the rest of this course

Volume
size does mattes
(thousands of TBs of data)

Veracity
data is often
incomplete/inconsistent

Variety
many data formats
(structured, semi-structured, etc.)

Velocity
data often arrives at fast speed
(updates are frequent)

Foundations of XML

XML at First Glance

XML = eXtensible Markup Language

- W3C standard for document markup since 1998
- Generic syntax to markup data with human- and machine-readable tags
- One of the most common data formats
- Several XML-related W3C standards
  - XML Schema: define the markup permitted in a document
  - XPath: navigation mechanism
  - XSLT: transformation language
  - XQuery: query language

An exciting toping for database theorists:

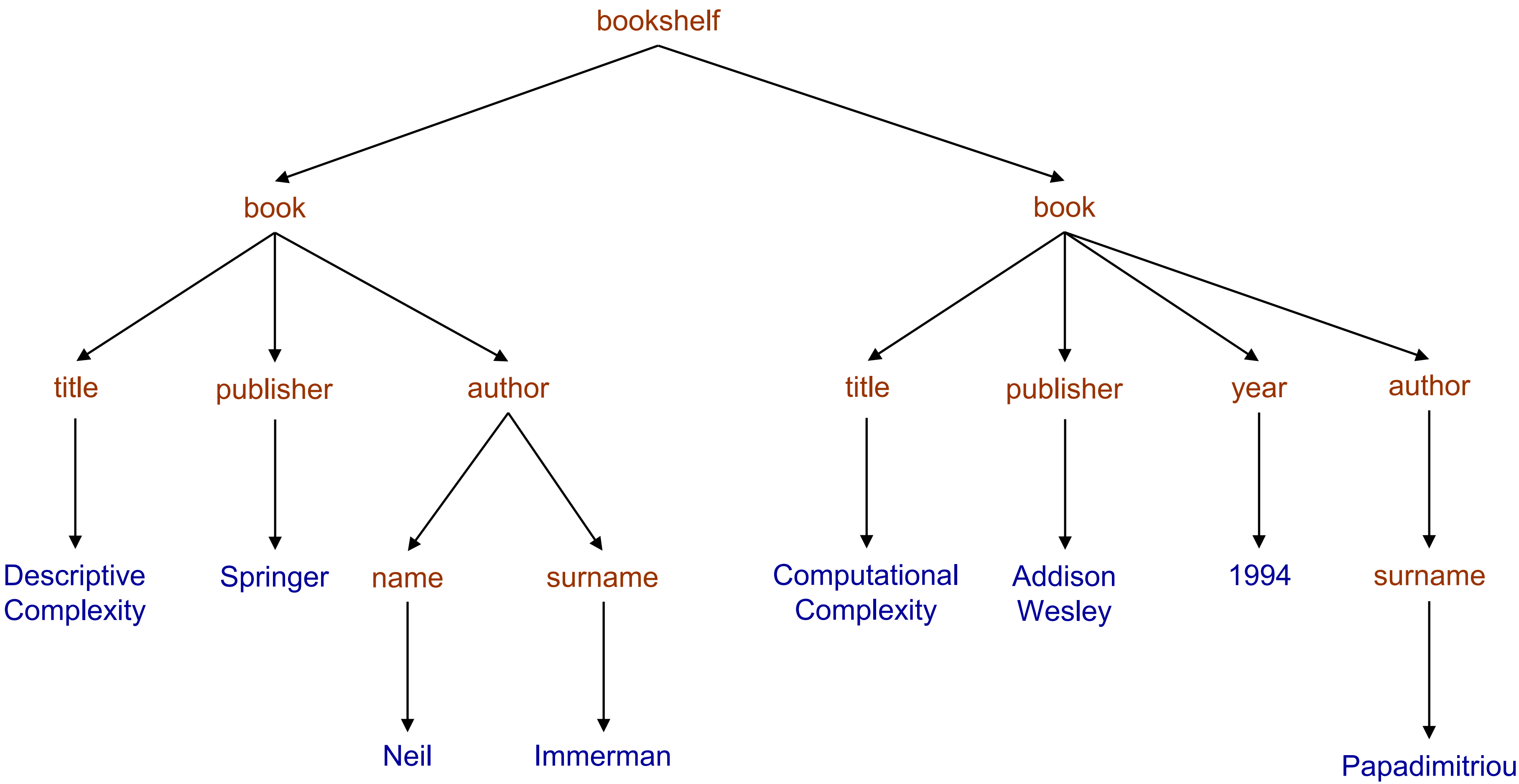
it brings techniques from formal language theory and merges them nicely with logic

XML at First Glance

```
root element

<bookshelf>
    <book>
        <title>Descriptive Complexity</title>            child elements of book
        <publisher>Springer</publisher>
        <author>
            <name>Neil</name>                            child elements of author
            <surname>Immerman</surname>
        </author>
    </book>
    <book>
        <title>Computational Complexity</title>          child elements of book
        <publisher>Addison Wesley</publisher>
        <year>1994</year>
        <author>
            <surname>Papadimitriou</surname>             child element of author
        </author>
    </book>
</bookshelf>
```

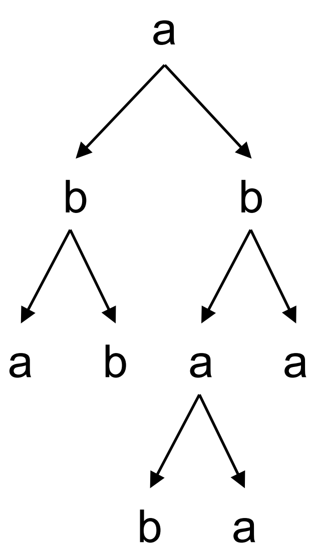
XML Documents as Trees



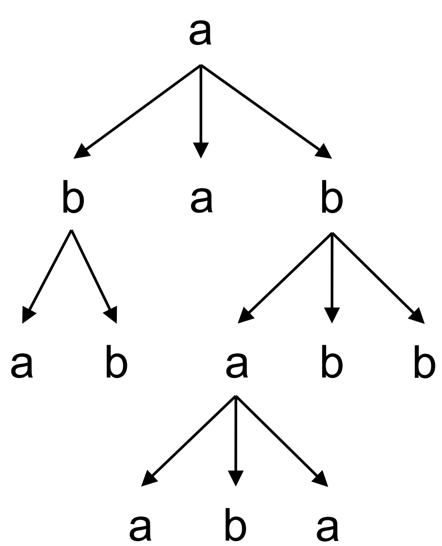
labeled ordered unranked tree

Ranked vs. Unranked Trees

Typically in computer science one works with ranked trees, e.g.,



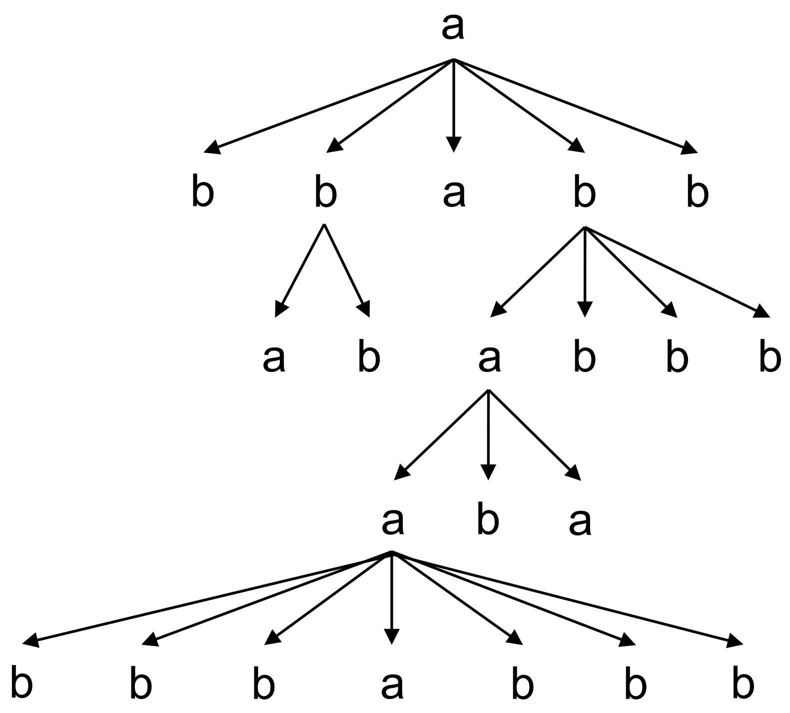
binary trees



ternary trees

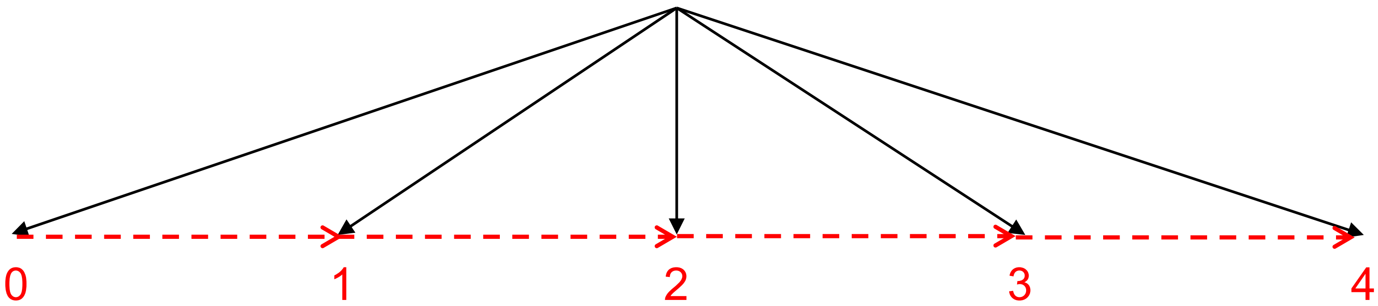
Ranked vs. Unranked Trees

But for XML we need unranked trees – nodes can have arbitrarily many children



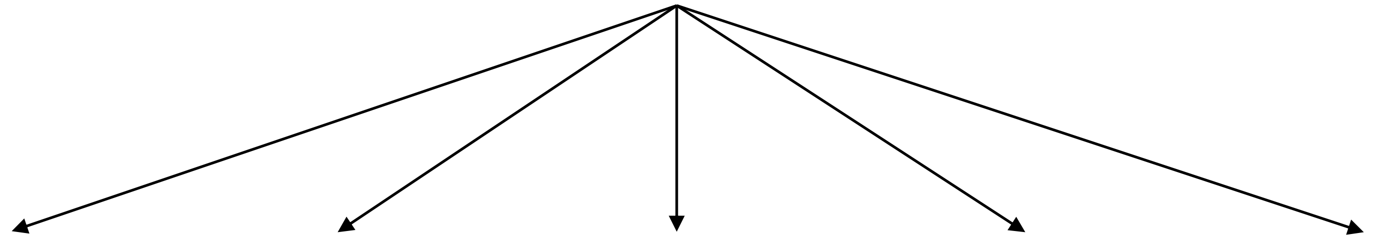
Ordered vs. Unordered Trees

In ordered trees, siblings are ordered (from the oldest to the youngest)



a “build-in” binary relation provides access to this ordering

In unordered trees, such an order among siblings does not exist



XML Development

- Clean and simple model – labeled ordered unranked trees
- Declarative languages – XPath
  - Flavour of traditional first-order logic, or
  - Temporal logics for describing navigation
- Procedural languages – automata-theoretic constructions
- Key advantages (like the relational model)
  - Simple and clean mathematical model (based on logic)
  - Separation of declarative and procedural

