Approximation of Conjunctive Queries

Advanced Topics in Foundations of Databases, University of Edinburgh, 2016/17

Possible Approaches

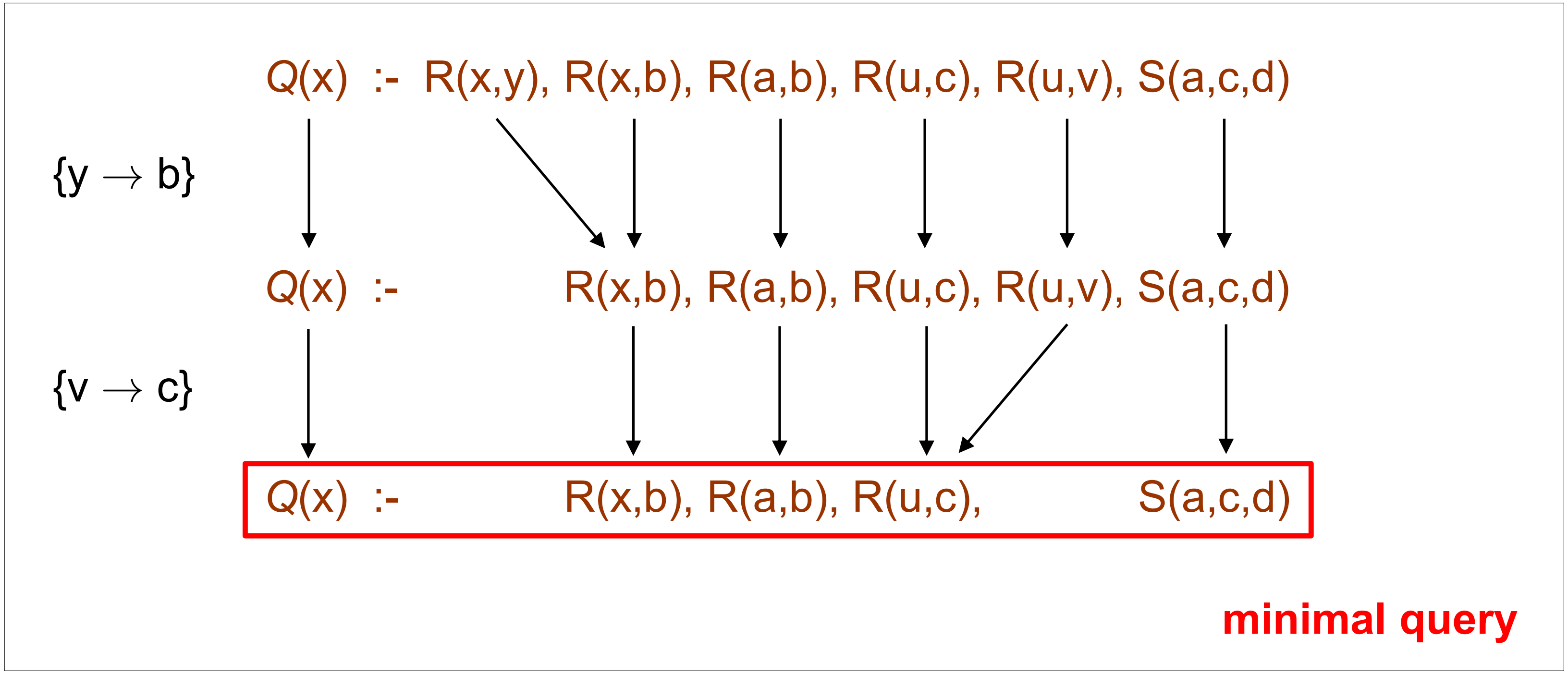
…to address the challenges raised by the volume of big data

- Scale Independence – find queries than can be answered regardless of scale ✓

- Replace the query with one that is much faster to execute

Minimizing Conjunctive Queries

- Database theory has developed principled methods for optimizing CQs:
  - Find an equivalent CQ with minimal number of atoms (the core)
  - Provides a notion of “true” optimality



