On Scale Independence for Querying Big Data

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ABSTRACT

To make query answering feasible in big datasets, practitioners have been looking into the notion of scale independence of queries. Intuitively, such queries require only a relatively small subset of the data, whose size is determined by the query and access methods rather than the size of the dataset itself. This paper aims to formalize this notion and study its properties. We start by defining what it means to be scale-independent, and provide matching upper and lower bounds for checking scale independence, for queries in various languages, and for combined and data complexity. Since the complexity turns out to be rather high, and since scale-independent queries cannot be captured syntactically, we develop sufficient conditions for scale independence. We formulate them based on access schemas, which combine indexing and constraints together with bounds on the sizes of retrieved data sets. We then study two variations of scale-indexing and constraints together with bounds on the sizes of retrieved data sets. These variations are captured and studied using a framework based on access schemas. We prove lower bounds for checking scale independence, for queries in SQL, and provide matching upper and lower bounds for checking scale independence for a large class of queries.

Categories and Subject Descriptors: H.2.1 [Database Management]: Logical Design – Data Models; H.2.1 [Database Management]: Systems – Query Processing

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1. INTRODUCTION

Big data introduces challenges to the scalability of query answering. Given a query Q and a dataset D, it is often prohibitively costly to compute the answers Q(D) to Q in D when D is big, e.g., of PetaByte (1015 bytes) or ExaByte (1018) size. To this end, one may want to use heuristics, “quick and dirty” algorithms which return approximate answers. However, in many applications it is a must to find exact query answers.

To cope with these, practitioners have been studying scale independence (e.g., [4–6]). A query Q is said to be scale-independent in a dataset D w.r.t. M if Q(D) can be computed by accessing a set DQ of at most M tuples in D, independent of the size of the underlying dataset D. Here M is a non-negative integer, indicating the capacity of our available resources, such as time and space.

The need for scale independence is evident in practice. It allows us to answer Q in big D within our available resources. Moreover, if Q is scale-independent in all datasets, we can answer Q without performance degradation when D grows, i.e., make Q scalable.

Scale independence per se is not easy to achieve, nor is it easy to test for, as we shall show. Nonetheless, there are many practical scenarios where scale independence is achievable, roughly classified into three groups below.

(1) Additional information about accessing information in a dataset, which is typically provided in the form of indices and/or cardinality constraints, can make rather expressive classes of queries scale-independent. Such access information is in fact commonly available in many large datasets being used in the real world.

(2) Even if a query Q is not scale-independent, we may still make it feasible to query big data incrementally, i.e., to evaluate Q incrementally in response to changes ∆D to D, by accessing an M-fraction of the dataset D. That is, we compute Q(D), once and offline, as precomputation, and then incrementally answer Q on demand.

(3) Additionally, we can sometimes achieve scale independence using views, i.e., when a set V of views is defined, we rewrite Q into Q’ using V, such that for any dataset D, we can compute Q(D) by using Q’, which accesses materialized views V(D) and only a bounded amount of data from D (this is subject to the storage and maintenance costs of V(D)).

We now illustrate these three scenarios by examples.

Example 1.1: Some real-life queries are actually scale-independent. For example, below are (slightly modified) queries taken from Graph Search of Facebook [11].

(a) Query Q1 is to find all friends of a person p who live in NYC, from a dataset D1. Here D1 consists of two relations specified by person(id, name, city) and friend(id1, id2), recording the basic information of people (with a key id) and their friends, respectively. Query Q1 can be written as follows: Q1(p, name) = ∃id (friend(p, id) ∧ person(id, name, NYC)).

In personalized social searches we do not want to compute the entire answer to Q1, but rather do it for a specified
person $p_0$; that is, given $p_0$, we want to find all values of name so that $(p_0, \text{name})$ is in the answer to $Q_1$.

The dataset $D_1$ is often big in real life: Facebook has more than 1 billion users with 140 billion friend links [10]. A naive computation of the answer to $Q_1$, even if $p_0$ is known, may fetch the entire $D_1$, and is cost prohibitive.

 Nonetheless, we can compute $Q_1(p_0, D_1)$ by accessing only a small subset $D_{Q_1}$ of $D_1$. Indeed, Facebook has a limit of 5000 friends per user (cf. [5]), and is a key of person. Thus by using indices on id attributes, we can identify $D_{Q_1}$, which consists of a subset $D_1$ of friend including all friends of $p_0$, and a set $D_p$ of person tuples $t$ such that $t[\text{id}] = t'[\text{id2}]$ for some tuple $t'$ in $D_1$. Then $Q_1(p_0, D_{Q_1}) = Q_1(p_0, D_1)$.

Moreover, $D_{Q_1}$ contains at most 10000 tuples of $D_1$, and is much smaller than $D_1$. Thus $Q_1$ is scale-independent in $D_1$ w.r.t. $M \geq 10000$.

This illustrates the key ingredients of the recipe for scale independence. First, we may have to fix values of some parameters of the query $(p_0, p)$ above. Second, we may need access information telling us that based on some key values, tuples can be fetched efficiently, and there is an upper limit on the number of tuples fetched.

(b) Query $Q_2$ is to find from $D_2$ all restaurants rated $A$ in NYC, where $p$'s friends in NYC have been. Dataset $D_2$ consists of four relations: person and friend as above, and relations specified by $\text{restr}(\text{rid, name, city, rating})$ (with rid as a key) and $\text{visit}(\text{id, rid})$. Here $Q_2$ is $Q_2(p, \text{rn}) = \exists \text{id, rid, pn}(\text{friend}(p, \text{id}) \land \text{visit}(\text{id, rid}) \land \text{person}(\text{id, pn, NYC}) \land \text{restr}(\text{rid, rn, NYC, A}))$.

Again, we want to find the answers for a given person $p_0$. However, unlike $D_1$, dataset $D_2$ imposes no restriction on the number of restaurants in NYC or on the number of restaurants which a person visits.

Nonetheless, in this case we can incrementally evaluate $Q_2$ scale-independently, leveraging the old output $Q_2(p_0, D_2)$. Given a set $\Delta D_2$ of insertions to visit, one can compute $Q_2(D_2 \cup \Delta D_2)$ by fetching at most $3|\Delta D_2|$ tuples from $D_2$, where $|S|$ denotes the number of tuples in $S$. Indeed, for each tuple $(\text{id, rid})$ in $\Delta D_2$, we fetch the restaurant identified by the key rid, the friend of $p_0$ identified by id, and person tuple identified by id to verify that the friend lives in NYC, via indexing. Let $S$ be the set of the names of the restaurants that are fetched and visited by NYC friends. Then $S \cup Q_2(D_2) = Q_2(D_2 \cup \Delta D_2)$. Note that $\Delta D_2$ is often small in practice. If $3|\Delta D_2| \leq M$, then the incremental evaluation of $Q_2$ is scale-independent w.r.t. $M$.

(c) Assume that two views are defined: $V_1$ contains all restaurants in NYC, and $V_2$ is a subset of visit(id, rid) such that id lives in NYC (restaurants visited by NYC locals). Using $V_1$ and $V_2$, query $Q_2$ can be rewritten as:

$Q_2(p, \text{rn}) = \exists \text{id, rid, pn}(\text{friend}(p, \text{id}) \land V_2(\text{id, rid}) \land V_1(\text{rid, rn, A}))$.

Then for all datasets $D_2$, by using materialized views $V_1(D_2)$ and $V_2(D_2)$, $Q_2$ needs to fetch at most 5000 friend tuples from $D_2$ for $p = p_0$, i.e., for a fixed $p$, $Q_2$ is scale-independent in all $D_2$ by using $V_1$ and $V_2$. Here we assume that the views are cached in memory and can be efficiently retrieved. That is, for scale independence using views to be effective, the materialized views should be of small size.

All three examples thus show that it is feasible to answer a query $Q$ in a big dataset $D$ by accessing a bounded amount of data. To make practical use of scale independence, however, several questions have to be answered. Given $Q$ and $D$, can we decide whether $Q$ is scale-independent in $D$? If such an identification is expensive, can we find sufficient conditions for scale independence, perhaps using indices and other access information? If $Q$ is determined to be scale-independent in $D$, can we effectively identify a small $D_Q \subseteq D$ such that $Q(D) = Q(D_Q)$ and $|D_Q| \leq M$, by using available indices? And can we achieve reasonable time bounds for finding this set and for evaluating the query over it? Similar questions also arise for incremental scale independence and scale independence using views.

**Contributions.** Our goal is to give a formal definition of scale independence (this has not been previously done) and study the above questions along the following three lines.

**Complexity of scale independence.** We look at the problem of deciding, given a query $Q$, a dataset $D$, and a bound $M$, whether $Q$ is scale-independent in $D$ w.r.t. $M$, i.e., $Q(D)$ can be computed by accessing at most $M$ tuples in $D$. We call it QDSI (with QD emphasizing that both $Q$ and $D$ are inputs). We establish both combined and data complexity bounds for QDSI, when $Q$ is a conjunctive query (CQ), a union of conjunctive queries (UCQ), or in first-order logic (FO). We also study a special case when $M$ is a constant, i.e., when the capacity of our resources is fixed. Moreover, we study another problem, QSI, to check whether $Q$ is scale-independent in *all instances* $D$ of a relational schema.

