Advanced Combinatorics - 2016 Fall
Exercise 3

You should be able to attempt the following problems after lecture 3.
Comments and corrections are welcome.

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1. Recall the second proof of Turán's Theorem in class. Use Erdős's theorem to show that if a K_{r+1}-free graph G of order n has $t_r(n)$ many edges, then it is isomorphic to $T_r(n)$.
   (Hint: go through the proof and examine what happens when every equality holds.)

2. Recall the third proof of Turán's Theorem in class.
   Let G be a K_{r+1}-free graph that maximizes the number of edges. In class we have shown that if $uw \notin E$, $vw \notin E$, then $uv \notin E$.

   Fill in the details to determine G's structure.
   (Hint: it should be the Turán graph.)

3. We know that trees have maximum number of edges in a graph of order n without any cycle, which have $n-1$ edges. Determine:

   (a) The maximum number of edges in a graph of order n without any cycle of odd length.

   (b) The maximum number of edges in a graph of order n without any cycle of even length.

4. Let A be the graph in Figure 1a and B be the graph in Figure 1b.

(a) A (b) B

Figure 1: The graphs A and B

   (a) If G is a graph on $n \geq 4$ vertices with $e(G) \geq \lfloor n^2/4 \rfloor + 1$, then G contains A as a subgraph.
   (Hint: G cannot be triangle-free. Do induction with this information.)

   (b) If G is a graph on $n \geq 5$ vertices with $e(G) \geq \lfloor n^2/4 \rfloor + 2$, then G contains B as a subgraph.
   (Hint: use (a).)

   (c) Determine $ex(n, A)$ and $ex(n, B)$.