Q(x) :- R(x,y), R(x,b), R(a,b), R(u,c), R(u,v), S(a,c,d)

{y → b}

Q(x) :- R(x,b), R(a,b), R(u,c), R(u,v), S(a,c,d)

{v → c}

Q(x) :- R(x,b), R(a,b), R(u,c), S(a,c,d)

minimal query

Minimizing Conjunctive Queries

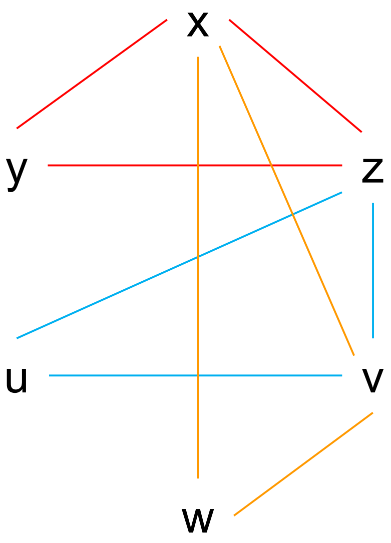
- But, a minimal equivalent CQ might not be easier to evaluate – query evaluation remains NP-hard

- However, we know “good” classes of CQs for which query evaluation is tractable (in combined complexity):
  - Graph-based
  - Hypergraph-based

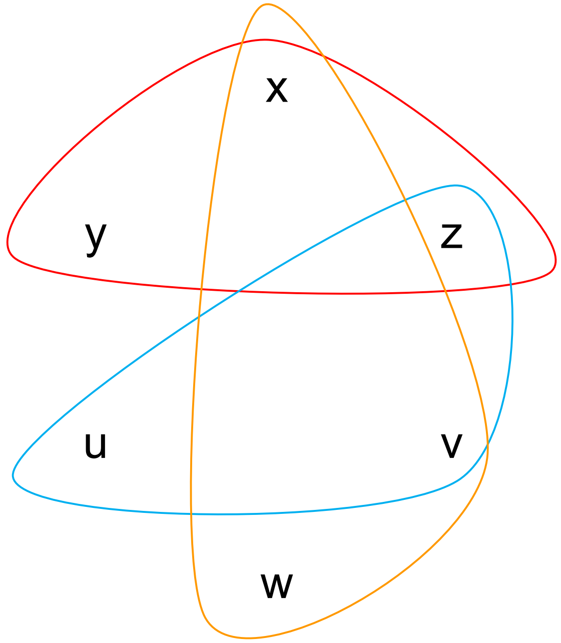
(Hyper)graph of Conjunctive Queries

Q :- R(x,y,z), R(z,u,v), R(v,w,x)

graph of *Q* - *G*(*Q*)



hypergraph of *Q* - *H*(*Q*)



“Good” Classes of Conjunctive Queries

measures how close a graph is to a tree

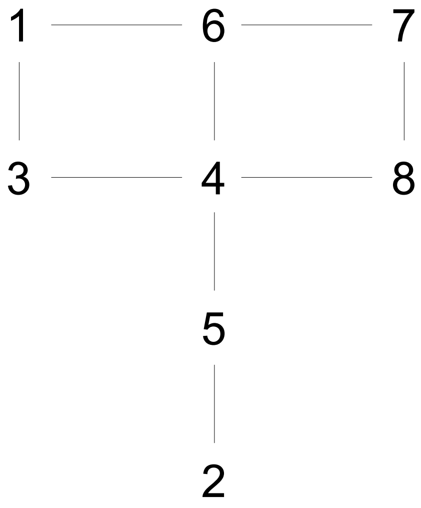
- Graph-based
  - CQs of bounded treewidth – their graph has bounded treewidth

