Getting Performance out of Process Algebra

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16th August 2005

Outline

Introduction

Interplay: Process Algebra and Markov Process

Applications

Conclusions

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Applications

Conclusions

The PEPA project

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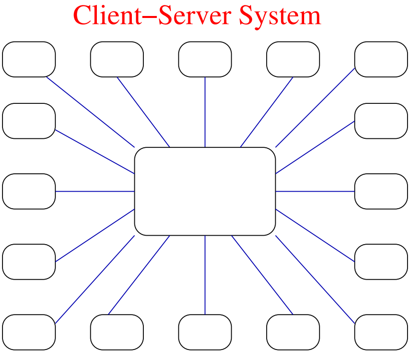
- Performance Evaluation Process Algebra (PEPA) sought to address these problems by the introduction of a suitable process algebra.
- We have sought to investigate and exploit the interplay between the process algebra and the continuous time Markov chain (CTMC).

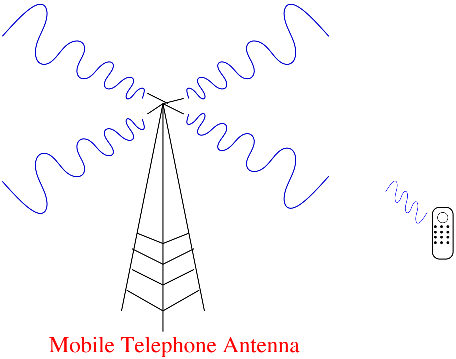
Performance Modelling: Motivation



Scalability issues

- How many clients can the existing server support and maintain reasonable response times?

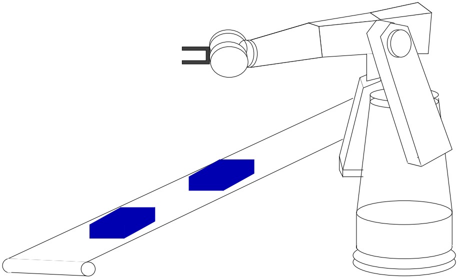
Performance Modelling: Motivation



Frequency Division Multiplexing

- How many frequencies do you need to keep blocking probabilities low?

Performance Modelling: Motivation

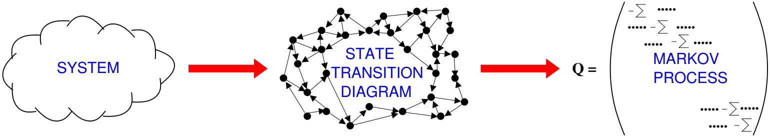


Flexible Manufacturing System

System Tuning

- What speed of conveyor belt will minimize robot idle time and maximize throughput whilst avoiding lost widgets?

Performance Modelling using CTMC



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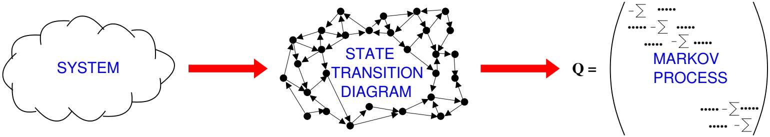

A stochastic process $X(t)$ is a Markov process iff for all $t_0 < t_1 < \ldots < t_n < t_{n+1}$, the joint probability distribution of $(X(t_0), X(t_1), \ldots, X(t_n), X(t_{n+1}))$ is such that

$$\Pr(X(t_{n+1}) = s_{i_{n+1}} \mid X(t_0) = s_{i_0}, \ldots, X(t_n) = s_{i_n}) = \Pr(X(t_{n+1}) = s_{i_{n+1}} \mid X(t_n) = s_{i_n})$$

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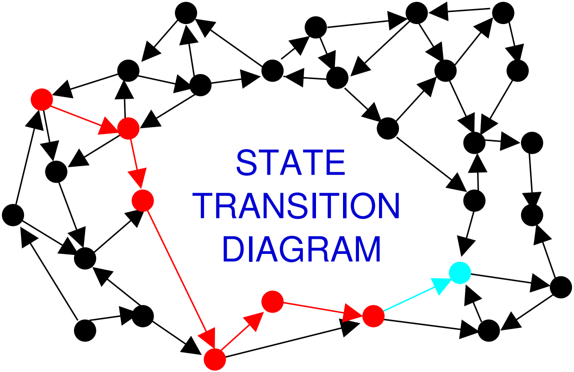

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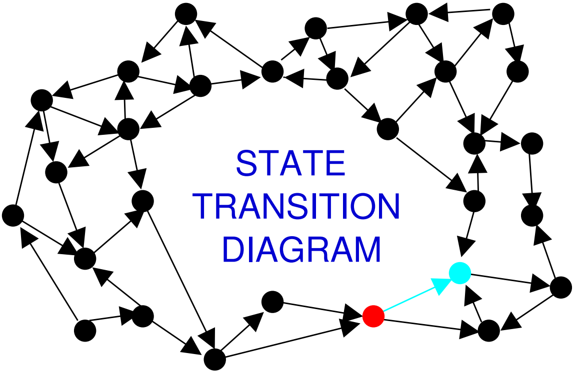

