

# Online Learning of Large Margin HMMs for Automatic Speech Recognition

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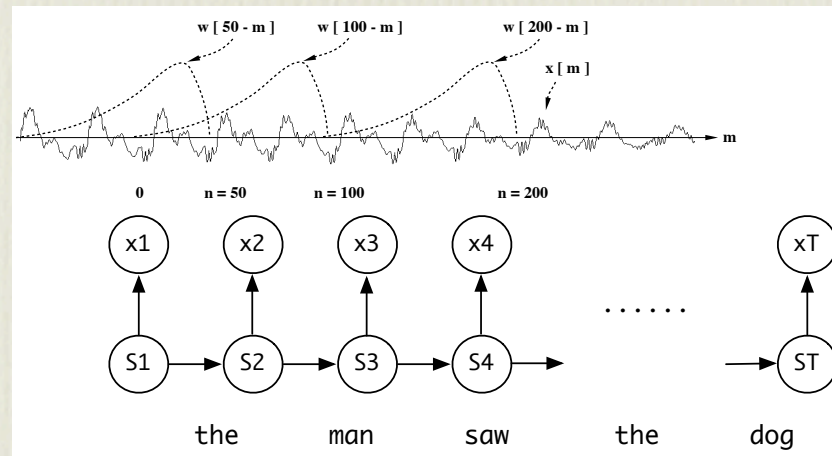
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# Speech recognition since 1980s



- **Hidden Markov models (HMMs)**

- ▶ Hidden states: phone/word classes ( $s_1, s_2, \dots, s_T$ )
- ▶ Observations: acoustic feature vectors ( $x_1, x_2, \dots, x_T$ )

- **Inference and Learning**

- ▶ Viterbi algorithm for decoding
- ▶ Forward-backward algorithms for sufficient statistics

# Types of learning

- **Maximum likelihood estimation** (ML)

- + simple updates, monotonic convergence
- model mismatch, wrong objective

$$p(x|s)$$

- **Discriminative training**

- + minimize error rates
- more complicated, expensive

$$p(s|x) = \frac{p(x|s)p(s)}{\sum_{s'} p(x|s')p(s')}$$

- **Online learning**

- + scales well to large data sets
- potential instability

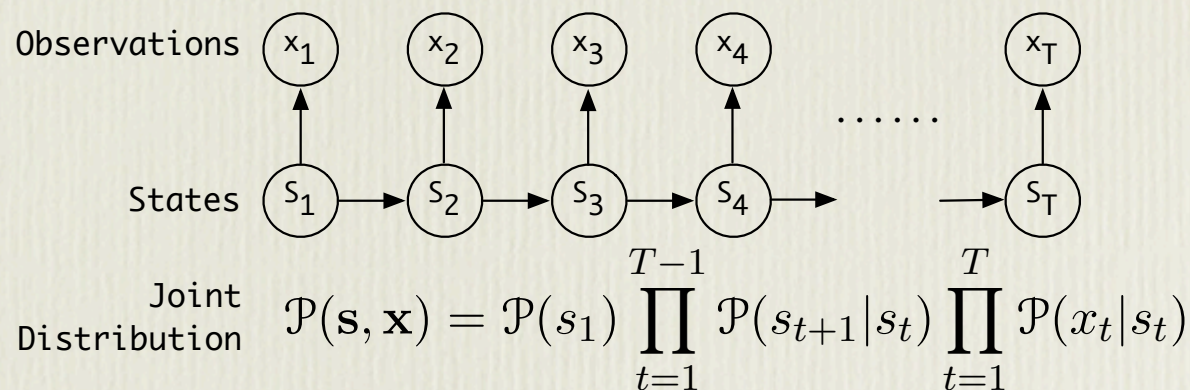
perceptron training of  
**discrete** HMMs  
(Collins, 2002)

# Outline

- Motivation and overview
- **Mistake-driven learning in CD-HMMs**
- Large margins: do they help?
- Acoustic feature adaptation
- What's next?

# Continuous-density HMMs

- **Joint distribution**



- **Emission densities** are parameterized by Gaussian mixture models (GMMs):

$$\mathcal{P}(x|s) = \sum_c \frac{\omega_{sc}}{\sqrt{(2\pi)^d |\Sigma_{sc}|}} e^{-\frac{1}{2}(x-\mu_{sc})^\top \Sigma_{sc}^{-1}(x-\mu_{sc})}$$

- **Maximum likelihood estimation (MLE)**

$$\Theta^{\text{MLE}} = \operatorname{argmax}_{\Theta} \sum_{n=1}^N \log \mathcal{P}(\mathbf{s}_n, \mathbf{x}_n | \Theta)$$