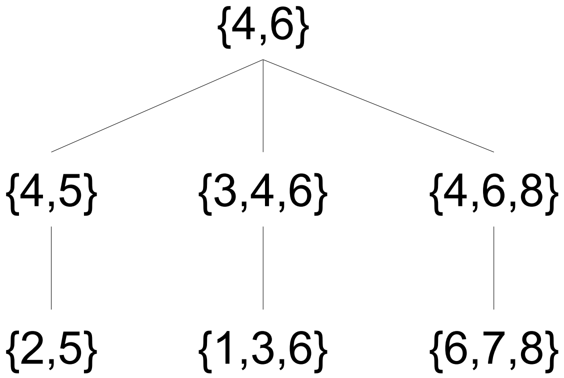
measures how close a hypergraph is to an acyclic one

- Hypergraph-based:
  - CQs of bounded hypertree width – their hypergraph has bounded hypertree width
  - Acyclic CQs – their hypegraph has hypertree width 1

Treewidth of a Graph

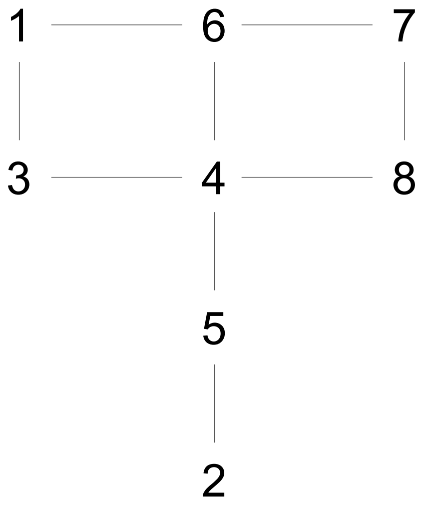
- A tree decomposition of a graph **G** = (V,E) is a labeled tree **T** = (N,F,λ), where λ : N → 2^V such that:
  1. For each node u ∈ V of **G**, there exists n ∈ N such that u ∈ λ(n)
  2. For each edge (u,v) ∈ E, there exists n ∈ N such that {u,v} ⊆ λ(n)
  3. For each node u ∈ V of **G**, the set {n ∈ N | u ∈ λ(n)} induces a *connected* subtree of **T**

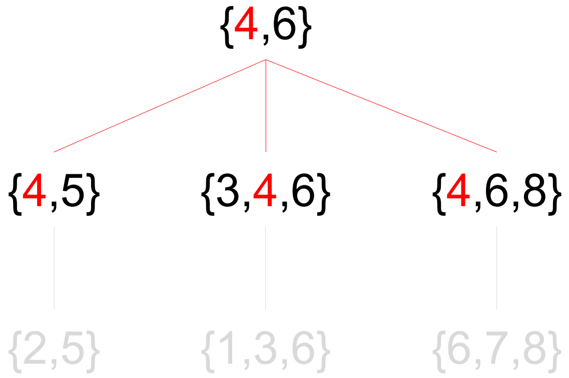




Treewidth of a Graph

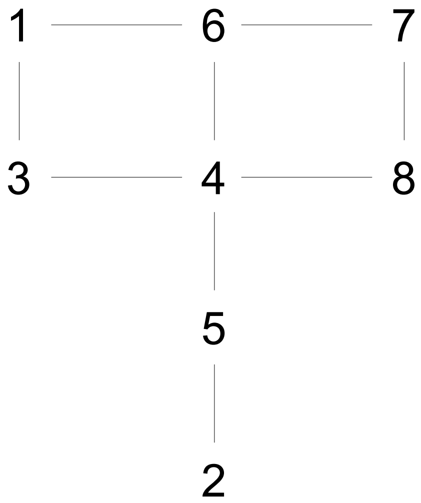
- A tree decomposition of a graph **G** = (V,E) is a labeled tree **T** = (N,F,λ), where $\lambda : N \rightarrow 2^V$ such that:
  1. For each node $u \in V$ of **G**, there exists $n \in N$ such that $u \in \lambda(n)$
  2. For each edge $(u,v) \in E$, there exists $n \in N$ such that $\{u,v\} \subseteq \lambda(n)$
  3. For each node $u \in V$ of **G**, the set $\{n \in N \mid u \in \lambda(n)\}$ induces a *connected* subtree of **T**

