the rest of this course

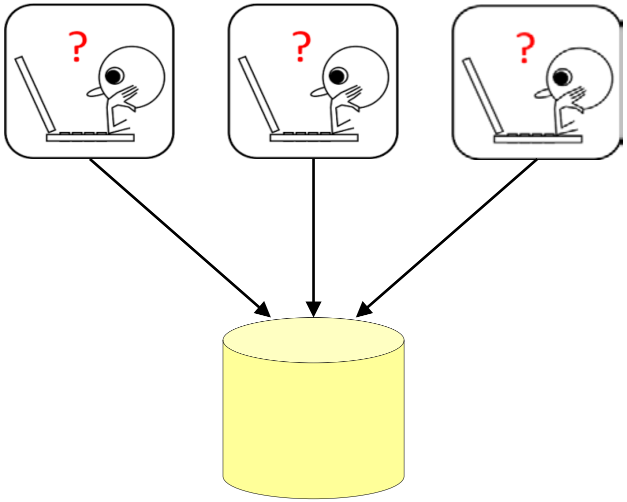
Volume
size does mattes
(thousands of TBs of data)
✓

Veracity
data is often
incomplete/inconsistent

Variety
many data formats
(structured, semi-structured, etc.)
✓

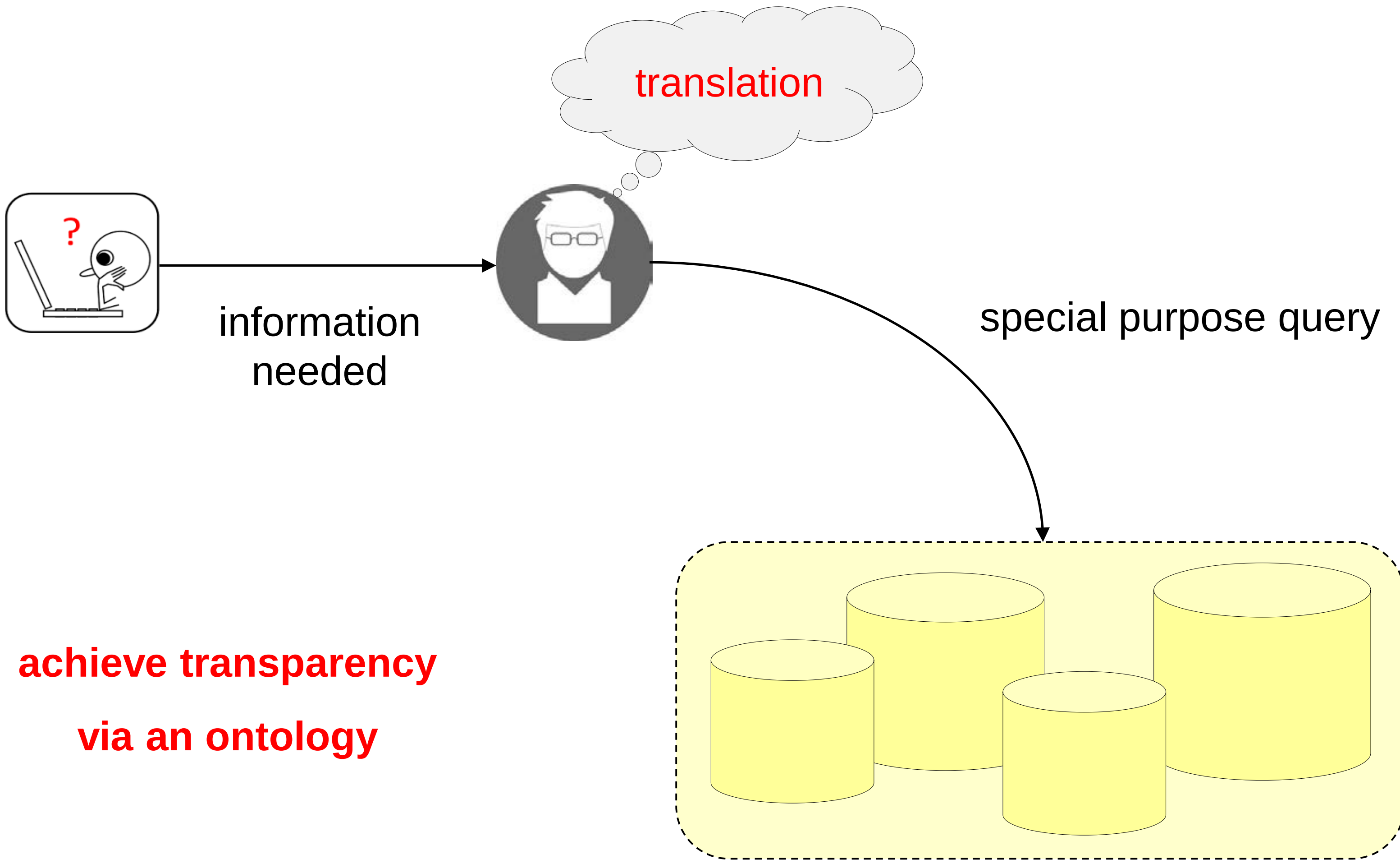
Velocity
data often arrives at fast speed
(updates are frequent)

Ontology-Based Data Access



ideal information system

in many modern organizations



achieve transparency

via an ontology

What is an Ontology?

An engineering artifact; its objective is to provide

an explicit specification of a conceptualization

an abstract model of (some aspect of) the world

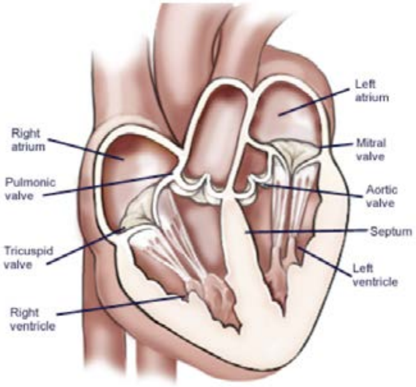
using unambiguous language, typically logic

What is an Ontology?

1. Introduces vocabulary relevant to a domain

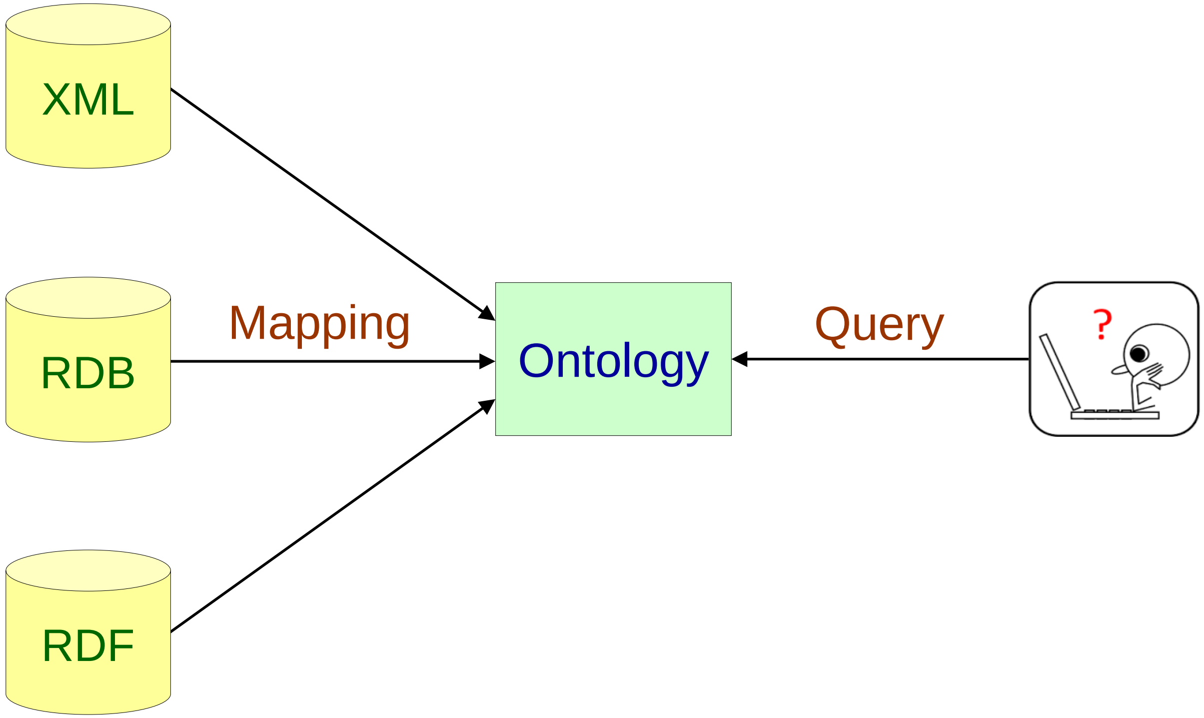
2. Specifies the meaning (semantics) of the terms

Heart is a muscular organ that is part of the circulatory system



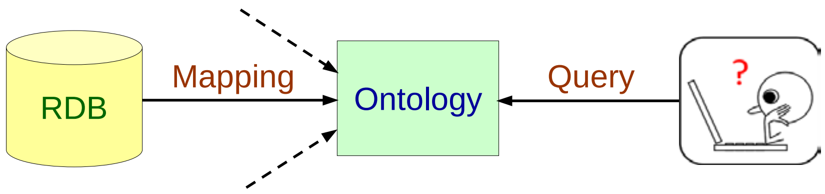
$$\forall x \, (\mathrm{Heart}(x) \rightarrow \mathrm{MuscularOrgan}(x) \land \exists y \, (\mathrm{isPartOf}(x,y) \land \mathrm{CirculatorySystem}(y)))$$

Ontology-Based Data Access (OBDA)



use an ontology as a mediator

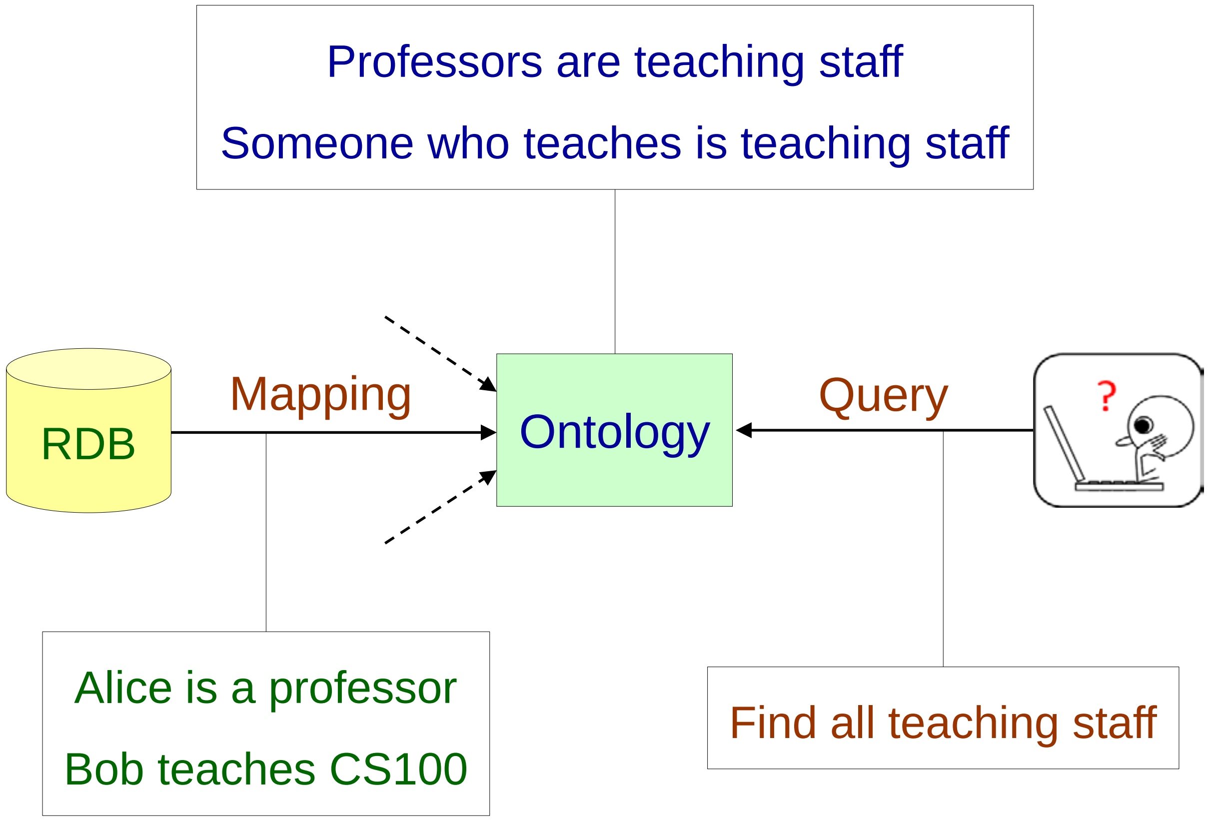
What are Ontologies Good For?



1. **Integrate different data sources (variety)**
    - Conceptual “global view” of the data
    - Access data in a uniform and transparent way

2. **Support automated reasoning (incompleteness)**
    - Implicit consequences are taken into account
    - More complete answers

Incomplete Data Sources



Expected answer = {Alice, Bob}

Ontology-Based Data Access: Example

Ontology Σ - high level representation of the domain of interest

$$\forall x\, (\mathrm{Researcher}(x) \rightarrow \exists y\, (\mathrm{worksFor}(x,y) \wedge \mathrm{Project}(y)))$$

$$\forall x\, (\mathrm{Project}(x) \rightarrow \exists y\, (\mathrm{worksFor}(y,x) \wedge \mathrm{Researcher}(y)))$$

$$\forall x \forall y\, (\mathrm{worksFor}(x,y) \rightarrow \mathrm{Researcher}(x) \wedge \mathrm{Project}(y))$$

$$\forall x\, (\mathrm{Project}(x) \rightarrow \exists y\, (\mathrm{ProjectName}(x,y)))$$

Ontology-Based Data Access: Example

Relational database *D* - a single database that represents the sources

worksIn	SSN	Name
	100	AAA
	200	BBB
	300	CCC

Ontology-Based Data Access: Example

Relational database *D* - a single database that represents the sources

worksIn	SSN	Name
	100	AAA
	200	BBB
	300	CCC

the researcher with SSN 100 works for the project with name “AAA”

Ontology-Based Data Access: Example

Mapping *M* - semantically link data at the sources with the ontology

SELECT SSN, Name FROM worksIn	$\subseteq$	Researcher(person(SSN)) $\wedge$ Project(proj(Name)) $\wedge$ worksFor(person(SSN), proj(Name)) $\wedge$ ProjectName(proj(Name), Name)

Ontology-Based Data Access: Example

Mapping *M* - semantically link data at the sources with the ontology

SELECT SSN, Name
FROM worksIn

$\subseteq$

Researcher(person(SSN)) $\wedge$
Project(proj(Name)) $\wedge$
worksFor(person(SSN), proj(Name)) $\wedge$
ProjectName(proj(Name), Name)

- Constructors to create objects from tuples of values in the database
- The constructors are simply Skolem functions

