

Advanced Combinatorics - 2016 Fall

Exercise 2

You should be able to attempt the following problems after lecture 2.
Comments and corrections are welcome.

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1. Prove that if G is a graph of order $n \geq 3$ such that $d(u) + d(v) \geq n$ for any $(u, v) \notin E(G)$, then G is Hamiltonian.
2. Show that Dirac's theorem is the best possible in terms of minimal degrees. In other words, for any odd $n \geq 3$, construct a graph G such that $\delta(G) = \frac{n-1}{2}$ and G is not Hamiltonian.
3. Let $k < n$ be two positive integers. Let G be a connected graph of order n such that $\delta(G) \geq \frac{k}{2}$. Prove that G must have a cycle of length $\lceil \frac{k}{2} \rceil + 1$, and this is tight.