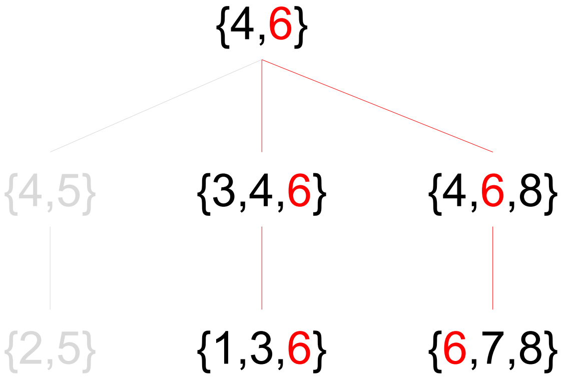




Treewidth of a Graph

- A tree decomposition of a graph **G** = (V,E) is a labeled tree **T** = (N,F,λ), where λ : N → 2^V such that:
  1. For each node u ∈ V of **G**, there exists n ∈ N such that u ∈ λ(n)
  2. For each edge (u,v) ∈ E, there exists n ∈ N such that {u,v} ⊆ λ(n)
  3. For each node u ∈ V of **G**, the set {n ∈ N | u ∈ λ(n)} induces a *connected* subtree of **T**





Treewidth of a Graph

- A tree decomposition of a graph **G** = (V,E) is a labeled tree **T** = (N,F,λ), where $\lambda : N \rightarrow 2^V$ such that:
  1. For each node $u \in V$ of **G**, there exists $n \in N$ such that $u \in \lambda(n)$
  2. For each edge $(u,v) \in E$, there exists $n \in N$ such that $\{u,v\} \subseteq \lambda(n)$
  3. For each node $u \in V$ of **G**, the set $\{n \in N \mid u \in \lambda(n)\}$ induces a *connected* subtree of **T**

- The width of a tree decomposition **T** = (N,F,λ) is $\max_{n \in N} \{|\lambda(n)| - 1\}$

  -1 so that the treewidth of a tree is 1

- The treewidth of **G** is the minimum width over all tree decompositions of **G**