Ontology-Based Data Access: Example

Virtual data layer *M*(*D*)

worksIn	SSN	Name
	100	AAA
	200	BBB
	300	CCC

SELECT SSN, Name FROM worksIn	$\subseteq$	Researcher(person(SSN)) $\wedge$ Project(proj(Name)) $\wedge$ worksFor(person(SSN), proj(Name)) $\wedge$ ProjectName(proj(Name), Name)

Researcher(person(100)), Project(proj(AAA)), worksFor(person(100), proj(AAA)),

ProjectName(proj(AAA), AAA),

Ontology-Based Data Access: Example

Virtual data layer *M*(*D*)

worksIn	SSN	Name
	100	AAA
	200	BBB
	300	CCC

SELECT SSN, Name
FROM worksIn $\subseteq$ Researcher(person(SSN)) $\wedge$ Project(proj(Name)) $\wedge$ worksFor(person(SSN), proj(Name)) $\wedge$ ProjectName(proj(Name), Name)

Researcher(person(100)), Project(proj(AAA)), worksFor(person(100), proj(AAA)),

ProjectName(proj(AAA), AAA),

Researcher(person(200)), Project(proj(BBB)), worksFor(person(200), proj(BBB)),

ProjectName(proj(BBB), BBB),

Ontology-Based Data Access: Example

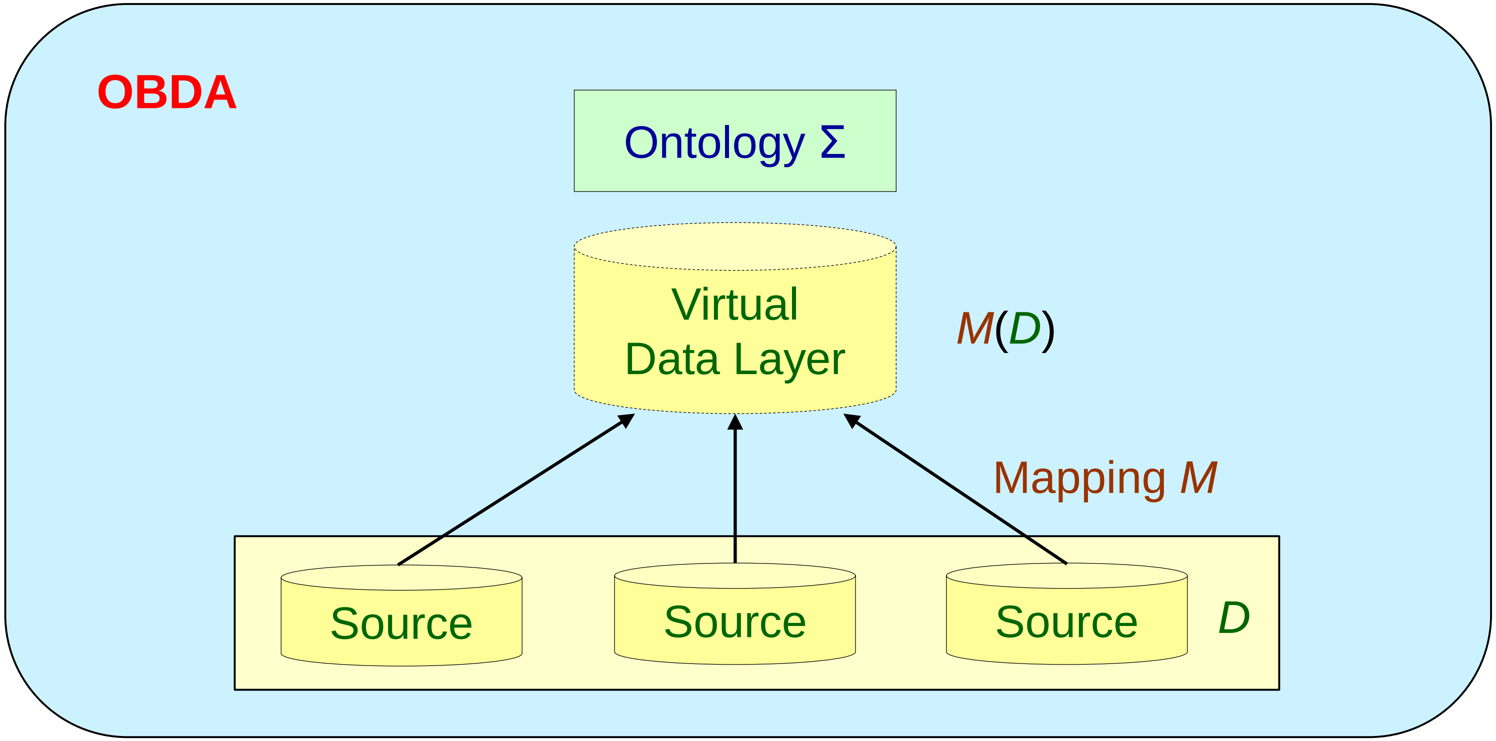
Virtual data layer *M*(*D*)

worksIn	SSN	Name
	100	AAA
	200	BBB
	300	CCC

SELECT SSN, Name
FROM worksIn $\subseteq$ Researcher(person(SSN)) $\wedge$
Project(proj(Name)) $\wedge$
worksFor(person(SSN), proj(Name)) $\wedge$
ProjectName(proj(Name), Name)

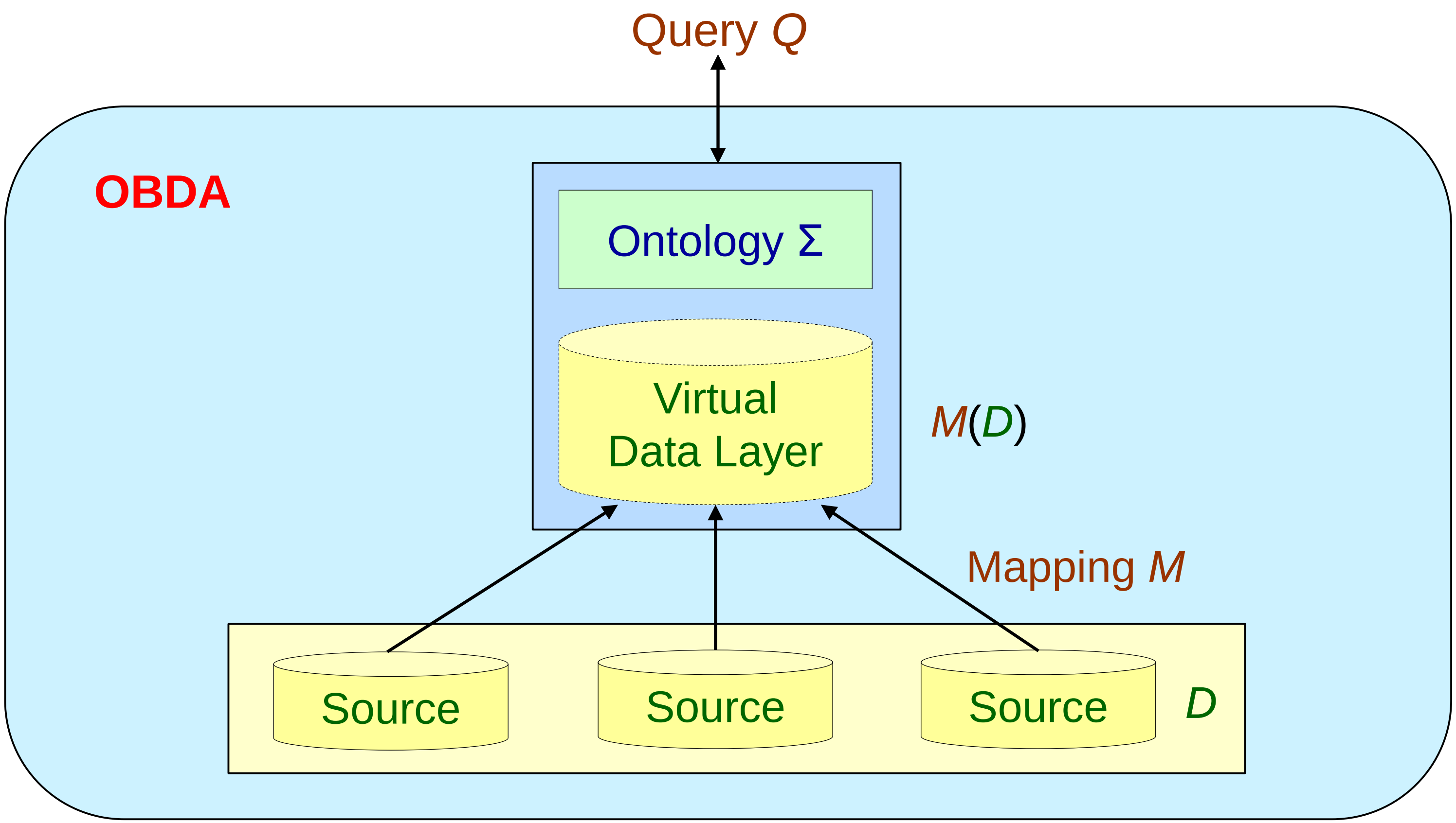
Researcher(person(100)), Project(proj(AAA)), worksFor(person(100), proj(AAA)),
ProjectName(proj(AAA), AAA),
Researcher(person(200)), Project(proj(BBB)), worksFor(person(200), proj(BBB)),
ProjectName(proj(BBB), BBB),
Researcher(person(300)), Project(proj(CCC)), worksFor(person(300), proj(CCC)),
ProjectName(proj(CCC), CCC)

Query Answering in OBDA



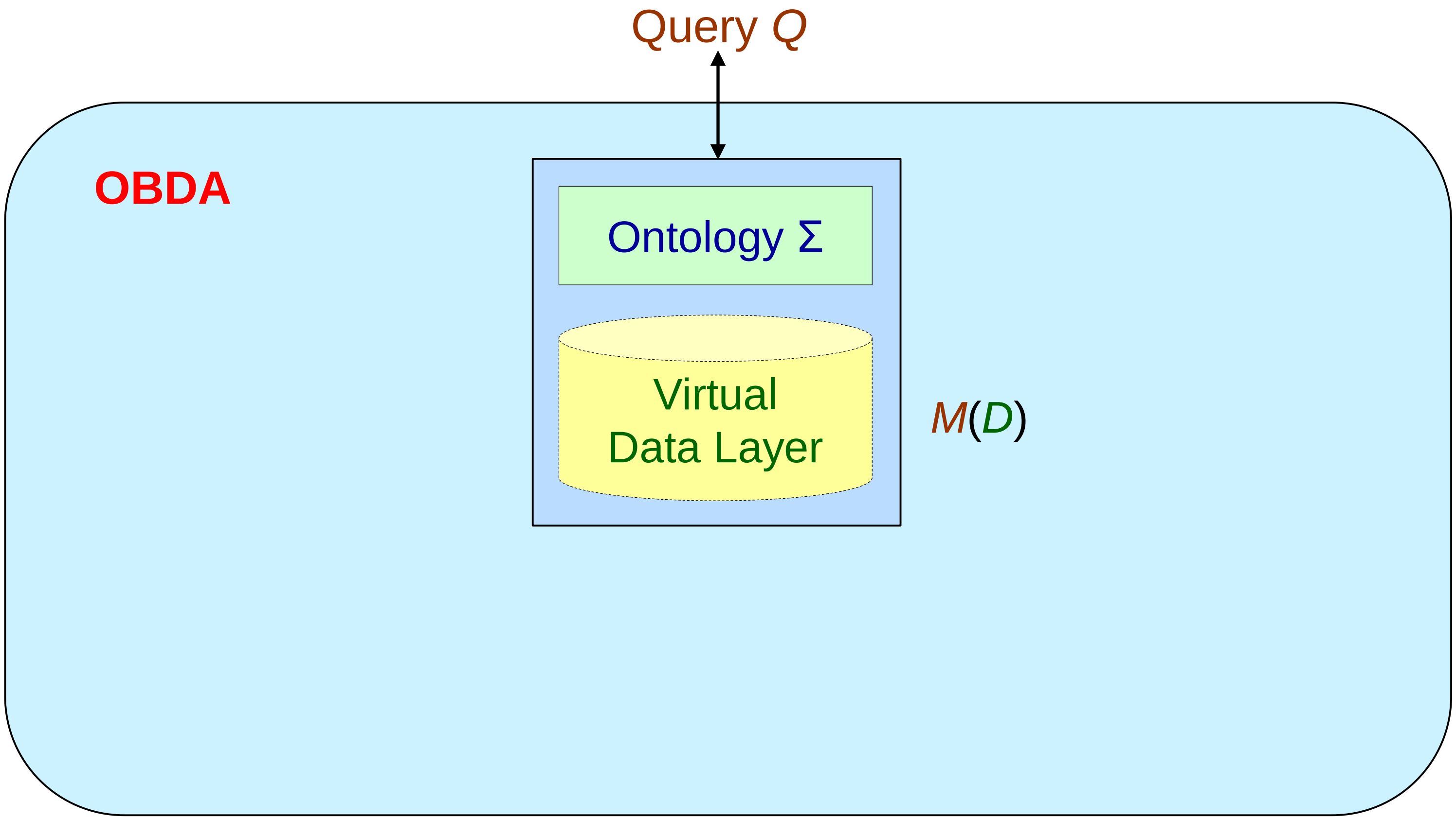
- The sources and the mapping define a virtual data layer $M(D)$

Query Answering in OBDA



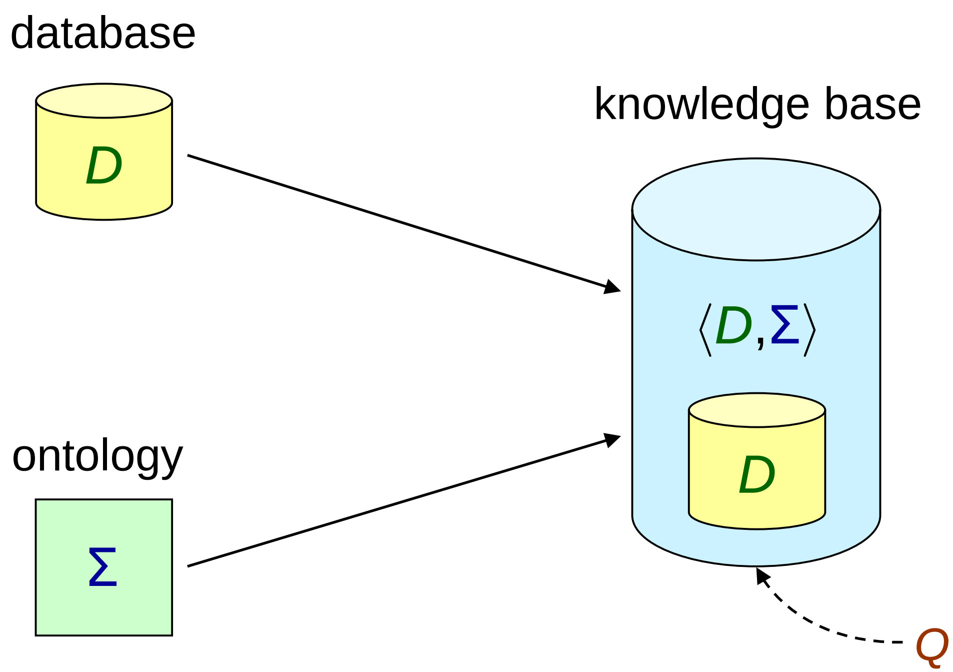
- The sources and the mapping define a virtual data layer $M(D)$
- Queries are answered against the knowledge base $\langle M(D), \Sigma \rangle$