Ordered Unranked Trees: Definition

Fix a finite alphabet Λ

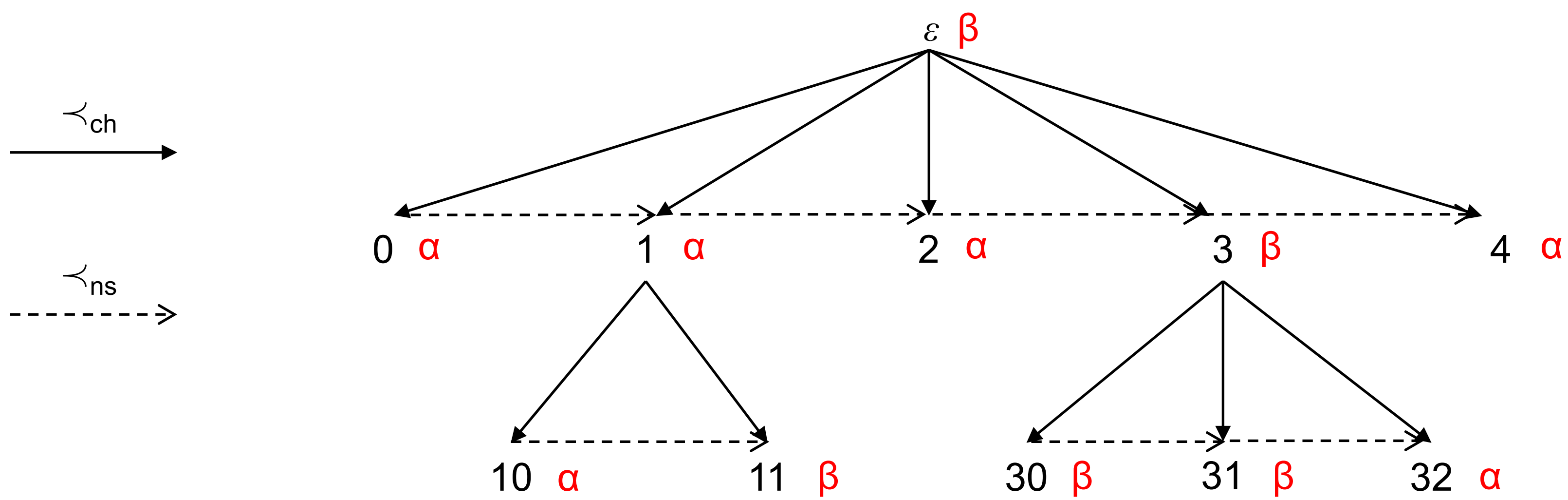
An ordered unranked tree T is a structure

$$(\mathbf{D}, \prec_{ch^*}, \prec_{ns^*}, \{P_s\}_{s \in \Lambda})$$

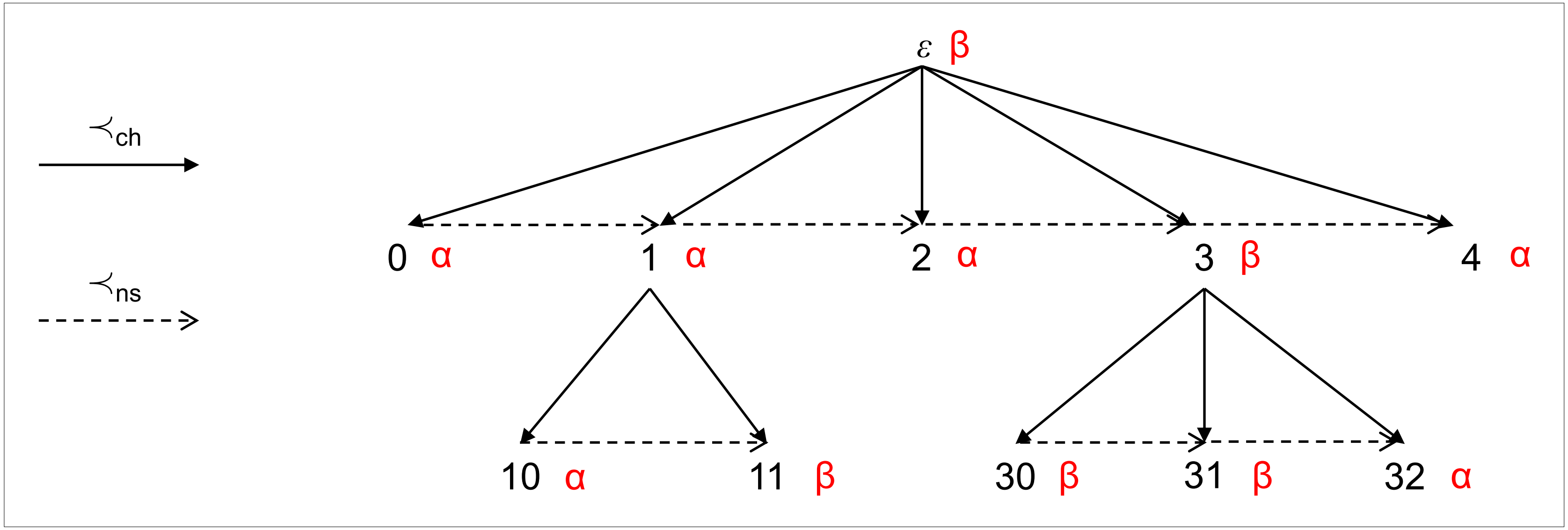
- **D** is a finite prefix-closed subset of N^* such that $s \cdot i \in \mathbf{D} \Rightarrow s \cdot j \in \mathbf{D}$ for every $j < i$
- $\prec_{ch^*}$ is the descendant relation
- $\prec_{ns^*}$ is the sibling relation
- P_s's are interpreted as disjoint sets whose union is the entire domain **D**

Ordered Unranked Trees: Example

- Let $\Lambda = \{\alpha,\beta\}$
- Consider the ordered unranked tree $T = (\mathbf{D}, \prec_{ch^*}, \prec_{ns^*}, \{P_s\}_{s \in \Lambda})$, where
  - $\mathbf{D} = \{\varepsilon, 0, 1, 2, 3, 4, 10, 11, 30, 31, 32\}$
  - $\prec_{ch} = \{(\varepsilon,0), (\varepsilon,1), (\varepsilon,2), (\varepsilon,3), (\varepsilon,4), (1,10), (1,11), (3,30), (3,31), (3,32)\}$
  - $\prec_{ns} = \{(0,1), (1,2), (2,3), (3,4), (10,11), (30,31), (31,32)\}$
  - $P_\alpha = \{0, 1, 2, 4, 10, 32\}$
  - $P_\beta = \{\varepsilon, 3, 11, 30, 31\}$



Ordered Unranked Trees: Basic Predicates



In T = (**D**, $\prec_{ch^*}$, $\prec_{ns^*}$, $\{P_s\}_{s \in \Lambda}$) we use the transitive closures of $\prec_{ch}$ and $\prec_{ns}$

- They are not definable in first-order logic
- However, if the adopted logic is powerful enough to define them, then we can simply use $\prec_{ch}$ and $\prec_{ns}$

Ordered Unranked Trees: Querying

Check that in a tree *T* over the alphabet {α,β} every α-labeled node always has a β-labeled descendant

$$Q = \forall x(P_\alpha(x) \rightarrow \exists y(x \prec_{ch^*} y \wedge P_\beta(y))$$

Ordered Unranked Trees: Querying

Select the nodes in a tree *T* over the alphabet {α,β,γ} that are

(i) labeled α,

(ii) have a descendant d labeled b, and

(iii) d has a younger sibling labeled γ

$$Q(x) \;=\; P_\alpha(x) \;\wedge\; \exists y \exists z (x \prec_{ch^*} y \;\wedge\; P_\beta(y) \;\wedge\; y \prec_{ns^*} z \;\wedge\; P_\gamma(z))$$

Ordered Unranked Trees: Querying

Check that in a tree *T* over the alphabet {α,β} every α-labeled node always has a β-labeled descendant, but using only $\prec_{ch}$

any set of nodes that contains x and is closed under the $\prec_{ch}$ relation, also contains y

$$Q = \forall x(P_\alpha(x) \rightarrow \exists y(desc(x,y) \wedge P_\beta(y))$$

$$desc(x,y) = \forall S((x \in S \wedge \forall z \forall w((z \in S \wedge z \prec_{ch} w) \rightarrow w \in S)) \rightarrow y \in S)$$

second-order quantifier ranging over sets of nodes

first-order quantifiers ranging over nodes

monadic second-order logic (MSO)

Ordered Unranked Trees: Querying

Compute the pairs of nodes (x,y) such that y is a descendant of x

and the path between them is of odd length

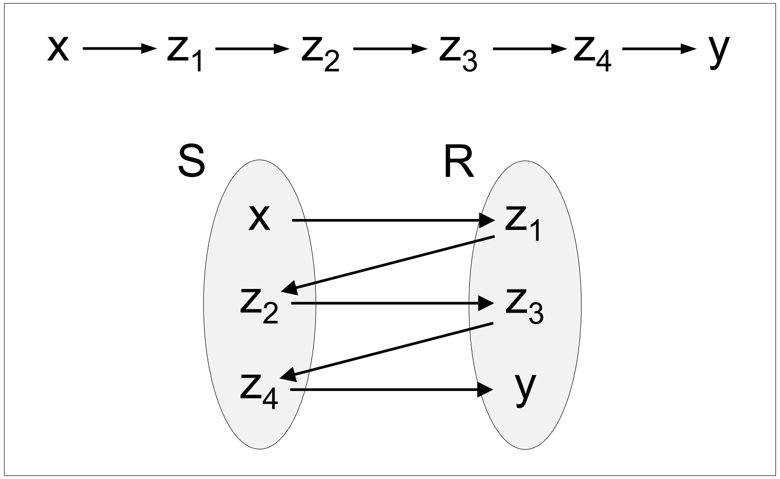
$\equiv$

There exist two sets of nodes S and R that

(i) partition the path from x to y

(ii) $x \in S$ and $y \in R$

(iii) the successor element of each element in S is in R, and vice versa



Ordered Unranked Trees: Querying

Compute the pairs of nodes (x,y) such that y is a descendant of x

and the path between them is of odd length

$$Q(x,y) \;=$$

$$\exists S \exists R \left(\begin{array}{ll} & \forall z((x \prec_{ch^*} z \prec_{ch^*} y) \rightarrow (z \in S \leftrightarrow \neg(z \in R))) \\ \wedge & (x \in S \wedge y \in R) \\ \wedge & \forall z \forall w((x \prec_{ch^*} z \prec_{ch} w \prec_{ch^*} y) \rightarrow ((z \in S \rightarrow w \in R) \wedge (z \in R \rightarrow w \in S))) \end{array} \right)$$

Ordered Unranked Trees: Querying

- For querying labeled ordered unranked trees we use:
  - **First-order logic (FO)**
    - Boolean connectives $\vee$, $\wedge$, $\neg$
    - Quantifiers $\exists x$ and $\forall x$ that range over nodes of trees
  - **Monadic second-order logic (MSO)**
    - FO plus quantifiers $\exists S$ and $\forall S$ that range over sets of nodes
    - New formulae $x \in S$
- Most commonly they define:
  - Boolean (yes/no) queries – in fact, they define sets of trees
  - Unary queries that select nodes in trees