CQs of Bounded Treewidth

Theorem: For a fixed k ≥ 0, BQE($\mathbf{CQTW_k}$) is in PTIME

{*Q* ∈ **CQ** | the treewidth of *G*(*Q*) is at most k}

Actually, if *G*(*Q*) has treewidth k ≥ 0, then *Q* can be evaluated in time

$O(|D|^k)$ + time to compute a tree decomposition for *G*(*Q*) of optimal width,

which is feasible in linear time

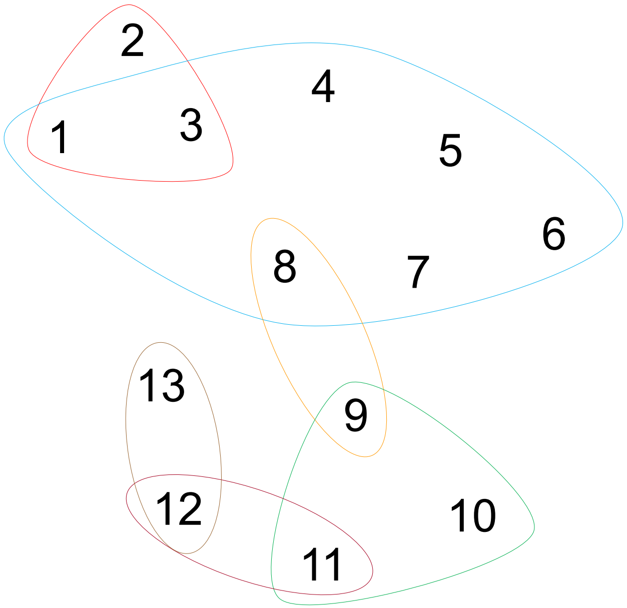
“Good” Classes of Conjunctive Queries

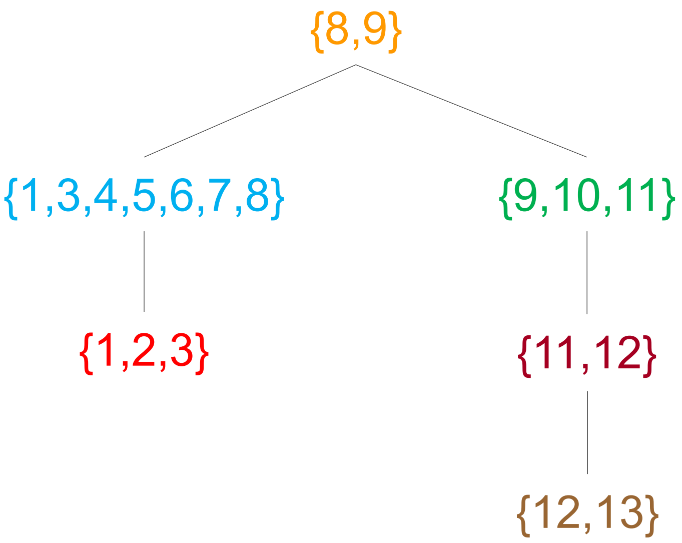
- Graph-based
  - CQs of bounded treewidth – their graph has bounded treewidth
    - Evaluation is feasible in polynomial time

- Hypergraph-based:
  - CQs of bounded hypertree width – their hypergraph has bounded hypertree width
  - Acyclic CQs – their hypegraph has hypertree width 1

Acyclic Hypergraphs

- A join tree of a hypergraph **H** = (V,E) is a labeled tree **T** = (N,F,λ), where λ : N → E such that:
    1. For each hyperedge e ∈ E of **H**, there exists n ∈ N such that e = λ(n)
    2. For each node u ∈ V of **H**, the set {n ∈ N | u ∈ λ(n)} induces a *connected* subtree of **T**





{8,9}

{1,3,4,5,6,7,8}

{9,10,11}

{1,2,3}

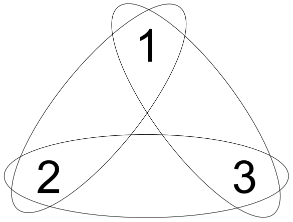
{11,12}

{12,13}

Acyclic Hypergraphs

- A join tree of a hypergraph **H** = (V,E) is a labeled tree **T** = (N,F,λ), where λ : N → E such that:
  1. For each hyperedge e ∈ E of **H**, there exists n ∈ N such that e = λ(n)
  2. For each node u ∈ V of **H**, the set {n ∈ N | u ∈ λ(n)} induces a *connected* subtree of **T**

- **Definition:** A hypergraph is acyclic if it has a join tree

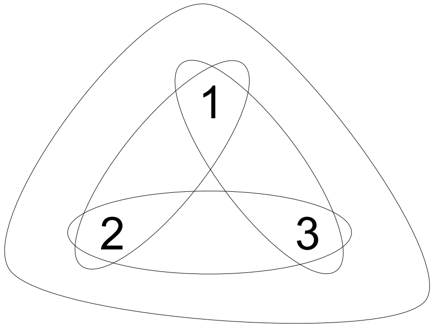


prime example of a cyclic hypergraph

Acyclic Hypergraphs

- A join tree of a hypergraph **H** = (V,E) is a labeled tree **T** = (N,F,λ), where λ : N → E such that:
  1. For each hyperedge e ∈ E of **H**, there exists n ∈ N such that e = λ(n)
  2. For each node u ∈ V of **H**, the set {n ∈ N | u ∈ λ(n)} induces a *connected* subtree of **T**

- **Definition:** A hypergraph is acyclic if it has a join tree



but this is acyclic

Acyclic CQs

Theorem: BQE(**ACQ**) is in PTIME

$\{Q \in \textbf{CQ} \mid H(Q) \text{ is acyclic}\}$

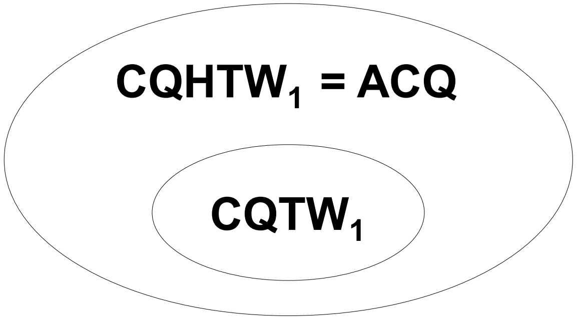
Actually, if $H(Q)$ is acyclic, then Q can be evaluated in time $O(|D| \cdot |Q|)$,