Query Answering in OBDA



Ontology-Based Query Answering

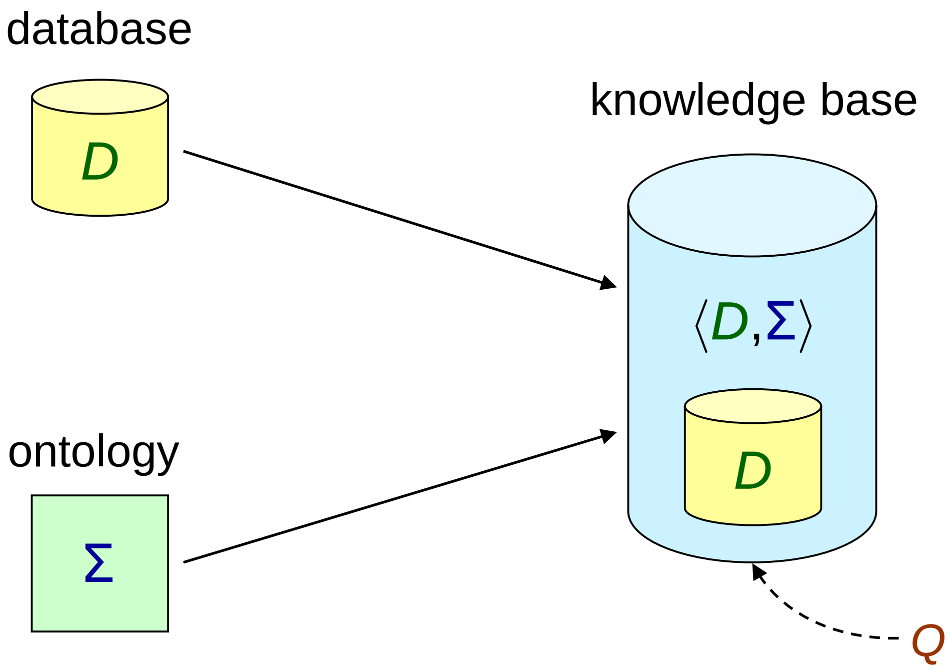
Ontology-Based Query Answering (OBQA)



$$\text{certain-answers}(Q, \langle D,\Sigma\rangle) \quad = \quad \bigcap_{J \in \text{models}(D \wedge \Sigma)} Q(J)$$

(formal definitions later - once we fix the languages)

Ontology-Based Query Answering (OBQA)



NOTE: OBQA is not OBDA, but a crucial task in OBDA

We should talk about OBDA only in the presence of external sources and mappings

Issues in Ontology-Based Query Answering

What is the right ontology language?

- A wide spectrum of languages that differ in expressive power and computational complexity (e.g., description logics, existential rules)
- Scalability to very large amounts of data is a key

What is the right query language?

- Well-known languages from database theory (e.g., conjunctive queries)

Few Words on Description Logics (DLs)

- DLs are well-behaved fragments of first-order logic
- Several DL-based languages exist (from lightweight to very expressive logics)
- Strongly influenced the W3C standard Web Ontology Language OWL

- **Syntax:** We start from a vocabulary with
  - Concept names: atomic classes or unary predicates, e.g., Parent, Person
  - Role names: atomic relations or binary predicates, e.g., hasParent

  and we build axioms
  - $\text{Person} \sqsubseteq \exists \text{hasParent.Parent}$ - each person has a parent
  - $\text{Parent} \sqsubseteq \text{Person}$ - each parent is a person

- **Semantics:** Via first-order interpretations

DL-Lite Family

DL-Lite: Popular family of DLs - at the basis of the OWL 2 QL profile of OWL 2

DL-Lite Axioms	First-order Representation
$A \sqsubseteq B$	$\forall x\ (A(x) \rightarrow B(x))$
$A \sqsubseteq \exists R$	$\forall x\ (A(x) \rightarrow \exists y\ R(x,y))$
$\exists R \sqsubseteq A$	$\forall x \forall y\ (R(x,y) \rightarrow A(x))$
$\exists R \sqsubseteq \exists P$	$\forall x \forall y\ (R(x,y) \rightarrow \exists z\ P(x,z))$
$A \sqsubseteq \exists R.B$	$\forall x\ (A(x) \rightarrow \exists y\ (R(x,y) \wedge B(y)))$
$R \sqsubseteq P$	$\forall x \forall y\ (R(x,y) \rightarrow P(x,y))$
$A \sqsubseteq \neg B$	$\forall x\ (A(x) \wedge B(x) \rightarrow \bot)$

The Description Logic EL

EL: Popular DL for biological applications - at the basis of the OWL 2 EL profile

EL Axioms	First-order Representation
$A \sqsubseteq B$	$\forall x\ (A(x) \rightarrow B(x))$
$A \sqcap B \sqsubseteq C$	$\forall x\ (A(x) \wedge B(x) \rightarrow C(x))$
$A \sqsubseteq \exists R.B$	$\forall x\ (A(x) \rightarrow \exists y\ (R(x,y) \wedge B(y)))$
$\exists R.B \sqsubseteq A$	$\forall x \forall y\ (R(x,y) \wedge B(y) \rightarrow A(x))$

…several other, more expressive, description logics exist

…but, in what follows we focus on existential rules

an alternative way for representing ontologies

A Simple Example

$$\forall x\ (\mathrm{Researcher}(x) \rightarrow \exists y\ (\mathrm{worksFor}(x,y) \wedge \mathrm{Project}(y)))$$

$$\forall x\ (\mathrm{Project}(x) \rightarrow \exists y\ (\mathrm{worksFor}(y,x) \wedge \mathrm{Researcher}(y)))$$

$$\forall x \forall y\ (\mathrm{worksFor}(x,y) \rightarrow \mathrm{Researcher}(x) \wedge \mathrm{Project}(y))$$

$$\forall x\ (\mathrm{Project}(x) \rightarrow \exists y\ (\mathrm{ProjectName}(x,y)))$$

Some Terminology

- Our basic vocabulary:
  - A countable set **C** of constants - domain of a database
  - A countable set **N** of (labeled) nulls - globally ∃-quantified variables
  - A countable set **V** of (regular) variables - used in rules and queries
- A term is a constant, null or variable
- An atom has the form $R(t_1, \ldots, t_n)$ - R is an n-ary relation and t_i's are terms
- An instance is a (possibly infinite) set of atoms with constants and nulls
- A database is a finite instance with only constants

Syntax of Existential Rules

An existential rule is an expression

$$\forall \mathbf{x} \forall \mathbf{y}\ (\underbrace{\varphi(\mathbf{x},\mathbf{y})}_{\text{body}} \rightarrow \exists \mathbf{z}\ \underbrace{\psi(\mathbf{x},\mathbf{z})}_{\text{head}})$$

- $\mathbf{x}$,$\mathbf{y}$ and $\mathbf{z}$ are tuples of variables of **V**
- $\varphi(\mathbf{x},\mathbf{y})$ and $\psi(\mathbf{x},\mathbf{z})$ are (constant-free) conjunctions of atoms

…a.k.a. tuple-generating dependencies and Datalog$^{\pm}$ rules

Semantics of Existential Rules

- An instance J is a model of the rule

$$\sigma \ = \ \forall \mathbf{x} \forall \mathbf{y} \ (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z} \ \psi(\mathbf{x},\mathbf{z}))$$

  written as $J \vDash \sigma$, if the following holds:

  whenever there exists a homomorphism h such that $h(\varphi(\mathbf{x},\mathbf{y})) \subseteq J$,

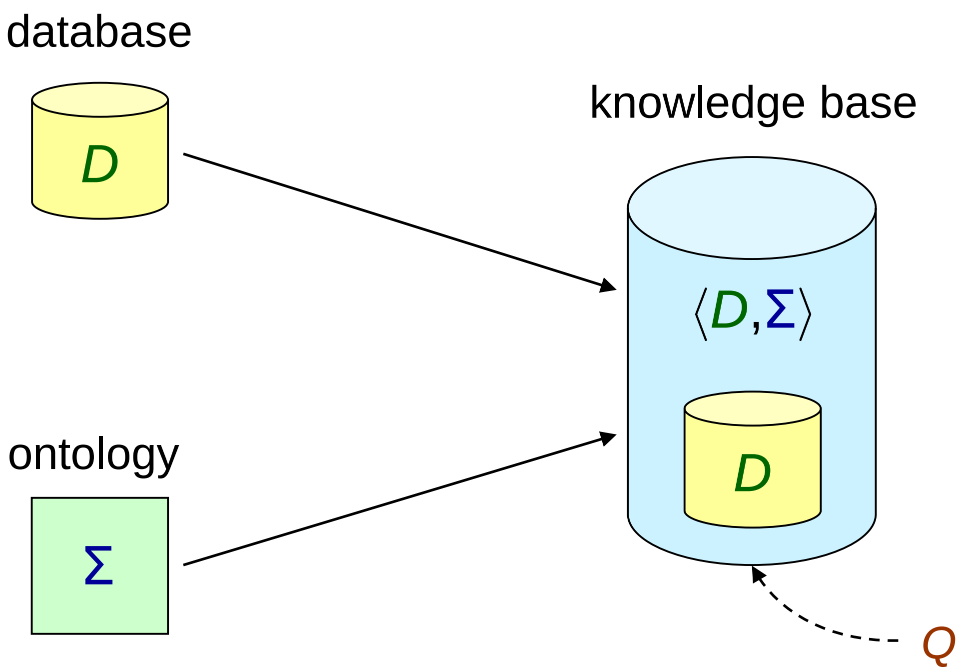
  then there exists $g \supseteq h_{|\mathbf{x}}$ such that $g(\psi(\mathbf{x},\mathbf{z})) \subseteq J$

  $\{t \rightarrow h(t) \mid t \in \mathbf{x}\}$ - the restriction of h to **x**

- Given a set Σ of existential rules, J is a model of Σ, written as $J \vDash \Sigma$, if the following holds: for each $\sigma \in \Sigma$, $J \vDash \sigma$

- $J \vDash \Sigma$ iff $\mathfrak{J}$ is a model of the first-order theory $\bigwedge_{\sigma \in \Sigma} \sigma$

Ontology-Based Query Answering (OBQA)



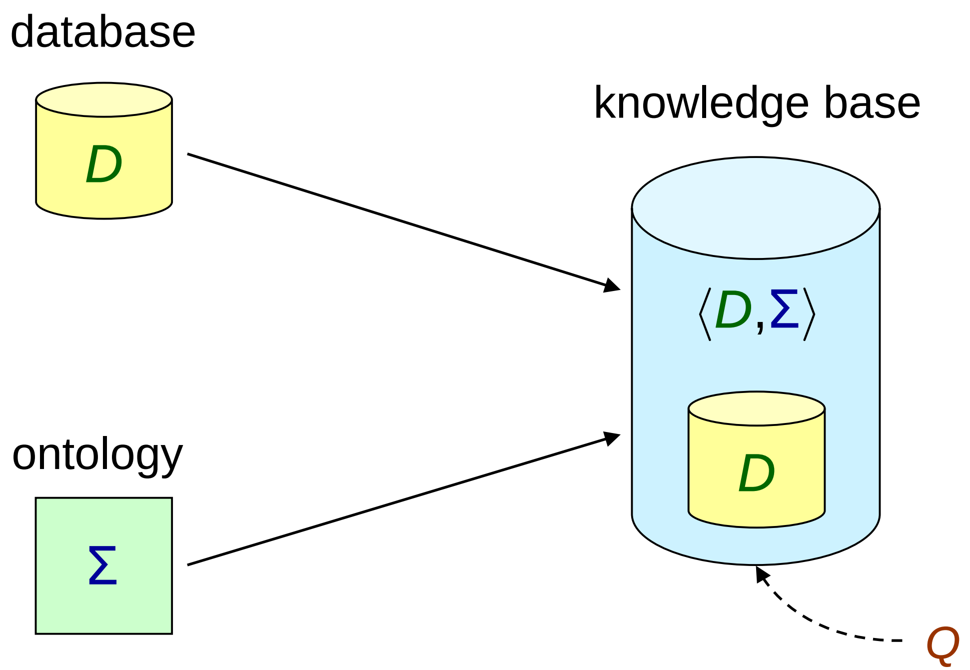
existential rules

$$\forall \mathbf{x} \forall \mathbf{y}\ (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\ \psi(\mathbf{x},\mathbf{z}))$$

conjunctive queries

$$Q(\mathbf{x}) \ \text{:-}\ R_1(\mathbf{v_1}),\ldots,R_m(\mathbf{v_m})$$

Ontology-Based Query Answering (OBQA)



$$\text{certain-answers}(Q, \langle D,\Sigma\rangle) = \bigcap_{J \in \text{models}(D \wedge \Sigma)} Q(J)$$

$$\{J \mid J \supseteq D \text{ and } J \models \Sigma\}$$

Exercise: Compute the Certain Answers

D = {Person(john), Person(bob), Person(tom),
hasFather(john,bob), hasFather(bob,tom)}

Σ = $\{\forall x\,(\mathrm{Person}(x) \rightarrow \exists y\, \mathrm{hasFather}(x,y)),$
$\forall x \forall y\,(\mathrm{hasFather}(x,y) \rightarrow \mathrm{Person}(x) \wedge \mathrm{Person}(y))\}$

Q_1(x,y) :- hasFather(x,y)

Q_2(x) :- hasFather(x,y)

Q_3(x) :- hasFather(x,y), hasFather(y,z), hasFather(z,w)

Q_4(x,w) :- hasFather(x,y), hasFather(y,z), hasFather(z,w)

Exercise: Compute the Certain Answers

D = {Person(john), Person(bob), Person(tom),

hasFather(john,bob), hasFather(bob,tom)}

Σ = $\{\forall x\,(\text{Person}(x) \rightarrow \exists y\ \text{hasFather}(x,y)),$

$\forall x \forall y\,(\text{hasFather}(x,y) \rightarrow \text{Person}(x) \wedge \text{Person}(y))\}$

$Q_1(x,y)$:- hasFather(x,y)

{(john,bob), (bob,tom)}

Exercise: Compute the Certain Answers

D = {Person(john), Person(bob), Person(tom),

hasFather(john,bob), hasFather(bob,tom)}

Σ = $\{\forall x\ (\text{Person}(x) \rightarrow \exists y\ \text{hasFather}(x,y)),$

$\forall x \forall y\ (\text{hasFather}(x,y) \rightarrow \text{Person}(x) \land \text{Person}(y))\}$

$Q_2(x)$:- hasFather(x,y)

{(john), (bob), (tom)}

Exercise: Compute the Certain Answers

D = {Person(john), Person(bob), Person(tom),
hasFather(john,bob), hasFather(bob,tom)}

$$\Sigma = \{\forall x\,(\text{Person}(x) \rightarrow \exists y\ \text{hasFather}(x,y)),\ \forall x \forall y\,(\text{hasFather}(x,y) \rightarrow \text{Person}(x) \wedge \text{Person}(y))\}$$