# Recognition with CD-HMMs

- **Discriminant function:**

$$\mathcal{D}(\mathbf{x}, \mathbf{s}) = \log \mathcal{P}(s_1) + \sum_{t=1}^{T-1} \log \mathcal{P}(s_{t+1}|s_t) + \sum_{t=1}^T \log \mathcal{P}(x_t|s_t)$$

- **Correct recognition if:**

$$\forall \mathbf{s} \neq \mathbf{y}, \quad \mathcal{D}(\mathbf{x}, \mathbf{y}) > \mathcal{D}(\mathbf{x}, \mathbf{s})$$

$\mathbf{y}$  : correct transcription of the observation  $\mathbf{x}$

$\mathbf{s}$  : arbitrary transcription

# Online Updating

- For each  $\mathbf{x}_n$  in the training set
  - ▶ compute **Viterbi** decoding sequence  $\mathbf{s}_n^*$ 
$$\mathbf{s}_n^* = \operatorname{argmax}_{\mathbf{s}} \mathcal{D}(\mathbf{x}_n, \mathbf{s})$$
  - ▶ compare to **ground truth** sequence  $\mathbf{y}_n$
  - ▶ **update** if  $\mathbf{s}_n^* \neq \mathbf{y}_n$

$$\Theta \leftarrow \Theta + \eta \frac{\partial}{\partial \Theta} [\mathcal{D}(\mathbf{x}_n, \mathbf{y}_n) - \mathcal{D}(\mathbf{x}_n, \mathbf{s}_n^*)]$$

- Iterate until algorithm converges or no longer reduces recognition errors

# Devil in the details

- How to parameterize CD-HMMs for online learning?
- How to enforce constraints on parameters?
- How to dampen fluctuations in decision boundary?



# GMMs – a closer look

- Conventionally parameterized in terms of **means**, **covariance matrices**, and **mixture weights**.
- **Gradient-based learning** for component  $c$  of state  $s$  :

$$\begin{pmatrix} \nu \\ \mu \\ \Sigma \end{pmatrix}_{sc} \leftarrow \begin{pmatrix} \nu \\ \mu \\ \Sigma \end{pmatrix}_{sc} + \begin{pmatrix} \eta_\nu & 0 & 0 \\ 0 & \eta_\mu & 0 \\ 0 & 0 & \eta_\Sigma \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial \nu} \\ \frac{\partial}{\partial \mu} \\ \frac{\partial}{\partial \Sigma} \end{pmatrix}_{sc} [\mathcal{D}(\mathbf{x}_n, \mathbf{y}_n) - \mathcal{D}(\mathbf{x}_n, \tilde{\mathbf{s}}_n^*)]$$

Empirically difficult to tune multiple learning rates:  
many gradient-based systems only adapt GMM means.

# Reparameterization

- **Change of variables**

For each mixture component, aggregate Gaussian parameters into a single positive semidefinite matrix:

$$\Phi = \begin{bmatrix} \Sigma^{-1} & -\Sigma^{-1}\mu \\ -\mu^T \Sigma^{-1} & \gamma \end{bmatrix} \quad \text{where} \quad \gamma = \log[(2\pi)^d |\Sigma|] + \mu^T \Sigma^{-1} \mu$$

The matrix  $\Phi$  is a  $(d+1) \times (d+1)$  symmetric matrix. The top-left  $d \times d$  block is  $\Sigma^{-1}$ , the top-right  $d \times 1$  block is  $-\Sigma^{-1}\mu$ , the bottom-left  $1 \times d$  block is  $-\mu^T \Sigma^{-1}$ , and the bottom-right  $1 \times 1$  block is  $\gamma$ . Arrows above the matrix indicate the dimensions:  $d+1$  for the full matrix and  $d$  for the  $\Sigma^{-1}$  block.

- **Likelihood computation**

$$\log \mathcal{P}(x|s) = -\frac{1}{2} z^T \Phi_s z \quad \text{where} \quad z = \begin{bmatrix} x \\ 1 \end{bmatrix}$$

# Reparameterized Update

$$\Phi_{sc} \leftarrow \Phi_{sc} + \eta \frac{\partial}{\partial \Phi_{sc}} [\mathcal{D}(\mathbf{x}_n, \mathbf{y}_n) - \mathcal{D}(\mathbf{x}_n, \mathbf{s}_n^*)]$$

- **Problem:**

Update can violate **positive semidefiniteness** of matrix  $\Phi_{sc}$ .

- **Solution:**

Follow each update **by projecting**  $\Phi_{sc}$  back to cone of positive semidefinite matrices.

# Reparameterized Update

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- **Problem:**

Update can violate **positive semidefiniteness** of matrix  $\Phi_{sc}$ .

- **Solution:**

Follow each update **by projecting**  $\Phi_{sc}$  back to cone of positive semidefinite matrices.