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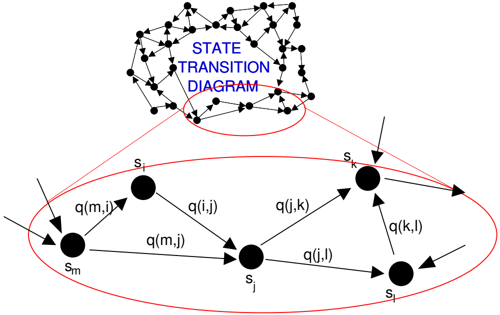
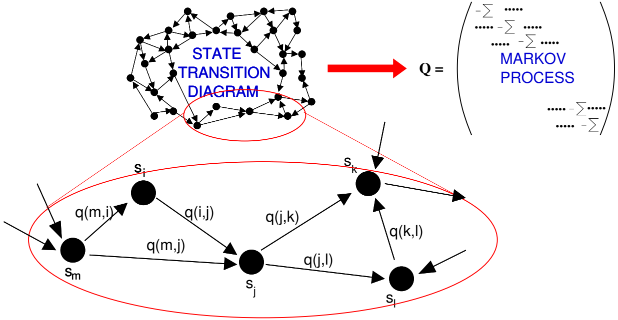


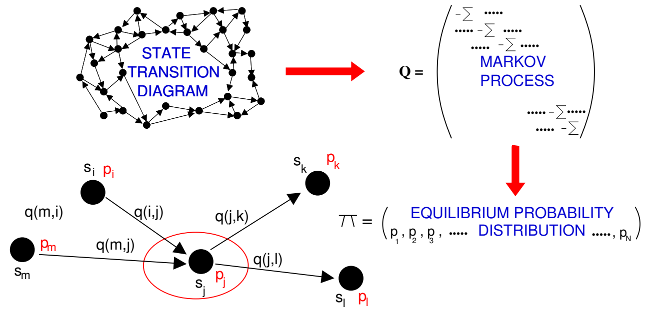
A negative exponentially distributed duration is associated with each transition.

Performance Modelling using CTMC



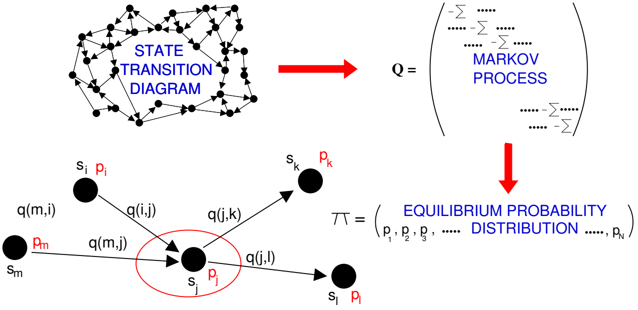
these parameters form the entries of the infinitesimal generator matrix Q

Performance Modelling using CTMC



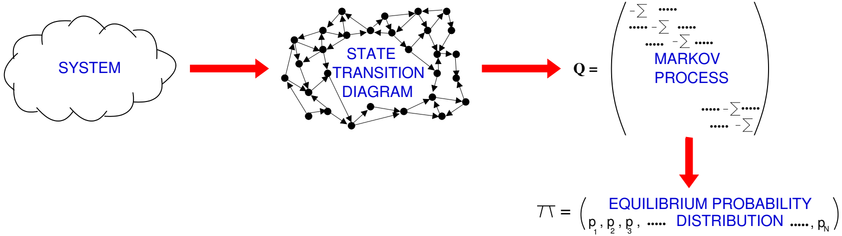
In steady state the probability flux out of a state is balanced by the flux in.

Performance Modelling using CTMC



"Global balance equations" captured by $\pi Q = 0$ solved by linear algebra

Performance Modelling using CTMC



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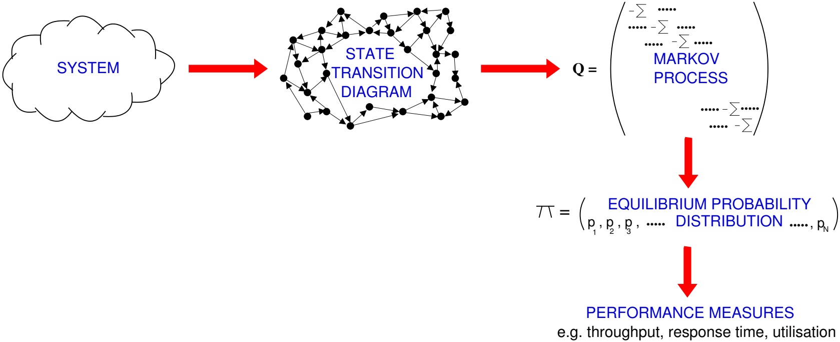