$Q_3(x)$:- hasFather(x,y), hasFather(y,z), hasFather(z,w)

{(john), (bob), (tom)}

Exercise: Compute the Certain Answers

D = {Person(john), Person(bob), Person(tom),

hasFather(john,bob), hasFather(bob,tom)}

Σ = $\{\forall x\,(\text{Person}(x) \rightarrow \exists y\ \text{hasFather}(x,y)),$

$\forall x \forall y\,(\text{hasFather}(x,y) \rightarrow \text{Person}(x) \wedge \text{Person}(y))\}$

Q_4(x,w) :- hasFather(x,y), hasFather(y,z), hasFather(z,w)

{ }

OBQA: Formal Definition

ontology language based on existential rules

OBQA(**L**)

Input: database D, existential rules $\Sigma \in \mathbf{L}$, CQ $Q(\mathbf{x})$, tuple $\mathbf{t} \in \text{adom}(D)^{|\mathbf{X}|}$

Question: $\mathbf{t} \in \text{certain-answers}(Q,\langle D,\Sigma\rangle) \;=\; \bigcap_{J \in \text{models}(D \wedge \Sigma)} Q(J)$?

$$\mathbf{t} \in \text{certain-answers}(Q,\langle D\ \Sigma\rangle) \iff \forall J \in \text{models}(D \wedge \Sigma),\ \mathbf{t} \in Q(J)$$

$$\iff \forall J \in \text{models}(D \wedge \Sigma),\ () \in Q_{\mathbf{t}}(J), \text{ where } Q_{\mathbf{t}} = Q(\mathbf{t})$$

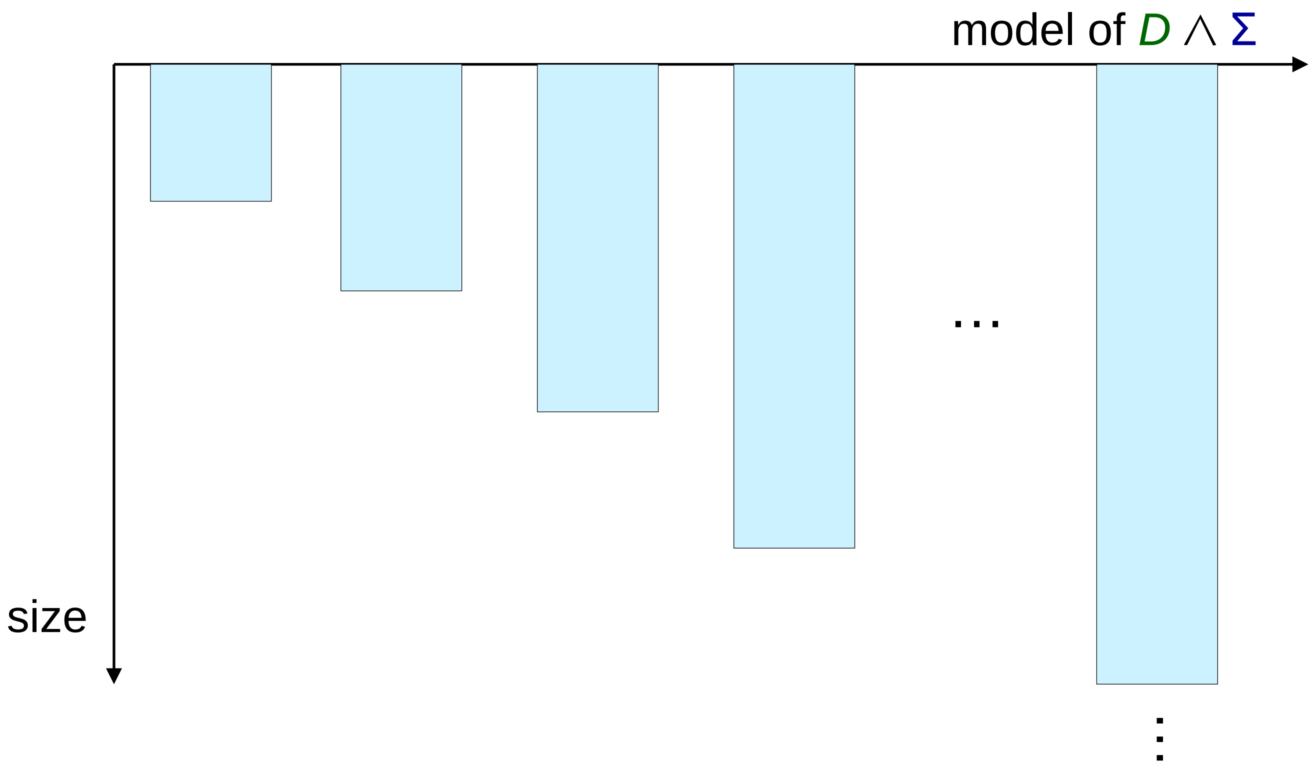
Boolean CQ - no output variables

Why is OBQA technically challenging?

What is the right tool for tackling this problem?

The Two Dimensions of Infinity

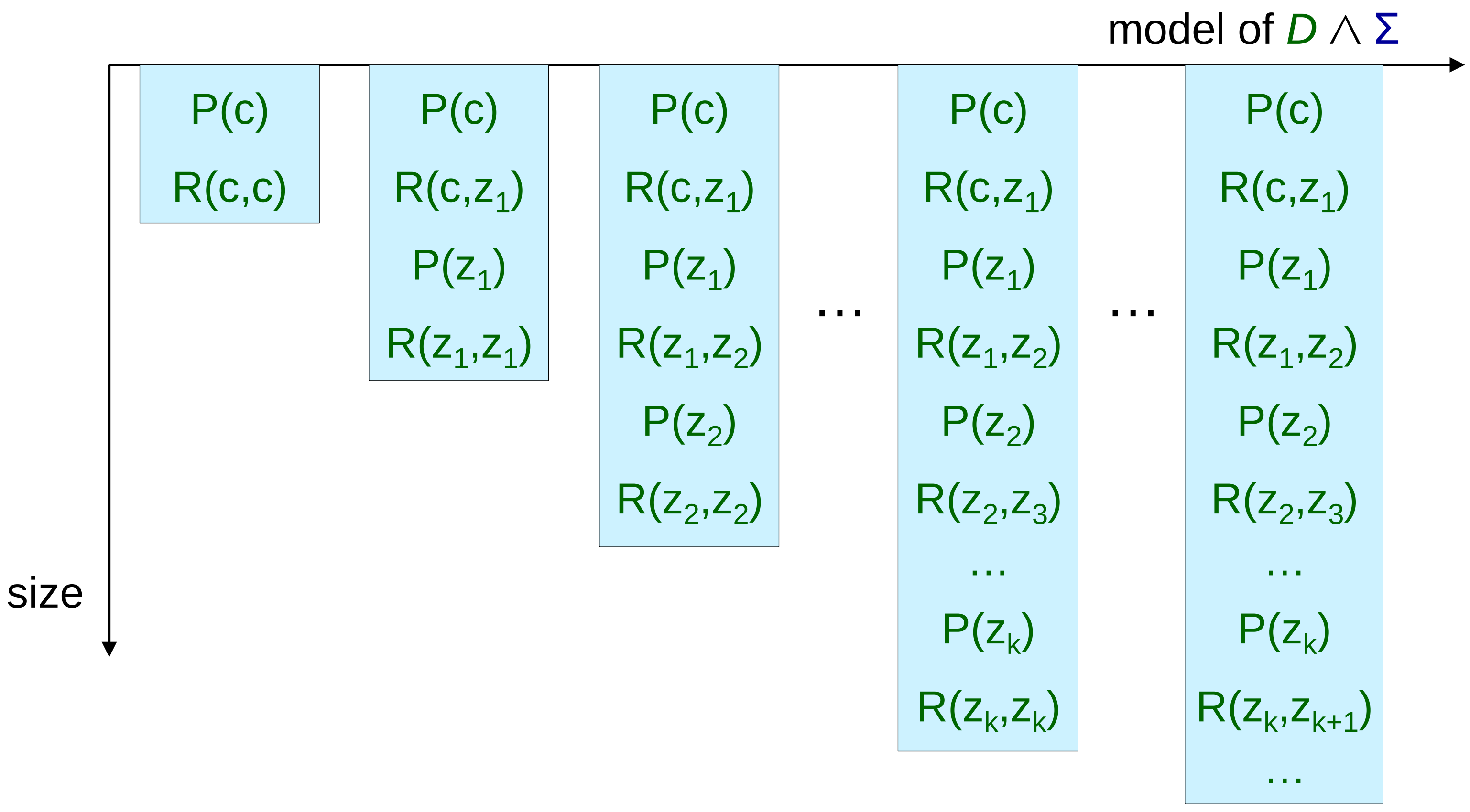
Consider the database *D*, and the set of existential rules Σ



$D \wedge \Sigma$ admits infinitely many models, of possibly infinite size

The Two Dimensions of Infinity

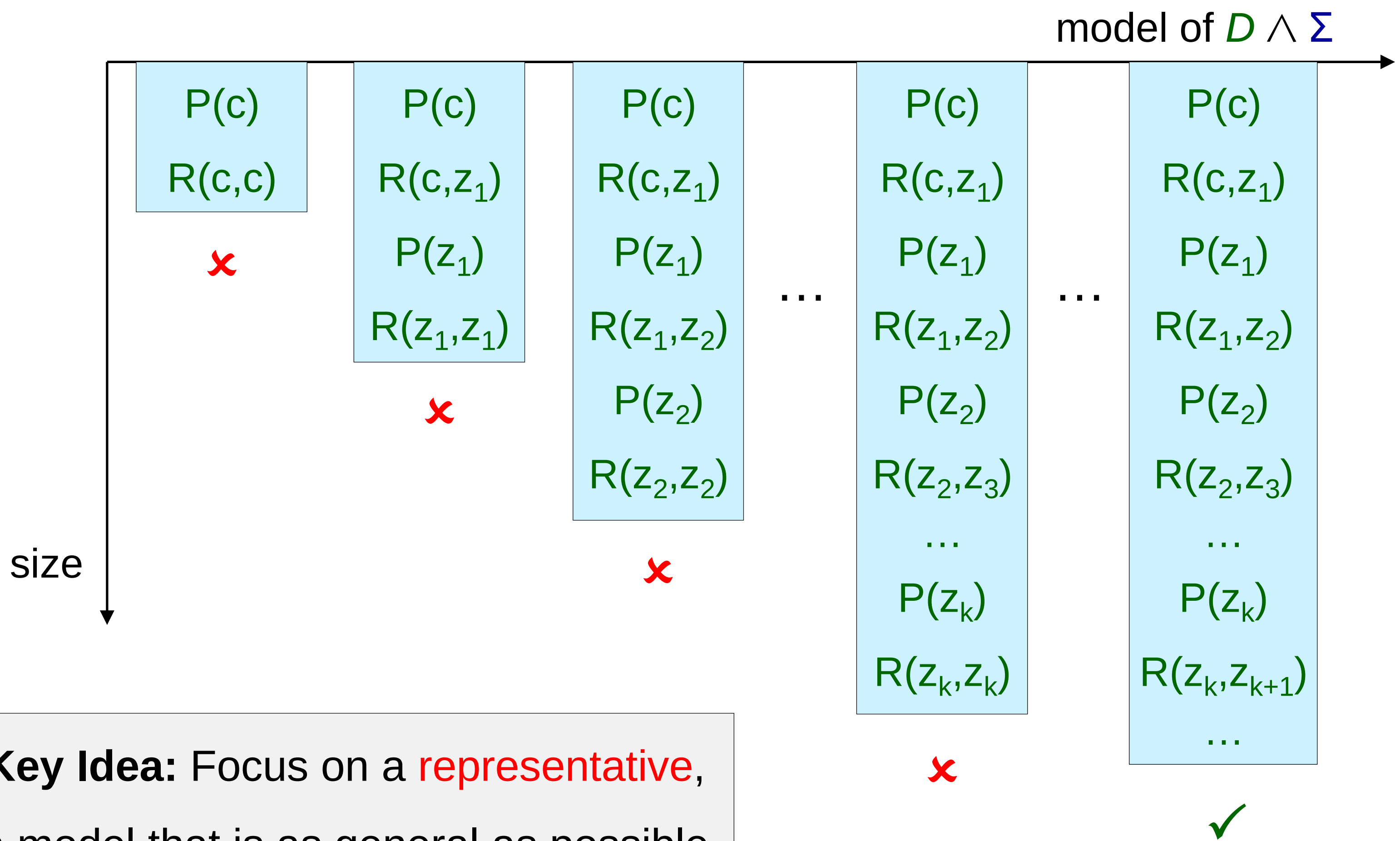
D = $\{P(c)\}$ $\Sigma = \{\forall x\, (P(x) \rightarrow \exists y\, (R(x,y) \wedge P(y)))\}$



z_1, z_2, z_3, … are nulls of **N**

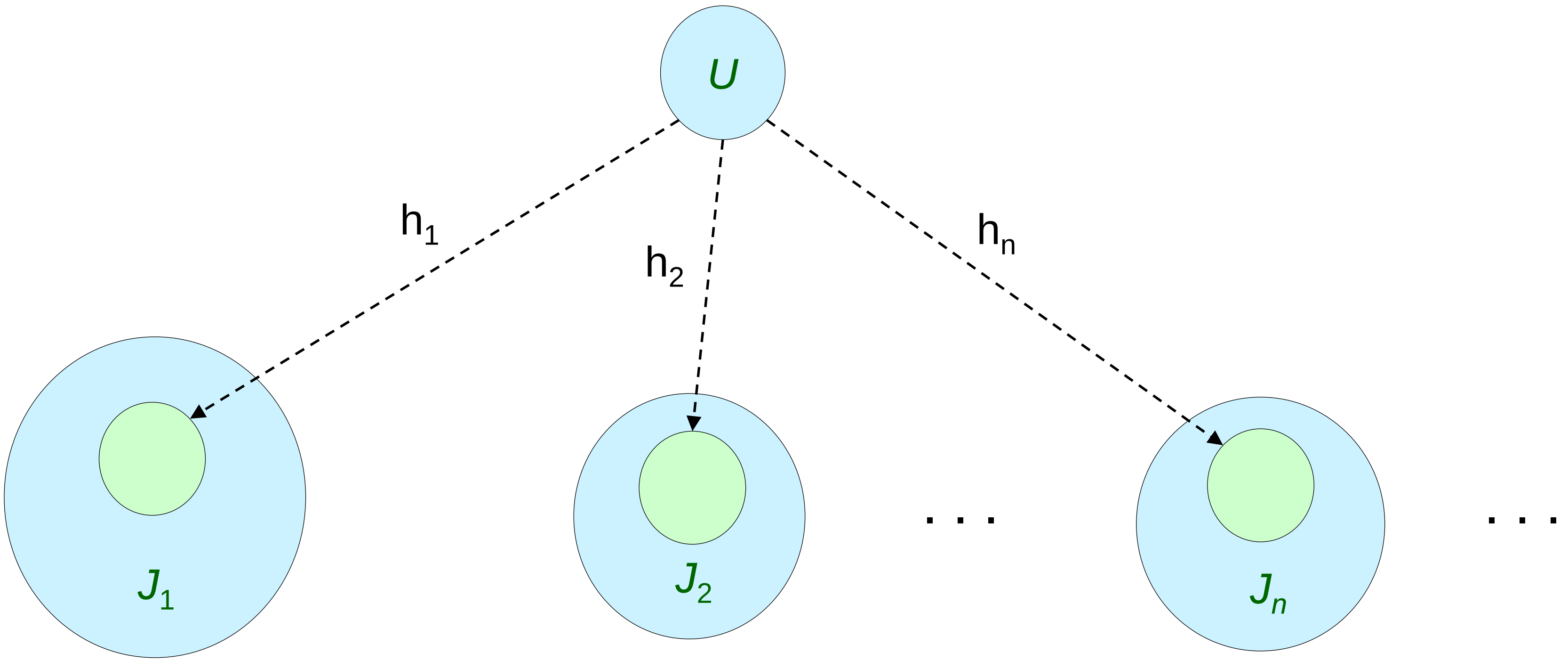
Taming the First Dimension of Infinity

D = $\{P(c)\}$ $\Sigma = \{\forall x\,(P(x) \rightarrow \exists y\,(R(x,y) \land P(y)))\}$