Ordered Unranked Trees: Definability in Logic

Let **L** be some logic (such as **FO** or **MSO**)

- A Boolean query Q (i.e., a set of trees $\mathbf{T}$) is **L**-definable if there is a sentence φ of **L** such that $T \in \mathbf{T} \;\Leftrightarrow\; T \models \varphi$

- A unary query $Q(x)$ is **L**-definable if there is a formula $\varphi(x)$ of **L** such that for every tree T and node v in T, $v \in Q(T) \;\Leftrightarrow\; T \models \varphi(v)$

the set of nodes in T selected by Q

Unranked Tree Automata

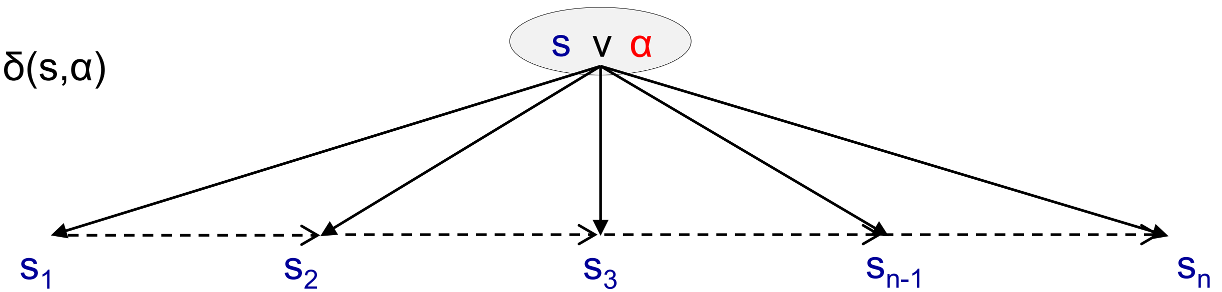
- A nondeterministic unranked tree automaton (NUTA) over Λ-labeled trees is a triple

$$A \;=\; (S, F, \delta)$$

  - S is a finite set of states
  - $F \subseteq S$ is the set of final states
  - $\delta : S \times \Lambda \rightarrow 2^{S^*}$ such that $\delta(s,\alpha)$ is a regular language over S

- A run of A on a tree T with domain **D** is a function λ_A : **D** $\rightarrow$ S such that: if v is a node with n children, and is labeled α, then the string $\lambda_A(v\cdot 0)\ldots\lambda_A(v\cdot(n\text{-}1)) \in \delta(\lambda_A(v),\alpha)$

$s_1\ldots s_n \in \delta(s,\alpha)$



Unranked Tree Automata

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- A run of A on a tree T with domain **D** is a function $\lambda_A : \mathbf{D} \rightarrow S$ such that: if v is a node with n children, and is labeled α, then the string $\lambda_A(v \cdot 0) \ldots \lambda_A(v \cdot (n-1)) \in \delta(\lambda_A(v),\alpha)$

- A run is accepting if $\lambda_A(\varepsilon) \in F$, i.e., the root is in an accepting state

- A tree T is accepted by A if there exists an accepting run of A on T

- We denote by L(A) the set of all trees accepted by A – a set of trees accepted by an NUTA is called regular

Unranked Tree Automata: Example

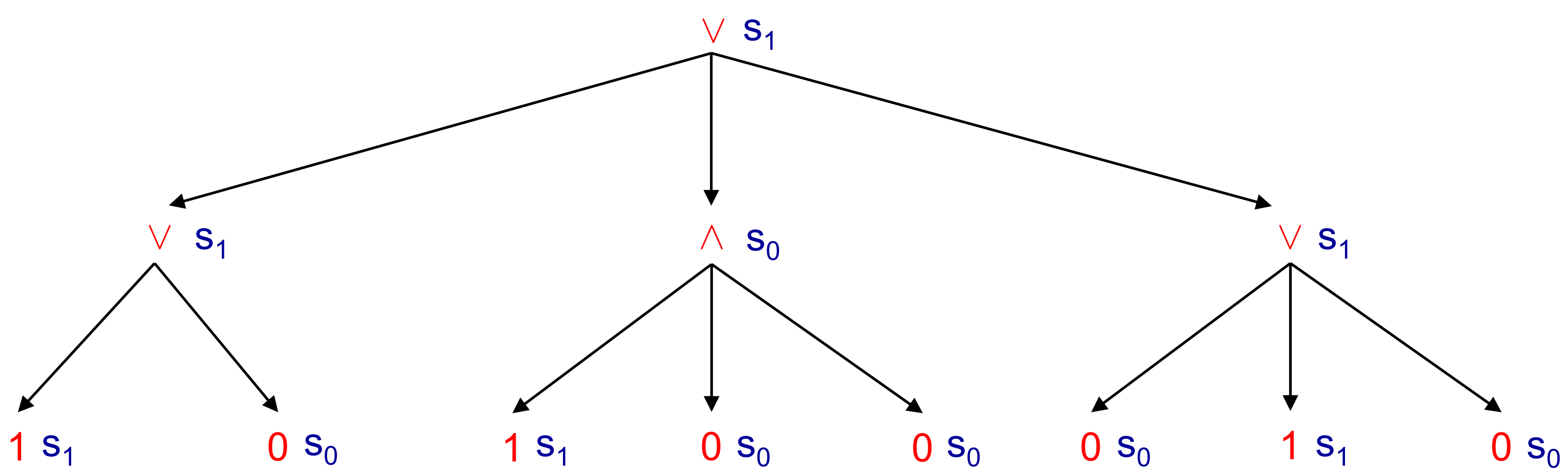
- Let $\Lambda = \{\wedge, \vee, 0, 1\}$, and consider Λ-labeled trees where 0,1 appear only at leaves, while $\wedge, \vee$ can appear everywhere except at leaves

- We define $A = (\{s_0, s_1\}, \{s_1\}, \delta)$, where

$$\delta(s_0,0) = \delta(s_1,1) = \{\varepsilon\} \qquad \delta(s_1,\wedge) = s_1^*$$

$$\delta(s_0,1) = \delta(s_1,0) = \emptyset \qquad \delta(s_0,\vee) = s_0^*$$

$$\delta(s_0,\wedge) = (s_0 \cup s_1)^* \cdot s_0 \cdot (s_0 \cup s_1)^* \qquad \delta(s_1,\vee) = (s_0 \cup s_1)^* \cdot s_1 \cdot (s_0 \cup s_1)^*$$



MSO = NUTA

We can now present an interesting result:

Theorem: Consider a set **T** of labeled ordered unranked trees. Then:

T is **MSO**-definable $\Leftrightarrow$ **T** is regular

…but, what about unary queries?

we need an extended automata model

Query Automata

- A nondeterministic query automaton (NQA) over Λ-labeled trees is a pair

$$A \quad = \quad (B, \mathrm{P})$$

an NUTA (S, F, δ)   a subset of S $\times$ Λ

- Such an automaton defines a unary query Q_A over unranked trees:

$$v \in Q_A(T) \quad \Leftrightarrow \quad (\lambda_B(v),\mathrm{label}(v)) \in \mathrm{P}, \text{ for some accepting run } \lambda_B \text{ of } B \text{ on } T$$

MSO = NQA

We have similar characterization for unary queries:

Theorem: Consider a unary query Q on labeled ordered unranked trees. Then:

Q is **MSO**-definable $\Leftrightarrow$ Q is of the form Q_A for some NQA A

Ordered Unranked Trees: Recap

- XML documents are modeled as labeled ordered unranked trees

- **MSO** is the yardstick logic for querying ordered unranked trees

- Most commonly we consider:
  - Boolean queries that they define sets of trees: **MSO** = NUTA
  - Unary queries that select nodes in trees: **MSO** = NQA

…but, what about the complexity of **MSO** over trees?

Complexity of MSO

BQE(**MSO**)

Input: a labeled ordered unranked tree T, an **MSO** sentence φ

Question: $T \vDash \varphi$?

- The same problem can be defined for unary formulas
  - Given a tree T, a unary formula $\varphi(x)$, and a node v: does $T \vDash \varphi(v)$?
- As usual, we consider the data and combined complexity
  - Data complexity: T is input, φ is fixed
  - Combined complexity: both T and φ are part of the input

Complexity of MSO

Theorem: It holds that:

- BQE(**MSO**) is in PTIME in data complexity (in fact, linear time)
- BQE(**MSO**) is non-elementary in combined complexity

Proof idea: By translation to automata:

- Convert the given sentence φ into a NUTA A_φ such that $T \vDash \varphi \Leftrightarrow T \in L(A_\varphi)$
- To decide whether $T \in L(A_\varphi)$ is feasible in time $O(|T| \cdot |A_\varphi|^2)$

non-elementary

$$\left.2^{2^{2^{\cdot^{\cdot^{\cdot^{2^{2^{|\varphi|}}}}}}}}\right\} \text{depends on } \varphi$$

Complexity of MSO

Theorem: It holds that:

- BQE(**MSO**) is in PTIME in data complexity (in fact, linear time)
- BQE(**MSO**) is non-elementary in combined complexity

Proof idea: By translation to automata:

- Convert the given sentence φ into a NUTA A_φ such that $T \models \varphi \Leftrightarrow T \in L(A_\varphi)$
- To decide whether $T \in L(A_\varphi)$ is feasible in time $O(|T| \cdot |A_\varphi|^2)$

Even a bigger problem: there is no algorithm (even if we avoid automata) for checking whether $T \models \varphi$ that runs in time $O(|T| \cdot f(|\varphi|))$ and f is an elementary function (unless P = NP)

Complexity of MSO

Theorem: It holds that:

- BQE(**MSO**) is in PTIME in data complexity (in fact, linear time)
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Proof idea: By translation to automata:

- Convert the given sentence φ into a NUTA A_φ such that $T \models \varphi \Leftrightarrow T \in L(A_\varphi)$
- To decide whether $T \in L(A_\varphi)$ is feasible in time $O(|T| \cdot |A_\varphi|^2)$

We need logics that have the same power as **MSO**, but permit faster evaluation algorithms

Alternative Logics for MSO

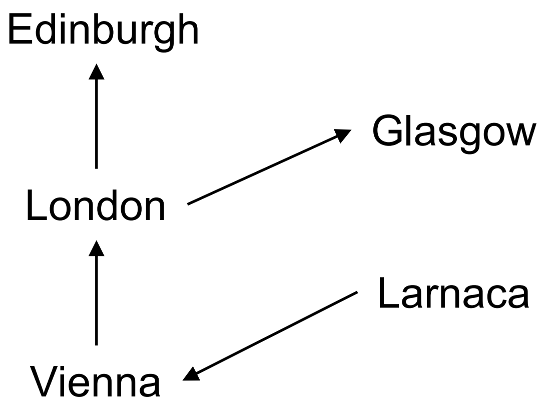
- Efficient Tree Logic (ETL) – obtained by posing some syntactic restrictions on **MSO** formulae, and at the same time adding new constructors for formulae that are not in **MSO**, but are **MSO**-definable

- μ-calculus – extension of a temporal logic with the least fixed-point operator

- Monadic Datalog – fragment of Datalog, a database query language that essentially extends existential positive **FO** with the least fixed-point operator

Reachability

Is Glasgow reachable from Vienna?