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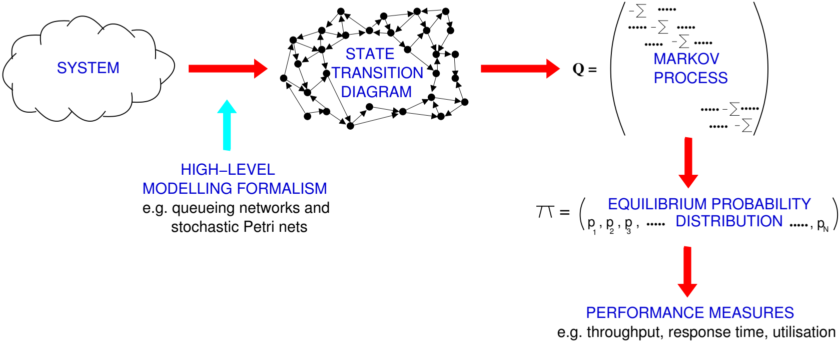
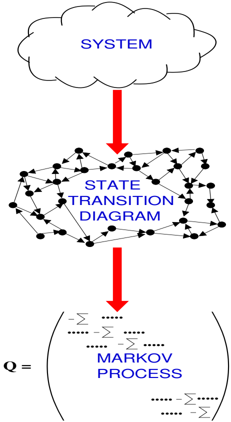


Performance Modelling using CTMC

Model Construction

- describing the system using a high level modelling formalism
- generating the underlying CTMC



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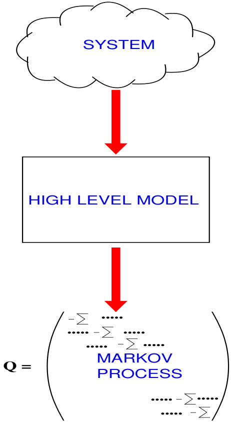

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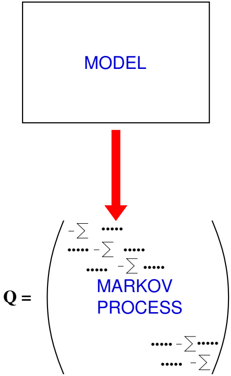
- model simplification
- model aggregation



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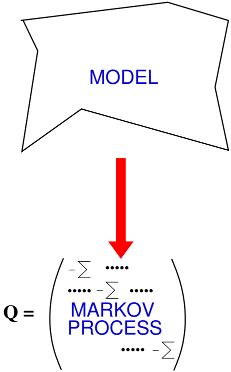

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Model Solution

- solving the CTMC to find steady state probability distribution
- deriving performance measures

Process Algebra

- Models consist of agents which engage in actions.

$$\alpha.P$$

action type or name

agent/ component

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Process algebra model $\xrightarrow{\text{SOS rules}}$ Labelled transition system

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Its clients might be web browsers, in a domain with a local cache of frequently requested pages. Thus any display request might result in an access to the server or in a page being loaded from the cache.

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A simple version of the Web can be considered to be the interaction of these components:

$$WEB \stackrel{def}{=} (Browser \parallel Browser) \mid Server$$

Qualitative Analysis

- The labelled transition system underlying a process algebra model can be used for functional verification e.g.: reachability analysis, specification matching and model checking.

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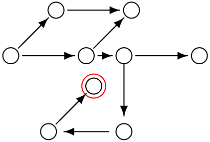
Will the system arrive in a particular state?



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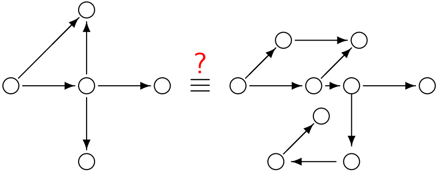
Does system behaviour match its specification?



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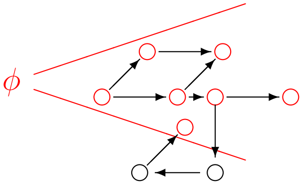

Performance Evaluation Process Algebra

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PEPA
MODEL

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HIDING:	P/L	abstraction $\alpha \in L \Rightarrow \alpha \to \tau$

Example revisited

The behaviour of the server is the same but now quantitative information is recorded for each operation:

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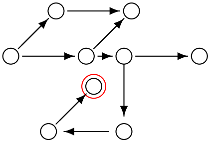
Reachability analysis

How long will it take for the system to arrive in a particular state?



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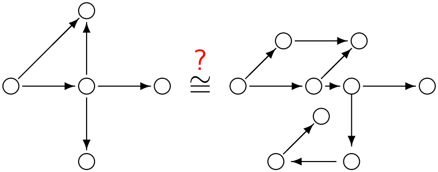
Specification matching

With what probability does system behaviour match its specification?



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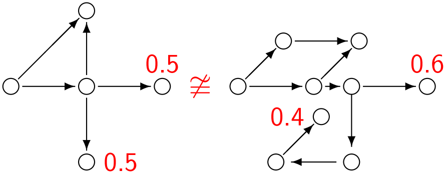
Specification matching

Does the "*frequency profile*" of the system match that of the specification?