Key Idea: Focus on a representative, a model that is as general as possible

Universal Models (a.k.a. Canonical Models)



An instance U is a universal model of $D \land \Sigma$ if the following holds:

1. U is a model of $D \land \Sigma$

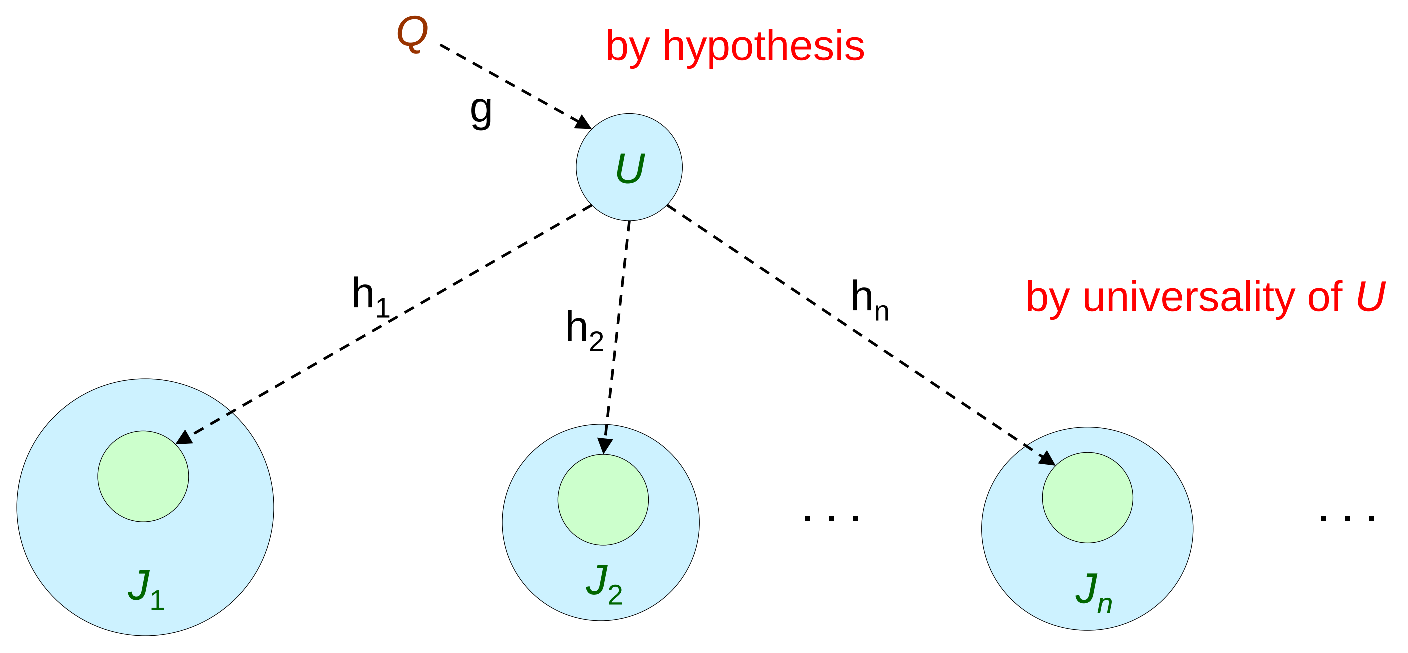
2. $\forall J \in$ models($D \land \Sigma$), there exists a homomorphism h_J such that $h_J(U) \subseteq J$

Query Answering via Universal Models

Theorem: $D \wedge \Sigma \models Q$ iff $U \models Q$, where U is a universal model of $D \wedge \Sigma$

Proof: ($\Rightarrow$) Trivial since, for every $J \in$ models($D \wedge \Sigma$), $J \models Q$

($\Leftarrow$) By exploiting the universality of U



$\forall J \in$ models($D \wedge \Sigma$), $\exists h_J$ such that $h_J(g(Q)) \subseteq J$ $\Rightarrow$ $\forall J \in$ models($D \wedge \Sigma$), $J \models Q$

$\Rightarrow$ $D \wedge \Sigma \models Q$

The Chase Procedure

- Fundamental algorithmic tool used in databases

- It has been applied to a wide range of problems:
  - Checking containment of queries under constraints
  - Computing data exchange solutions
  - Computing certain answers in data integration settings
  - …

… what's the reason for the ubiquity of the chase in databases?

it constructs universal models

The Chase Procedure

D

person(john)

Σ

$\forall x\ (\text{Person}(x) \rightarrow \exists y\ (\text{hasParent}(x,y) \wedge \text{Person}(y)))$

chase(D,Σ) = D $\cup$

The Chase Procedure

D

person(john)

Σ

$\forall x\,(\mathrm{Person}(x) \rightarrow \exists y\,(\mathrm{hasParent}(x,y) \wedge \mathrm{Person}(y)))$

$\mathrm{chase}(D,\Sigma) = D \cup \{\mathrm{hasParent}(\mathrm{john},\ z_1),\ \mathrm{Person}(z_1)$

The Chase Procedure

D

person(john)

Σ

$\forall x\,(\mathrm{Person}(x) \rightarrow \exists y\,(\mathrm{hasParent}(x,y) \wedge \mathrm{Person}(y)))$

$\mathrm{chase}(D,\Sigma) = D \cup \{\mathrm{hasParent}(\mathrm{john}, z_1), \mathrm{Person}(z_1),$

$\mathrm{hasParent}(z_1, z_2), \mathrm{Person}(z_2)$

The Chase Procedure

D

person(john)

Σ

$\forall x\, (Person(x) \rightarrow \exists y\, (hasParent(x,y) \wedge Person(y)))$

$chase(D,\Sigma) = D \cup \{hasParent(john, z_1), Person(z_1),$

$hasParent(z_1, z_2), Person(z_2),$

$hasParent(z_2, z_3), Person(z_3)$

The Chase Procedure

D

person(john)

Σ

$\forall x\ (\mathrm{Person}(x) \rightarrow \exists y\ (\mathrm{hasParent}(x,y) \wedge \mathrm{Person}(y)))$

$\mathrm{chase}(D,\Sigma) = D \cup \{\mathrm{hasParent}(\mathrm{john}, z_1), \mathrm{Person}(z_1),$

$\mathrm{hasParent}(z_1, z_2), \mathrm{Person}(z_2),$

$\mathrm{hasParent}(z_2, z_3), \mathrm{Person}(z_3), \ldots$

infinite instance

The Chase Procedure: Formal Definition

- Chase rule - the building block of the chase procedure

- A rule $\sigma = \forall \mathbf{x} \forall \mathbf{y}\, (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\, \psi(\mathbf{x},\mathbf{z}))$ is applicable to instance J if:

  1. There exists a homomorphism h such that $h(\varphi(\mathbf{x},\mathbf{y})) \subseteq J$
  2. There is no $g \supseteq h_{|\mathbf{x}}$ such that $g(\psi(\mathbf{x},\mathbf{z})) \subseteq J$

J = {R(a), P(a,b)}

h = {x→ a}

g = {x→ a, y→ b}

$\forall x\, (R(x) \rightarrow \exists y\, P(x,y))$

✗

J = {R(a), P(b,a)}

h = {x→ a}

×

$\forall x\, (R(x) \rightarrow \exists y\, P(x,y))$

✓

The Chase Procedure: Formal Definition

- Chase rule - the building block of the chase procedure

- A rule $\sigma = \forall \mathbf{x} \forall \mathbf{y}\ (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\ \psi(\mathbf{x},\mathbf{z}))$ is applicable to instance J if:
  1. There exists a homomorphism h such that $h(\varphi(\mathbf{x},\mathbf{y})) \subseteq J$
  2. There is no $g \supseteq h_{|\mathbf{x}}$ such that $g(\psi(\mathbf{x},\mathbf{z})) \subseteq J$

- Let $J_+ = J \cup \{g(\psi(\mathbf{x},\mathbf{z}))\}$, where $g \supseteq h_{|\mathbf{z}}$ and $g(\mathbf{z})$ are "fresh" nulls not in J

- The result of applying σ to J is J_+, denoted $J\langle\sigma,h\rangle J_+$ - single chase step

The Chase Procedure: Formal Definition

- A finite chase of D w.r.t. Σ is a finite sequence

$$D\langle\sigma_1,h_1\rangle J_1\langle\sigma_2,h_2\rangle J_2\langle\sigma_3,h_3\rangle J_3 \;\ldots\; \langle\sigma_n,h_n\rangle J_n$$

and chase(D,Σ) is defined as the instance J_n

all applicable rules will eventually be applied

- An infinite chase of D w.r.t. Σ is a fair finite sequence

$$D\langle\sigma_1,h_1\rangle J_1\langle\sigma_2,h_2\rangle J_2\langle\sigma_3,h_3\rangle J_3 \;\ldots\; \langle\sigma_n,h_n\rangle J_n \ldots$$

and chase(D,Σ) is defined as the instance $\cup_{k \geq 0} J_k$ (with $J_0 = D$)

least fixpoint of a monotonic operator - chase step

Chase: A Universal Model

Theorem: chase(D,Σ) is a universal model of $D \wedge \Sigma$

the result of the chase after k applications of the chase step

Proof:

- By construction, chase(D,Σ) $\in$ models($D \wedge \Sigma$)
- It remains to show that chase(D, Σ) can be homomorphically embedded into every other model of $D \wedge \Sigma$
- Fix an arbitrary instance $J \in$ models($D \wedge \Sigma$). We need to show that there exists h such that h(chase(D,Σ)) $\subseteq J$
- **By induction on the number of applications of the chase step, we show that for every $k \geq 0$, there exists h_k such that $h_k(\text{chase}^{[k]}(D,\Sigma)) \subseteq J$, and h_k is compatible with h_{k-1}**
- Clearly, $\cup_{k \geq 0} h_k$ is a well-defined homomorphism that maps chase(D,Σ) to J
- The claim follows with $h = \cup_{k \geq 0} h_k$

Chase: Uniqueness Property

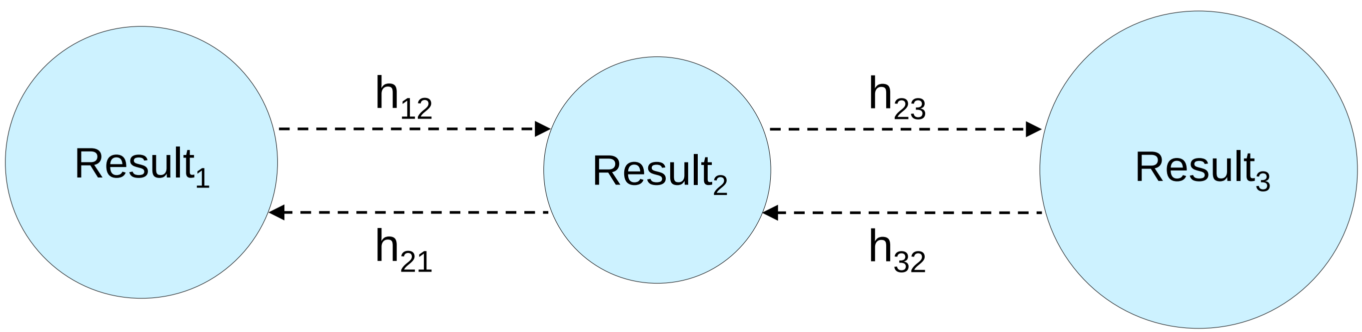
- The result of the chase is not unique - depends on the order of rule application

$D = \{P(a)\}$ $\sigma_1 = \forall x\,(P(x) \rightarrow \exists y\, R(y))$ $\sigma_2 = \forall x\,(P(x) \rightarrow R(x))$

$\text{Result}_1 = \{P(a), R(z), R(a)\}$ σ_1 then σ_2

$\text{Result}_2 = \{P(a), R(a)\}$ σ_2 then σ_1

- But, it is unique up to homomorphic equivalence



- Thus, it is unique for query answering purposes

Query Answering via the Chase

Theorem: $D \wedge \Sigma \models Q$ iff $U \models Q$, where U is a universal model of $D \wedge \Sigma$

&

Theorem: chase(D, Σ) is a universal model of $D \wedge \Sigma$

$\Downarrow$

Corollary: $D \wedge \Sigma \models Q$ iff chase(D,Σ) $\models Q$

- We can tame the first dimension of infinity by exploiting the chase procedure
- What about the second dimension of infinity? - the chase may be infinite

Can we tame the second dimension of infinity?

Undecidability of OBQA

arbitrary existential rules

Theorem: OBQA(∃**RULES**) is undecidable

Proof Idea : By simulating a deterministic Turing machine with an empty tape

Deterministic Turing Machine (DTM)

$S \setminus \{s_{acc}\} \times \Lambda \rightarrow S \times \Lambda \times \{-1,0,+1\}$

accepting state

$$M \;=\; (S, \Lambda, \sqcup, \delta, s_0, s_{acc})$$

states

tape symbols

blank symbol

initial state

$\delta(s_1, \alpha) = (s_2, \beta, +1)$

IF at some time instant τ the machine is in sate s_1, the cursor points to cell κ, and this cell contains α

THEN at instant τ+1 the machine is in state s_2, cell κ contains β, and the cursor points to cell κ+1

Undecidability of OBQA

arbitrary existential rules

Theorem: OBQA(∃**RULES**) is undecidable

Proof Idea : By simulating a deterministic Turing machine with an empty tape.

Encode the computation of a DTM M with an empty tape using a database D,

a set Σ of existential rules, and a BCQ Q such that $D \wedge \Sigma \models Q$ iff M accepts

How we ensure decidability of OBQA?

Gaining Decidability

By restricting the database

- {Start(c)} $\wedge\ \Sigma \models Q$ iff the DTM M accepts
- The problem is undecidable already for singleton databases
- No much to do in this direction

By restricting the query language

- $D \wedge \Sigma \models Q$:- Accept(x) iff the DTM M accepts
- The problem is undecidable already for atomic queries
- No much to do in this direction

By restricting the ontology language

- Achieve a good trade-off between expressive power and complexity
- Field of intense research
- Any ideas?