- **Problem:**

Projected gradient methods converge **much slower** than unconstrained methods.

# Matrix factorization

- **Yet another reparametrization**

Remove constraint via matrix square root:

$$\Phi_{sc} = \Lambda_{sc} \Lambda_{sc}^T$$

- **New update rule:**

$$\Lambda_{sc} \leftarrow \Lambda_{sc} + \eta \frac{\partial}{\partial \Lambda_{sc}} [\mathcal{D}(\mathbf{x}_n, \mathbf{y}_n) - \mathcal{D}(\mathbf{x}_n, \mathbf{s}_n^*)]$$

+ unconstrained update

– local minima?

– which matrix square root?

# Dampening fluctuations

- **Cumulative averaging**

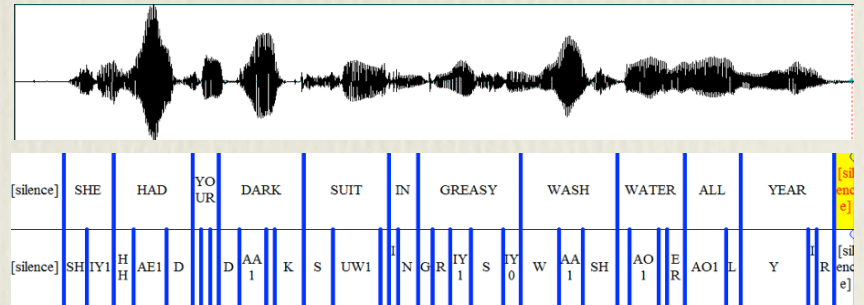
Borrow idea from “averaged” perceptrons:

$$\tilde{\Phi}^{(i)} = \frac{1}{i} \sum_j \Phi^{(j)}$$

- **Smoothed parameter trajectories**

- ▶ averaged  $\tilde{\Phi}$  changes more slowly than non-averaged  $\Phi$
- ▶ used only for testing, not training

# Experiments



- **Phonetic transcription on TIMIT corpus**
  - ▶ 39 phone classes
  - ▶ Frames of speech:
    - 1.1M training, 120K development, 56K test
- **Evaluation**

Compare **recognized** vs **manual** transcriptions:

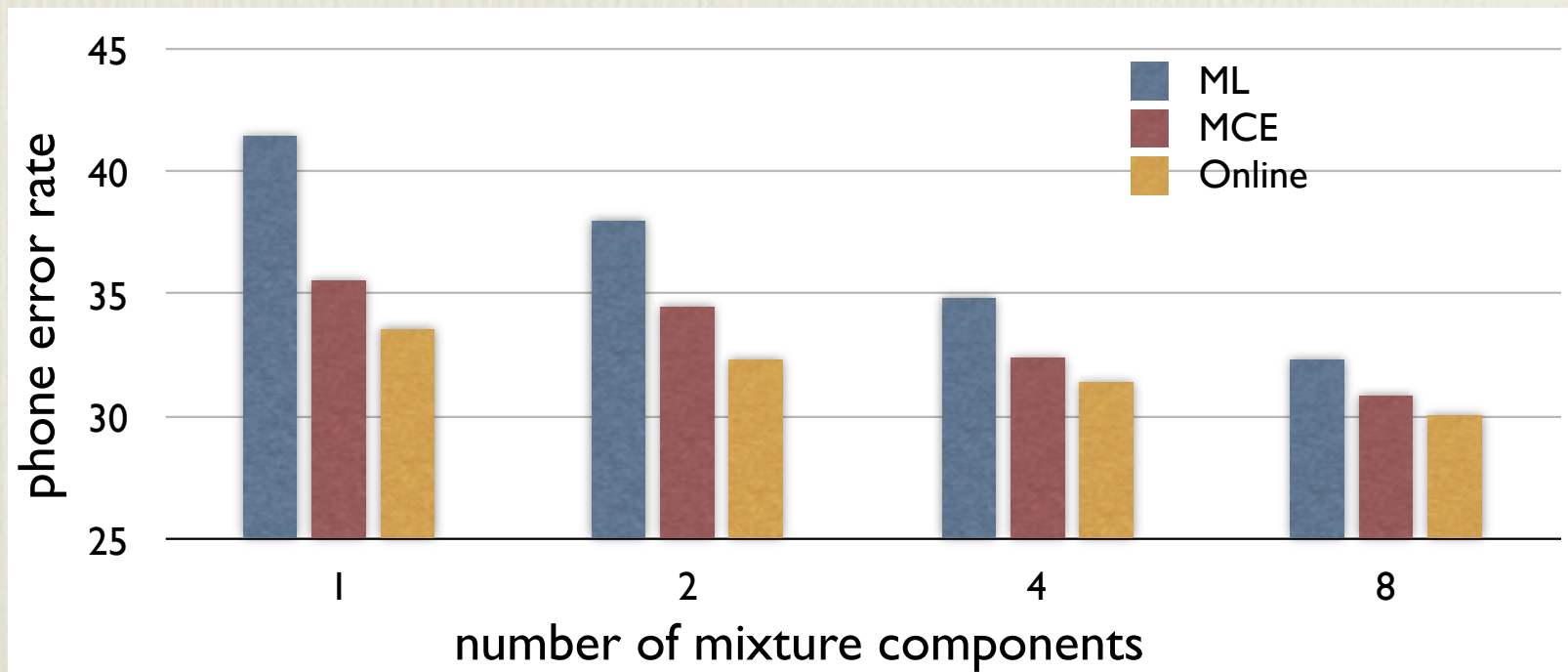
  - ▶ **Frame error rate (FER)**: % of misclassified frames
  - ▶ **Phone error rate (PER)**: edit distance by alignment