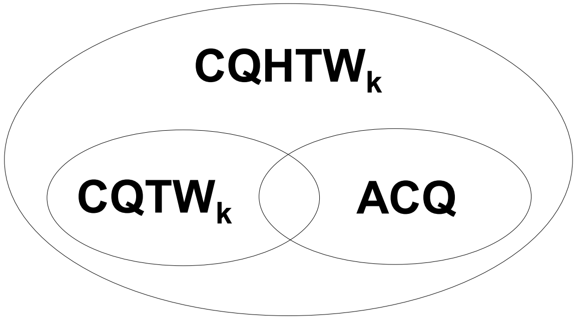
i.e., linear time in the size of D and Q

“Good” Classes of Conjunctive Queries: Recap

- Graph-based
  - CQs of bounded treewidth – their graph has bounded treewidth
    - Evaluation is feasible in polynomial time

- Hypergraph-based:
  - CQs of bounded hypertree width – their hypergraph has bounded hypertree width
    - Evaluation is feasible in polynomial time
  - Acyclic CQs – their hypegraph has hypertree width 1
    - Evaluation is feasible in linear time





Back to Our Goal

Replace a given CQ with one that is much faster to execute

or

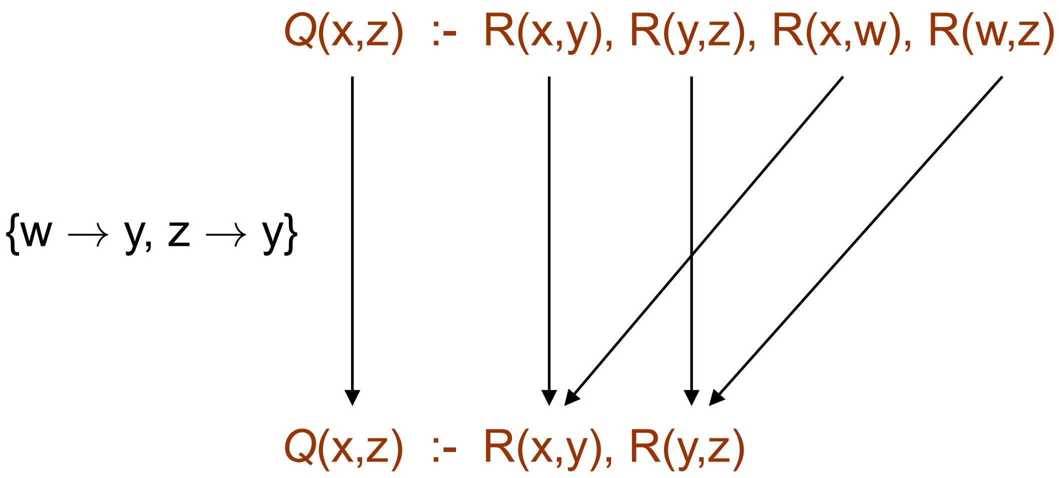
Replace a given CQ with one that falls in “good” class of CQs

preferably, with an acyclic CQ

since evaluation is in linear time

Semantic Acyclicity

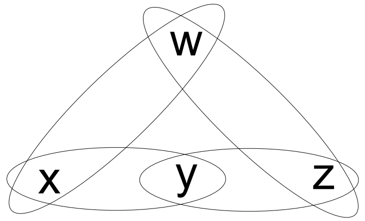
Definition: A CQ Q is semantically acyclic if there exists an acyclic CQ Q' such that $Q \equiv Q'$

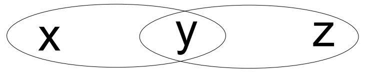


Q(x,z) :- R(x,y), R(y,z), R(x,w), R(w,z)