What is the Source of Non-termination?

D

person(john)

Σ

$\forall x\ (Person(x) \rightarrow \exists y\ (hasParent(x,y) \wedge Person(y)))$

$chase(D,\Sigma) = D \cup \{hasParent(john, z_1), Person(z_1),$

$hasParent(z_1, z_2), Person(z_2),$

$hasParent(z_2, z_3), Person(z_3), \ldots$

1. **Existential quantification**
2. **Recursive definitions**

Termination of the Chase

- Drop the existential quantification
  - We obtain the class of **full** existential rules
  - Very close to Datalog

- Drop the recursive definitions
  - We obtain the class of **acyclic** existential rules
  - A.k.a. non-recursive existential rules

Full Existential Rules

- A full existential rule is an existential rule of the form

$$\forall \mathbf{x} \forall \mathbf{y}\ (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \psi(\mathbf{x}))$$

- We denote **FULL** the class of full existential rules

- A local property - we can inspect one rule at a time

  $\Rightarrow$ given Σ, we can decide in linear time whether $\Sigma \in$ **FULL**

  $\Rightarrow$ closed under union - $\Sigma_1 \in$ **FULL**, $\Sigma_2 \in$ **FULL** $\Rightarrow (\Sigma_1 \cup \Sigma_2) \in$ **FULL**

- But, is this a reasonable ontology language?

FULL and OWL 2 RL

- The acronym RL reflects its relation to rules
- **FULL** captures OWL 2 RL

$$\text{Parent} \sqcap \text{Male} \sqsubseteq \text{Father}$$

$$\forall x\ (\text{Parent}(x) \land \text{Male}(x) \rightarrow \text{Father}(x))$$

$$\exists \text{parentOf}.\exists \text{parentOf}.\text{T} \sqsubseteq \text{Grandfather}$$

$$\forall x \forall y\ (\text{parentOf}(x,y) \land \text{parentOf}(y,z) \rightarrow \text{Grandfather}(x))$$

$$\text{MetalDevice} \sqsubseteq \forall \text{hasPart}.\text{Metal}$$

$$\forall x \forall y\ (\text{MetalDevice}(x) \land \text{hasPart}(x,y) \rightarrow \text{Metal}(y))$$

FULL and OWL 2 RL

- The acronym RL reflects its relation to rules
- **FULL** captures OWL 2 RL

$\text{childOf} \circ \text{childOf} \sqsubseteq \text{grandchildOf}$

$\forall x \forall y \forall z\ (\text{childOf}(x,y) \land \text{childOf}(y,z) \rightarrow \text{grandchildOf}(x,z))$

$\text{Person} \sqsubseteq \exists_{\leq 1}\, \text{hasPassport.Valid}$

$\forall x \forall y \forall z\ (\text{Person}(x) \land \text{hasPassport}(x,y) \land \text{Valid}(y) \land$

$\text{hasPassport}(x,z) \land \text{Valid}(z) \rightarrow y = z)$

Disj(childOf, parentOf)

$\forall x \forall y\ (\text{childOf}(x,y) \land \text{parentOf}(x,y) \rightarrow \bot)$

Full Existential Rules

- A full existential rule is an existential rule of the form

$$\forall \mathbf{X} \forall \mathbf{Y} \, (\varphi(\mathbf{X},\mathbf{Y}) \rightarrow \psi(\mathbf{X}))$$

- We denote **FULL** the class of full existential rules

- A local property - we can inspect one rule at a time

  $\Rightarrow$ given Σ, we can decide in linear time whether $\Sigma \in$ **FULL**

  $\Rightarrow$ closed under union - $\Sigma_1 \in \mathbf{FULL}$, $\Sigma_2 \in \mathbf{FULL} \Rightarrow (\Sigma_1 \cup \Sigma_2) \in \mathbf{FULL}$

- But, is this a reasonable ontology language? **OWL 2 RL**

Full Existential Rules

- Consider a database D and a set $\Sigma \in$ **FULL**

- $\text{chase}(D,\Sigma) \subseteq \{P(c_1,\ldots,c_n) \mid (c_1,\ldots,c_n) \in \text{adom}(D)^n \text{ and } P \in \text{sch}(\Sigma)\}$

active domain - constants occurring in D

schema - predicates occurring in Σ

- $$|\text{chase}(D,\Sigma)| \leq \underbrace{|\text{sch}(\Sigma)| \cdot \overbrace{(|\text{adom}(D)|)^{\text{maxarity}}}^{\text{maximum number of tuples with terms of adom}(D)}}_{\text{maximum number of atoms with predicates of sch}(\Sigma)\text{ and terms of adom}(D)}$$

maxarity $= \max_{P \in \text{sch}(\Sigma)} \{\text{arity}(P)\}$

Complexity Measures for OBQA

OBQA(**L**)

Input: database D, existential rules $\Sigma \in \mathbf{L}$, CQ $Q(\mathbf{x})$, tuple $\mathbf{t} \in \mathrm{adom}(D)^{|\mathbf{X}|}$

Question: $\mathbf{t} \in \text{certain-answers}(Q,\langle D,\Sigma\rangle) = \bigcap_{J \in \text{models}(D \wedge \Sigma)} Q(J)$?

- Data complexity: is calculated by considering only the database as part of the input, while the ontology and the query are fixed - $\text{OBQA}_{\Sigma,Q}(\mathbf{L})$

- Combined complexity: is calculated by considering, apart from the database, also the ontology and the query as part of the input

Data Complexity of **FULL**

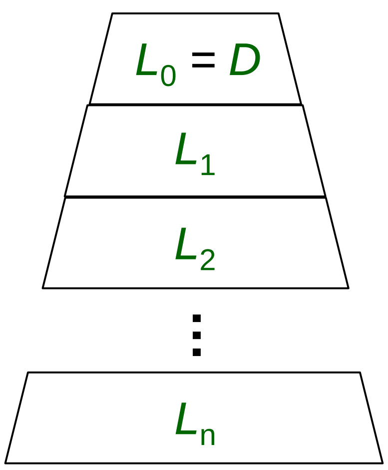
Theorem: $\text{OBQA}_{\Sigma,Q}$(**FULL**) is in PTIME

Proof: Consider a database D, a set $\Sigma \in$ **FULL**, and a (Boolean) CQ Q

We apply the naïve algorithm:

1. Construct chase(D,Σ)
2. Check for the existence of a homomorphism h such that h(Q) $\subseteq$ chase(D,Σ)

Step 1: We construct the chase level-by-level



- **From L_k to L_{k+1}**: for each $\sigma \in \Sigma$, find all the homomorphisms h such that h(body(σ)) $\subseteq L_k$, and add to L_k the set of atoms h(head(σ))
- **Stop** when $L_k = L_{k+1}$

$$|\Sigma| \cdot (|\text{adom}(D)|)^{\text{maxvariables}(\Sigma)} \cdot \text{maxbody}(\Sigma) \cdot |L_k|$$

Data Complexity of **FULL**

Theorem: $\mathrm{OBQA}_{\Sigma,Q}$(**FULL**) is in PTIME

Proof: Consider a database D, a set $\Sigma \in$ **FULL**, and a (Boolean) CQ Q

We apply the naïve algorithm:

1. Construct chase(D,Σ)
2. Check for the existence of a homomorphism h such that $h(Q) \subseteq \mathrm{chase}(D,\Sigma)$

Step 1: We construct the chase level-by-level in time

$$(k-1) \cdot |\Sigma| \cdot (|\mathrm{adom}(D)|)^{\mathrm{maxvariables}(\Sigma)} \cdot \mathrm{maxbody}(\Sigma) \cdot |L|$$

$$\text{where} \quad k, |L| \leq |\mathrm{chase}(D,\Sigma)| \leq |\mathrm{sch}(\Sigma)| \cdot (|\mathrm{adom}(D)|)^{\mathrm{maxarity}}$$

Data Complexity of **FULL**

Theorem: $\text{OBQA}_{\Sigma,Q}$(**FULL**) is in PTIME

Proof: Consider a database D, a set $\Sigma \in$ **FULL**, and a (Boolean) CQ Q

We apply the naïve algorithm:

1. Construct chase(D,Σ)
2. Check for the existence of a homomorphism h such that h(Q) $\subseteq$ chase(D,Σ)

Step 2: By applying similar analysis, we can show that the existence of h can be checked in time

$$(|\text{adom}(D)|)^{\#\text{variables}(Q)} \cdot |Q| \cdot |\text{chase}(D,\Sigma)|$$

$$\text{where} \quad |\text{chase}(D,\Sigma)| \leq |\text{sch}(\Sigma)| \cdot (|\text{adom}(D)|)^{\text{maxarity}}$$

Data Complexity of **FULL**

Theorem: $\mathrm{OBQA}_{\Sigma,Q}$(**FULL**) is in PTIME

Proof: Consider a database D, a set $\Sigma \in$ **FULL**, and a (Boolean) CQ Q

We apply the naïve algorithm:

1. Construct chase(D,Σ)
2. Check for the existence of a homomorphism h such that $h(Q) \subseteq \mathrm{chase}(D,\Sigma)$

Consequently, in the worst case, the naïve algorithm runs in time

$$(|\mathrm{sch}(\Sigma)| \cdot (|\mathrm{adom}(D)|)^{\mathrm{maxarity}})^2 \cdot |\Sigma| \cdot (|\mathrm{adom}(D)|)^{\mathrm{maxvariables}(\Sigma)} \cdot \mathrm{maxbody}(\Sigma)$$

$$+$$

$$(|\mathrm{adom}(D)|)^{\#\mathrm{variables}(Q)} \cdot |Q| \cdot |\mathrm{sch}(\Sigma)| \cdot (|\mathrm{adom}(D)|)^{\mathrm{maxarity}}$$

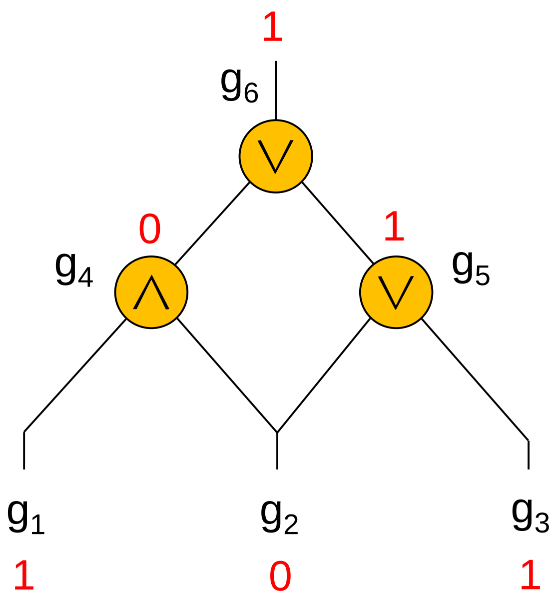
Data Complexity of **FULL**

We cannot do better than the naïve algorithm

Theorem: $\mathrm{OBQA}_{\Sigma,Q}$(**FULL**) is PTIME-hard

Proof : By a LOGSPACE reduction from Monotone Circuit Value problem

Data Complexity of **FULL**



Does the circuit evaluate to *true*?

encoding of the circuit as a database *D*

$T(g_1) \quad T(g_3)$

$AND(g_4,g_1,g_2) \quad OR(g_5,g_2,g_3) \quad OR(g_6,g_4,g_5)$

evaluation of the circuit via a *fixed* set Σ

$\forall x \forall y \forall z\ (T(x) \wedge OR(z,x,y) \rightarrow T(z))$

$\forall x \forall y \forall z\ (T(y) \wedge OR(z,x,y) \rightarrow T(z))$

$\forall x \forall y \forall z\ (T(x) \wedge T(y) \wedge AND(z,x,y) \rightarrow T(z))$

Circuit evaluates to *true* iff $D \wedge \Sigma \vDash T(g_6)$

Combined Complexity of FULL

Theorem: OBQA(**FULL**) is in EXPTIME

Proof: Consider a database D, a set $\Sigma \in$ **FULL**, and a BCQ Q

We apply the naïve algorithm:

1. Construct chase(D,Σ)
2. Check for the existence of a homomorphism h such that h(Q) $\subseteq$ chase(D,Σ)

Consequently, in the worst case, the naïve algorithm runs in time

$$(|sch(\Sigma)| \cdot (|adom(D)|)^{maxarity})^2 \cdot |\Sigma| \cdot (|adom(D)|)^{maxvariables(\Sigma)} \cdot maxbody(\Sigma)$$

$$+$$

$$(|adom(D)|)^{\#variables(Q)} \cdot |Q| \cdot |sch(\Sigma)| \cdot (|adom(D)|)^{maxarity}$$

Combined Complexity of **FULL**

We cannot do better than the naïve algorithm

Theorem: OBQA(**FULL**) is in EXPTIME-hard

Proof : By simulating a deterministic exponential time Turing machine

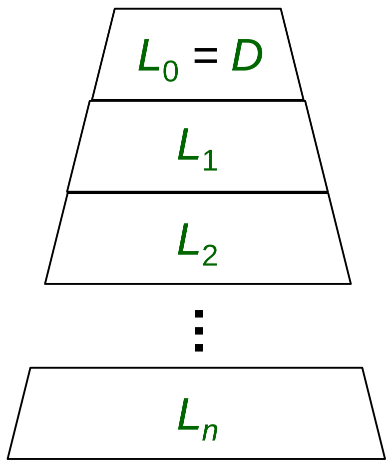
Termination of the Chase

- Drop the existential quantification
  - We obtain the class of **full** existential rules
  - Very close to Datalog ✓

- Drop the recursive definitions
  - We obtain the class of **acyclic** existential rules
  - A.k.a. non-recursive existential rules

…the naïve algorithm is not clever enough

The Naïve Algorithm for **ACYCLIC**



$|L_0| = 2$

$|L_1| = (|L_0|)^2$

		L_1
0	0	z_{00}
0	1	z_{01}
1	0	z_{10}
1	1	z_{11}

$D = \{P_0(0), P_0(1)\}$

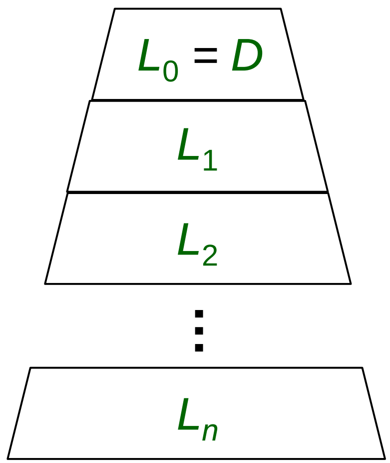
$$\Sigma = \{\forall x \forall y\ (P_0(x) \wedge P_0(y) \rightarrow \exists z\ (S_1(x,y,z) \wedge P_1(z)))$$

$$\forall x \forall y\ (P_1(x) \wedge P_1(y) \rightarrow \exists z\ (S_2(x,y,z) \wedge P_2(z)))$$

$$\dots$$

$$\forall x \forall y\ (P_{n-1}(x) \wedge P_{n-1}(y) \rightarrow \exists z\ (S_n(x,y,z) \wedge P_n(z)))\}$$