# Batch versus Online

**ML** = maximum likelihood estimation (batch)

**MCE** = minimum classification error (batch)

**Online** (best configuration)





# Devil in the details

Training	FER (%)
Batch ML	30.7
Online (w/o reparametrization)	33.9
Online (w/o factorization)	30.9
Online (Cholesky)	31.4
Online (w/o averaging)	35.2
Online (w/o MLE initialization)	36.2
Online (init+SVD+averaging)	<b>28.8</b>

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# Large Margin Training

- **Goal**

Attempt to separate scores of correct and incorrect transcriptions by a large margin.

- **Motivation**

Balance minimization of empirical error rate versus generalization on unseen data.

- **Large margin criterion**

$$\forall \mathbf{s} \neq \mathbf{y}, \quad \mathcal{D}(\mathbf{x}, \mathbf{y}) > \mathcal{D}(\mathbf{x}, \mathbf{s}) + \rho \mathcal{H}(\mathbf{s}, \mathbf{y})$$

$\mathcal{H}(\mathbf{s}, \mathbf{y})$  Hamming distance

$\rho > 0$  margin scaling factor

# Online update rule

- For each  $\mathbf{x}_n$  in the training set
  - ▶ compute the **margin-based decoding** sequence  $\tilde{\mathbf{s}}_n^*$

$$\tilde{\mathbf{s}}_n^* = \operatorname{argmax}_{\mathbf{s}} [\mathcal{D}(\mathbf{x}_n, \mathbf{s}) + \rho \mathcal{H}(\mathbf{s}, \mathbf{y})]$$

- ▶ compare to **ground truth** sequence  $\mathbf{y}_n$
- ▶ **update** if  $\tilde{\mathbf{s}}_n^* \neq \mathbf{y}_n$

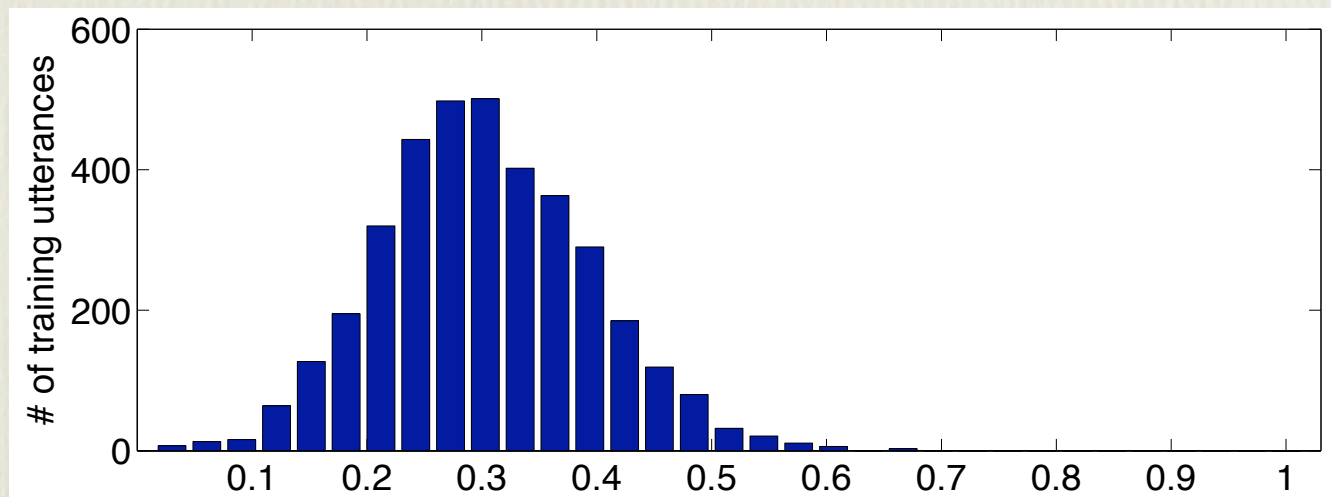
$$\Theta \leftarrow \Theta + \eta \frac{\partial}{\partial \Theta} [\mathcal{D}(\mathbf{x}_n, \mathbf{y}_n) - \mathcal{D}(\mathbf{x}_n, \tilde{\mathbf{s}}_n^*)]$$