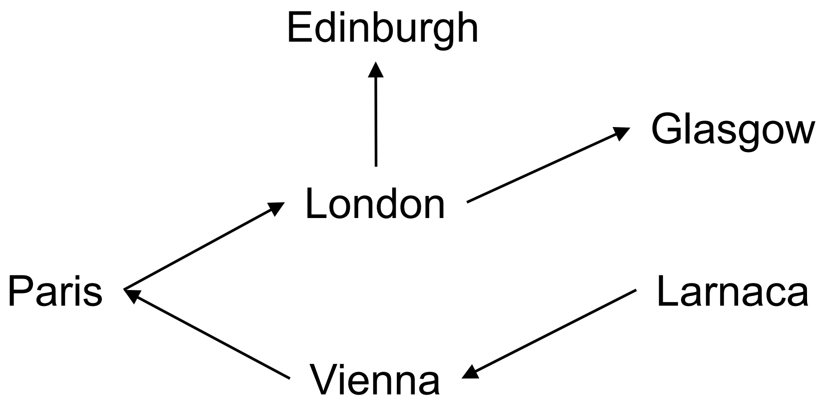


$\exists x\ (Flight(Vienna,x) \wedge Flight(x,Glasgow))$

✓

Reachability

Is Glasgow reachable from Vienna?

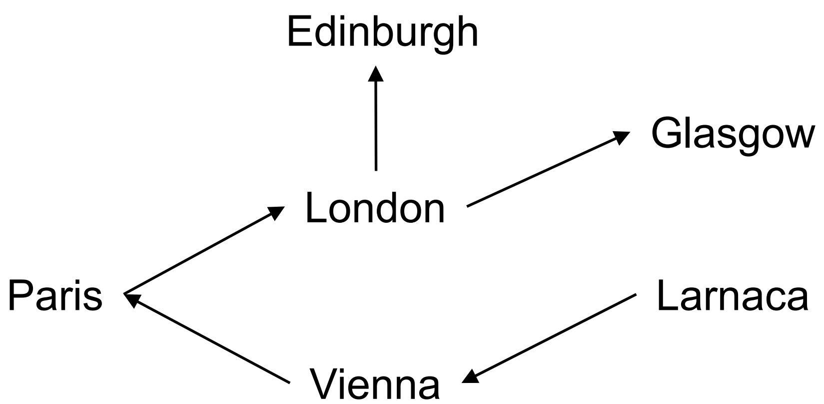


$\exists x\,(\text{Flight}(\text{Vienna},x) \wedge \text{Flight}(x,\text{Glasgow}))$

✘

Reachability

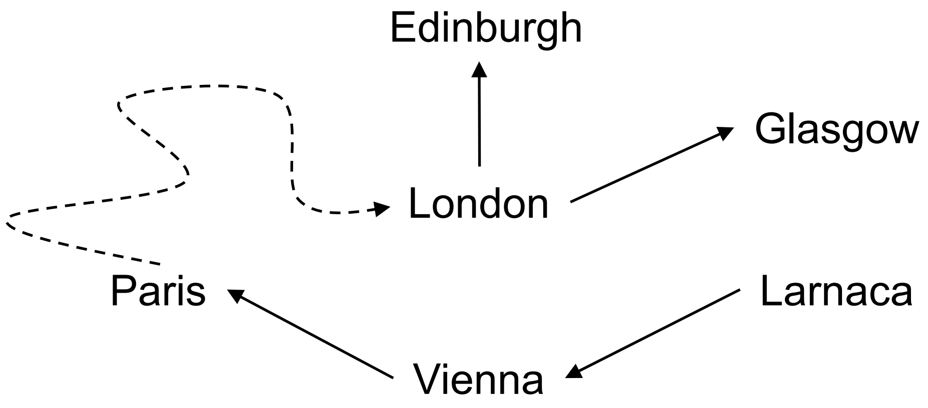
Is Glasgow reachable from Vienna?



$\exists x \exists y\ (\text{Flight}(\text{Vienna},x) \wedge \text{Flight}(x,y) \wedge \text{Flight}(y,\text{Glasgow}))$ ✓

Reachability

Is Glasgow reachable from Vienna?

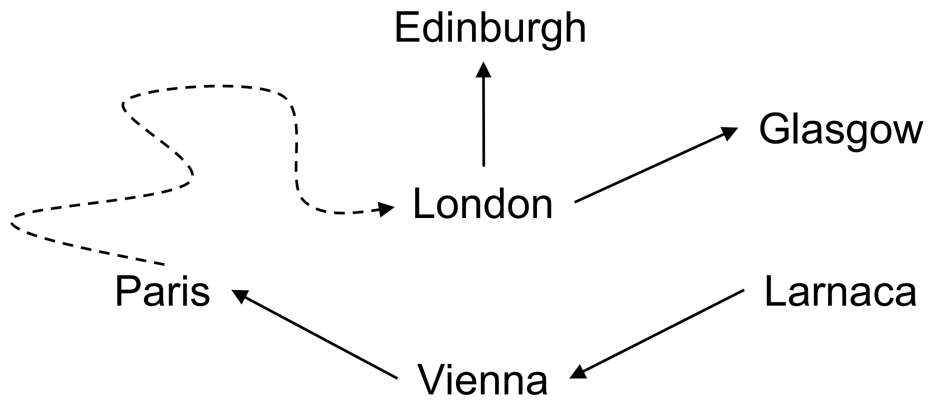


$\exists x \exists y\ (Flight(Vienna,x) \wedge Flight(x,y) \wedge Flight(y,Glasgow))$

✘

Reachability

Is Glasgow reachable from Vienna?

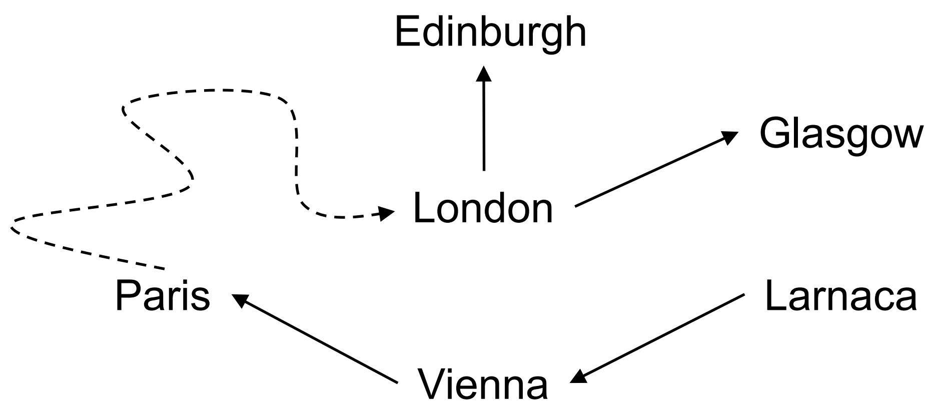


Here is a possible strategy:

- Compute all the pairs of cities (c_1,c_2) such that c_2 is reachable from c_1
- Check if there is a pair (Vienna,Glasgow)

Reachability

Is Glasgow reachable from Vienna?



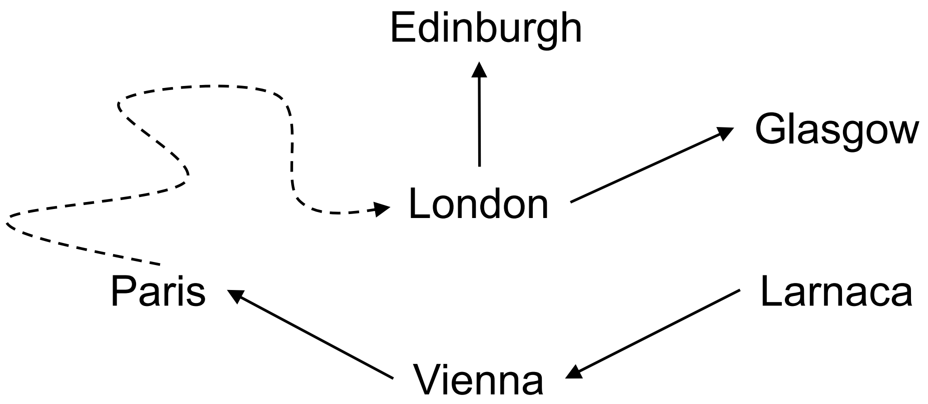
Here is a possible strategy:

- Compute all the pairs of cit… …uch that c_2 is reachable from c1
- Check if th… …ir (Vienna,Glasgow)

not possible via an **FO** query – we need recursion

Reachability

Is Glasgow reachable from Vienna?



Reachable(x,y) :- Flight(x,y)

Reachable(x,z) :- Flight(x,y), Reachable(y,z)

Goal :- Reachable(Vienna,Glasgow)

Reachability

DATALOG

essentially, positive **FO** with least fixed-point

Reachable(x,y)	:-	Flight(x,y)
Reachable(x,z)	:-	Flight(x,y), Reachable(y,z)
Goal	:-	Reachable(Vienna,Glasgow)

Monadic Datalog

all the introduced (or intentional) predicates are unary

Select all nodes v such that their descendants (including v) are labeled α

$$\mathrm{Goal}(x) \;:\!-\; P_\alpha(x),\ \mathrm{Leaf}(x)$$

$$\mathrm{Goal}(x) \;:\!-\; P_\alpha(x),\ x \prec_{fc} y,\ \mathrm{Mark}(y)$$

Mark($\cdot$) collects all the nodes v such that

- Goal(v) holds
- For every u such that $v \prec_{ns^*} u$, Goal(u) holds

$$\mathrm{Mark}(x) \;:\!-\; \mathrm{LastChild}(x),\ \mathrm{Goal}(x)$$

$$\mathrm{Mark}(x) \;:\!-\; \mathrm{Goal}(x),\ x \prec_{ns} y,\ \mathrm{Mark}(y)$$

Monadic Datalog

R = $\{\prec_{fc}$, Leaf, LastChild, Root, $\{P_s\}_{s \in \Lambda}\}$

Theorem: Consider a unary query Q on labeled ordered unranked trees. Then:

Q is **MSO**-definable $\Leftrightarrow$ Q is definable in Monadic Datalog over **R**

Theorem: A Monadic Datalog query Q can be evaluated on a tree T in time $O(|Q| \cdot |T|)$

Monadic Datalog is heavily used in Web data extraction:
real-life languages are based on Monadic Datalog, which
combines expressiveness and good evaluation properties

XML Schemas

- Usually, we are not interested in documents containing arbitrary elements, but only in documents that satisfy some specific constraints

- Schema – the markup permitted in an XML document

- Many different XML schema languages available:
  - Document Type Definitions (DTDs)
  - W3C XML Schema
  - REgular LAnguage for XML Next Generation (RELAX NG)
  - Schematron
  - …

DTDs: An Example

```
<bookshelf>
  <book>
    <title>Descriptive Complexity</title>
    <publisher>Springer</publisher>
    <author>
      <name>Neil</name>
      <surname>Immerman</surname>
    </author>
  </book>
  <book>
    <title>Computational Complexity</title>
    <publisher>Addison Wesley</publisher>
    <year>1994</year>
    <author>
      <surname>Papadimitriou</surname>
    </author>
  </book>
</bookshelf>
```

the XML document is valid w.r.t. the DTD

```
<!DOCTYPE  bookshelf  [
  <!ELEMENT bookshelf (book+)>
  <!ELEMENT book (title, publisher, year?, author+)>
  <!ELEMENT title (#PCDATA)>
  <!ELEMENT publisher (#PCDATA)>
  <!ELEMENT year (#PCDATA)>
  <!ELEMENT author (name?, surname)>
  <!ELEMENT name (#PCDATA)>
  <!ELEMENT surname (#PCDATA)>
]>
```