The Naïve Algorithm for **ACYCLIC**



$|L_0| = 2$

$|L_1| = (|L_0|)^2$

$|L_2| = (|L_1|)^2$

$D = \{P_0(0), P_0(1)\}$

$\Sigma = \{\forall x \forall y\ (P_0(x) \land P_0(y) \rightarrow \exists z\ (S_1(x,y,z) \land P_1(z)))$

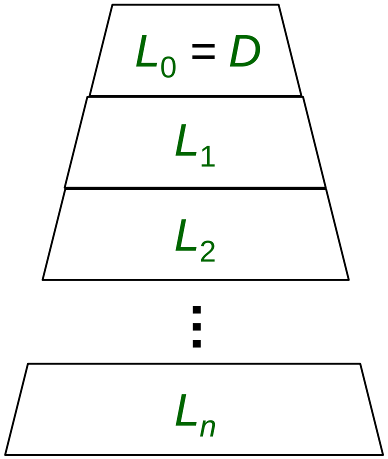
$\forall x \forall y\ (P_1(x) \land P_1(y) \rightarrow \exists z\ (S_2(x,y,z) \land P_2(z)))$

$\dots$

$\forall x \forall y\ (P_{n-1}(x) \land P_{n-1}(y) \rightarrow \exists z\ (S_n(x,y,z) \land P_n(z)))\}$

		L_2
z_{00}	z_{00}	z_{0000}
z_{00}	z_{01}	z_{0001}
z_{00}	z_{10}	z_{0010}
z_{00}	z_{11}	z_{0011}
z_{01}	z_{00}	z_{0100}
z_{01}	z_{01}	z_{0101}
z_{01}	z_{10}	z_{0110}
z_{01}	z_{11}	z_{0111}
z_{10}	z_{00}	z_{1000}
z_{10}	z_{01}	z_{1001}
z_{10}	z_{10}	z_{1010}
z_{10}	z_{11}	z_{1011}
z_{11}	z_{00}	z_{1100}
z_{11}	z_{01}	z_{1101}
z_{11}	z_{10}	z_{1110}
z_{11}	z_{11}	z_{1111}

The Naïve Algorithm for **ACYCLIC**



$|L_0| = 2$

$|L_1| = (|L_0|)^2$

$|L_2| = (|L_1|)^2$

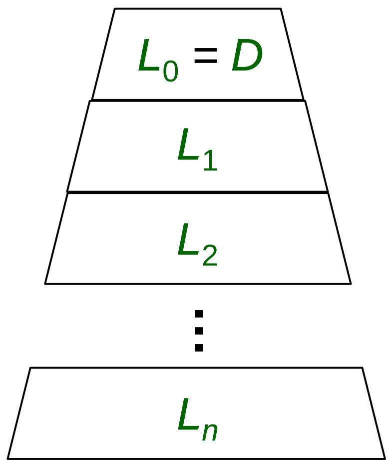
$|L_n| = (|L_{n-1}|)^2$

		L_n
$z_{\mathbf{0...0}}$	$z_{\mathbf{0...0}}$	$z_{0...00...0}$
…	…	…
$z_{1...1}$	$z_{1...1}$	$z_{1...11...1}$

$D = \{P_0(0), P_0(1)\}$

$$
\begin{aligned}
\Sigma = \{ & \forall x \forall y\, (P_0(x) \wedge P_0(y) \rightarrow \exists z\, (S_1(x,y,z) \wedge P_1(z))) \\
& \forall x \forall y\, (P_1(x) \wedge P_1(y) \rightarrow \exists z\, (S_2(x,y,z) \wedge P_2(z))) \\
& \qquad \ldots \\
& \forall x \forall y\, (P_{n-1}(x) \wedge P_{n-1}(y) \rightarrow \exists z\, (S_n(x,y,z) \wedge P_n(z)))\}
\end{aligned}
$$

The Naïve Algorithm for **ACYCLIC**



$|L_0| = 2$

$|L_1| = (|L_0|)^2$

$|L_2| = (|L_1|)^2$

$|L_n| = (|L_{n-1}|)^2$

$$\mathbf{|L_n| = 2^{(2\wedge n)}}$$

$D = \{P_0(0), P_0(1)\}$

$\Sigma = \{\forall x \forall y\ (P_0(x) \wedge P_0(y) \rightarrow \exists z\ (S_1(x,y,z) \wedge P_1(z)))$

$\forall x \forall y\ (P_1(x) \wedge P_1(y) \rightarrow \exists z\ (S_2(x,y,z) \wedge P_2(z)))$

…

$\forall x \forall y\ (P_{n-1}(x) \wedge P_{n-1}(y) \rightarrow \exists z\ (S_n(x,y,z) \wedge P_n(z)))\}$

Complexity of **ACYCLIC**

- The naïve algorithm shows OBQA(**ACYCLIC**) is
  - in PTIME w.r.t. the data complexity
  - in 2EXPTIME w.r.t. the combined complexity

…however, we can do better than the naïve algorithm

Theorem: It holds that

- $\mathrm{OBQA}_{\Sigma,Q}$(**FULL**) is in LOGSPACE (data complexity)
- OBQA(**FULL**) is NEXPTIME-complete (combined complexity)

Our Simple Example

D

person(john)

Σ

$\forall x\ (Person(x) \rightarrow \exists y\ (hasParent(x,y) \wedge Person(y)))$

$chase(D,\Sigma) = D \cup \{hasParent(john, z_1), Person(z_1),$

$hasParent(z_1, z_2), Person(z_2),$

$hasParent(z_2, z_3), Person(z_3), \ldots$

Existential quantification & recursive definitions

are key features for modelling ontologies

Research Challenge

We need classes of existential rules such that

- Existential quantification and recursive definition coexist

  $\Rightarrow$ the chase may be infinite

- OBQA is decidable, and tractable w.r.t. the data complexity

$$\Downarrow$$

Tame the infinite chase:

Deal with infinite structures without explicitly building them

Linear Existential Rules

- A linear existential rule is an existential rule of the form

$$\forall \mathbf{x} \forall \mathbf{y}\, (P(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\, \psi(\mathbf{x},\mathbf{z}))$$

single atom

- We denote **LINEAR** the class of linear existential rules

- A local property - we can inspect one rule at a time

  $\Rightarrow$ given Σ, we can decide in linear time whether $\Sigma \in$ **LINEAR**

  $\Rightarrow$ closed under union

- But, is this a reasonable ontology language?

LINEAR vs. DL-Lite

DL-Lite: Popular family of DLs - at the basis of the OWL 2 QL profile of OWL 2

DL-Lite Axioms	First-order Representation
$A \sqsubseteq B$	$\forall x\,(A(x) \rightarrow B(x))$
$A \sqsubseteq \exists R$	$\forall x\,(A(x) \rightarrow \exists y\, R(x,y))$
$\exists R \sqsubseteq A$	$\forall x \forall y\,(R(x,y) \rightarrow A(x))$
$\exists R \sqsubseteq \exists P$	$\forall x \forall y\,(R(x,y) \rightarrow \exists z\, P(x,z))$
$A \sqsubseteq \exists R.B$	$\forall x\,(A(x) \rightarrow \exists y\,(R(x,y) \wedge B(y)))$
$R \sqsubseteq P$	$\forall x \forall y\,(R(x,y) \rightarrow P(x,y))$
$A \sqsubseteq \neg B$	$\forall x\,(A(x) \wedge B(x) \rightarrow \perp)$

Linear Existential Rules

- A linear existential rule is an existential rule of the form

$$\forall \mathbf{X} \forall \mathbf{Y} \, (P(\mathbf{X},\mathbf{Y}) \rightarrow \exists \mathbf{Z} \, \psi(\mathbf{X},\mathbf{Z}))$$

single atom

- We denote **LINEAR** the class of linear existential rules

- A local property - we can inspect one rule at a time

  $\Rightarrow$ given Σ, we can decide in linear time whether $\Sigma \in$ **LINEAR**

  $\Rightarrow$ closed under union

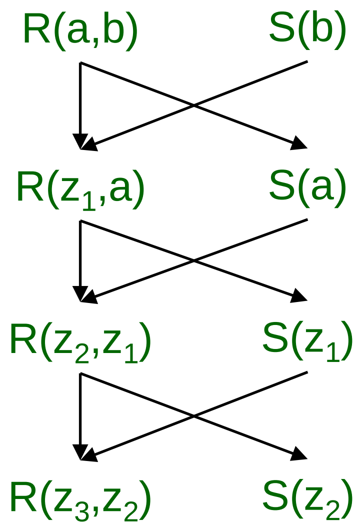
- But, is this a reasonable ontology language? **OWL 2 QL**

Chase Graph

The chase can be naturally seen as a graph - chase graph

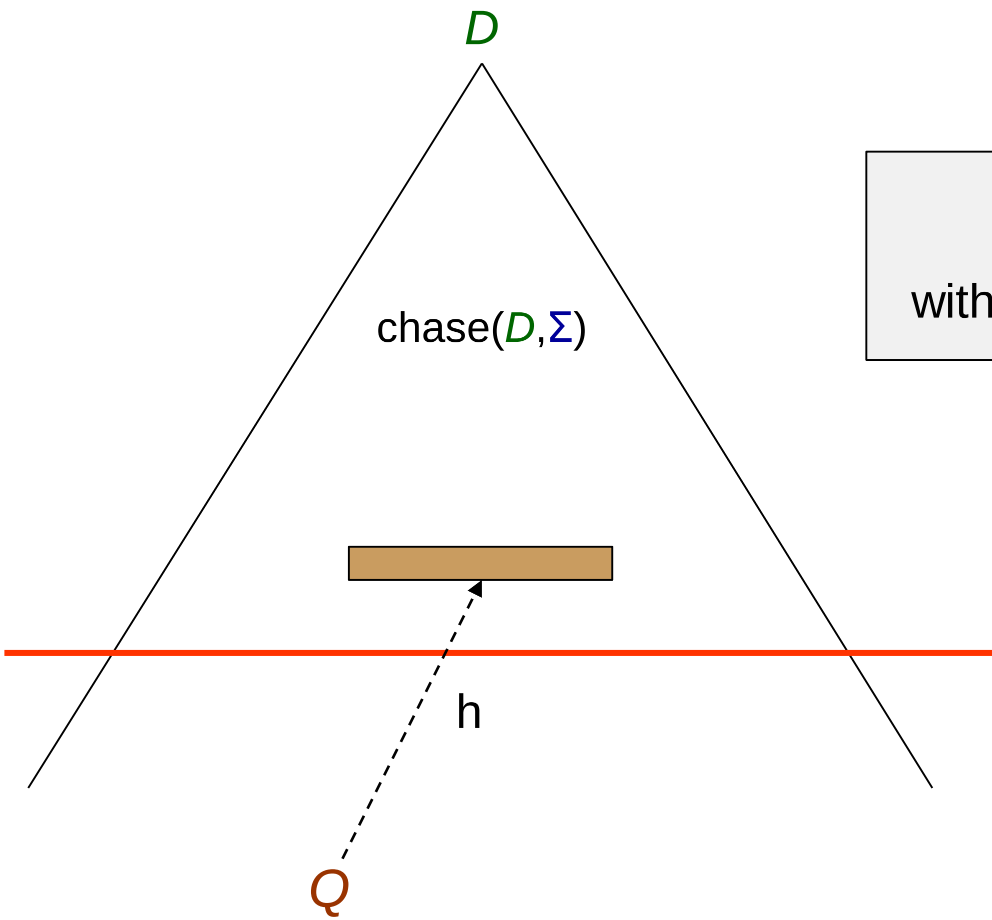
D = {R(a,b), S(b)}

$$\Sigma = \begin{cases} \forall x \forall y\ (R(x,y) \wedge S(y) \rightarrow \exists z\ R(z,x)) \\ \forall x \forall y\ (R(x,y) \rightarrow S(x)) \end{cases}$$



For **LINEAR** the chase graph is a forest

Bounded Derivation-Depth Property



depth k that does not depend on D

For **LINEAR**, $k = |Q| \cdot m$

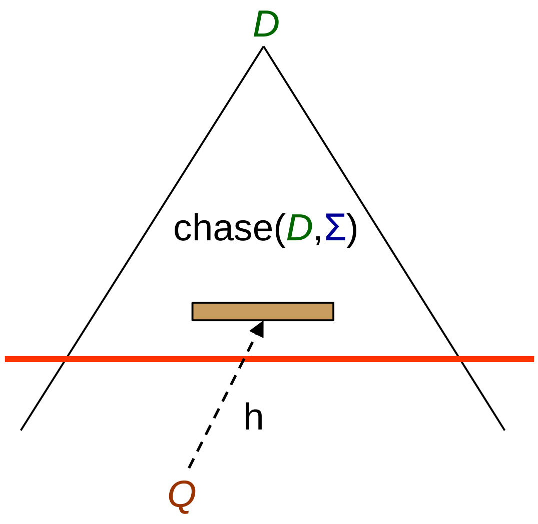
with $m = |sch(\Sigma)| \cdot (2 \cdot \text{maxarity})^{\text{maxarity}}$

chase graph up to depth k

$$\text{chase}(D,\Sigma) \models Q \quad \Rightarrow \quad \text{chase}^k(D,\Sigma) \models Q$$

The Blocking Algorithm for **LINEAR**

- The blocking algorithm shows that OBQA(**LINEAR**) is
  - in PTIME w.r.t. the data complexity
  - in 2EXPTIME w.r.t. the combined complexity