- iterate until the algorithm converges or no longer reduces recognition errors

# Margin-based decoding

$$\mathbf{s}_n^* = \operatorname{argmax}_{\mathbf{s}} \mathcal{D}(\mathbf{x}_n, \mathbf{s})$$

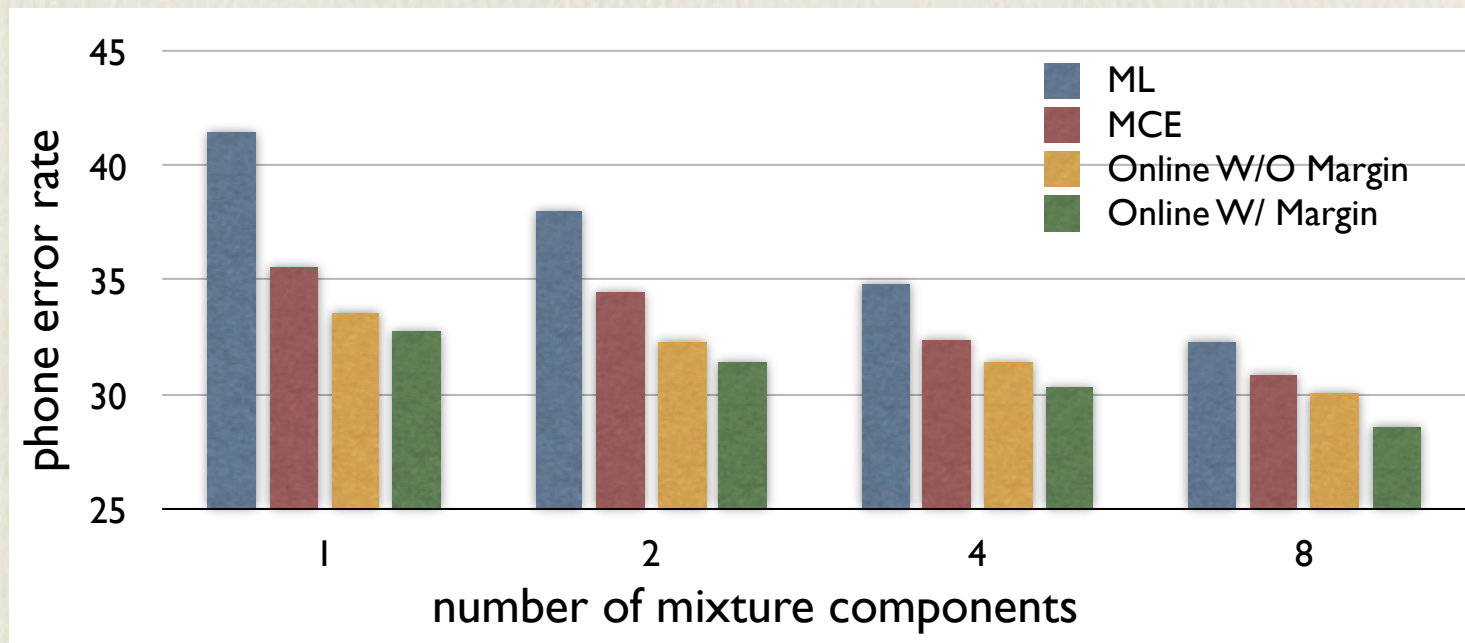
$$\tilde{\mathbf{s}}_n^* = \operatorname{argmax}_{\mathbf{s}} [\mathcal{D}(\mathbf{x}_n, \mathbf{s}) + \rho \mathcal{H}(\mathbf{s}, \mathbf{y})]$$



Normalized Hamming Distance  $\mathcal{H}(\mathbf{s}^*, \tilde{\mathbf{s}}^*)/\text{length}(\mathbf{s}^*)$

**Yields very different competing transcriptions!**

# Do large margins help? **Yes.**



- **MLE** = maximum likelihood estimation (batch)
- **MCE** = minimum classification error (batch)
- **Online w/o margin** = online algorithm for CD-HMMs
- **Online w/ margin** = online algorithm for large margin training

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# Acoustic features



- **Standard front end**

Compute 13 cepstral features from each 30 ms window of speech.

