Learning from Data, Tutorial Sheet for Week 2

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1. Calculate the length of the vector $(1, -1, 2)$.

2. Show that $\sum_i w_i x_i = |\boldsymbol{w}||\boldsymbol{x}| \cos\theta$, where θ is the angle between the two vectors $\boldsymbol{w}$ and $\boldsymbol{x}$.

3. Let A and $\boldsymbol{v}$ be defined as

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Calculate $A\boldsymbol{v}$. Is $\boldsymbol{v}$ an eigenvector of A, and if so what is the corresponding eigenvalue?

4. Partial derivatives. Find the partial derivatives of the function $f(x, y, z) = (x + 2y)^2 \sin(xy)$.

5. Probability. Lois knows that on average radio station RANDOM-FM plays 1 out of 4 of her requests. If she makes 3 requests, what is the probability that at least one request is played?

6. Let X be distributed according to a uniform distribution, i.e. $f(x) = 1$ for $0 \leq x \leq 1$, and 0 otherwise. Show that X has mean $\frac{1}{2}$ and variance $\frac{1}{12}$.

7. Find the (unconstrained) minimum of the function

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A\boldsymbol{x} - \boldsymbol{b}^T\boldsymbol{x}$$

where A is a positive definite symmetric matrix.

Find the minimum of $f(\boldsymbol{x})$ along the line $\boldsymbol{x} = \boldsymbol{a} + t\boldsymbol{v}$ where t is a real parameter.

Calculate explicitly the unconstrained minimum in the case that

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Additional question 8. Consider the integral

$$I = \int_{\infty}^{\infty} e^{-\frac{1}{2}x^2} dx$$

By considering

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

and converting to polar coordinates, r, θ, show that

$$I = \sqrt{2\pi}$$