Most of the results are negative: for FO queries, QSI is undecidable, and few sensible CQ queries are scale-independent in all instances, while the complexity of QDSI also tends to be rather high. This naturally brings us to the next theme of our investigation.

**Sufficient conditions for scale independence.** Not only is the complexity of QDSI and QSI rather high, it is also impossible to capture scale-independent queries syntactically. Thus, we need to find sufficient conditions for scale independence, with additional access information.

Such additional information comes in the form of access schemas $A$ that specify what parts of data can be efficiently retrieved from $D$ by using indices, as practiced in real life, and in addition give cardinality restrictions on such retrieved sets of tuples. We then provide a sufficient syntactic condition for an FO query to be scale-independent under $A$. The class of queries we define is compositional: it is given by a set of rules forming new queries from existing ones, and we show that each rule is optimal, i.e., it cannot get any tighter. We then show that the syntactic class of queries guarantees scale independence under $A$. For instance, the query $Q_1$ we have seen above is an example of such a query: our conditions say that when the person is fixed, and if the Facebook restriction on the number of friends applies, then the query can be executed scale-independently.

Furthermore, we introduce embedded access schemas to incorporate constraints commonly found in practice, such as functional dependencies. We show that some queries that are not scale-independent may become so under embedded $A$ with simple constraints.

**Two variants of scale independence.** Finally, we extend our study to incremental scale independence and scale independence using views, which have been used in practice [6]. We investigate the following problems.
(a) \(\Delta QSI\) is to decide whether for all small updates \(\Delta D\) to \(D\), the answer to \(Q\) on the updated database can be incrementally computed from \(Q(D)\) by using at most \(M\) additional tuples from base relations in \(D\).

(b) \(VQSI\) is to decide whether a query \(Q\) can be rewritten to another query \(Q'\) using a set \(V\) of views, such that for all datasets \(D\), \(Q(D)\) can be computed by using \(Q'\), which accesses materialized views \(V(D)\) and at most \(M\) tuples in the data source \(D\).

We provide complexity bounds for these problems, and sufficient conditions for queries to be incrementally scale-independent and scale-independent using views.

To the best of our knowledge, this work is the first effort to give a formal treatment of scale independence, a notion recently proposed and implemented [4-6]. Our results provide a comprehensive picture of complexity bounds for the problem, help us identify a bounded amount of data from a large dataset for query evaluation, and suggest what indices to build on our datasets. The lower bounds also justify the adoption of approximate query answering.

**Related work.** The notion of scale independence was proposed in [5], to guarantee that a bounded amount of work (key/value store operations) is required to execute all queries in an application, regardless of the size of the underlying data. An extension to SQL was developed in [4] to enforce scale independence, which allows users to specify bounds on the amount of data accessed and the size of intermediate results; when the data required exceeds the bounds, only top-k items are retrieved to meet the bounds. View selection and maintenance were studied in [6], such that a bounded amount of work is needed to answer queries by query rewriting using views and materialized (precomputed) views.

The goal of this work is to give a precise notion of scale independence and study its properties. We identify problems fundamental to scale independence, provide matching complexity bounds, propose access schemas to formulate data access via indexing and constraints, and give sufficient conditions for scale independence (incrementally, or using views). The results tell us what is doable and what is not. To the best of our knowledge, no prior work has studied these.

Related to our notion of access schemas is the notion of access patterns. Access patterns require that a relation can only be accessed by providing certain combinations of attribute values. Query processing under limited access patterns has been extensively studied, e.g., [7, 9, 22, 24]. In contrast to the prior work, we use access schemas to combine indexing and the amount of data retrieved, and embed cardinality constraints in an access schema. Our goal is to provide a sufficient condition for identifying what queries are scale-independent with indices and constraints, rather than to study the complexity or executable plans for answering queries under access patterns [7, 9, 22, 24].

There has been a host of work on incremental query answering (surveyed in [15]) and query rewriting using views (surveyed in [16, 19]). The prior work has mostly focused on improving performance by making maximum use of precomputed query answers or views. In contrast, incremental scale independence and scale independence using views aim to access a minimum amount of data in data sources, to cope with the sheer volume of big data. There has also been prior work on bounded incremental computation [28], self-maintainable views [27], queries independent of updates [21] and view complements [25], which also access limited source data or no source data at all. We will clarify the difference between those previous works and ours in Sections 5 and 6.

Related to problem QDSI is the relatively complete database problem (RCDP) studied in [12]. Given a query \(Q\), a database \(D\), master data \(D_m\), a set \(\Sigma\) of containment constraints on \(D\) and \(D_m\), RCDP is to decide whether \(D\) has complete information to answer \(Q\) relative to \(D_m\) and \(\Sigma\), i.e., for all extensions \(D'\) of \(D\), if \(D'\) and \(D_m\) satisfy \(\Sigma\), then \(Q(D) = Q(D')\). In contrast, QDSI is to decide whether there exists a \(D_0 \subseteq D\) such that \(Q(D) = Q(D_0)\) and \(|D_0|\) is below a bound \(M\). The two problems are quite different, from complexity bounds to proofs. For instance, when \(Q\) is in CQ and constraints in \(\Sigma\) are expressed in CQ, RCDP is NEXPTIME-complete, while QDSI is \(\Sigma_2^p\)-complete.

There has also been recent work on querying big data, e.g., on the communication complexity of parallel query evaluation [17, 18], the complexity of query processing in terms of MapReduce rounds [2, 30], and the study of query classes that are tractable on big data [13]. In contrast, this work studies whether it is feasible to compute query answers on big data by accessing a small subset of the data, and if so, how to efficiently identify this subset.

**Organization.** Section 2 presents notations. Section 3 establishes the complexity bounds for QDSI and QSI, and Section 4 deals with conditions for scale independence under access schemas. Sections 5 and 6 investigate \(\Delta QSI\) and \(VQSI\), respectively. Conclusions are in Section 7.

## 2. PRELIMINARIES

A relational schema \(R\) consists of a collection of relation names \((R_1, \ldots, R_n)\), with each \(R_i\) having a fixed set of attributes. We assume a countably infinite set \(U\) from which elements populating databases are drawn. That is, an instance \(D\) of \(R\) associates with each \(R \in R\) having \(m\) attributes an \(m\)-ary relation \(R_i^D\) over \(U\), i.e., a subset of \(U^m\). When there is no confusion, we omit the superscript \(D\). The set of all elements of \(U\) present in relations in \(D\) is called the active domain of \(D\) and is denoted by \(adom(D)\).

We shall use logical languages for expressing queries declaratively. The languages \(L\) used here are standard relational languages (see [1] for details). We list them now, together with their relational algebra equivalents.

- Conjunctive queries (CQ) are built up from relation atoms \(R_i(\bar{x})\) (for \(R_i \in R\)), and equality atoms \(x = y\) or \(x = c\) (for constant \(c\), by closing them under conjunction \(\wedge\) and existential quantifier \(\exists\) (i.e., the class \(SPJ\) of select-project-join queries);
- Unions of conjunctive queries (UCQ) are queries of the form \(Q_1 \cup \cdots \cup Q_k\), where each \(Q_i\) is in CQ for \(i \in [1, k]\) (equivalently, \(SPJU\) queries);
- First-order logic queries (FO) are built from atomic formulas by using \(\wedge\), \(\lor\), negation \(\neg\), and quantifiers \(\exists\) and \(\forall\) (equivalently, the full relational algebra).

If \(\bar{x}\) is the tuple of free variables of \(Q\), we shall also write \(Q(\bar{x})\). Given a query \(Q(\bar{x})\) with \(|\bar{x}| = m\) and a database \(D\), the answer to \(Q\) in \(D\), denoted by \(Q(D)\), is the set of tuples \(\{\bar{a} \in adom(D)^m \mid D \models Q(\bar{a})\}\).
Often we need to fix values for some free variables. For a query \( Q(\bar{x}, \bar{y}) \) with \(|\bar{y}| = m \) and a tuple \( \bar{a} \) of values for \( \bar{x} \), \( Q(\bar{a}, D) \) denotes \( \{ b \in \text{dom}(D)^m \mid D \models Q(\bar{a}, b) \} \).

If \( Q \) is a sentence (i.e., it has no free variables), we refer to it as a Boolean query; such a query returns true or false. To distinguish queries that have free variables, we shall call them data selecting queries; for such a query \( Q \), the answer \( Q(D) \) is a set of (nonempty) tuples.

**Remark.** All our complexity results for \( \text{CQ} \) also hold for \( \text{UCQ} \). Hence, in what follows, we will only mention \( \text{CQ} \) and \( \text{FO} \) when reporting complexity bounds.

## 3. SCALE-INDEPENDENT QUERIES

The key idea behind scale independence is that we can find a small subset of a database \( D \) so that a query \( Q \) can be answered over that subset, rather than over the entire \( D \). In this section, we define the notion of scale independence formally, study its basic properties, and establish the complexity of problems associated with it.

Let \( \mathcal{R} \) be a relational schema, \( D \) a database of this schema, \( Q \) a query in language \( \mathcal{L} \), and \( M \) a non-negative integer. Let \(|D|\) denote the size of \( D \), measured as the total number of tuples in relations of \( D \).

We say that \( Q \) is scale-independent in \( D \) w.r.t. \( M \) if there exists a subset \( D_Q \subseteq D \) such that

- \(|D_Q| \leq M \)
- \( Q(D_Q) = Q(D) \).