Integrated analysis

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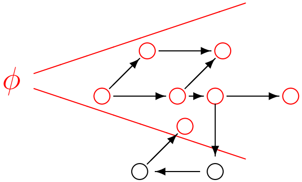
Model checking

Does a given property ϕ hold within the system with a given probability?

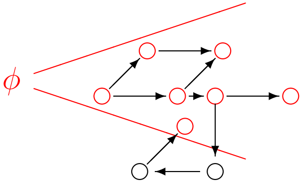


Integrated analysis

- Qualitative verification can now be complemented by quantitative verification:

Model checking

For a given starting state
how long is it until
a given property ϕ holds?



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- From the process algebra side the Markov chain had a profound influence on the design of the language and in particular on the interactions between components.
- From the Markov chain perspective the process algebra structure has been exploited to find aspects of independence even between interacting components.

SPA Languages

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integrated time

SPA

orthogonal time

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    - exponential only
    - exponential + instantaneous
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The Importance of Being Exponential

$$\begin{tikzcd}
 & (\alpha, r).Stop \parallel (\beta, s).Stop \arrow[dl, "(\alpha, r)"'] \arrow[dr, "(\beta, s)"] & \\
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The memoryless property of the negative exponential distribution means that residual times do not need to be recorded.

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\end{tikzcd}$$

We retain the expansion law of classical process algebra:

$$(\alpha, r).Stop \parallel (\beta, s).Stop = \\ (\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)$$

only if the negative exponential distribution is assumed.

Parallel Composition

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- In classical process algebra is it often associated with communication.
- When the activities of the process algebra have a duration the definition of parallel composition becomes more complex.

Who Synchronises...?

Even within classical process algebras there is variation in the interpretation of parallel composition:

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CCS-style

- Actions are partitioned into input and output pairs.
- Communication or synchronisation takes places between conjugate pairs.
- The resulting action has silent type τ.

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Most stochastic process algebras adopt CSP-style synchronisation.

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- The issue of what it means for two timed activities to synchronise is a vexed one....

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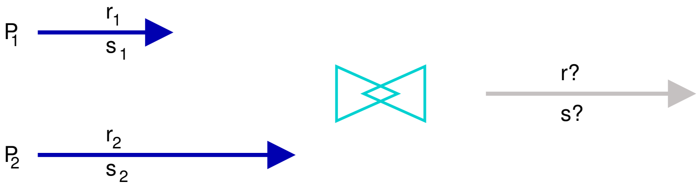

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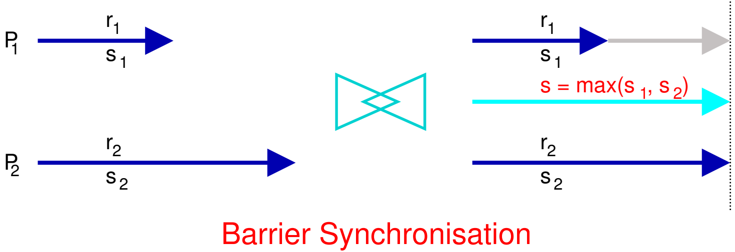

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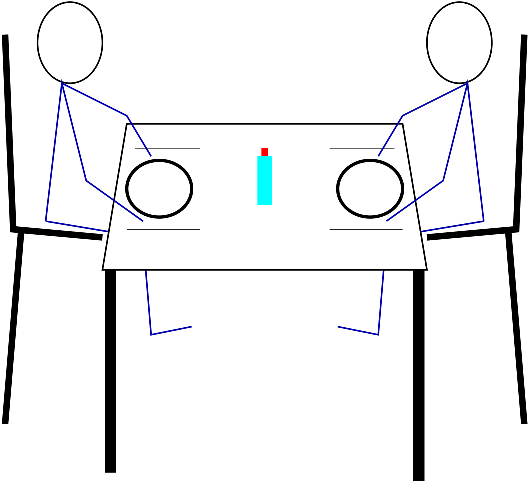

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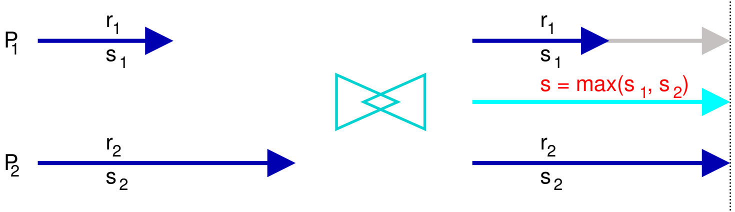

s is no longer exponentially distributed

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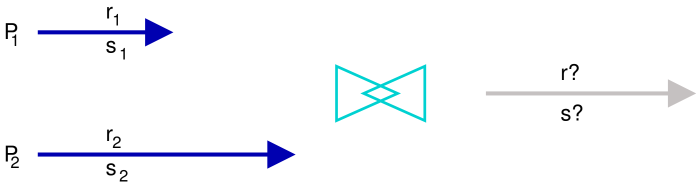