- **Context modeling**

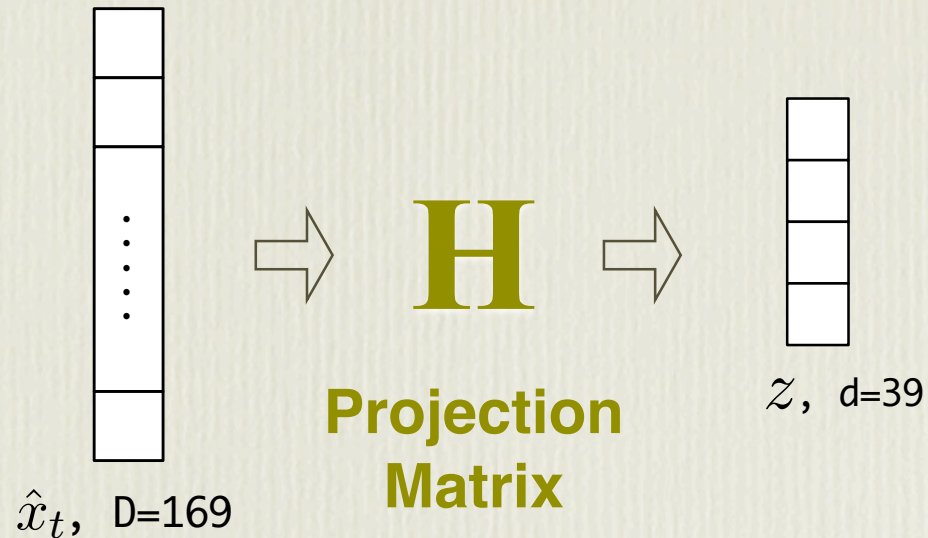
Incorporate features from adjacent windows into observations of CD-HMMs.

- **Scaling of model size**

$$(\# \text{ GMM parameters}) \sim (\# \text{ features})^2$$



# Acoustic feature adaptation



- **Incorporating context**

- ▶ Concatenate features from 13 adjacent windows.
- ▶ Project into a lower dimensional subspace.

- **End-to-end learning**

How to adapt GMM parameters  $\Phi_{sc}$  along with projection  $H$ ?

# Online Optimization

- **Approach**

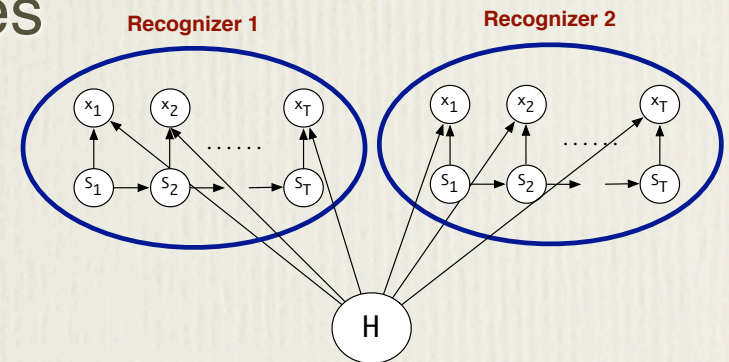
Maximize margin by alternatively updating projection matrix  $H$  and GMM parameters  $\Phi_{sc}$ .

- **Problem**

Small changes in  $H$  (from one utterance) result in big changes to recognizer (across all phonemes).