That is, to answer \( Q \) in \( D \), we need only to fetch at most \( M \) tuples from \( D \), regardless of how big \( D \) is.

We refer to \( D_Q \) as a witness for scale independence of \( Q \) in \( D \) w.r.t. \( M \). We write \( \text{SQ}_\mathcal{L}(D, M) \) for the set of all \( \mathcal{L} \) queries that are scale-independent in \( D \) w.r.t. \( M \).

We say that \( Q \) over schema \( \mathcal{R} \) is scale-independent w.r.t. \( M \) if \( Q \) is scale-independent in \( D \) w.r.t. \( M \) for all databases \( D \) of \( \mathcal{R} \), and write \( \text{SQ}_{\mathcal{L}, \mathcal{R}}(M) \) for the set of all \( \mathcal{L} \) queries that are scale-independent w.r.t. \( M \).

For instance, for \( Q_1(\text{p.name}) \) and \( D_1 \) given in Example 1.1, the query \( Q_1(\text{p0.name}) \) with a given person \( \text{p0} \) is in both \( \text{SQ}_{\mathcal{L}}(D_1, 10000) \) and \( \text{SQ}_{\mathcal{L}, \mathcal{R}}(10000) \), under the constraints that limit 5000 friends per person and \( \text{i} \) is a key of person.

These two notions lead to two problems of determining scale independence, i.e., whether there exists a witness at all, denoted by \( \text{QDSI} \) and \( \text{QSI} \). They are stated as follows.

- **Problem \( \text{QDSI}(\mathcal{L}) \):**
  - INPUT: A relational schema \( \mathcal{R} \), an instance \( D \) of \( \mathcal{R} \), a query \( Q \in \mathcal{L} \) over \( \mathcal{R} \), and \( M \geq 0 \).
  - QUESTION: Is \( Q \) in \( \text{SQ}_{\mathcal{L}}(D, M) \)?

- **Problem \( \text{QSI}(\mathcal{L}) \):**
  - INPUT: A schema \( \mathcal{R} \), a query \( Q \in \mathcal{L} \) over \( \mathcal{R} \), and \( M \geq 0 \).
  - QUESTION: Is \( Q \) in \( \text{SQ}_{\mathcal{L}, \mathcal{R}}(M) \)?

Problem \( \text{QDSI} \) tests query-database scale independence. It is highly relevant in practice since one often wants to know whether a query is scale-independent for the database at hand. The need for this is particularly evident for e.g., Facebook: it maintains a single dataset \( D \) for its social data (when \( D \) is updated, we only need to consider incremental scale independence). Problem \( \text{QSI} \) is “stronger”: it tests query scale independence for all instances of a schema.

For problem \( \text{QDSI}(\mathcal{L}) \) we also have two versions to study: data complexity, when schema and query are fixed, but database and \( M \) may vary; and combined complexity, when everything \((\mathcal{R}, Q, D, M)\) is a parameter.

We now study the complexity of these problems. **All the lower bounds of this paper also hold when schema \( \mathcal{R} \) is fixed.** We also study the case when \( M \) is fixed too.

**Quer-y-database scale independence.** We first deal with the problem \( \text{QDSI} \) in which both the query and the database are part of the input. We start with combined complexity. The first result is for data-selecting queries, and we show that the problem is necessarily in the polynomial hierarchy even for simple classes of queries, and it is in \( \text{PSPACE} \) for \( \text{FO} \).

**Theorem 3.1:** For data selecting queries, the combined complexity of \( \text{QDSI}(\mathcal{L}) \) is

- \( \Sigma_3^p \)-complete when \( \mathcal{L} \) is \( \text{CQ} \); and
- \( \text{PSPACE} \)-complete when \( \mathcal{L} \) is \( \text{FO} \).

**Proof sketch.** (1) Upper bounds. We first consider a simpler witness problem. That problem asks, given \( Q, D, M \) and \( D' \subseteq D \) with \(|D'| \leq M \), whether \( Q(D) = Q(D') \), i.e., it checks whether a given \( D' \) witnesses scale independence. It can be verified that the witness problem is \( \Pi^p_2 \)-complete for \( \text{CQ} \) and \( \text{PSPACE} \)-complete for \( \text{FO} \). Observe that solving \( \text{QDSI} \) just adds an existential guess of \( D' \) on top of the witness problem, thus taking us to the third level of the polynomial hierarchy for \( \text{CQ} \) and staying in \( \text{PSPACE} \) for \( \text{FO} \).

(2) Lower bounds. \( \text{QDSI}(\mathcal{CQ}) \) is verified by reduction from the \( \exists^p \forall^* \exists^*3\text{CNF} \) problem, which is known to be \( \Sigma_3^p \)-complete [29]. The latter problem is to decide, given a sentence \( \psi = \exists X \forall Y \exists Z \psi(X, Y, Z) \), whether \( \psi \) is true, where \( \psi(X, Y, Z) \) is an instance of \( 3\text{SAT} \), i.e., \( \psi \) is \( C_1 \land \cdots \land C_r \), and each \( C_i \) is a disjunction of three literals (variables or negations of variables in \( X, Y \) or \( Z \)).

To show that \( \text{QDSI}(\text{FO}) \) is \( \text{PSPACE} \)-hard, we use reduction from \( 3\text{SAT} \), which is \( \text{PSPACE} \)-complete (cf. [26]). Given a sentence \( \psi = P_1 x_1 \land \cdots \land P_m x_m \land 
\psi(x_1, \ldots, x_m) \), \( 3\text{SAT} \) is to decide whether \( \psi \) is true, where \( P_i \) is either 3 or \( \forall \), and \( \psi \) is an instance of \( 3\text{SAT} \). The reduction uses a Boolean FO query and a constant \( M \).

**Boolean queries.** For Boolean queries, \( \text{QDSI} \) becomes much simpler for \( \text{CQ} \). Indeed, if \( Q(D) \) is true, then \( Q(D_{\bar{a}}) \) is true for some \( D_{\bar{a}} \) such that \(|D_{\bar{a}}| \leq |Q| \). For a \( \text{CQ} \), we measure \(|Q| \) as the size of the tableau of \( Q \). This follows from the standard homomorphism semantics of \( \text{CQ} \) (see [1]). For a \( \text{UCQ} \) \( Q = Q_1 \cup \cdots \cup Q_k \), we define \(|Q| \) to be \( \max \{|Q_i| \mid i \in [1, k] \} \). In practice, typically \(|Q| \ll |M| \ll |D| \).

Since the \( \text{PSPACE} \) lower bound of Theorem 3.1 was verified by using a Boolean FO query, in the case of \( \text{FO} \) there is no lowering of the complexity. Thus, we have:

**Corollary 3.2:** For Boolean queries \( Q \), \( \text{QDSI}(\mathcal{L}) \) is

- in constant time (if \(|Q| \leq M \) when \( \mathcal{L} \) is \( \text{CQ} \); and
- \( \text{PSPACE} \)-complete when \( \mathcal{L} \) is \( \text{FO} \).

for the combined complexity.

A similar analysis gives a bound for \( M \) when \( Q \) is a data-selecting CQ query. Since for each tuple \( \bar{a} \) in \( Q(D) \)
Table 1: Complexity bounds for QDSI. (O(1) cases hold when |Q| ≤ M)

<table>
<thead>
<tr>
<th>Query languages</th>
<th>Data selecting</th>
<th>Boolean</th>
</tr>
</thead>
<tbody>
<tr>
<td>CQ, UCQ</td>
<td>combined (Th 3.3)</td>
<td>data (Th 3.3)</td>
</tr>
<tr>
<td>FO</td>
<td>${\Sigma}_2^P$-complete</td>
<td>NP-complete</td>
</tr>
<tr>
<td>CQ, UCQ</td>
<td>combined (Prop 3.4)</td>
<td>data (Prop 3.4)</td>
</tr>
<tr>
<td>FO</td>
<td>${\Pi}_2^P$-complete</td>
<td>PTIME</td>
</tr>
</tbody>
</table>

When $M$ is fixed. When we have a fixed set of resources, the bound $M$ is a constant. Fixing $M$ simplifies the analysis of QDSI. The combined complexity drops one level in the polynomial hierarchy for CQ, but it remains intact for FO. Data complexity becomes tractable, even for data-selecting FO queries, since only fixed-size subsets need to be checked.

Proposition 3.4: When $M$ is fixed, the combined complexity of QDSI($\mathcal{L}$) is

- $\Pi_2^P$-complete for data-selecting queries, and is in O(1)-time for Boolean queries if $|Q| \leq M$, for CQ;
- PSPACE-complete for both data-selecting queries and Boolean queries in FO.

The data complexity of QDSI($\mathcal{L}$) is the same as in Theorem 3.3 except that it becomes PTIME for FO.

Proof sketch. (1) Upper bounds. When $M$ is fixed, we give a $\Sigma_2^P$ algorithm to check whether $Q$ is in SQ$_2(D,M)$ when $Q$ is a data-selecting query in CQ, and a PTIME algorithm when $Q$ is a fixed data-selecting FO query. More precisely, we have a $\Sigma_2^P$ algorithm for the complement problem in which the guess of $D'$ and the guess of a witness of $Q(D') \neq Q(D)$ are combined. The O(1) and PSPACE bounds are inherited from Corollary 3.2 and Theorem 3.1, respectively.