{w → y, z → y}

Q(x,z) :- R(x,y), R(y,z)





Semantic Acyclicity

Theorem: A CQ Q is semantically acyclic iff its core is acyclic

Theorem: Deciding whether a CQ Q is semantically acyclic is NP-complete

Proof idea (upper bound):

- We can show the following: if Q is semantically acyclic, then there exists an acyclic CQ Q' such that $|Q'| \leq |Q|$ and $Q \equiv Q'$
- Then, we can guess in polynomial time:
  - An acyclic CQ Q' such that $|Q'| \leq |Q|$
  - A mapping h_1 : terms(Q) $\rightarrow$ terms(Q')
  - A mapping h_2 : terms(Q') $\rightarrow$ terms(Q)
- And verify in polynomial time that h_1 is a query homomorphism from Q to Q' (i.e., $Q' \subseteq Q$), and h_2 is a query homomorphism from Q' to Q (i.e., $Q \subseteq Q'$)

Semantic Acyclicity

Theorem: A CQ Q is semantically acyclic iff its core is acyclic

Theorem: Deciding whether a CQ Q is semantically acyclic is NP-complete

But, semantic acyclicity is rather *weak*:

- Not many CQs are semantically acyclic

  ⇒ consider acyclic approximations of CQs

- Semantic acyclicity is not an improvement over usual optimization – both approaches are based on the core

  ⇒ exploit semantic information in the form of constraints

Acyclic Approximations of CQs

Acyclic Approximations

If our CQ Q is not semantically acyclic, we may target a CQ that is:

1. Easy to evaluate – acyclic
2. Provides sound answers – contained in Q
3. As “informative” as possible – “maximally” contained in Q

Definition: A CQ Q' is an acyclic approximation of Q if:

1. Q' is acyclic
2. $Q' \subseteq Q$
3. There is no acyclic CQ Q'' such that $Q' \subset Q'' \subseteq Q$

Do Acyclic Approximations Exist?

The cyclic CQ

Q :- R(x,y,z), R(z,u,v), R(v,w,x)

has several acyclic approximations

Q_1 :- R(x,y,z), R(z,u,y), R(y,v,x)

Q_2 :- R(x,y,z), R(z,u,v), R(v,w,x), R(x,z,v)

Q_3 :- R(x,y,x)

Existence, Size and Computation

Theorem: Consider a CQ Q. Then:

1. Q has an acyclic approximation
2. Each acyclic approximation of Q has size polynomial in Q
3. An acyclic approximation of Q can be found in time $2^{O(|Q| \cdot \log |Q|)}$
4. Q has at most exponentially many (non-equivalent) acyclic approximations

Evaluating Acyclic Approximations

- Recall that evaluating Q over D takes time $|D|^{O(|Q|)}$

- Evaluating an acyclic approximation Q' of Q over D takes time

$$2^{O(|Q|\cdot\log|Q|)} \quad + \quad |D| \cdot |Q|^{k}$$

time for computing Q'

time for evaluating Q'

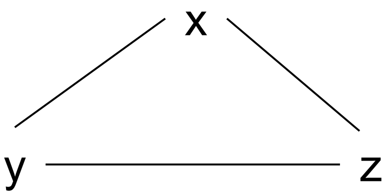
  - $|Q'| \leq |Q|^{k}$
  - Evaluation of an acyclic CQ Q_A is feasible in time $O(|D| \cdot |Q_A|)$

- Observe that $2^{O(|Q|\cdot\log|Q|)} + |D| \cdot |Q|^{k}$ is dominated by $|D| \cdot 2^{O(|Q|\cdot\log|Q|)}$