DTDs: Formal Definition

Fix a finite alphabet Λ

A document type definition (DTD) *D* is function-symbol pair

$$(f : \Lambda \rightarrow \text{regular expressions over } \Lambda\text{ , } s \in \Lambda)$$

For example, the previous DTD is written as (f, bookshelf), where

$$f(\text{bookshelf}) = \text{book} \cdot \text{book*}$$

$$f(\text{book}) = \text{title} \cdot \text{publisher} \cdot (\text{year} \cup \varepsilon) \cdot (\text{author} \cdot \text{author*})$$

$$f(\text{author}) = (\text{name} \cup \varepsilon) \cdot \text{surname}$$

$$f(\text{title}) = f(\text{publisher}) = f(\text{year}) = f(\text{name}) = f(\text{surname}) = \varepsilon$$

DTDs into Tree Automata (and MSO)

- The previous DTD is written as D = (f, bookshelf), where

$$f(\text{bookshelf}) = \text{book} \cdot \text{book}^*$$
$$f(\text{book}) = \text{title} \cdot \text{publisher} \cdot (\text{year} \cup \varepsilon) \cdot (\text{author} \cdot \text{author}^*)$$
$$f(\text{author}) = (\text{name} \cup \varepsilon) \cdot \text{surname}$$
$$f(\text{title}) = f(\text{publisher}) = f(\text{year}) = f(\text{name}) = f(\text{surname}) = \varepsilon$$

- Let $A_D = (\{s_{\text{bookshelf}}, s_{\text{book}}, s_{\text{title}}, s_{\text{publisher}}, s_{\text{year}}, s_{\text{author}}, s_{\text{name}}, s_{\text{surname}}\}, \{s_{\text{bookshelf}}\}, \delta)$, where

$$\delta(s_x, x) = \varepsilon, \text{ for every } x \in \{\text{title, publisher, year, name, surname}\}$$
$$\delta(s_{\text{bookshelf}}, \text{bookshelf}) = \text{book} \cdot \text{book}^*$$
$$\delta(s_{\text{book}}, \text{book}) = \text{title} \cdot \text{publisher} \cdot (\text{year} \cup \varepsilon) \cdot (\text{author} \cdot \text{author}^*)$$
$$\delta(s_{\text{author}}, \text{author}) = (\text{name} \cup \varepsilon) \cdot \text{surname}$$

$L(A_D) = \{T \mid T \text{ is valid w.r.t. } D\}$

Recap

- XML documents are modeled as labeled ordered unranked trees
- **MSO** is the yardstick logic for querying ordered unranked trees
  - Boolean queries that they define sets of trees: **MSO** = NUTA
  - Unary queries that select nodes in trees: **MSO** = NQA
- **MSO** over trees can be evaluated in linear time in data complexity, but the combined complexity is non-elementary
- Monadic Datalog – an alternative logic for MSO with good evaluation properties
- DTDs are captured by NUTA (**MSO**)

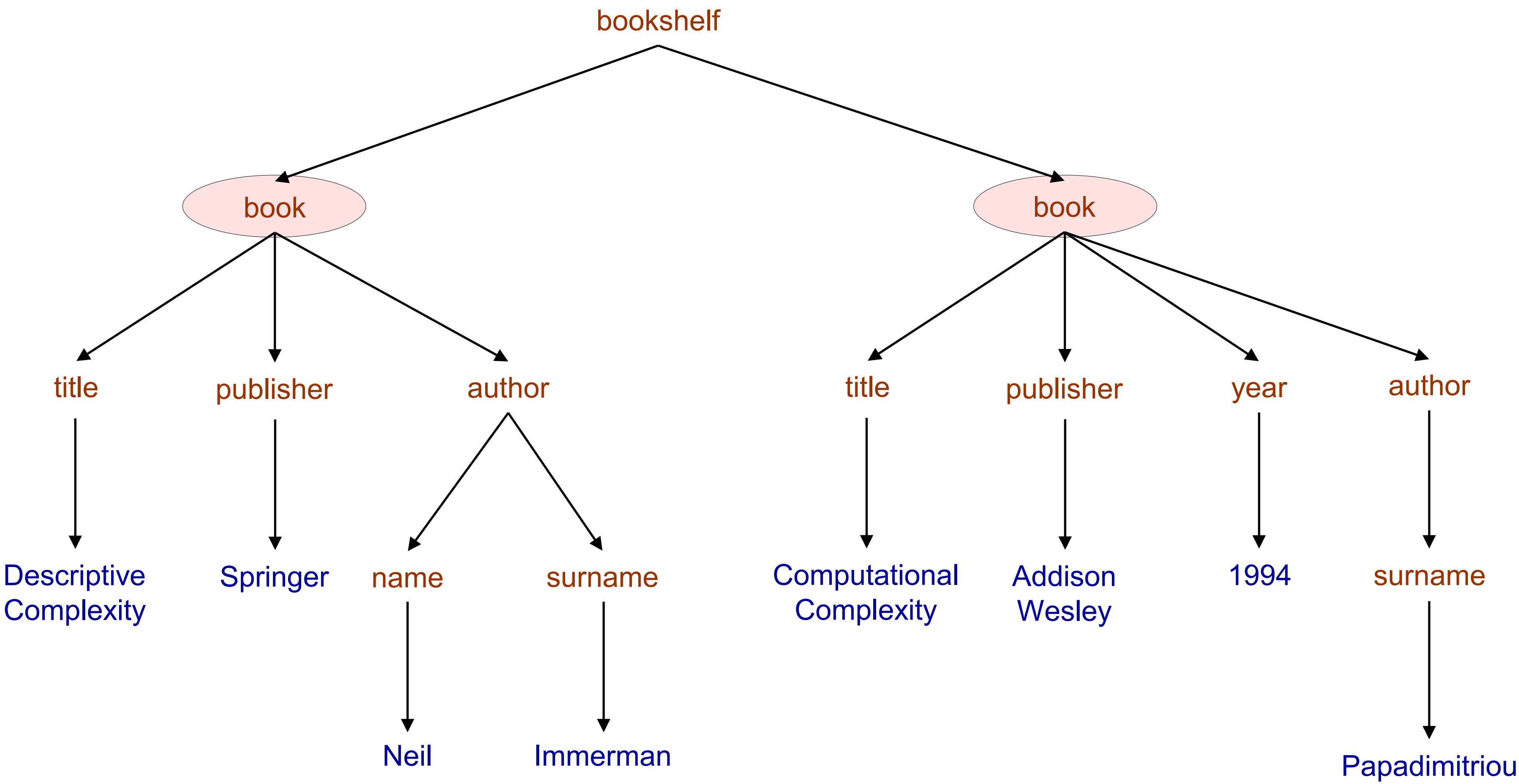
Ordered Unranked Trees: Querying

For querying labeled ordered unranked trees we use:

- **First-order logic (FO)** – often studied in connection with XPath
  - Boolean connectives $\vee$, $\wedge$, $\neg$
  - Quantifiers $\exists x$ and $\forall x$ that range over nodes of trees

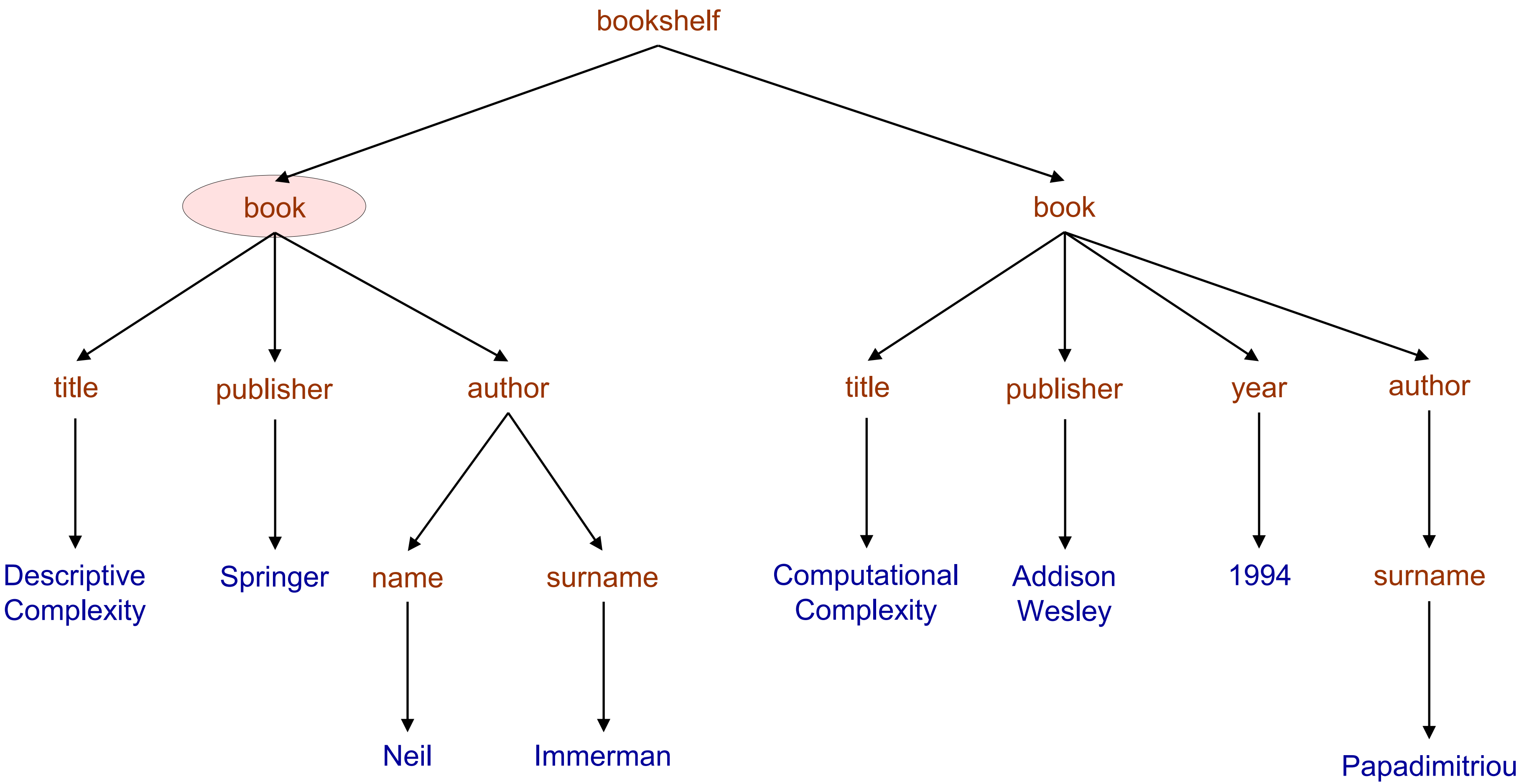
✓ • **Monadic second-order logic (MSO)** – the yardstick logic
  - **FO** plus quantifiers $\exists S$ and $\forall S$ that range over sets of nodes
  - New formulae $x \in S$

