Advanced Combinatorics - 2016 Fall Exercise 3

You should be able to attempt the following problems after lecture 3. Comments and corrections are welcome.

Heng Guo h.guo@qmul.ac.uk

1. Recall the second proof of Turán's Theorem in class. Use Erdős's theorem to show that if a K_{r+1} -free graph G of order n has $t_r(n)$ many edges, then it is isomorphic to $T_r(n)$.

(Hint: go through the proof and examine what happens when every equality holds.)

- Recall the third proof of Turán's Theorem in class. Let G be a K_{r+1}-free graph that maximizes the number of edges. In class we have shown that if uw ∉ E, vw ∉ E, then uv ∉ E. Fill in the details to determine G's structure. (Hint: it should be the Turán graph.)
- 3. We know that trees have maximum number of edges in a graph of order n without any cycle, which have n 1 edges. Determine:
 - (a) The maximum number of edges in a graph of order n without any cycle of odd length.
 - (b) The maximum number of edges in a graph of order n without any cycle of even length.
- 4. Let A be the graph in Figure 1a and B be the graph in Figure 1b.

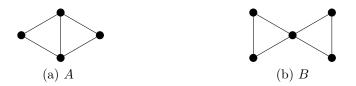


Figure 1: The graphs A and B

(a) If G is a graph on $n \ge 4$ vertices with $e(G) \ge \lfloor n^2/4 \rfloor + 1$, then G contains A as a subgraph.

(Hint: G cannot be triangle-free. Do induction with this information.)

- (b) If G is a graph on n ≥ 5 vertices with e(G) ≥ ⌊n²/4⌋ + 2, then G contains B as a subgraph.
 (Hint: use (a).)
- (c) Determine ex(n, A) and ex(n, B).