(2) Lower bounds. For data-selecting CQ, we show it is $\Pi_2^P$-hard by reduction from $\forall \exists$3CNF [29]. It is to decide, given a sentence $\varphi = \forall X \exists Y \psi(X,Y)$, whether $\varphi$ is true, where $\psi(X,Y)$ is an instance of 3SAT (see the proof of Theorem 3.1 for 3SAT). For Boolean FO queries, the proof of Theorem 3.1 already verified the PSPACE-hardness by using $M = 3$. □

Note that $Q \in$ SQ$_2(D,|D|)$ for every language $\mathcal{L}$ and every query $Q$: all this says is that $Q$ can be answered on $D$ itself. The question is whether the $|D|$ bound can be lowered, ideally to a constant. For general data-selecting queries this may not be doable over all databases, e.g., when the queries need to look at the entire input such as those that simply return the input database. But what about Boolean queries?

We say that a Boolean query $Q$ does not use its input fully if there is a function $f_Q : \mathbb{N} \to \mathbb{N}$ such that $f_Q(n) < n$ for all sufficiently large $n$ and $Q \in$ SQ$_2(D,|D|)$ for every $D$. Otherwise a query fully uses its input.

Clearly every Boolean CQ does not use its input fully: it only needs a portion of it of the size $|Q|$. But when it comes to FO, this is not the case. Indeed, one can easily find Boolean FO queries that fully use their input.

Proposition 3.6: There are FO Boolean queries that fully use their input.

The results above might look negative: testing scale independence is computationally hard in the presence of data (which may be of very large size), or undecidable when we want to check whether it works on all databases (or, worse yet, the answer is simply negative). Nonetheless, this is not an atypical situation in databases, and it simply tells us that we should look for meaningful restrictions on queries to achieve scale independence. This is what we do next.
4. QUERY ANSWERING WITH ACCESS SCHEMAS

The results of the previous section indicate that without additional knowledge about the class of databases on which queries are posed, it is hard to achieve scale independence and hard to test it. We now introduce additional restrictions that will allow us to define an expressive fragment of $\text{FO}$ admitting scale independence.

The motivation for the type of restrictions we want to use comes from access methods used in practice, and it is already implicit in Example 1.1. There are three reasons why $Q_1$ can be answered fast. First, Facebook imposes a limit on the number of friends. Second, for each person id, we can retrieve his/her friends (as well as other information) quickly, due to the presence of an index. And third, the query used constant person id $p_0$.

Thus, to be able to state that some queries are scale-independent, we need information about access to data: both on the speed of access, and on the amount of data that can be retrieved. We formalize this in terms of a notion independent, we need information about access to data

Access schemas and scale independence. For a relational schema $\mathcal{R} = \langle R_1, \ldots, R_n \rangle$, an access schema $\mathcal{A}$ over $\mathcal{R}$ is a set of tuples $(R, X, N, T)$ where
- $R$ is a relation name in $\mathcal{R}$,
- $X$ is a set of attributes of $R$, and
- $N, T \in \mathbb{N}$.

A database $D$ conforms to the access schema $\mathcal{A}$ if for each condition $\sigma$ holds for each $(R, X, N, T) \in \mathcal{A}$:
1. for each tuple of values $\bar{a}$ of attributes of $X$, the set $\sigma_{X=\bar{a}}(R)$ has at most $N$ tuples; and
2. $\sigma_{X=\bar{a}}(R)$ can be retrieved in time at most $T$.

That is, one has an index on $X$ that allows efficient retrieval of tuples from the database, and in addition there is a bound $N$ on the number of such tuples (in the simplest case, when $X$ is a key, the bound is 1). Moreover, the $N$ tuples can be retrieved in $T$ time units by using the index.

In our Facebook example, we would have a tuple (friend, id, 5000, T) for some value $T$ in the access schema, indicating that if id is provided, at most 5000 tuples with such an id exist in friend, and it takes time $T$ to retrieve those. In addition, we would have a tuple (person, id, 1, T'), saying that id is a key for person with a known time $T'$ for retrieving the tuple for a given id.

Given a relation schema $\mathcal{R}$, an access schema $\mathcal{A}$, and a query $Q(\bar{x}, \bar{y})$, we say that $Q$ is $\bar{x}$-scale-independent under $\mathcal{A}$ if for each database $D$ that conforms to $\mathcal{A}$ and each tuple $\bar{a}$ of values for $\bar{x}$, the answer $Q(\bar{a}, D)$ can be found in time that depends only on $\mathcal{A}$ and $Q$, but not on $D$. In analogy to data complexity, when the query $Q$ is fixed but the access schema $\mathcal{A}$ may vary, we say that $Q$ is efficiently $\bar{x}$-scale-independent under $\mathcal{A}$ if the time to answer $Q(\bar{a}, D)$ is polynomial in $\mathcal{A}$.

Consider again the Facebook example:

$Q_1(p, name) = \exists \text{friend}(p, id) \land \exists \text{person}(id, name, \text{NYC})$. Then, under the access schema given above, $Q_1$ is $p$-scale-independent: for each given person $p_0$, the answers to $Q_1$ can be found in time determined by the access schema alone.

Controllability and scale independence. It is undecidable whether a query is $\bar{x}$-scale-independent under access schema $\mathcal{A}$, even if $\mathcal{A}$ is empty. Even more, the set of scale-independent queries is not recursively enumerable, as we have seen in Proposition 3.5, which rules out the existence of an effective syntax for it.

However, the lack of effective syntactic characterizations of classes of queries is common in databases. It is typically overcome by finding good and practically relevant sufficient conditions that guarantee desired properties of queries.

This is exactly what we do now: we define a syntactic class of $\bar{x}$-controlled queries for a given access schema, where $\bar{x}$ is a subset of free variables of a query, and show that each $\bar{x}$-controlled query under $\mathcal{A}$ is efficiently $\bar{x}$-scale-independent under $\mathcal{A}$. That is, an $\bar{x}$-controlled query becomes scale-independent under $\mathcal{A}$ once we fix values $\bar{a}$ for $\bar{x}$.

We now inductively define the class of $\bar{x}$-controlled $\mathcal{A}$-queries under an access schema $\mathcal{A}$. We also say that $Q(\bar{x})$ is controlled if it is $\bar{x}$-controlled, i.e., controlled by providing values for all its free variables. The rules for $\bar{x}$-controlled formulae are as follows:

- **atoms**: if $(R, X, N, T)$ is in $\mathcal{A}$, then $R(\bar{y})$ is $\bar{x}$-controlled under $\mathcal{A}$, where $\bar{x}$ is the subtuple of $\bar{y}$ corresponding to attributes in $X$;
- **conditions**: if $Q(\bar{x})$ is a Boolean combination of equalities among variables in $\bar{x}$, then $Q$ is $\bar{x}$-controlled;
- **disjunction**: if $Q_i(\bar{y})$ is $\bar{x}_i$-controlled under $\mathcal{A}$ for $i = 1, 2$, then $Q_1(\bar{y}) \lor Q_2(\bar{y})$ is ($\bar{x}_1 \cup \bar{x}_2$)-controlled;
- **conjunction**: if $Q_i(\bar{x}_i, \bar{y}_i)$ is $\bar{x}_i$-controlled under $\mathcal{A}$ for $i = 1, 2$, then $Q_1(\bar{x}_1, \bar{y}_1) \land Q_2(\bar{x}_2, \bar{y}_2)$ is controlled under $\mathcal{A}$ by both $\bar{x}_1 \cup (\bar{x}_2 - \bar{y}_1)$ and $\bar{x}_2 \cup (\bar{x}_1 - \bar{y}_2)$;
- **safe negation**: if $Q(\bar{y})$ is $\bar{x}$-controlled under $\mathcal{A}$, and $Q'(\bar{z})$ with $\bar{z} \subseteq \bar{y}$ is controlled under $\mathcal{A}$, then $Q \land \neg Q'$ is $\bar{x}$-controlled under $\mathcal{A}$;
- **existential quantification**: if $Q(\bar{y})$ is $\bar{x}$-controlled under $\mathcal{A}$ and $\bar{z}$ is a subtuple of $\bar{y} - \bar{x}$, then $\exists \bar{z} Q$ is $\bar{x}$-controlled under $\mathcal{A}$;
- **universal quantification**: if $Q(\bar{x}, \bar{y})$ is $\bar{x}$-controlled under $\mathcal{A}$ and $Q'(\bar{x})$ with $\bar{z} \subseteq \bar{x} \cup \bar{y}$ is controlled under $\mathcal{A}$, then $\forall \bar{y} (Q(\bar{x}, \bar{y}) \rightarrow Q'(\bar{x}))$ is $\bar{x}$-controlled under $\mathcal{A}$;
- **expansion**: if $Q(\bar{y})$ is $\bar{x}$-controlled under $\mathcal{A}$ and $\bar{z} \subseteq \bar{x}' \subseteq \bar{y}$, then $Q$ is $\bar{x}'$-controlled under $\mathcal{A}$.