XPath at First Glance



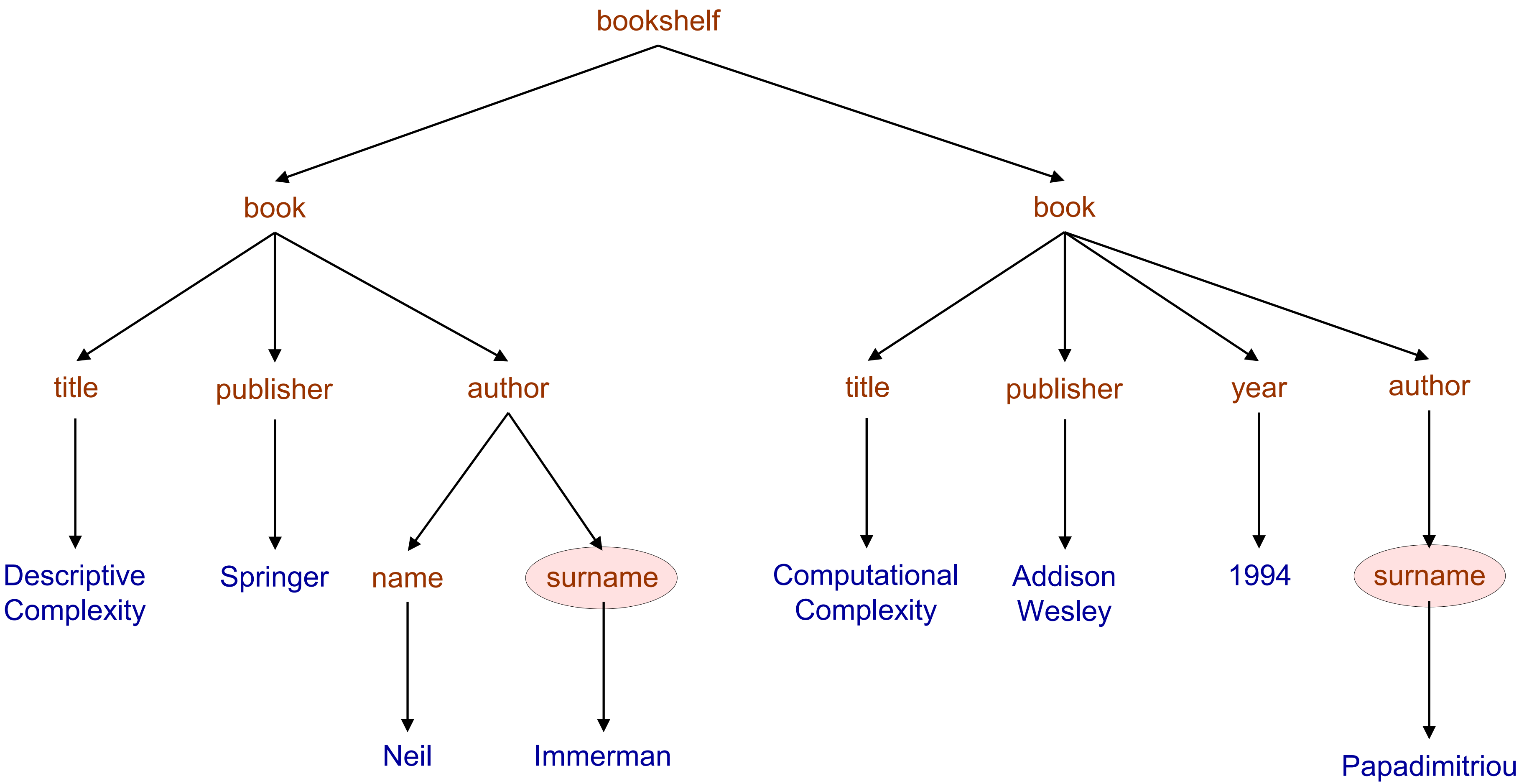
/child::bookshelf/child::book

XPath at First Glance



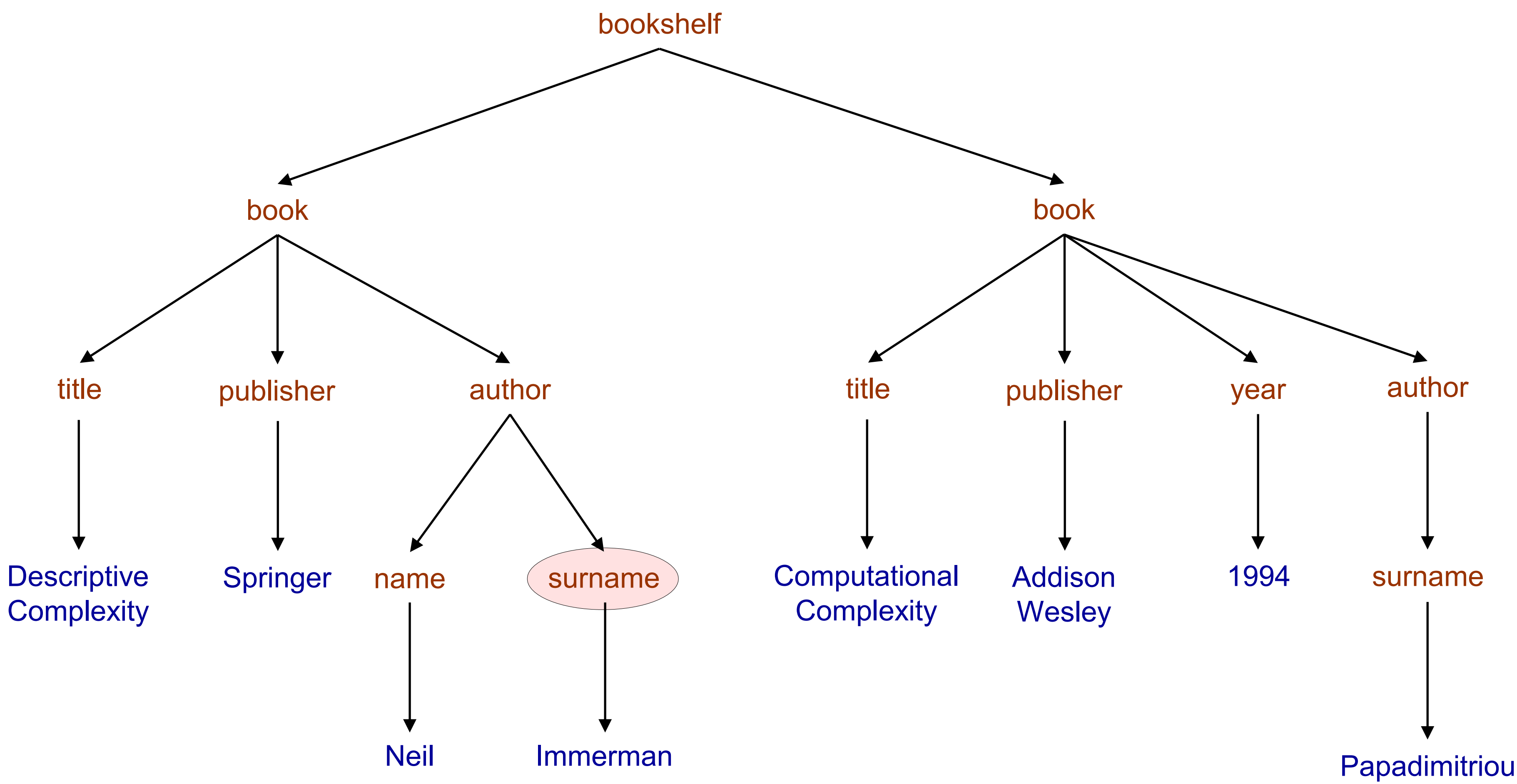
/child::bookshelf/child::book[position() = 1]