algebraic considerations limit choices

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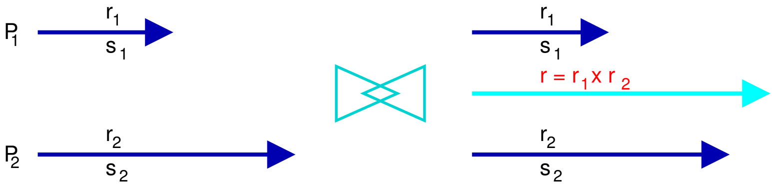
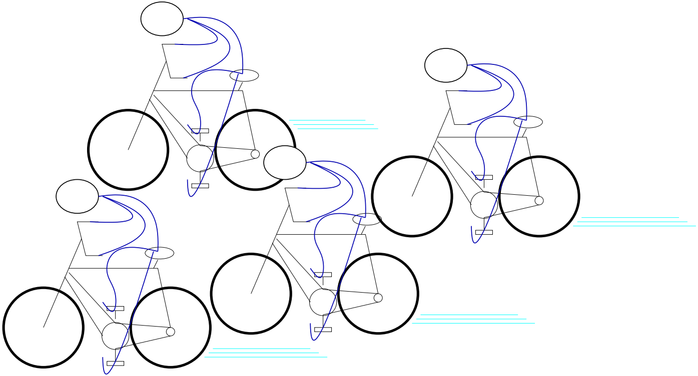


TIPP: new rate is product of individual rates

Timed Synchronisation



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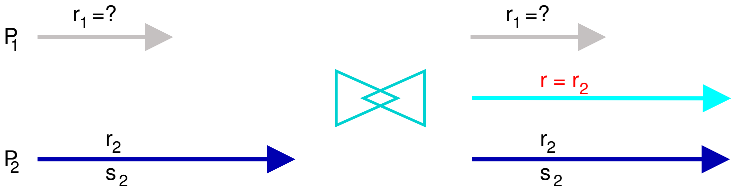
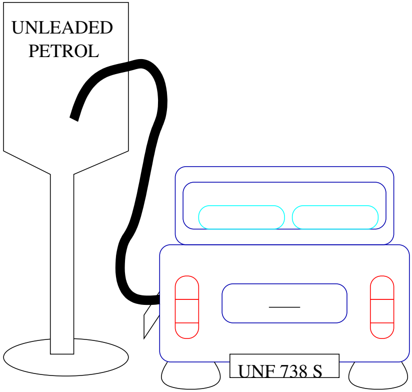


EMPA: one participant is passive

Timed Synchronisation



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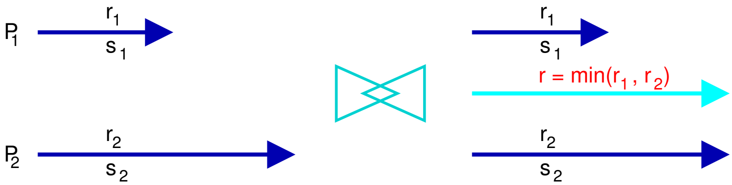

bounded capacity: new rate is the minimum of the rates

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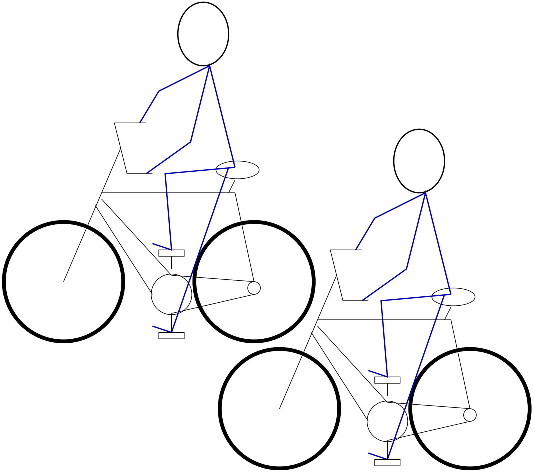

Cooperation in PEPA

- In PEPA each component has a bounded capacity to carry out activities of any particular type, determined by the apparent rate for that type.

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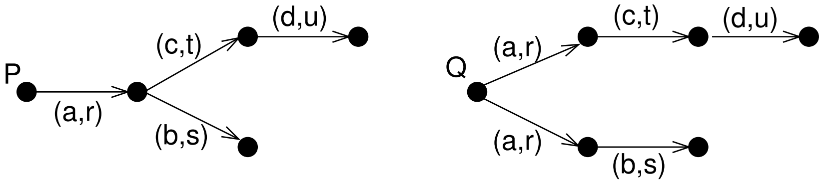
- Synchronisation, or cooperation cannot make a component exceed its bounded capacity.
- Thus the apparent rate of a cooperation is the minimum of the apparent rates of the co-operands.