  $\Rightarrow$ fixed-parameter tractable

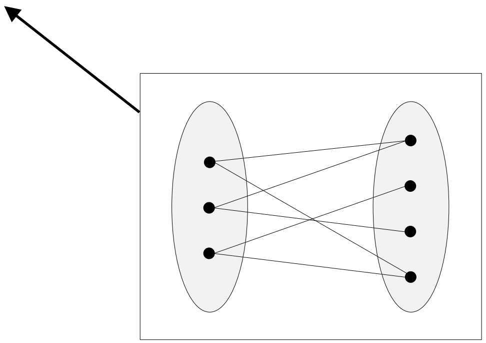
Poor Approximations

Q :- E(x,y), E(y,z), E(z,x)



has only one acyclic approximation, that is, Q’ :- E(x,x)

Proposition: Consider a Boolean CQ Q that contains a single binary relation E(.,.). If *G*(Q) is not bipartite, then the only acyclic approximation of Q is Q’ :- E(x,x)



Acyclic Approximations: Recap

- Acyclic approximations are useful when the CQ is not semantically acyclic
- Always exist, but are not unique
- Have polynomial size, and can be computed in exponential time
- Can be evaluated “efficiently” (fixed-parameter tractability)
- In some cases, acyclic approximations are not very informative

Back to Semantic Acyclicity

But, semantic acyclicity is rather *weak*:

- Not many CQs are semantically acyclic

  ⇒ consider acyclic approximations of CQs ✓

- Semantic acyclicity is not an improvement over usual optimization – both approaches are based on the core

  ⇒ exploit semantic information in the form of constraints

Associated Papers

- Pablo Barceló, Leonid Libkin, Miguel Romero: Efficient Approximations of Conjunctive Queries. SIAM J. Comput. 43(3): 1085-1130 (2014)

  Eligible topics include static analysis of approximations

- Pablo Barceló, Miguel Romero, Moshe Y. Vardi: Semantic Acyclicity on Graph Databases. SIAM J. Comput. 45(4): 1339-1376 (2016)

  Semantic acyclicity for CQs

- Hubie Chen, Víctor Dalmau: Beyond Hypertree Width: Decomposition Methods Without Decompositions. CP 2005: 167-181

  Complexity of semantic acyclicity for CQs (in a different context)

- Víctor Dalmau, Phokion G. Kolaitis, Moshe Y. Vardi: Constraint Satisfaction, Bounded Treewidth, and Finite-Variable Logics. CP 2002: 310-326

  Evaluation of semantically acyclic CQ (in a different context)

Associated Papers

- Joerg Flum, Martin Grohe: Fixed-Parameter Tractability, Definability, and Model-Checking. SIAM J. Comput. 31(1): 113-145 (2001)

  A different way of measuring complexity, and its full analysis

- Joerg Flum, Markus Frick, Martin Grohe: Query evaluation via tree- decompositions. Journal of the ACM 49(6): 716-752 (2002)

  Using tree decompositions to get faster query evaluation

- Markus Frick, Martin Grohe: Deciding first-order properties of locally tree-decomposable structures. Journal of the ACM 48(6): 1184-1206 (2001)

  How to improve performance of relational queries on databases with special properties

Associated Papers

- Minos N. Garofalakis, Phillip B. Gibbons: Approximate Query Processing: Taming the TeraBytes. VLDB 2001

  Approximation techniques that take into account both data and queries

- Georg Gottlob, Nicola Leone, Francesco Scarcello: The complexity of acyclic conjunctive queries. Journal of the ACM 48(3):431-498 (2001)

  An in-depth study of acyclicity

- Georg Gottlob, Nicola Leone, Francesco Scarcello: Hypertree Decompositions and Tractable Queries. J. Comput. Syst. Sci. 64(3):579-627 (2002)

  A hierarchy of classes of efficient CQs, the bottom level of which is acyclic queries

Associated Papers

- Martin Grohe, Thomas Schwentick, Luc Segoufin: When is the evaluation of conjunctive queries tractable? STOC 2001: 657-666

  Characterizing efficiency of CQs via the notion of bounded treewidth

- Yannis E. Ioannidis: Approximations in Database Systems. ICDT 2003: 16-30

  Approximation techniques that take into account both data and queries

- Mihalis Yannakakis: Algorithms for Acyclic Database Schemes. VLDB 1981: 82-94

  Notion of acyclicity of CQs and fast evaluation scheme based on it