XPath at First Glance



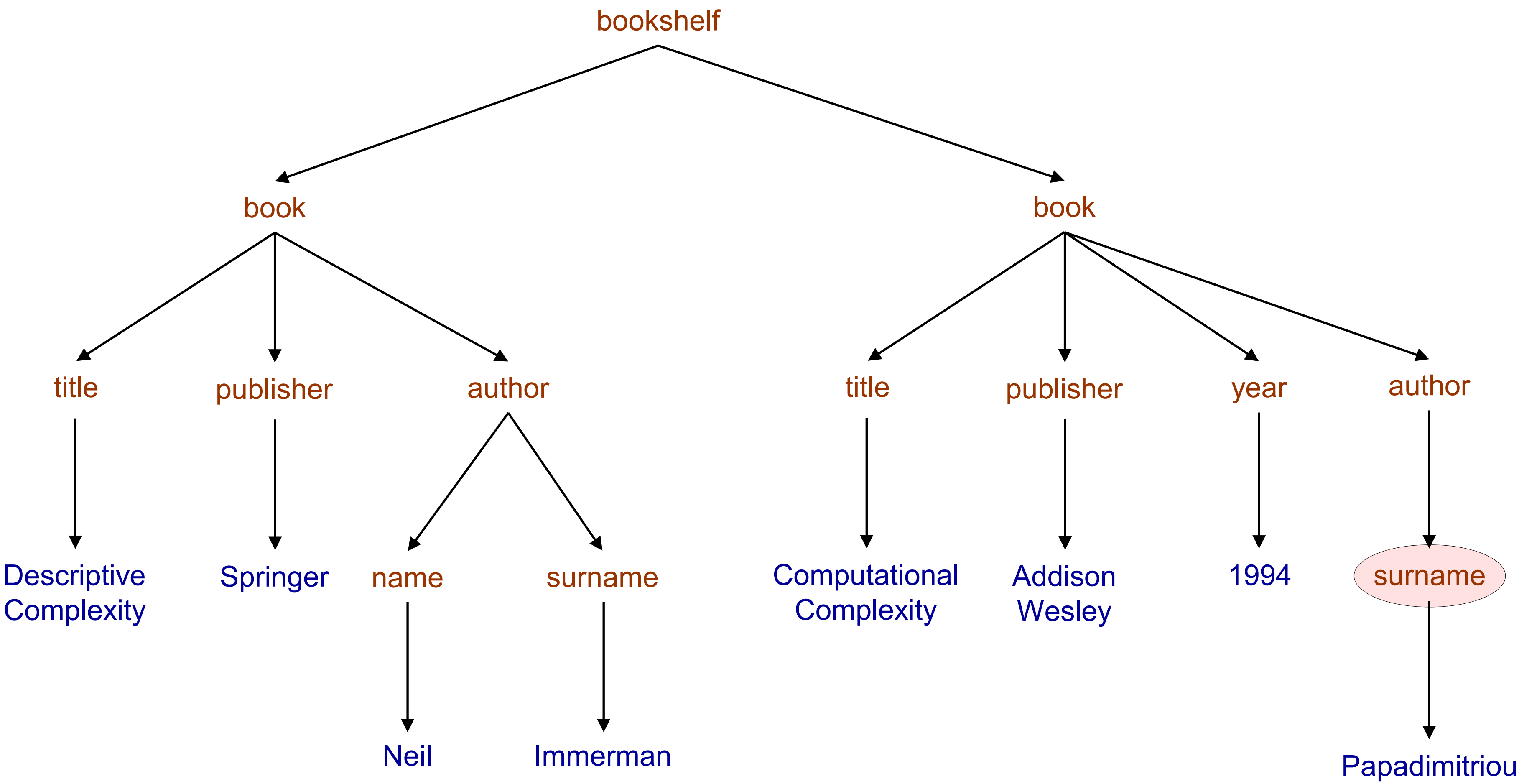
/descendant::author/child::surname

XPath at First Glance



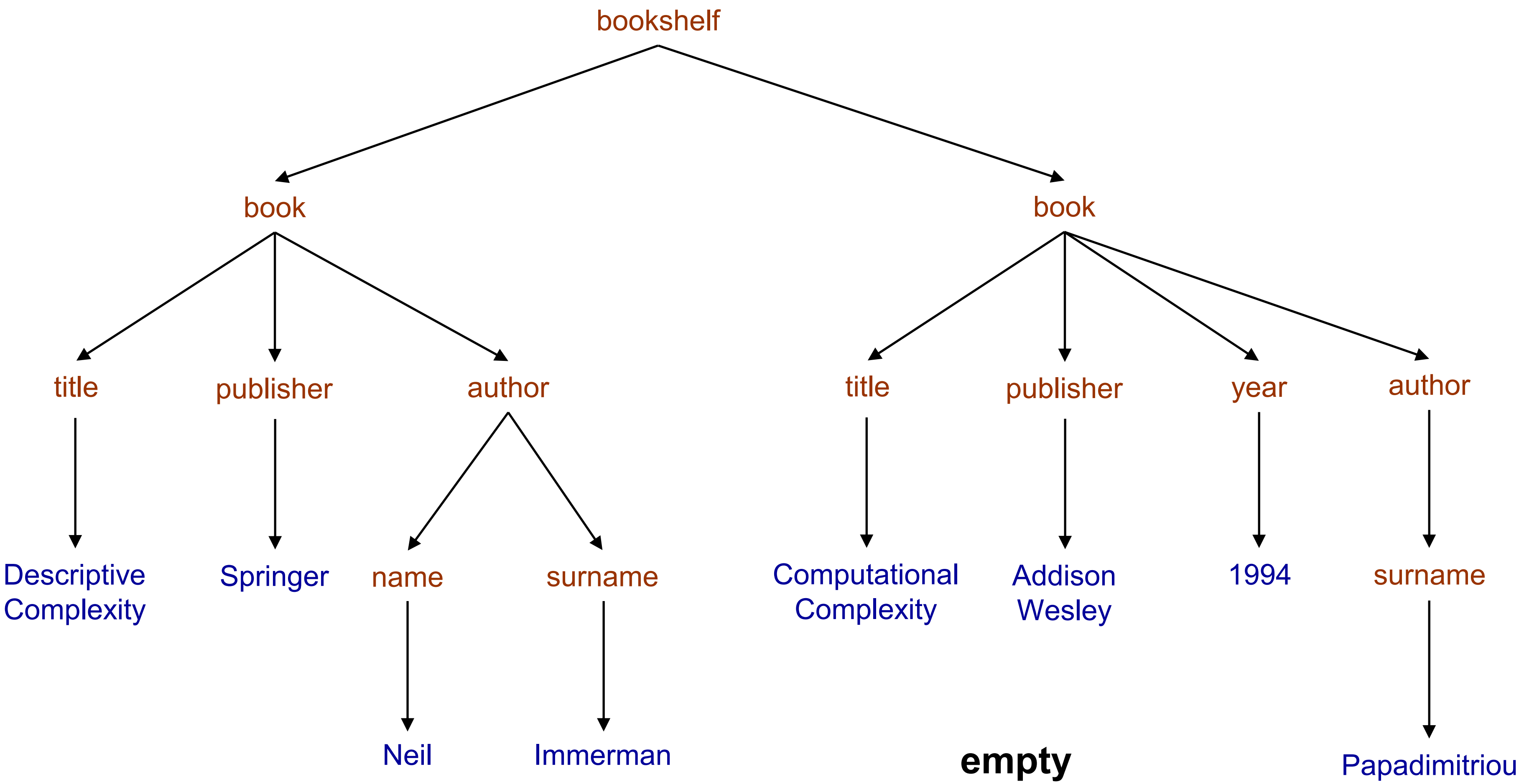
/descendant::book/child::author[child::name]/child::surname

XPath at First Glance



/descendant::book/child::author[position() = 2]/child::surname

XPath at First Glance



/descendant::book/child::author[position() = 2][child::name]

Location Paths

- XPath uses location paths to select nodes in a tree
- A location path is a series of location steps separated by the symbol /
- Each location step has the form

axis::node-test[expression-1][expression-2]…

defines the relationship
to be followed

defines what kind
of nodes must be selected

zero or more predicates,
which filter the selected
nodes according to
arbitrary selection criteria

The Anatomy of a Location Path

$$\underbrace{\underbrace{\text{child}}_{\text{axis}}::\underbrace{\text{bookshelf}}_{\text{node-test}}}_{\text{location step}}/\underbrace{\underbrace{\text{child}}_{\text{axis}}::\underbrace{\text{book}}_{\text{node-test}}\underbrace{[\text{position() = 1}]}_{\text{predicate}}}_{\text{location step}}$$

$$\underbrace{\text{child::bookshelf/child::book[position() = 1]}}_{\text{location path}}$$

NOTE: The first location step does not have a predicate

FO over Ordered Unranked Trees

- **First-order logic (FO)** – often studied in connection with XPath
  - Boolean connectives $\vee$, $\wedge$, $\neg$
  - Quantifiers $\exists$x and $\forall$x that range over nodes of trees

- The navigational features of XPath can be described in **FO**

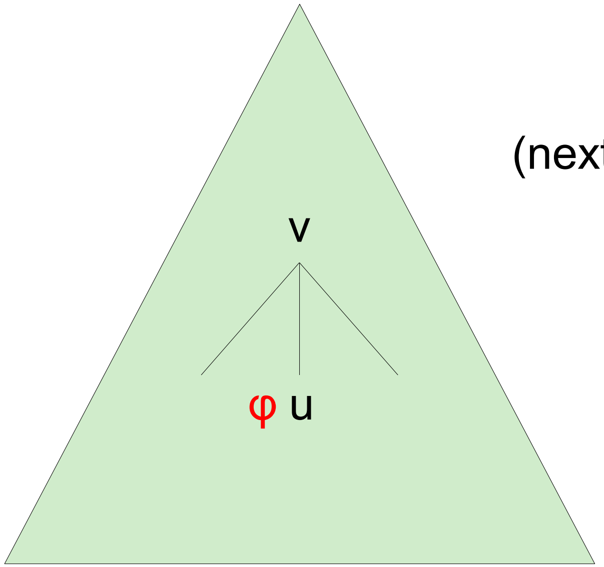
- Can we define alternative logics for **FO** over ordered unranked trees with good evaluation properties?
  - LTL-like logics
  - CTL-like logics

Tree Temporal Logic – TL^{tree}

Syntax: with $d \in \{ch, ns\}$

$$\varphi, \varphi' \;:=\; \alpha,\ \alpha \in \Lambda \mid \varphi \vee \varphi' \mid \neg\varphi \mid \mathbf{X}_d\, \varphi \mid \mathbf{invX}_d\, \varphi \mid \varphi\, \mathbf{U}_d\, \varphi' \mid \varphi\, \mathbf{S}_d\, \varphi'$$

$(\mathit{T}, v) \models \mathbf{X}_{ch}\, \varphi$



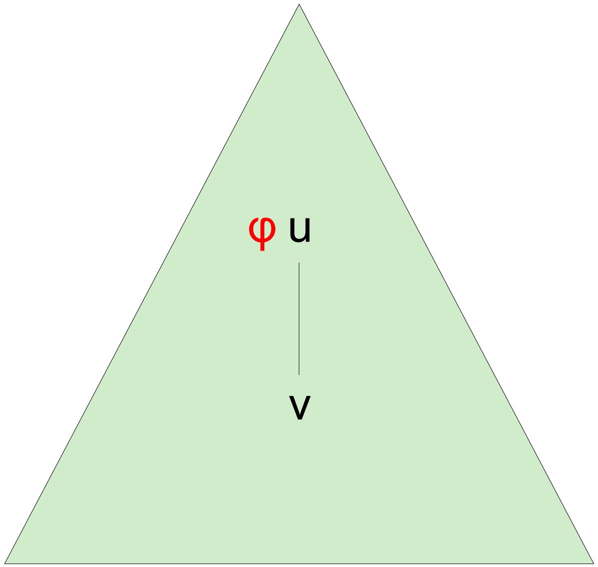
(next)

Tree Temporal Logic – TL^{tree}

Syntax: with $d \in \{ch,ns\}$

$$\varphi, \varphi' := \alpha, \alpha \in \Lambda \mid \varphi \vee \varphi' \mid \neg\varphi \mid \mathbf{X}_d\,\varphi \mid \mathbf{invX}_d\,\varphi \mid \varphi\ \mathbf{U}_d\ \varphi' \mid \varphi\ \mathbf{S}_d\ \varphi'$$

$(\mathit{T},v) \vDash \mathbf{invX}_{ch}\,\varphi$

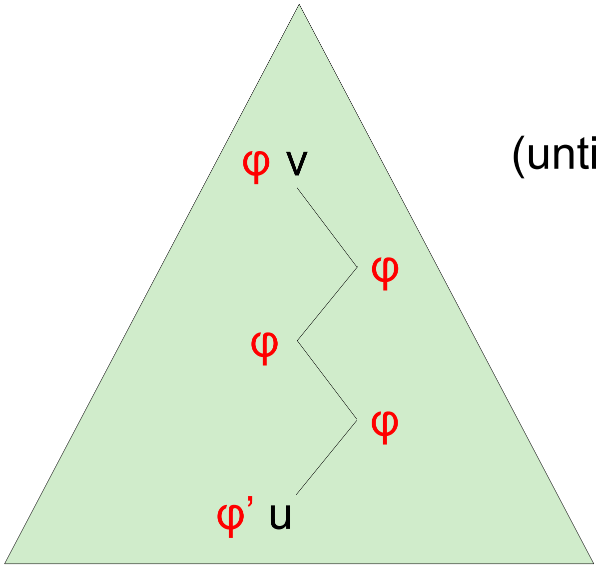


Tree Temporal Logic – TL^{tree}

Syntax: with $d \in \{ch, ns\}$

$$\varphi, \varphi' \ := \ \alpha, \alpha \in \Lambda \mid \varphi \vee \varphi' \mid \neg\varphi \mid \mathbf{X}_d\, \varphi \mid \mathbf{invX}_d\, \varphi \mid \varphi\, \mathbf{U}_d\, \varphi' \mid \varphi\, \mathbf{S}_d\, \varphi'$$

$(\mathit{T}, v) \models \varphi\, \mathbf{U}_{ch}\, \varphi'$



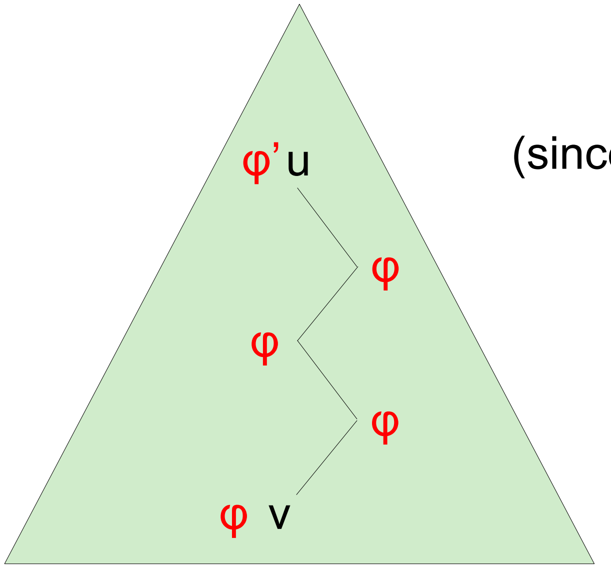
(until)

Tree Temporal Logic – TL^{tree}

Syntax: with $d \in \{ch, ns\}$

$$\varphi, \varphi' := \alpha, \alpha \in \Lambda \mid \varphi \vee \varphi' \mid \neg\varphi \mid \mathbf{X}_d\, \varphi \mid \mathbf{invX}_d\, \varphi \mid \varphi\, \mathbf{U}_d\, \varphi' \mid \varphi\, \mathbf{S}_d\, \varphi'$$

$(T, v) \models \varphi\, \mathbf{S}_{ch}\, \varphi'$



(since)

(analogously for $\mathbf{X}_{ns}\, \varphi \mid \mathbf{invX}_{ns}\, \varphi \mid \varphi\, \mathbf{U}_{ns}\, \varphi' \mid \varphi\, \mathbf{S}_{ns}\, \varphi'$)

Tree Temporal Logic – TL^{tree}

Syntax: with $d \in \{ch, ns\}$

$$\varphi, \varphi' := \alpha, \alpha \in \Lambda \mid \varphi \vee \varphi' \mid \neg\varphi \mid \mathbf{X}_d\, \varphi \mid \mathbf{invX}_d\, \varphi \mid \varphi\, \mathbf{U}_d\, \varphi' \mid \varphi\, \mathbf{S}_d\, \varphi'$$

Theorem: Consider a Boolean/unary query Q on labeled ordered unranked trees. Then:

Q is **FO**-definable $\Leftrightarrow$ Q is $\mathbf{TL^{tree}}$-definable

Important Algorithmic Problems for XPath

XPathSAT

Input: an XPath expression E, a DTD D

Question: is there a tree T valid w.r.t. D so that E selects at least one node in it?