Remark. We use set-theoretic operations for tuples of free variables to avoid cumbersome notations, as the meaning of those is clear from the context: for instance, $\bar{x} \cup \bar{y}$ is the tuple of all the variables used in $\bar{x}$ and $\bar{y}$, while $\bar{x} - \bar{y}$ is the subtuple of $\bar{x}$ from which variables occurring on $\bar{y}$ are eliminated.

**Example 4.1.** For example, under the access schema shown earlier, the query $Q_1(p, name)$ is $p$-controlled. Indeed, the access schema tells us that friend(p, id) is $p$-controlled and person(id, name, NYC) is id-controlled, and hence their conjunction is $p$-controlled. After adding an existential quantifier over id, the whole $Q_1$ is still $p$-controlled.

As another example, consider query $Q_2$, which revises $Q_2$ of Example 1.1 to find all restaurants in NYC that are rated $A$ and were visited in a given year by $p_0$’s friends who lived in NYC. Relation visit(id, rid) is extended by including attributes $yy$, $mm$, $dd$, indicating that person id visited restaurant rid on a given date. Then:
and then it provides genuinely new information.

A verification rule can be used in conjunction with another query, controllability statements. For instance, the universal quantification $\exists$ can be evaluated in a scale-independent way. This shows that the whole conjunction $Q$ is $\exists$-controlled under an access schema $A$, then it is efficiently $\exists$-scale-independent under $A$.

Proof sketch. We show by induction on $Q(\bar{x}, \bar{y})$ how to retrieve a set $D_Q(\bar{a}) \subseteq D$ on which $Q(\bar{a}, \cdot)$ can be evaluated, i.e., $Q(\bar{a}, D) = Q(\bar{a}, D_Q(\bar{a}))$, for a given set of values $\bar{a}$ for $\bar{x}$, and provide polynomial bounds for its size and query evaluation time. The base case is provided by the access schema. We now sketch the conjunction case; others are similar. Assume we have $Q_i(\bar{x}_i, \bar{y}_i)$ which are $\bar{x}_i$-controlled for $i = 1, 2$, also let $\bar{x}_2'$ stand for the subtuple of $\bar{x}_2$ consisting of variables that occur in $\bar{y}_1$, and $\bar{x}_2''$ for the remaining variables of $\bar{x}_2$, i.e., $\bar{x}_2' = \bar{x}_2 - \bar{y}_1$. Given values $\bar{a}_1$ and $\bar{a}_2''$ for $\bar{x}_1$ and $\bar{x}_2''$, proceed as follows. Find $D_{Q_1}(\bar{a}_1)$ by using, e.g., indices. Since $Q(\bar{a}_1, D) = Q(\bar{a}_1, D_Q(\bar{a}_1))$, the number of tuples $\bar{b}$ in $Q(\bar{a}_1, D)$ is bounded too. For each such tuple $\bar{b}$, let $\bar{b}_2'$ stand for the part of it corresponding to the variables in $\bar{x}_2$. Then for each tuple $(\bar{b}_2', \bar{a}_2'')$ we can find $Q_2(\bar{b}_2', \bar{a}_2'', D)$ effectively by constructing a subset $D_{Q_2}(\bar{b}_2', \bar{a}_2'')$ and evaluating the query on it; this shows that the whole conjunction (join) is scale-independent once $\bar{a}_1$ and $\bar{a}_2''$ are known.

Intuitively, Theorem 4.2 suggests the following. First, guided by an access schema $A$, we can build up indices on certain attributes of relations in an instance of schema $R$. Second, capitalizing on the indices, for all instances $D$ of $R$, we can answer FO queries $Q$ that are $\bar{x}$-controlled under $A$ by retrieving a small subset $D_Q \subseteq D$, such that $Q(D) = Q(D_Q)$ and $|D_Q| \leq M$, where $M$ can be derived from the $N$-values in $A$. Furthermore, there exists an effective plan for identifying $D_Q$, which can be derived from $A$ and an inductive analysis of the structure of $Q$. In particular, this confirms our intuition that for a fixed $p_0$, the query $Q_1$ in our example can be evaluated in a scale-independent way.

Even though the rules might look rather easy and perhaps limited in some cases (e.g., the universal quantification rule only guarantees controllability with all free variables), it is the combination of rules that lets us derive nontrivial controllability statements. For instance, the universal quantification rule can be used in conjunction with another query, and then it provides genuinely new information.

To give an example, assume that we have a schema with relations $R(A, B)$, $S(A, B, C)$, and $T(A, B, C)$, and suppose that $(R, A, N, T)$ is in the access schema for some $N$ and $T$. Now we are given a query


What are the conditions on $S$ and $T$ that will make this query scale-independent? Using our rules we can quickly answer this. The query is equivalent to $R(x, y) \land (x = 1) \land \forall z \left( S(x, y, z) \rightarrow T(x, y, z) \right)$. Applying the conjunction and universal quantification rules, we see that it suffices for $S$ to be $(A, B)$-controlled, and $T$ to be controlled by any set of attributes (in particular, $(A, B, C)$)-controlled for the whole query to be scale-independent. This suggests building an index on $A, B$ for $S$ and an arbitrary index for $T$.

Optimality of the rules. One may wonder whether the rules for controllability for FO queries are optimal, i.e., can they be relaxed so that they provide us with a "tighter" notion of controllability in terms of the number of values needed to guarantee scale independence? The answer is negative. Indeed, the rules – for queries of that syntactic shape – are optimal. In general, each rule is a template that applies to many queries of the same shape. Each of those templates has many instances of the form: under an access schema $A$, if $Q(\bar{x})$ is $\bar{x}$-controlled and $Q'(\bar{y})$ is $\bar{y}$-controlled, then some query $Q''(\bar{z})$, built from $Q$ and $Q'$, is controlled by tuples $\bar{z}_1, \ldots, \bar{z}_k$ for some of them, $Q''$ is not needed.

We say that such a rule is optimal if there exists an instance of it in which the query $Q''$ is not controlled by any subtuple $\bar{z}$ of a minimal tuple among the $\bar{z}_i$’s. That is, in full generality, we cannot achieve smaller controlling tuples. Then the following can be easily verified.

Proposition 4.3: Each of the rules for defining the classes of controlled queries is optimal.

Complexity of controllability. As controllability, unlike scale independence, is a purely syntactic condition, one can expect it to be decidable. In fact we pinpoint the exact complexity of it. The conjunction rule above involves two possibilities, indicating that some guessing is needed while looking for tuples controlling a query. This intuition is confirmed by NP-completeness of two problems shown below. For the second one, we say that $Q$ is minimally controlled by $\bar{x}$ if it is $\bar{x}$-controlled but not $\bar{x}'$-controlled for any subtuple $\bar{x}'$ of $\bar{x}$.

- **Problem QCntl**: 
  - **Input**: An access schema $A$, a number $K > 0$, an FO query $Q(\bar{y})$.
  - **Question**: Is there $\bar{x}$ with $|\bar{x}| \leq K$ so that $Q$ is $\bar{x}$-controlled?

- **Problem QCntl_{min}**: 
  - **Input**: An access schema $A$, an FO query $Q(\bar{y})$, a variable $x$.
  - **Question**: Is $Q$ minimally controlled by some $\bar{x}$ containing $x$?
Theorem 4.4: The problems $\text{QCnt}$ and $\text{QCnt}_{\text{min}}$ are NP-complete. They remain NP-hard for CQ.

Proof sketch. We use reductions from problems related to candidate keys and prime attributes, cf. [1].

Embedded controllability and query answering under constraints. So far we have looked at access schemas, which tell us that, given values of certain attributes, there is a bound on the set of tuples having those attribute values. But we often have cases when such constraints are embedded, i.e., they do not apply to the whole set of attributes. For instance, consider a relation $\text{visit}(\text{id}, \text{restaurant}, \text{yy}, \text{dd}, \text{mm})$ indicating that a person $\text{id}$ visited a restaurant on a given date. Then we can add to the access schema information stating that for every year $\text{yy}$, there is a limit on the number of retrieved months ($\text{mm}$) and days ($\text{dd}$) (namely 366), and those values can be efficiently found.

Another reason to consider such embedded statements is that they make it possible to incorporate constraints such as functional dependencies (FDs) into access schemas: an FD $X \rightarrow Y$ says that once $X$ is fixed, we have just one possibility for the values of $Y$.

Formally, embedded constraints in an access schema are tuples $(R, X[Y], N, T)$ with $X \subseteq Y$ being sets of attributes of $R$, indicating that for a given tuple $\bar{a}$ of values of $X$, the result of $\pi_Y(\sigma_{x=a}(R))$ has at most $N$ tuples and can be found in time $T$. Note that previous statements $(R, X, N, T)$ are just a special case when $Y = \text{attr}(R)$, the set of all attributes in $R$. An FD $X \rightarrow Y$ with a time guarantee $T$ to retrieve values of $Y$ for given values of $X$ is just $(R, X[X \cup Y], 1, T)$ in the access schema.

We can then extend the rules of controllability to define what it means for a query $Q(\bar{x})$ to be $\bar{x}[\bar{y}]-$controlled (under $A$) when $\bar{x} \subseteq \bar{y} \subseteq \bar{z}$. Most of the rules just mimic those for controllability except two that are similar to inference rules for FDs. Below we give two sample controllability rules (rules 1.2) and the two inference rules (rules 3.4).