- **Solutions**

1. Mini-batches of training utterances
2. Parameter-tying (of  $H$ ) across different **recognizers**



# Experiments

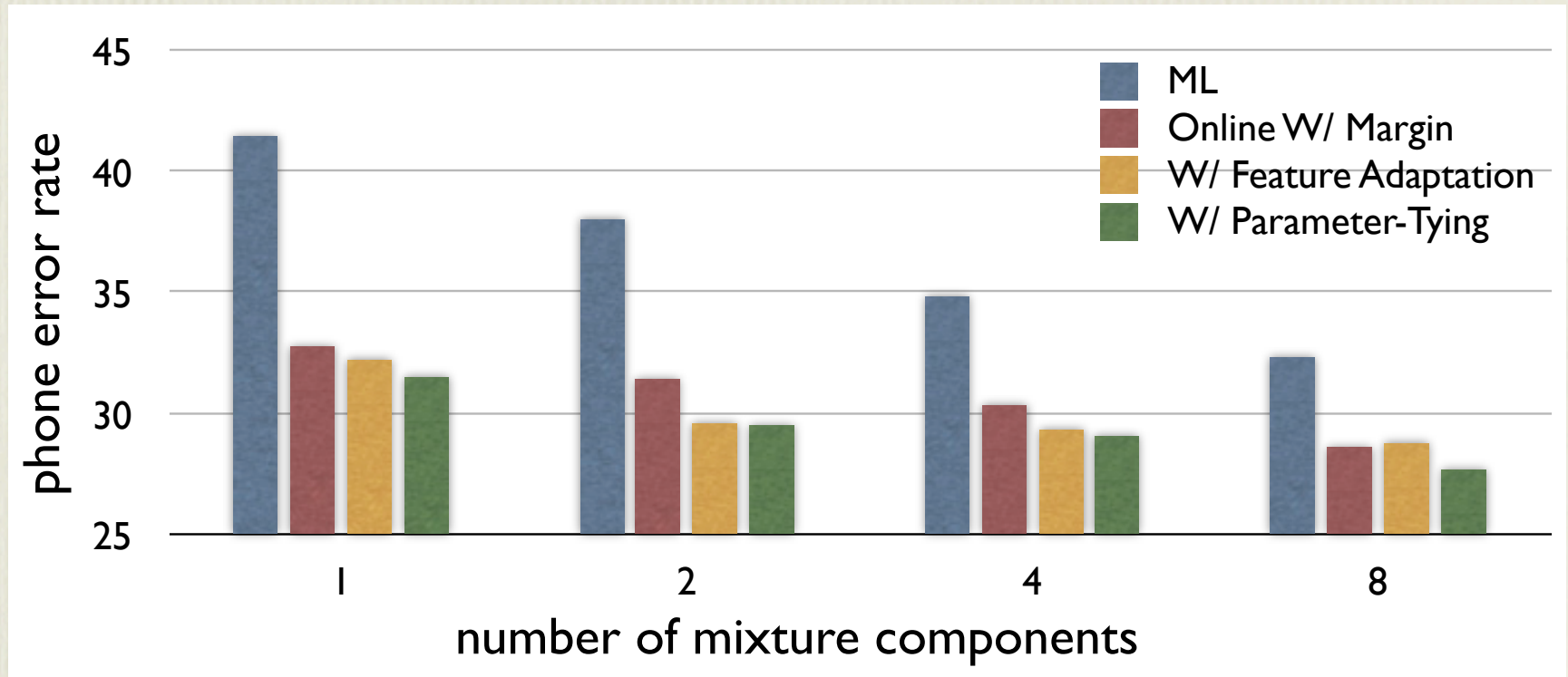
- **Acoustic features**

- ▶  $\hat{x}$  = 13 MFCCs across 13 consecutive frames ( $D=139$ )
- ▶  $z$  = lower-dimensional linear projection of  $\hat{x}$  ( $d=39$ )
- ▶  $H$  = projection matrix initialized to simulate differencing operations for 13 MFCCs +  $13\Delta$  +  $13\Delta\Delta$

- **End-to-end large-margin training**

- ▶ Initialize with maximum likelihood CD-HMMs
- ▶ Alternately optimize  $H$  and  $\Phi$

# Results



**Feature adaptation works best with parameter-tying**

# Summary

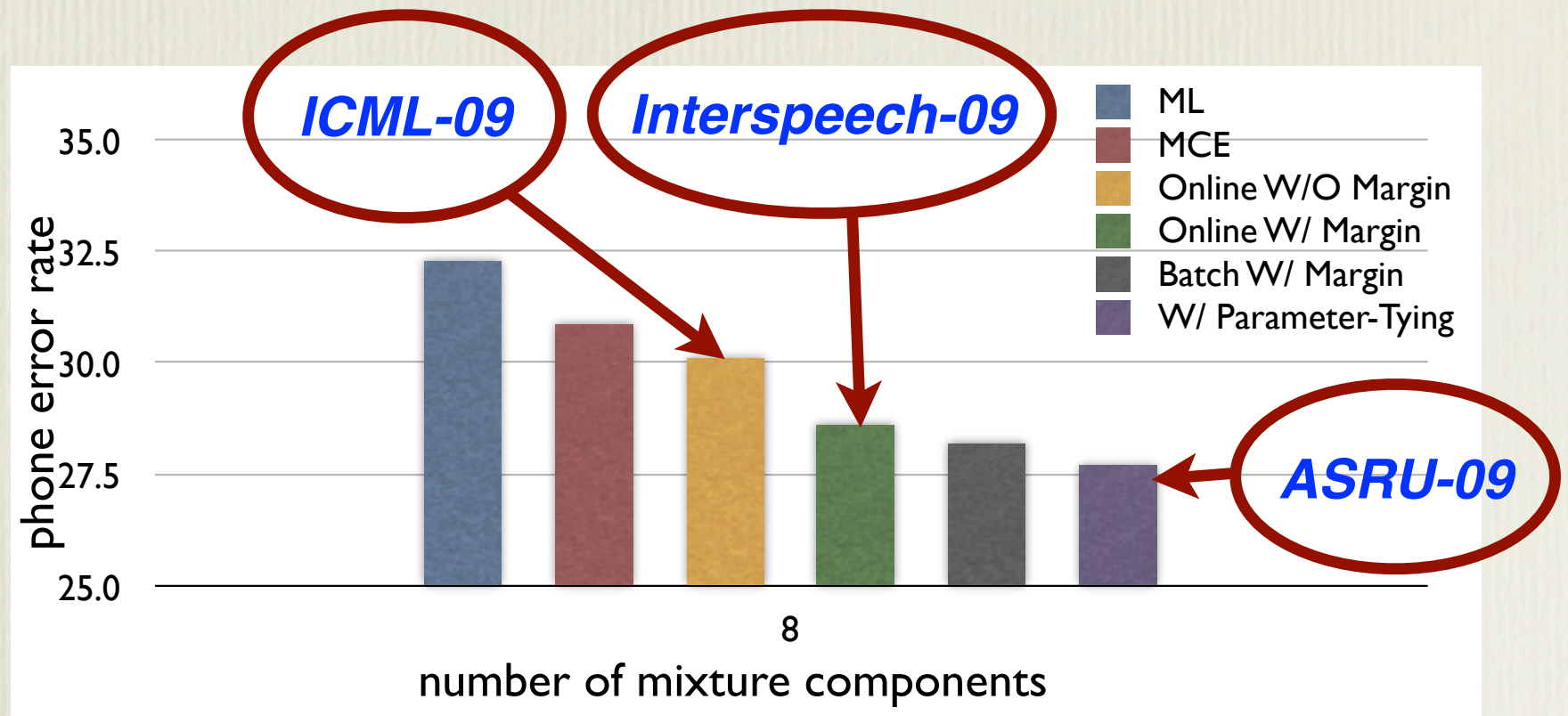
How to improve discriminative training of CD-HMMs with online updates?