XPathCONT

Input: two XPath expressions E, E' and a DTD D

Question: does $E \subseteq_D E'$, i.e., for every tree T valid w.r.t. D, each node selected by E is also selected by E'?

XPath Satisfiability

Theorem: Given an XPath expression E, and a DTD D, the problem of deciding whether E is satisfiable w.r.t. D is feasible in time $|D| \cdot 2^{O(|E|)}$

Proof idea: exploit automata

- Translate E into a query automaton A_E of exponential size in time $2^{O(|E|)}$
- Translate D into an automaton A_D in linear time
- Let $A = A_E \times A_D$ be the product of the two automata – exponential size
- Test A for emptiness – this can be done in polynomial time in the size of A

XPath Containment

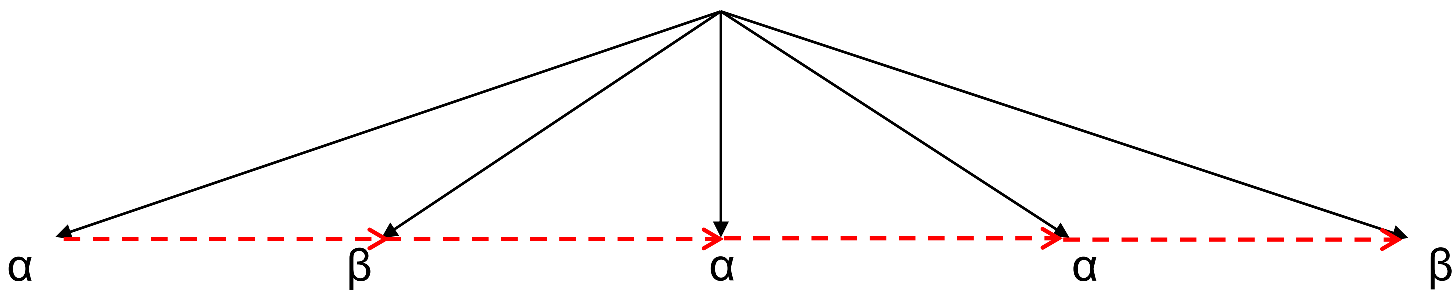
Theorem: Given two XPath expressions E, E' and a DTD D, the problem of deciding whether $E \subseteq_D E'$ is feasible in time $|D| \cdot 2^{O(|E| + |E'|)}$

Proof idea: exploit $\mathbf{TL^{tree}}$ and automata

- Translate E and E' into $\mathbf{TL^{tree}}$ formulae φ and ψ, respectively
- Construct a query automaton $A_{(\varphi \wedge \neg\psi)}$ for $\varphi \wedge \neg\psi$
- Translate D into an automaton A_D
- Let $A = A_{(\varphi \wedge \neg\psi)} \times A_D$ – a query automaton of size $|D| \cdot 2^{O(|E| + |E'|)}$
- Test A for emptiness – this can be done in polynomial time in the size of A

A Quick Note on Unordered Trees

- Like ordered trees but the sibling ordering ($\prec_{ns}$) is no longer available
- Without order, counting has to be introduced explicitly – order buys counting



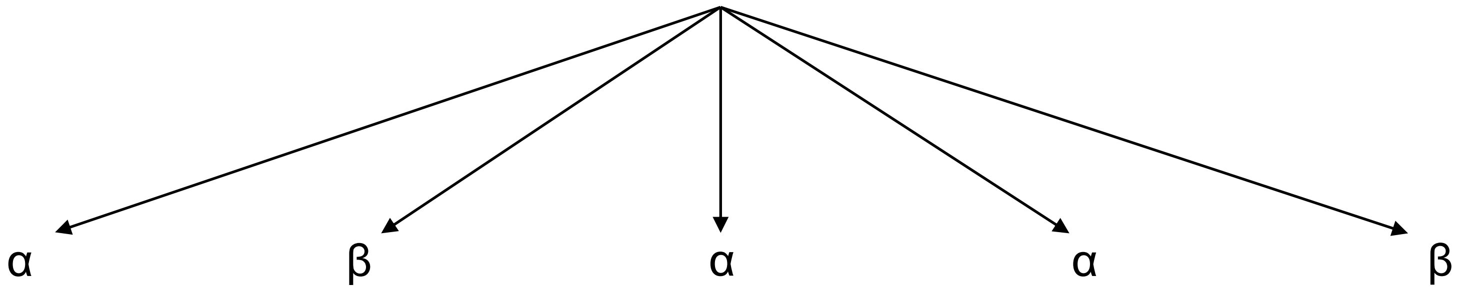
select the nodes with at least two children labeled α

$$Q(x) \quad = \quad \exists y \exists z (x \prec_{ch} y \;\wedge\; P_\alpha(y) \;\wedge\; y \prec_{ns^*} z \;\wedge\; P_\alpha(z))$$

A Quick Note on Unordered Trees

- Like ordered trees but the sibling ordering ($\prec_{ns}$) is no longer available

- Without order, counting has to be introduced explicitly – order buys counting



no way to say that there are at least two children labeled α

- We have counting NUTA and counting query automata

Associated Papers

- Frank Neven: Automata, Logic, and XML. CSL 2002: 2-26

  A survey of automata theoretic techniques in XML, particularly XML standards

- Leonid Libkin: Logics for Unranked Trees: An Overview. Logical Methods in Computer Science 2(3) (2006)

  A survey of logical techniques for languages used in XML (schema, navigation, querying)

- Georg Gottlob, Christoph Koch, Reinhard Pichler: Efficient algorithms for processing XPath queries. ACM Trans. Database Syst. 30(2): 444-491 (2005)

  How to evaluate XPath most efficiently

- Georg Gottlob, Christoph Koch, Reinhard Pichler, Luc Segoufin: The complexity of XPath query evaluation and XML typing. Journal of the ACM 52(2): 284-335 (2005)

  Pinpointing exact complexity of many problems related to XPath evaluation and XML schemas

Associated Papers

- Frank Neven, Thomas Schwentick: Query automata over finite trees. Theor. Comput. Sci. 275(1-2): 633-674 (2002)

  Extending automata to formalisms that select nodes in trees

- Georg Gottlob, Christoph Koch: Monadic datalog and the expressive power of languages for Web information extraction. Journal of the ACM 51(1): 74-113 (2004)

  Capturing MSO with a very efficient language, and applications in Web data extraction

- Pablo Barceló, Leonid Libkin: Temporal logics over unranked trees. LICS 2005: 31-40

  How XML languages are related to logics used in software/hardware verification

- Leonid Libkin, Cristina Sirangelo: Reasoning about XML with temporal logics and automata. J. Applied Logic 8(2): 210-232 (2010)

  ... and how to exploit the connection to verify properties of XML

Associated Papers

- Thomas Schwentick: XPath query containment. SIGMOD Record 33(1): 101-109 (2004)

  A survey of techniques for testing containment and equivalence of XPath queries

- Wenfei Fan, Leonid Libkin: On XML integrity constraints in the presence of DTDs. Journal of the ACM 49(3): 368-406 (2002)

  Explaining why unary keys and foreign keys are in NP for XML, and why beyond unary they are undecidable

- Marcelo Arenas, Wenfei Fan, Leonid Libkin: On the Complexity of Verifying Consistency of XML Specifications. SIAM J. Comput. 38(3): 841-880 (2008)

  Pushing this further to more expressive constraints used in XML, such as those in XML Schema

- Wim Martens, Frank Neven, Thomas Schwentick: Simple off the shelf abstractions for XML schema. SIGMOD Record 36(3): 15-22 (2007)

  Theoretical reconstructions of XML Schema

Associated Papers

- Wim Martens, Frank Neven, Thomas Schwentick, Geert Jan Bex: Expressiveness and complexity of XML Schema. ACM Trans. Database Syst. 31(3): 770-813 (2006)

  An automaton model for XML schema, and its use in efficient typing of documents

- Mikolaj Bojanczyk, Anca Muscholl, Thomas Schwentick, Luc Segoufin: Two-variable logic on data trees and XML reasoning. Journal of the ACM 56(3) (2009)

  Decidability/undecidability boundary for 2 vs 3 variables over data trees

- Tony Tan: Extending two-variable logic on data trees with order on data values and its automata. ACM Trans. Comput. Log. 15(1): 8 (2014)

  Pushing this to more expressive formalisms, with better (more readable) algorithms

- Claire David, Leonid Libkin, Tony Tan: Efficient reasoning about data trees via integer linear programming. ACM Trans. Database Syst. 37(3): 19 (2012)

  The NP bound for sets and linear constraints

Associated Papers

- Henrik Bjrklund, Wim Martens, Thomas Schwentick: Conjunctive query containment over trees. J. Comput. Syst. Sci. 77(3): 450-472 (2011)

  Extending CQ containment from relations to databases

- Wojciech Czerwinski, Wim Martens, Pawel Parys, Marcin Przybylko: The (Almost) Complete Guide to Tree Pattern Containment. PODS 2015: 117-130

  CQs over trees are essentially patterns: a detailed study of their containment

- Maarten Marx: Conditional XPath. ACM Trans. Database Syst. 30(4): 929-959 (2005)

  An XPath extension that captures all first-order queries over XML documents

- Balder ten Cate, Maarten Marx: Navigational XPath: calculus and algebra. SIGMOD Record 36(2): 19-26 (2007)

  Providing algebraic counterpart for XPath fragments

Associated Papers

- Loredana Afanasiev, Maarten Marx: An analysis of XQuery benchmarks. Inf. Syst. 33(2): 155-181 (2008)

  The title says it all

- Luc Segoufin, Cristina Sirangelo: Constant-Memory Validation of Streaming XML Documents Against DTDs. ICDT 2007: 299-313

  Analyzing which DTDs can be checked over streamed documents