Equivalence Relations

In process algebra equivalence relations are defined based on the notion of observability:



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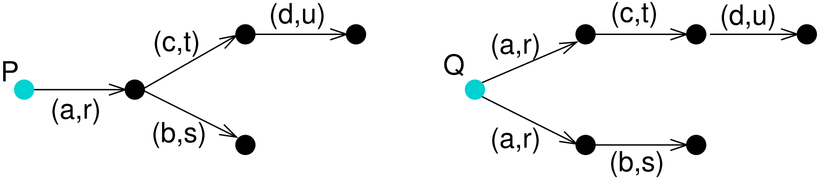

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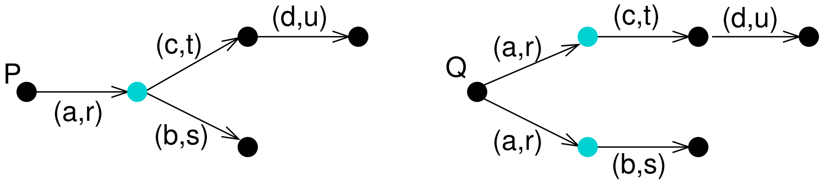

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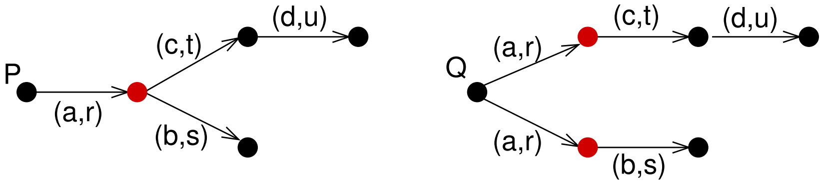

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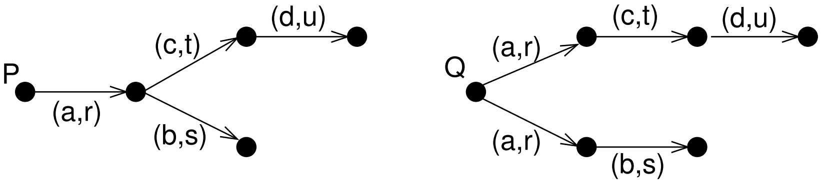

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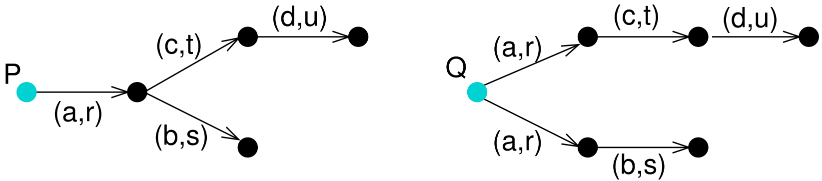

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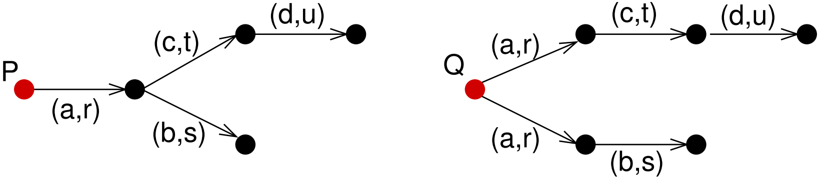

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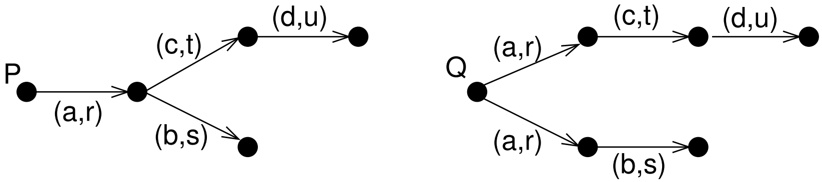

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The resulting equivalence relation is a bisimulation in the style of Larsen and Skou, and coincides with the Markov process notion of lumpability.

Aggregation and lumpability

Model aggregation: use a state-state equivalence to establish a partition of the state space of a model, and replace each set of states by one macro-state, i.e. take a different stochastic representation of the same model.

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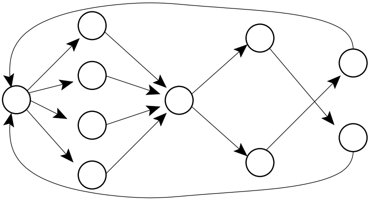

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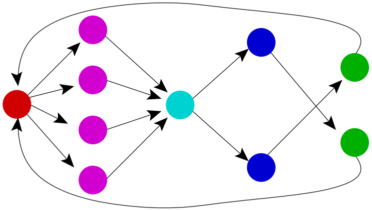