- **Best practices:**

- ▶ **Reparameterization**  $\Phi = \begin{bmatrix} \Sigma^{-1} & -\Sigma^{-1}\mu \\ -\mu^\top \Sigma^{-1} & \mu^\top \Sigma^{-1}\mu + \gamma \end{bmatrix}$
- ▶ **Factorization**  $\Phi = \Lambda \Lambda^\top$
- ▶ **Averaging**  $\tilde{\Phi}^{(i)} = \frac{1}{i} \sum_j \Phi^{(j)}$
- ▶ **Large margin**  $\tilde{s}_n^* = \operatorname{argmax}_s [\mathcal{D}(\mathbf{x}_n, \mathbf{s}) + \rho \mathcal{H}(\mathbf{s}, \mathbf{y})]$
- ▶ **Feature adaptation with parameter-tying**

- **Did we succeed?**

# Improvement over time



Online methods ultimately beat our best batch implementation.

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# What's next?

- **Scaling up**

- ▶ larger corpora
- ▶ word recognition (not phone recognition)
- ▶ context-dependent (triphone) HMMs
- ▶ word lattices for large-vocabulary ASR

- **Fast adaptation**

- ▶ new speakers
- ▶ infinite data (e.g., refreshed daily)



# What's next? (con't)

- **Other models and loss functions**

- ▶ Direct loss minimization (McAllester et al, 2010)
- ▶ Hidden-unit conditional random field (van der Maaten et al, 2011)
- ▶ Edit distance (versus Hamming distance)

# What's next? (con't)

- **Other models and loss functions**

- ▶ Direct loss minimization (McAllester et al, 2010)
- ▶ Hidden-unit conditional random field (van der Maaten et al, 2011)
- ▶ Edit distance (versus Hamming distance)

***See you at the next workshop ...***

# Publications

C.-C. Cheng, F. Sha, and L. K. Saul (2010). **Online Learning and Acoustic Feature Adaptation in Large Margin Hidden Markov Models**. In *IEEE Journal of Selected Topics in Signal Processing* 4(6): 926-942.

C.-C. Cheng, F. Sha, and L. K. Saul (2009). **Large Margin Feature Adaptation for Automatic Speech Recognition**. In *Proceedings of the IEEE Workshop on Automatic Speech Recognition and Understanding (ASRU-09)*, pages 87-92. Merano, Italy.

C.-C. Cheng, F. Sha, and L. K. Saul (2009). **A fast online algorithm for large margin training of continuous-density hidden Markov models**. In *Proceedings of the Tenth Annual Conference of the International Speech Communication Association (Interspeech-09)*, pages 668-671. Brighton, UK.

C.-C. Cheng, F. Sha, and L. K. Saul (2009). **Matrix updates for perceptron training of continuous-density hidden Markov models**. In *Proceedings of the Twenty Sixth International Conference on Machine Learning (ICML-09)*, pages 153- 160. Montreal, Canada.