1. if $(R, X[Y], N, T)$ is in $A$, then $R(\bar{x})$ is $\bar{x}[\bar{y}]-$controlled, where $\bar{x}$ and $\bar{y}$ are subtuples of $\bar{z}$ corresponding to attributes in $X$ and $Y$;
2. if $Q(\bar{x}, \bar{z})$ is $\bar{x}[\bar{y}]-$controlled and $\bar{x} = \bar{z} \cup \bar{y}$, then $Q$ is $\bar{z}[\bar{y}]-$controlled (and likewise for the symmetric case);
3. if $Q(\bar{x})$ is $\bar{x}[\bar{y}]-$controlled and $\bar{x} \subseteq \bar{z}$, then $Q$ is $\bar{z}[\bar{y}]-$controlled;
4. if $Q(\bar{x})$ is $\bar{x}[\bar{y}]-$controlled and $\bar{x} \subseteq \bar{y}$, then $Q$ is $\bar{y}[\bar{z}]-$controlled.

Then one adapts the proof of Theorem 4.2 to show:

Proposition 4.5: If $Q(\bar{x}, \bar{y}, \bar{z})$ is $\bar{x}[\bar{y} \cup \bar{z}]-$controlled under $A$, then $\exists \bar{x} Q(\bar{a}, \bar{y}, \bar{z})$ is efficiently scale-independent under $A$ for each $\bar{a}$.

By Proposition 4.5, we can process some queries scale-independently under constraints, as shown below.

Example 4.6: As we have seen in Example 4.1, query $Q_3$ is not scale-independent, even if $\text{p}$ and $\text{yy}$ are fixed. In contrast, below we show that $Q_3$ becomes scale-independent after adding two embedded statements to the access schema. One is $(\text{visit}, \text{yy}y[\text{yy}, \text{dd}, \text{mm}], 366, T)$, which simply says that a year has at most 366 days; and the other is an FD $\text{id}, \text{yy}, \text{dd}, \text{mm} \rightarrow \text{rid}$, saying that on a given day, each person $\text{id}$ dines out at most once, even in NYC (we assume the FD to be effective).

Given these two embedded statements, one can apply the rules to derive that subquery visit is $(\text{id}, \text{yy})$-controlled. Together with the assumption that $\text{id}$ forms an index for person and that $\text{rest}$ is city-controlled, this shows that query $Q_3$ is $(p, \text{yy})$-controlled. Thus, if we want to find, for instance, all $A$-rated NYC restaurants that were visited by a NYC friends of $p_0$ in 2013, we can pose the query $Q_3(\text{nn}, p_0, 2013)$, which is now scale-independent.

5. INCREMENTAL SCALE INDEPENDENCE

While some queries $Q$ may not be in $\text{SQ}_D(D, M)$ for a database $D$, they may be incrementally scale-independent in $D$ [6]. That is, we can compute $Q(D)$ once as precomputation, and then incrementally evaluate $Q$ on demand in response to changes to $D$, by accessing at most $M$ tuples from $D$. Incremental scale independence allows us to answer online queries efficiently, by making maximum use of previous computation.

In this section, we first formally specify incremental scale independence, and then establish its complexity. Finally, we identify sufficient conditions for a query to be scale-independent for incremental evaluation.

Incremental scale independence. We consider updates $\Delta D = (\Delta D, \nabla D)$ to $D$ that consist of a list of tuples $\Delta D$ to be inserted into $D$ and a list $\nabla D$ of tuples to be deleted. It is required that $\nabla D$ be contained in $D$ and $\Delta D$ be disjoint from $D$; in particular, $\Delta D \cap \nabla D = \emptyset$. We write $D \oplus \Delta D$ to denote the database obtained by applying $\Delta D$ to $D$, i.e., $(D - \nabla D) \cup \Delta D$ (where updates are applied relation-wise).

The setting of incremental query answering is as follows. We are given a query $Q$ and its answer $Q(D)$ on a database $D$. Now, for an update $\Delta D$, we want to compute $Q(D) \oplus \Delta D$, i.e., we want a pair of queries $\Delta Q = (\nabla Q, \Delta Q)$ that take $\Delta D$ and $D$ as inputs so that

$$Q(D) \oplus \Delta D = Q(D) \cup \Delta Q(\Delta D, D).$$

We use the notation $Q(D) \oplus \Delta Q(\Delta D, D)$ to denote $(Q(D) - \nabla Q(\Delta D, D)) \cup \Delta Q(\Delta D, D)$.

As shown in [14], for FO queries $Q$, we can effectively find $\nabla Q$ and $\Delta Q$ in $\text{FO}$ so that $\nabla Q(\Delta D, D) \subseteq Q(D)$ and $\Delta Q(\Delta D, D)$ is disjoint from $Q(D)$. Typically such queries are found by propagating changes through relational algebra expressions [15]. Such queries can often be evaluated much faster than computing $Q(D) \oplus \Delta D$ from scratch, if $\Delta D$ is small compared to $D$, as found in practice [15].

To give an example, recall $Q_2$ and $\Delta D_2$ from Example 1.1, where $\Delta D_2$ inserts tuples into visit, denoted by $\Delta \text{visit}(\text{id}_2, \text{rid})$. From $Q_2$ we derive $\Delta Q_2(\text{mn}, p)$ as:

$$\Delta Q_2(\text{mn}, p) = \exists \text{id}_2, \text{rid}(\text{friend}(\text{p}_0, \text{id}_2) \land \text{visit}(\text{id}_2, \text{rid}) \land \text{rest}(\text{rid}, \text{mn}, \text{NYC}, A)).$$

Here $\Delta Q_2$ finds tuples to be inserted into $Q_2(\text{D}_2)$, such that $Q_2(\Delta D_2 \oplus \Delta D_2) = Q_2(\text{D}_2) \cup \Delta Q_2(\Delta D_2, \text{D}_2)$.

We say that $Q$ is incrementally scale-independent in $D$ w.r.t. $(M, k)$ if for all updates $\Delta D$ to $D$ with $|\Delta D| \leq k$,
there exists a subset \( D_0 \subseteq D \) with \( |D_0| \leq M \) such that \( Q(D \uplus \Delta D) = Q(D) \uplus \Delta Q(\Delta D, D_0) \). That is, to incrementally answer \( Q \) in \( D \) in response to \( \Delta D \), we need to access no more than \( M \) tuples from \( D \), independent of the size of the underlying \( D \). We consider updates \( \Delta D \) with a bounded number of tuples since in real life updates are typically frequent but small, very often consisting of single tuple insertions or deletions.

We write \( \Delta QSI_L(D, M, k) \) for the set of all \( L \) queries that are incrementally scale-independent in \( D \) w.r.t. \((M, k)\). For instance, \( Q_2 \) above is in \( \Delta QSI_L(D_2, M, k) \) if \( M \geq 3k \), when some constraints state that only updates allowed to \( D_2 \) are insertions \( \Delta \text{visit}(id_2, rid) \).

We study the incremental scale-independence problem, denoted by \( \text{PROBLEM: } QSI(L) \), which is stated as follows.

- **Input**: A schema \( \mathcal{R} \), a query \( Q \in L \) over \( \mathcal{R} \), an instance \( D \) of \( \mathcal{R} \), and \( M, k \geq 0 \).
- **Question**: Is \( Q \) in \( \Delta QSI_L(D, M, k) \)?

**Complexity.** We next give the complexity of \( \Delta QSI \). For full \( \text{FO} \), \( \Delta QSI \) matches the bound of \( QDI \) for scale-independent query answering, even in simple settings, for combined complexity, and moves one level up in the polynomial hierarchy for data complexity.

**Theorem 5.1:** When \( L = \text{FO} \), \( \Delta QSI_L(L) \) is
- \( \text{PSPACE} \)-complete for combined complexity, even for Boolean queries, and even if \( M \) is fixed; and
- \( \Pi^P_2 \)-complete for data complexity (when both queries \( Q \) and \( \Delta Q \) are fixed), and \( \text{coNP} \)-complete if in addition, \( M \) is fixed.

**Proof sketch.** Since guessing is free in \( \text{PSPACE} \), and since the combined complexity of \( \text{FO} \) is in \( \text{PSPACE} \) as well, the definition of incremental scale independence naturally translates into a \( \text{PSPACE} \) algorithm. For the \( \text{PSPACE} \) hardness, we use reduction from \( \text{Q3SAT} \) for Boolean \( \text{FO} \) by using a fixed \( M \).

For the lower bounds for data complexity, we use reductions from the \( \forall \exists \exists \exists \text{3CNF} \) problem and the complement of \( \text{3SAT} \), when \( M \) is not fixed and fixed, respectively.

We next look at the case of \( \text{CQ} \). It turns out that we have rather high bounds.

**Theorem 5.2:** The complexity of \( \Delta QSI(\text{CQ}) \) is
- \( \Pi^P_2 \)-complete, or \( \Pi^P_3 \)-complete if \( M \) also is fixed, for combined complexity, assuming that the maintenance queries \( \Delta Q \) are \( \text{CQ} \) as well; and
- \( \Pi^P_2 \)-complete, or \( \text{coNP} \)-complete when \( M \) is in addition fixed, for data complexity.

**Proof sketch.** The upper bounds follow essentially from parsing the definitions of being incrementally scale-independent and using the tableau representation of a \( \text{CQ} \), except the case for fixed \( M \), which is more involved. For data complexity, the maintenance query is allowed to be in \( \text{FO} \), which also has \( \text{PTIME} \) data complexity.

