(i) Varying $|\mathsf{Q_b}|$ (Amazon) (j) Varying $|\mathsf{Q_b}|$ (Citation) (k) Varying $f_e(e)$ (Youtube) (l) Varying $|G|$ (synthetic)

Fig. 8: Performance evaluation

when graphs become dense, more redundant edges can be removed by the bottom-up strategy used in $\mathsf{MatchJoin_{min}}$. The results for $\mathsf{BMatchJoin_{min}}$ and $\mathsf{BMatchJoin_{nopt}}$ are consistent with Fig. 8(f) and are hence not shown.

Exp-3: Query containment. We evaluated the performance of pattern containment checking *w.r.t.* query complexity.

Efficiency of contain. We generated two sets of DAG and cyclic patterns, denoted by Q_{DAG} and Q_{Cyclic}, respectively. Fixing a set of synthetic views $\mathcal{V}$, we varied the pattern size from $(6, 6)$ to $(10, 20)$, where each size corresponds to *a set of patterns* with different structures and/or node labels. As shown in Figure 8(g), (1) contain is efficient, *e.g.*, it takes only 39 ms to decide whether a *cyclic* pattern with $|V_p|$=10 and $|E_p|$=20 is contained in $\mathcal{V}$; (2) contain takes more time over larger DAG and cyclic patterns, as expected; and (3) when pattern size is fixed, *cyclic* patterns cost more than DAG patterns for contain, due to a more time-consuming fixpoint process.

minimum *vs* minimal. To measure the performance of minimum and minimal, we define $R_1 = |\mathsf{T_{min}}|/|\mathsf{T_{mnl}}|$ as the ratio of the time used by minimum to that of minimal; and $R_2 = |\mathsf{Minimum}|/|\mathsf{Minimal}|$ for the ratio of the size of subsets of views found by minimum to that of minimal. Using the same view definitions and *cyclic* patterns as in Figure 8(g), we varied the size of pattens from (6,6) to (10, 20). As shown in Fig. 8(h), (1) minimum is efficient on all patterns used, *e.g.*, it takes about 0.4s over patterns with 10 nodes and 20 edges; (2) minimum is effective: while minimum takes up to 120% of the time of minimal (R_1), it finds *substantially smaller* sets of views, only about 40%-55% of the size of those found by minimal, as indicated by R_2. Both algorithms take more time over larger patterns, as expected.

Exp-4: Efficiency and scalability of BMatchJoin**.** In this set of experiment we evaluated (1) the efficiency of $\mathsf{BMatchJoin_{min}}$ vs. $\mathsf{BMatchJoin_{mnl}}$ and BMatch over the real-life datasets, by varying the size of pattern queries, and (2) the scalability of $\mathsf{BMatchJoin_{min}}$ over large synthetic graphs, by varying the size of patterns and data graphs.

Efficiency. We used the same patterns as for MatchJoin in Exp-1, except that the edge bounds of the patterns were set to be $f_e(e) = 2$ (resp. $f_e(e) = 3$) for queries over *Amazon* (resp. *Citation*). Figure 8(i) shows the results on *Amazon* in which the x-axis $(|V_p|, |E_p|, f_e(e))$ indicates the size of patterns $\mathsf{Q_s} = (V_p, E_p, f_e)$. From the results we find that $\mathsf{BMatchJoin_{min}}$ and $\mathsf{BMatchJoin_{mnl}}$ perform *much better* than BMatch: (1) $\mathsf{BMatchJoin_{min}}$ (resp. $\mathsf{BMatchJoin_{mnl}}$) needs only 10% (resp. 14%) of the time of BMatch; (2) when pattern size increases, the running time of $\mathsf{BMatchJoin_{min}}$ (resp. $\mathsf{BMatchJoin_{mnl}}$) grows slower than that of BMatch; and (3) $\mathsf{BMatchJoin_{min}}$ always outperforms $\mathsf{BMatchJoin_{mnl}}$. These are consistent with the result for *Citation*, shown in Fig. 8(j).

Fixing pattern size with $|V_p| = 4$ and $|E_p| = 8$, we varied $f_e(e)$ from 2 to 6. As shown in Fig. 8(k), (1) $\mathsf{BMatchJoin_{min}}$ *substantially outperforms* BMatch; when $f_e(e) = 3$, for example, $\mathsf{BMatchJoin_{min}}$ accounts for only 3% of the computational time of BMatch; (2) the larger $f_e(e)$ is, the more costly BMatch is, as it takes longer for BFS to identify ancestors or descendants of a node within the distance bound $f_e(e)$; and (3) $\mathsf{BMatchJoin_{min}}$ is more efficient than $\mathsf{BMatchJoin_{mnl}}$, as it uses less views.

Scalability. Fixing $|V_p| = 4$, $|E_p| = 6$ and $f_e(e) = 3$, we varied $|V|$ from $0.3M$ to $1M$ in $0.1M$ increments, while letting $|E| = 2|V|$. As shown in Fig 8(l), (1) $\mathsf{BMatchJoin_{min}}$ scales best with $|G|$; this is consistent with its complexity analysis; and

(2) $\mathsf{BMatchJoin}_{\mathsf{min}}$ takes only 6% of the computation time of BMatch, and the saving is more evident when G gets larger.

Summary. We find the following. (1) Answering (bounded) pattern queries using views is effective in querying large social graphs. For example, by using views, matching via bounded simulation takes only 3% of the time needed for computing matches directly in *YouTube*, and 6% on synthetic graphs. For graph simulation, the improvement is over 51% at least. (2) Our view-based matching algorithms scale well with the query and data size. Moreover, they are much less sensitive to the size of data graphs. (3) It is efficient to determine whether a (bounded) pattern query can be answered using views. In particular, our approximation algorithm for minimum containment effectively reduces redundant views, which in turn improves the performance of matching by 55% (resp. 94%) for (resp. bounded) pattern queries. (4) Better still, our optimization strategy further improves the performance of pattern matching using views by 46%.

VIII. Conclusion

We have studied graph pattern matching using views, from theory to algorithms. We have proposed a notion of pattern containment to *characterize* what pattern queries can be answered using views, and provided such an efficient matching algorithm. We have also identified three fundamental problems for pattern containment, established their complexity, and developed effective (approximation) algorithms. Our experimental results have verified the efficiency and effectiveness of our techniques. These results extend the study of query answering using views from relational and XML queries to graph pattern queries. Moreover, our techniques can be readily extended to strong simulation [28], retaining the same complexity.

The study of graph pattern matching using views is still in its infancy. One issue is to decide what views to cache such that a set of frequently used pattern queries can be answered by using the views. Techniques such as adaptive and incremental query expansion may apply. Another issue is to develop efficient algorithms for computing *maximally contained rewriting* using views, when a pattern query is not contained in available views [25]. A third problem concerns view-based pattern matching via subgraph isomorphism. The fourth topic is to find a subset $\mathcal{V}'$ of $\mathcal{V}$ such that $\mathcal{V}'(G)$ is minimum for all graphs G. Finally, to find a practical method to query "big" social data, one needs to combine techniques such as view-based, distributed, incremental, and compression methods.

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