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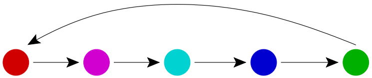
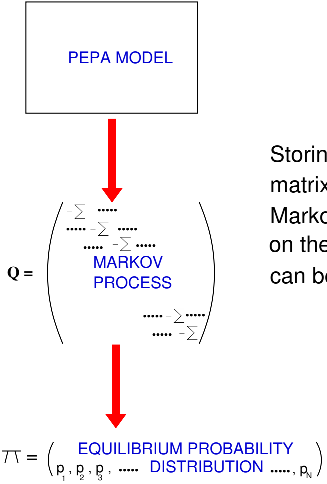


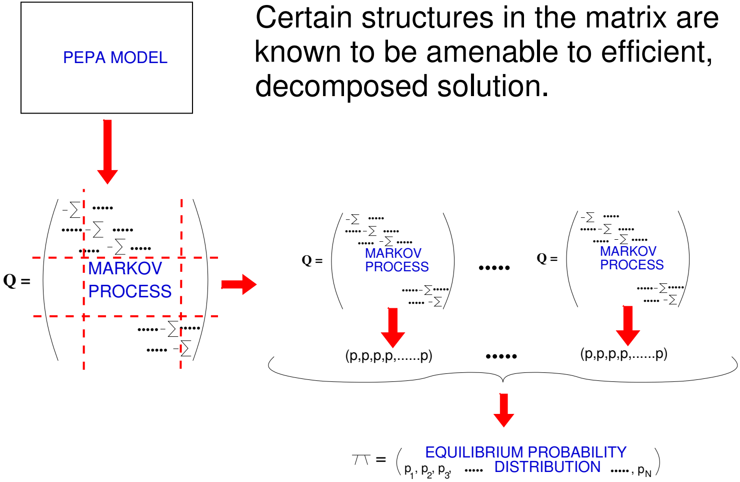
Characterising efficient solution



Storing and manipulating the matrix which represents the Markov process places limitations on the size of model which can be analysed.

Characterising efficient solution

Certain structures in the matrix are known to be amenable to efficient, decomposed solution.



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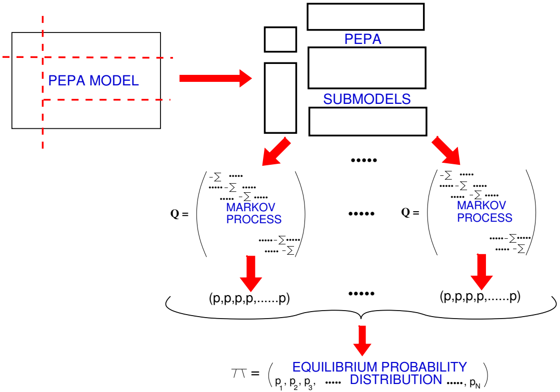

Finding the corresponding structures in the process algebra means that these techniques can be applied automatically, before the monolithic matrix is formed.

Outline

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Applications

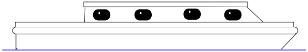
- Developing models of real applications has always been an integral part of the PEPA project.
- This allows us to demonstrate to ourselves and others that the theory we have developed is useful.
- It is also a valuable source of inspiration for new theory and future directions.

PEPA Case Studies (1)

- Multiprocessor access-contention protocols (Gilmore, Hillston and Ribaudo, Edinburgh and Turin)
- Protocols for fault-tolerant systems (Clark, Gilmore, Hillston and Ribaudo, Edinburgh and Turin)
- Multimedia traffic characteristics (Bowman et al, Kent)
- Database systems (The STEADY group, Heriot-Watt University)
- Software Architectures (Pooley, Bradley and Thomas, Heriot-Watt and Durham)
- Switch behaviour in active networks (Hillston, Kloul and Mokhtari, Edinburgh and Versailles)

PEPA Case Studies (2)

- Locks and movable bridges in inland shipping in Belgium (Knapen, Hasselt)



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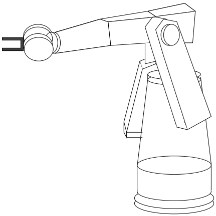

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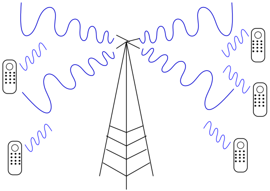

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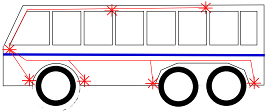
- Automotive diagnostic expert systems (Console, Picardi and Ribaudo, Turin)



New application domains: biochemical signalling pathways

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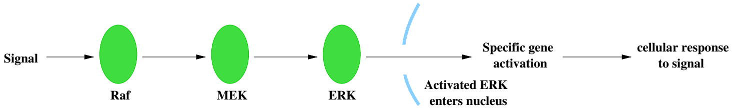
- Systems biology aims to develop a better understanding of the processes involved.
- Stochastic process algebras have found a new role in developing models for systems biology, allowing biologists to test hypotheses and prioritise experiments.