The lower bounds are verified by reductions from satisfiability problems: \( \forall \exists \forall \exists \exists \exists \text{3CNF} \) for \( \Pi^P_2 \), \( \forall \exists \exists \exists \text{3CNF} \) for \( \Pi^P_3 \), and the complement of \( \text{3SAT} \) for \( \text{coNP} \). The coding for the data complexity with variable \( M \) is more involved, to cope with various \( \Delta D \) and \( \nabla D \).

The condition that maintenance queries are in \( \text{CQ} \) is true for insertion-only updates; in fact in this case the maintenance query can be computed in polynomial time [14]. An example of arbitrary updates admitting \( \text{CQ} \) maintenance query is when \( Q \) is key-preserving [8], if the projection attributes of \( Q \) include a key of each occurrence of base relations that are involved in \( Q \).

**Under access schemas.** We now look at incremental scale independence under access schemas. Given a relational schema \( \mathcal{R} \), an access schema \( \mathcal{A} \), and a query \( Q(\bar{x}, \bar{y}) \), we say that \( Q \) is \( \bar{x} \)-incrementally scale-independent under \( \mathcal{A} \) if for each tuple \( \bar{a} \) of values for \( \bar{x} \), and each update \( \Delta D \), the answer to maintenance queries \( \Delta Q(\bar{a}, \Delta D, D) \) can be found in time that depends on \( Q, \mathcal{A} \), and \( \Delta D \) only, but not on \( D \). Note that \( \Delta Q \) has the same free variables \( \bar{x}, \bar{y} \) as \( Q \), but takes as its input both \( D \) and \( \Delta D \). Efficient incremental scale independence under \( \mathcal{A} \) is defined along the same lines as its counterpart given in Section 4.

From results of Section 4, we immediately obtain:

**Corollary 5.3:** If queries \( \Delta Q(\bar{x}, \bar{y}) \) are \( \bar{x} \)-controlled under an access schema \( \mathcal{A} \) for a query \( Q \) in \( \text{FO} \), then \( Q \) is efficiently incremental \( \bar{x} \)-scale-independent under \( \mathcal{A} \).

In the literature on view maintenance and incremental recomputation, queries \( \Delta Q \) are commonly derived for relational algebra queries. They are not queries that are written by the user, but rather automatically generated by the DBMS; hence it is better to produce them in their procedural version, to avoid an extra compilation stage [15]. We now show how to achieve incremental scale independence for relational algebra queries. As an intermediate step, we also show how to achieve scale independence for relational algebra.

The idea is as follows. The analog of a tuple \( \bar{x} \) of variables in \( \bar{x} \)-controlled queries is now a set of attributes \( X \) of an expression \( E \) of relational algebra. We then produce results on scale independence when the \( X \) attributes are fixed, i.e., on scale independence of queries \( \sigma_{X = \bar{a}}(E) \). For each expression \( E \) we next introduce expressions \( E^\forall \) and \( E^\exists \), and, for an access schema \( \mathcal{A} \), inductively generate the set \( \mathcal{R}_A \) of pairs \((E, X)\), where \( E \) is an expression, or an expression annotated with \( \Delta \) or \( \nabla \), and \( X \) is a set of its attributes. These will tell us whether the expression is (incrementally) scale-independent for fixed values of attributes in \( X \).

Formally, let \( \text{attr}(E) \) be the set of attributes of the output of a relational algebra expression \( E \). We assume that all selection conditions \( \theta \) in \( \sigma_{\theta} \) are conjunctions of equalities and inequalities. Then, for an access schema \( \mathcal{A} \), the set \( \mathcal{R}_A \) is defined inductively as follows:

**Relational algebra rules:***

- if \((R, X, N, T) \in \mathcal{A}\), then \((R, X) \in \mathcal{R}_A\);
- if \((E, X) \in \mathcal{R}_A\) and \(X \subseteq Y\), then \(\pi_Y(E, X) \in \mathcal{R}_A\);
- if \((E, X) \in \mathcal{R}_A\) and \(\theta \) is a condition, then \(\sigma_{\theta}(E, X \setminus X') \in \mathcal{R}_A\), where \(X' \) is the set of attributes \(A\) for which \(\theta\) implies that \(A = a\);
- if \((E_1, X_1), (E_2, X_2) \in \mathcal{R}_A\) and \(\text{attr}(E_1) = \text{attr}(E_2)\), then \((E_1 \cup E_2, X_1 \cup X_2) \in \mathcal{R}_A\);
- if \((E_1, X_1), (E_2, \text{attr}(E_2)) \in \mathcal{R}_A\) and \(\text{attr}(E_1) = \text{attr}(E_2)\), then \((E_1 - E_2, X_1) \in \mathcal{R}_A\);
Theorem 5.4: for maintenance queries, by applying the standard pushing incremental computation. We can also construct the set \((\text{attr})\) is controlled by any set of attributes (and thus by \(\sigma_Q\)). We now show rules for expressions \(E^\forall\) and \(E^\Delta\). For this, we use queries for propagating changes through relational expressions, and apply relational algebra rules to them. We use maintenance queries from [14] since, unlike others proposed in the literature, those guarantee that \(E^\forall \subseteq E\) and \(E^\Delta \cap E = \emptyset\). To give an example, consider the propagation expression \((E_1 - E_2)^\forall = (E_1^\forall - E_2^\forall) \cup (E_2^\Delta \cap E_1^\forall)\). If we know that \(E_1^\forall\) is controlled by a set \(X\) of attributes, and \(E_2\) is controlled by any set of attributes (and thus by \(\sigma(E_2)\)), then \(E_1^\forall - E_2^\forall\) is controlled by \(X\). Likewise, if \(E_2^\Delta\) is controlled by \(Z\) and \(E_1\) is controlled by anything (and thus by \(\sigma(E_1)\)), then \(E_2^\forall \cap E_1\) is controlled by \(Z\), and \((E_1 - E_2)^\forall\) by \(X \cup Z\). Now we list the rules for \(E^\forall\) and \(E^\Delta\).

Decrement rules:
- if \(R \in \mathcal{R}\), then \((R^\forall, \emptyset) \in \mathcal{RA}_A\);
- if \((E^\forall, X), (E, X)\) are in \(\mathcal{RA}_A\), and \(X \subseteq Y\), then \(((\pi_Y(E))^\forall, X) \in \mathcal{RA}_A\);
- if \((E^\forall, X) \in \mathcal{RA}_A\), then \(((\sigma_Q(E))^\forall, X) \in \mathcal{RA}_A\);
- if \((E^\forall, X), (E_1, \sigma_Q(E_2))\) are in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\forall, X) \cup \sigma_X = \emptyset \in \mathcal{RA}_A\);
- if \((E^\forall, X), (E_1, \sigma_X = \emptyset)\) are all in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\forall, X \cup Z) \in \mathcal{RA}_A\);
- if \((E^\forall, X, Y, (E_1, Y))\) are all in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\forall, X \cup X_2 \cup Y_1 - attr(E_2)) \cup (Y_2 - attr(E_1)) \in \mathcal{RA}_A\).

Increment rules:
- if \(R \in \mathcal{R}\), then \((R^\Delta, \emptyset) \in \mathcal{RA}_A\);
- if \((E^\Delta, X)\) and \((E, X)\) are in \(\mathcal{RA}_A\), and \(X \subseteq Y\), then \(((\pi_Y(E))^\Delta, X) \in \mathcal{RA}_A\);
- if \((E^\Delta, X) \in \mathcal{RA}_A\), then \(((\sigma_Q(E))^\Delta, X) \in \mathcal{RA}_A\);
- if \((E^\forall, X), (E_1, \sigma_Q(E_2))\) are all in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\Delta, X_2) \in \mathcal{RA}_A\);
- if \((E^\forall, X, Z), (E_1, \sigma_X = \emptyset)\) and \((E_1, attr(E_2))\) are all in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\Delta, X_1 \cup Z_2) \in \mathcal{RA}_A\);
- if \((E^\forall, X, Y, (E_1, Y))\) are all in \(\mathcal{RA}_A\) for \(i = 1, 2\), \(((E_1 \cup E_2)^\Delta, X_1 \cup X_2 \cup (Y_1 - attr(E_2)) \cup (Y_2 - attr(E_1)) \in \mathcal{RA}_A\).

These rules tell us when relational algebra expressions are scale-independent, both in the usual way and with respect to incremental computation. We can also construct the set \(D_Q\) for maintenance queries, by applying the standard pushing selection optimizations through the expressions.

Theorem 5.4: Assume that \(E\) is a relational algebra expression, and let \(\mathcal{A}\) be an access schema. Then
- if \((E, X) \in \mathcal{RA}_A\), then \(\sigma_{X \cdot \mathcal{A}}(E)\) is scale-independent under \(\mathcal{A}\);
- if \((E^\forall, X)\) and \((E^\forall, X)\) are in \(\mathcal{RA}_A\), then \(\sigma_{X \cdot \mathcal{A}}(E)\) is incrementally scale-independent under \(\mathcal{A}\).

Proof sketch. The first item is by induction on relational algebra expressions, similar to the proof of Theorem 4.2. For the second item, we use change propagation expressions for relational algebra queries given in [14], and analyze them to find controlling sets of attributes.