$k = |Q| \cdot |sch(\Sigma)| \cdot (2 \cdot \text{maxarity})^{\text{maxarity}}$

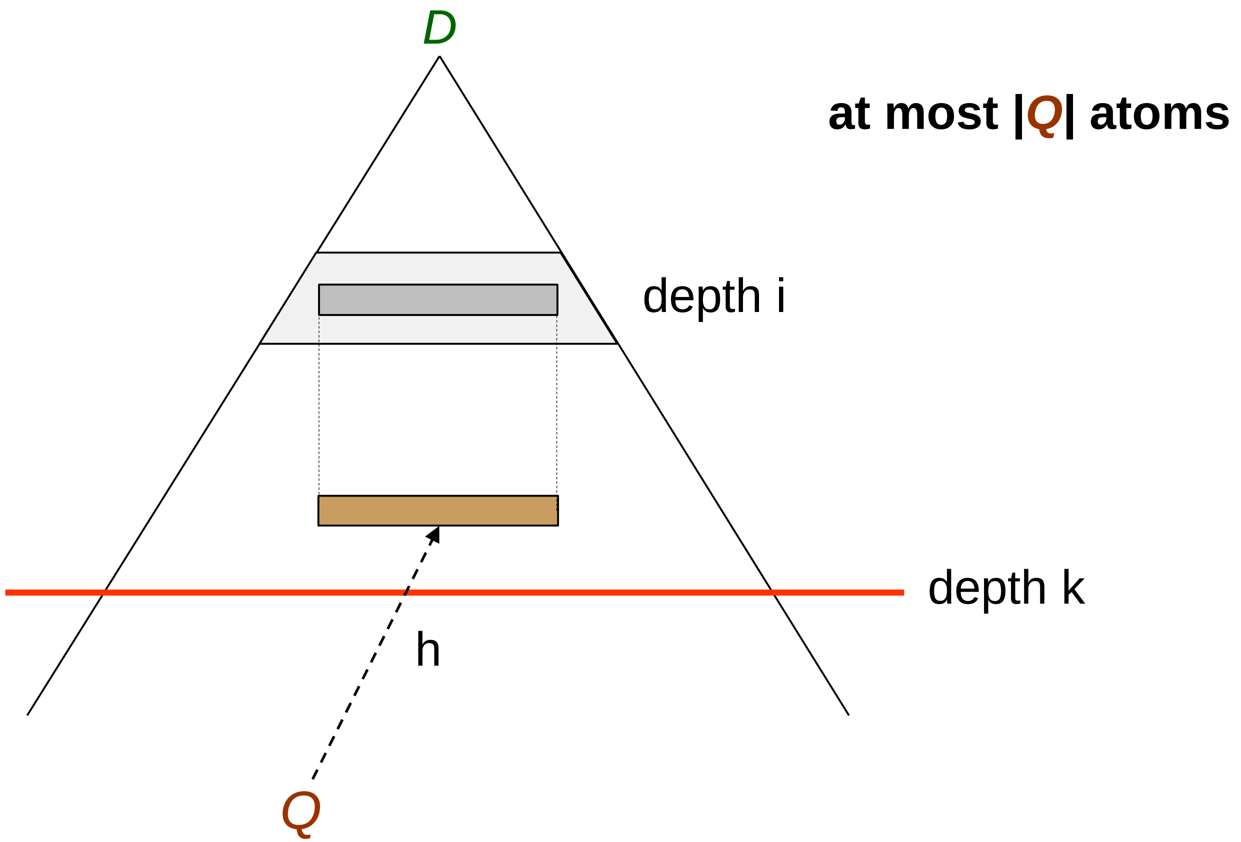
Complexity of **LINEAR**

…but, we can do better than the blocking algorithm

Theorem: It holds that

- $\mathrm{OBQA}_{\Sigma,Q}$(**LINEAR**) is in LOGSPACE (data complexity)
- OBQA(**LINEAR**) is PSPACE-complete (combined complexity)

Key Observation

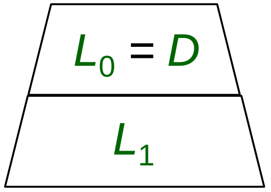


non-deterministic, level-by-level construction

Combined Complexity of **LINEAR**

Theorem: OBQA(**LINEAR**) is in PSPACE

Proof Idea:



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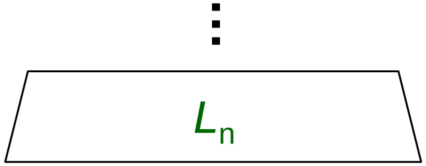


Combined Complexity of LINEAR

Theorem: OBQA(**LINEAR**) is in PSPACE

Proof Idea:

- At each step we need to maintain
  - $O(|Q|)$ atoms
  - A counter ctr $\leq |Q|^2 \cdot |\mathrm{sch}(\Sigma)| \cdot (2 \cdot \mathrm{maxarity})^{\mathrm{maxarity}}$
- Thus, we need polynomial space
- The claim follows since NPSPACE = PSPACE

Combined Complexity of **LINEAR**

We cannot do better than the previous algorithm

Theorem: OBQA(**LINEAR**) is PSPACE-hard

Proof : By simulating a deterministic polynomial space Turing machine

PSPACE-hardness of **LINEAR**

Our Goal: Encode the polynomial space computation of a DTM *M* on input string *I*

using a database *D*, a set $\Sigma \in$ **LINEAR**, and a (Boolean) CQ *Q* such that

$D \wedge \Sigma \models Q$ iff *M* accepts *I* using at most n = $|I|^k$ cells

PSPACE-hardness of **LINEAR**

- Assume that the tape alphabet is $\{0,1,\sqcup\}$
- Suppose that *M* halts on $I = \alpha_1 \ldots \alpha_m$ using $n = m^k$ cells, for $k > 0$

Initial configuration - the database *D*

$$\mathit{Config}(s_{init},\alpha_1,\ldots,\alpha_m,\underbrace{\sqcup,\ldots,\sqcup}_{n-m},1,\underbrace{0,\ldots,0}_{n-1})$$

PSPACE-hardness of **LINEAR**

- Assume that the tape alphabet is $\{0,1,\sqcup\}$
- Suppose that *M* halts on $I = \alpha_1 \ldots \alpha_m$ using $n = m^k$ cells, for $k > 0$

Transition rule - $\delta(s_1,\alpha) = (s_2,\beta,+1)$

for each $i \in \{1,\ldots,n\}$:

$$\forall \mathbf{x}\ (\mathrm{Config}(s_1,x_1,\ldots,x_{i-1},\alpha,x_{i+1},\ldots,x_n,\overbrace{0,\ldots,0}^{i-1},1,\overbrace{0,\ldots,0}^{n-i}) \rightarrow$$

$$\mathrm{Config}(s_2,x_1,\ldots,x_{i-1},\beta,x_{i+1},\ldots,x_n,\underbrace{0,\ldots,0}_{i},1,\underbrace{0,\ldots,0}_{n-i-1}))$$

PSPACE-hardness of **LINEAR**

- Assume that the tape alphabet is $\{0,1,\sqcup\}$
- Suppose that M halts on $I = \alpha_1 \ldots \alpha_m$ using $n = m^k$ cells, for $k > 0$

$$D \wedge \Sigma \models Q : - \mathrm{Config}(s_{acc}, \mathbf{X}) \quad \text{iff} \quad M \text{ accepts } I$$

…but, the rules are not constant-free

we can eliminate the constants by applying a simple trick

PSPACE-hardness of **LINEAR**

Initial configuration - the database *D*

auxiliary constants for the states

and the tape alphabet

$$\mathrm{Config}(s_{init},\alpha_1,\ldots,\alpha_m,\underbrace{\sqcup,\ldots,\sqcup}_{n-m},1,\underbrace{0,\ldots,0}_{n-1},s_1,\ldots s_\ell,0,1,\sqcup)$$

PSPACE-hardness of **LINEAR**

Transition rule - $\delta(s_1,0) = (s_2,\sqcup,+1)$

for each $i \in \{1,\ldots,n\}$:

$$\mathrm{Config}(s_1,x_1,\ldots,x_{i-1},z,x_{i+1},\ldots,x_n,\overbrace{z,\ldots,z}^{i-1},o,\overbrace{z,\ldots,z}^{n-i},s_1,\ldots,s_\ell,z,o,b) \rightarrow$$

$$\mathrm{Config}(s_2,x_1,\ldots,x_{i-1},b,x_{i+1},\ldots,x_n,\underbrace{z,\ldots,z}_{i},o,\underbrace{z,\ldots,z}_{n-i-1},s_1,\ldots,s_\ell,z,o,b)$$

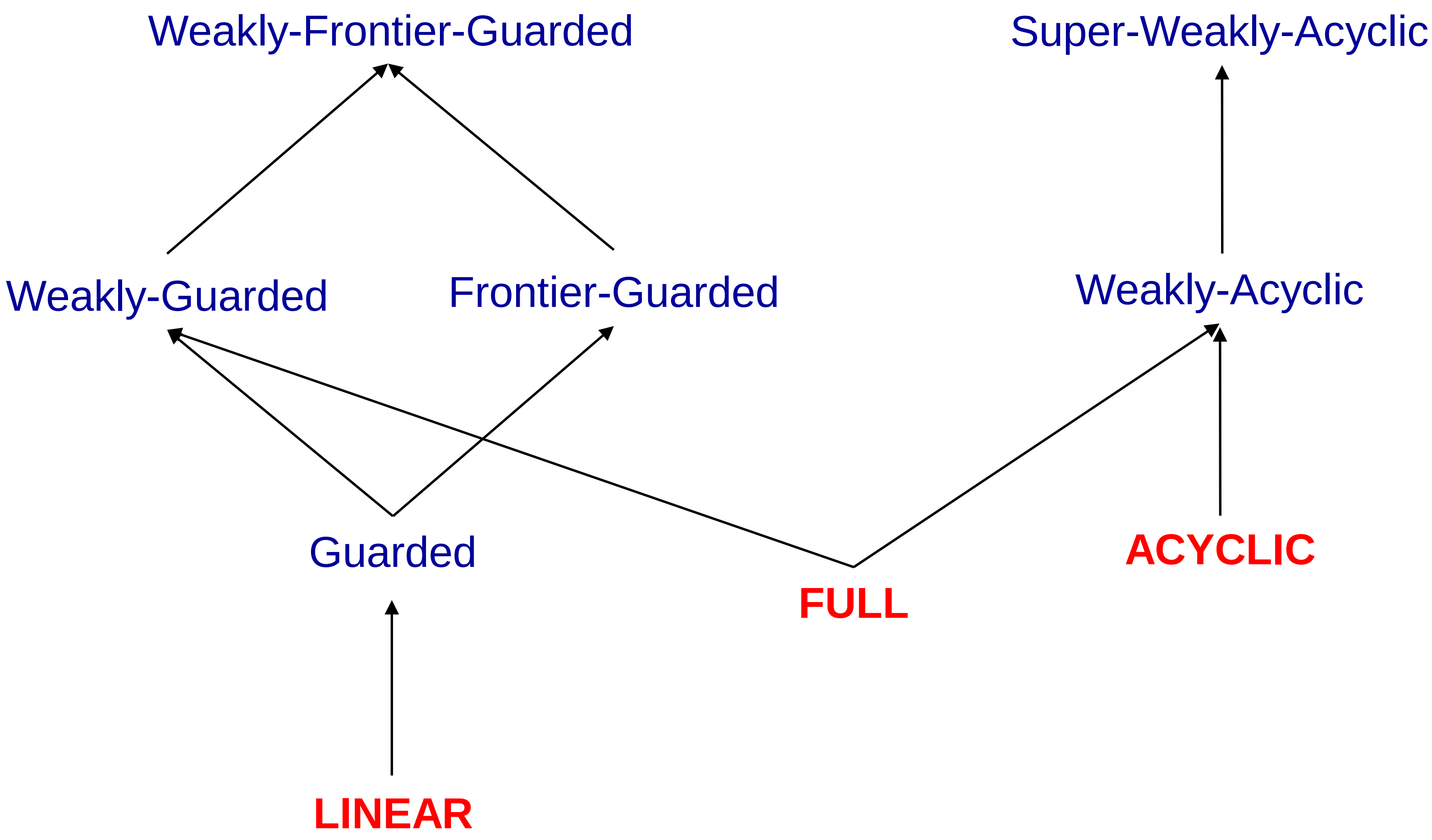
($\forall$-quantifiers are omitted)

Sum Up

	Data Complexity	
FULL	PTIME-c	Naïve algorithm
		Reduction from Monotone Circuit Value problem
ACYCLIC	in LOGSPACE	**Query rewriting**
LINEAR		

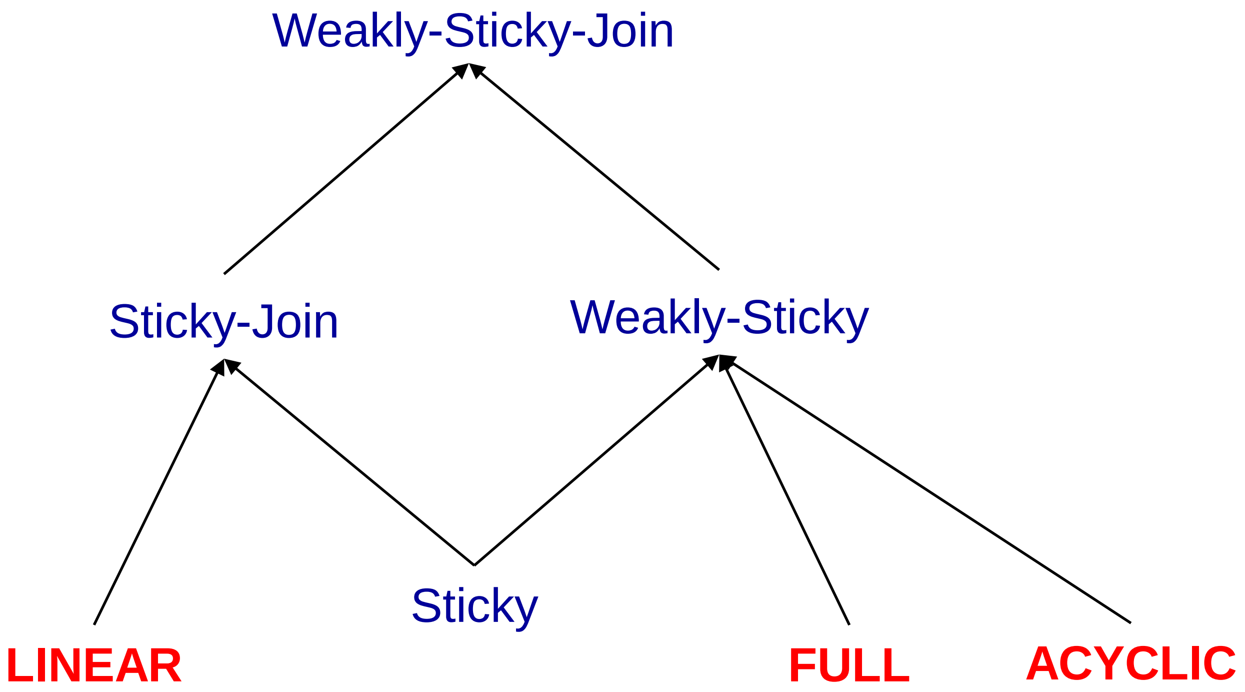
	Combined Complexity	
FULL	EXPTIME-c	Naïve algorithm
		Simulation of a deterministic exponential time TM
ACYCLIC	NEXPTIME-c	Small witness property
		Reduction from a Tiling problem
LINEAR	PSPACE-c	Level-by-level non-deterministic algorithm
		Simulation of a deterministic polynomial space TM

Several Other Languages Exist



Field of intense research

Several Other Languages Exist



Field of intense research

Additional Modelling Features

- Counting quantifiers - very little is known

$$\forall x\ (\mathrm{Professor}(x) \rightarrow \exists_{\leq 4} y\ (\mathrm{supervisorOf}(x,y) \wedge \mathrm{Student}(y))$$

- Default negation (or negation as failure) - relatively well-understood

$$\forall x\ (\mathrm{Number}(x) \rightarrow \exists y\ (\mathrm{hasSucc}(x,y) \wedge \mathrm{Number}(y))$$

$$\forall x\ (\mathrm{Number}(x) \wedge \text{not } \mathrm{Even}(x) \rightarrow \mathrm{Odd}(x))$$

$$\forall x\ (\mathrm{Number}(x) \wedge \text{not } \mathrm{Odd}(x) \rightarrow \mathrm{Even}(x))$$

- Disjunction - relatively well-understood

$$\forall x\ (\mathrm{Number}(x) \rightarrow \mathrm{Even}(x) \vee \mathrm{Odd}(x))$$