Extracellular signalling

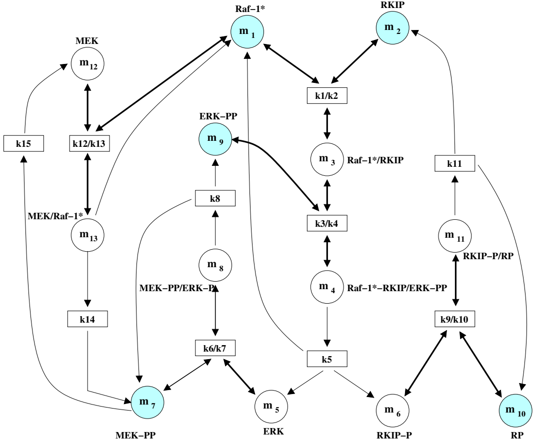
Extracellular signalling — communication between cells.

- signalling molecules released by one cell migrate to another;
- these molecules enter the cell and instigate a pathway, or series of reactions, which carries the information from the membrane to the nucleus;
- the Ras/Raf-1/MEK/ERK pathway conveys differentiation signals to the nucleus of a cell.



Special relevance to cancer research because when pathways operate abnormally cells divide uncontrollably.

The Ras/Raf-1/MEK/ERK pathway



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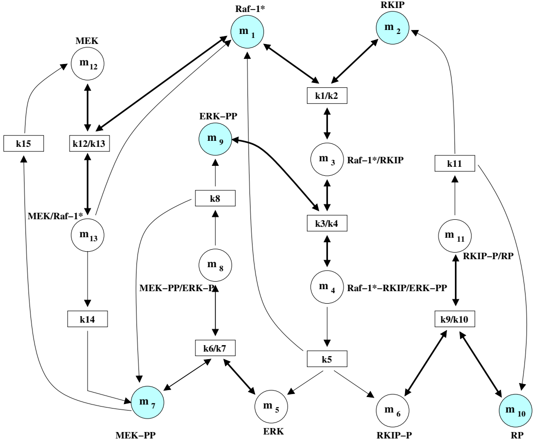

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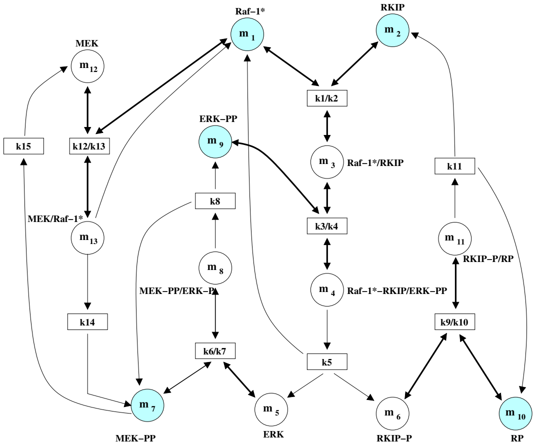

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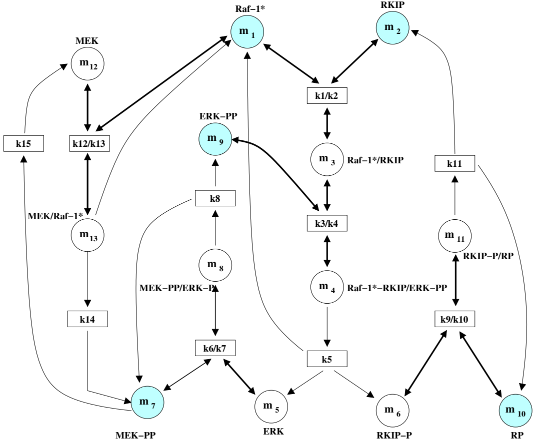

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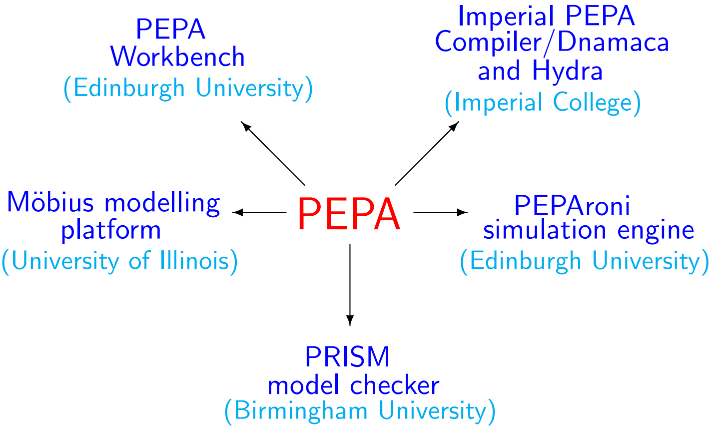
and shown them to be equivalent.

Tool Support



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The incorporation of stochastic quantitative information into process algebras has been extremely fruitful:

- For performance modelling:
  - the rigour of the formal description technique has had benefits for both practice and theory;
  - enhanced analysis capabilities.
- For theoretical computer science:
  - interesting new class of languages has been introduced, stimulating much new research.

Acknowledgements

The PEPA project has been funded by SERC, EPSRC and the CEC.

Thank you