Leveraging the special shape of maintenance queries, we have the following result for CQ. Let \(\mathcal{A}(R)\) be an access schema obtained from \(\mathcal{A}\) by adding \((R, \emptyset, 1, 1)\) stating that one can obtain the entire relation \(R\) in constant time. Note that it is easier to answer queries scale independently under \(\mathcal{A}(R)\) than under \(\mathcal{A}\) alone.

Proposition 5.5: Let \(Q(x, y)\) be a CQ, and \(\mathcal{A}\) an access schema. (1) If \(Q\) is \(\pi\)-scale-independent under \(\mathcal{A}(R)\), then \(Q\) is incrementally \(\pi\)-scale-independent under \(\mathcal{A}\) when updates are insertions into relation \(R\). (2) If it is also derivable by \(\mathcal{RA}_A\) rules that \(Q\) is controlled by all of its attributes, then \(Q\) is incrementally \(\pi\)-scale-independent under \(\mathcal{A}\) for arbitrary updates on \(R\).

Example 5.6: Recall query \(Q_2\) from Example 1.1 (also used earlier in this section). Query \(Q_2\) was not \(\pi\)-controllable, but when \((\text{visit}, \emptyset, 1, 1)\) is added to the access schema, \(\Delta Q_2\) becomes \(\pi\)-controllable, and therefore, \(\Delta Q_2(m, p_0)\) can be found scale independently for a fixed \(p_0\).

Connections with view maintenance. We conclude with a few remarks relating our notions here with several problems from incremental view maintenance.

One is the problem of self-maintainability. A query \(Q\) is self-maintainable if there exists a set \(V\) of views such that given updates \(\Delta D\) to \(D\), both \(Q(D \cup \Delta D)\) and \(V(D \cup \Delta D)\) can be computed from \(Q(D)\), \(V(D)\) and \(\Delta D\), without accessing the underlying database \(D\) [27]. This notion is stronger than incremental scale independence, but only in the presence of views that need to be maintained themselves.

Even stronger than self-maintainability is the notion of queries independent of updates [21]: i.e., whether for all databases \(D\) and \(\Delta D\), we have \(Q(D) = Q(D \cup \Delta D)\). In fact, [21] looks at more complex updates generated by other queries. This implies incremental scale independence even when \(M = 0\), but is much more restrictive.

Finally, to analyze incremental algorithms, [28] proposes to use \(|\text{CHANGED}| = |\Delta D| + |\Delta S|\), the size of the changes in the input and output. An incremental algorithm is said to be bounded if its cost can be expressed as a function of only \(|\text{CHANGED}|\), not of \(|D|\). In contrast to \(\Delta Q\), bounded incremental query answering concerns the size of \(\Delta Q(\Delta D, D)\) rather than the number of tuples in \(D\) accessed. Nonetheless, if \(Q\) is in \(\Delta Q\), and \(\Delta Q(D, M, k)\), the cost of computing \(\Delta Q(\Delta D, D)\) is a function of \(|\Delta D|\) and \(M\) for all \(\Delta D\) with \(|\Delta D| \leq k\).

6. SCALE INDEPENDENCE USING VIEWS

We may also approach a query \(Q\) that is not scale-independent by using views [6]. The idea is to maintain a set \(V(D)\) of views of a database \(D\), and compute \(Q(D)\) from \(V(D)\) and at most \(M\) additional tuples from \(D\).

This section studies scale independence using views. We state the problem, provide its complexity, and give sufficient conditions for scale independence by using views.

Scale independence using views. Assume a set \(V\) of views defined over a schema \(R\), in a query language \(L\). Consider an \(L\) query \(Q\). Let \(Q'\) be a query over \(R\) expanded with
Complexity bounds of database to achieve scale independence by using views, when $M$ views

$D$ and $V$ are boolean queries and a constant

$|Q|$ size relations that hold the views from $Q$. We denote by $\text{PROBLEM}$ undecidable when $NP$– $L$ is a constant. Moreover, the reductions $Q$ and $Q'$ are in UCQ.

Based on this, we develop an NP algorithm to check whether $Q$ is in $\text{VSQ}_L(V, M)$, for UCQ $Q$ and $V$.

Conditions for scale independence using views. Following Section 4, we say that a query $Q$ is $x$-scale-independent under an access schema $A$ using $V$ if there exists a rewriting $Q'(x)$ of $Q(x)$ using $V$ such that $\{a\}$ all the distinguished variables in $\bar{x}$ are constrained in $Q'(x)$ and $|Q'(x)| \leq M$. For Boolean $Q$, condition (b) alone suffices. The same characterization remains intact when $Q$, $V$ and $Q'$ are in UCQ.

Consider a rewriting $Q'$ of a $CQ$ $Q(x)$ using $V$, where $Q'(x) = \exists \bar{x} (Q'_i \land Q'_j)$ (see the proof of Theorem 6.1). We say that $Q'(x)$ is $\bar{y}$-controlled under $A$ using $V$ if its base part $Q'_i$ is $\bar{y}$-controlled under $A$, following the rules for the controllability of $CQ$ given in Section 4. Then the characterization given in the proof of Theorem 6.1 tells us that under $A$, $Q_i$ is $\bar{y}$-scale-independent using $V$ if $Q'_i(x)$ is $\bar{y}$-controlled under $A$ using $V$ and $\bar{y}$ includes all the unconstrained distinguished variables in $x$.

We also give a sufficient condition for an FO query $Q$ to be $x$-scale independence using FO views. Given an FO rewriting $Q'$ of $Q$, we define the expansion $Q'_e$ of $Q'$ to be the query that unfolds $Q'$ by substituting the definition of view $V$ for each occurrence of $V \in V$ in $Q'$.

From Theorem 4.2 and the proof of Theorem 6.1, one can readily get the following corollary.

Corollary 6.2: (1) For a query $Q$ and a set $V$ of views in FO, if $Q$ has a rewriting $Q'$ whose expansion $Q'_e$ is $\bar{x}$-controlled under an access schema $A$, then $Q$ is $\bar{x}$-scale-independent under $A$ using $V$. (2) For $Q$ and $V$ in $CQ$, if $Q$ has a $\bar{y}$-controlled rewriting $Q'(x)$ under $A$ using $V$ and if $\bar{y}$ contains all unconstrained variables of $Q'$ in $x$, then $Q$ is $\bar{y}$-scale-independent of $A$ using $V$.

Example 6.3: For instance, consider again the rewriting $Q_2(p, r)$ using views $V = \{V_1, V_2\}$ given in Example 1.1.
Under an access schema $A$ that limits 5000 friends per person, its base part $\text{friend}(p, \text{id})$ is $p$-controlled using $\mathcal{V}$, and hence, so is $Q'_2$. As a result, $Q_2$ is $p$-scale-independent under $A$ using $\mathcal{V}$. However, under the same $A$, $Q_2$ is not $p$-scale-independent in the absence of $\mathcal{V}$. □

Connections. Finally, we relate scale independence using views with prior work on the study of views.

A complete rewriting of $Q$ using $\mathcal{V}$ is a rewriting of $Q$ built up with only literals in $\mathcal{V}$ and equality predicates [20]. Obviously, if $Q$ has a complete rewriting using $\mathcal{V}$, then $Q$ is scale-independent using $\mathcal{V}$ with $M = 0$. That is, the former is a special case of the latter.

Given a set $\mathcal{V}$ of views over a schema $\mathcal{R}$, the study of view complements (e.g., [25]) is to find a (minimal) set $\mathcal{V}_c$ of views such that for all instances $D$ of $\mathcal{R}$, $D$ can be reconstructed from $\mathcal{V}(D)$ and $\mathcal{V}_c(D)$. When both $\mathcal{V}$ and $\mathcal{V}_c$ are available, all queries $Q$ is scale-independent using $\mathcal{V}$ and $\mathcal{V}_c$, with $M = 0$, without accessing $D$. However, it is not very practical to approach scale independence by using views and their complements: $\mathcal{V}(D)$ and $\mathcal{V}_c(D)$ are no smaller than $D$, and it is nontrivial to compute a minimal set $\mathcal{V}_c$ of view complements.

7. CONCLUSION

We have given a formal treatment of scale independence, incremental scale independence and scale independence using views. We have established complexity bounds of QDSI, QSI, $\Delta$QSI and VQSI for CQ, UCQ and FO, for Boolean and data selecting queries, and for combined and data complexity. We have also provided sufficient conditions for FO queries to be scale-independent under (embedded) access schemas, specifying indices and constraints. The conditions also suggest how we can efficiently identify a small subset $D_Q$ of a database $D$ for a query $Q$ such that $Q(D) = Q(D_Q)$.

In the future we want to study, when $Q$ is not scale-independent in $D$ w.r.t. $M$, what the best performance ratio is if we approximately compute $Q(D)$ by accessing at most $M$ tuples from $D$. We want to strike a balance between the accuracy of the inexact query answers and the bound on the amount of data retrieved. Another question concerns the design of access schemas. In particular, we would like to see how to optimally design access schemas for a given query workload. Furthermore, an interesting extension is to consider query languages that support grouping and aggregation. We believe that sufficient conditions for scale independence can also be developed to that setting. Finally, we want to investigate how the techniques from [3] can be used for incremental scale-independent queries.

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8. REFERENCES


