Compositional dynamic modelling: a Computer Science perspective

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encounters
INSPACE

Models of Computer Systems

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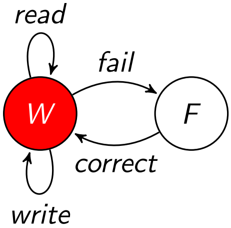

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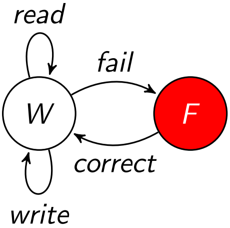

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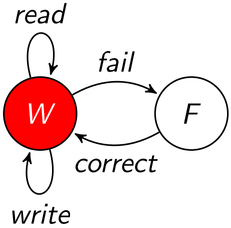
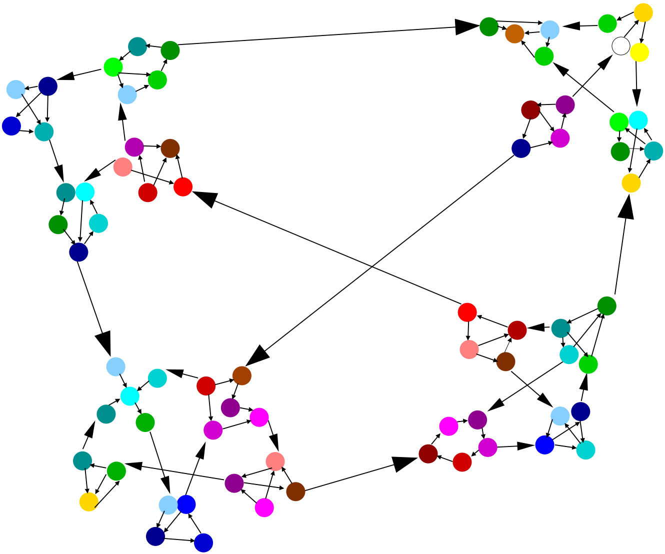
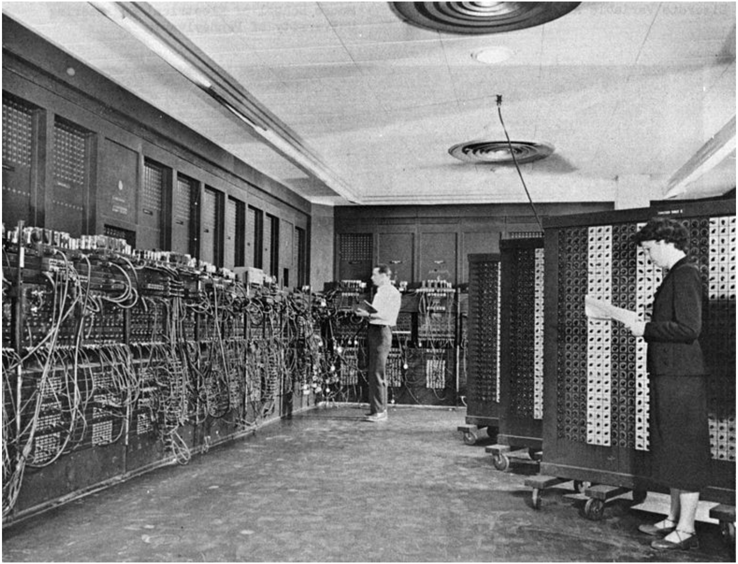


Detailed view of Behaviour — State Machine



Advancing Computer Technology

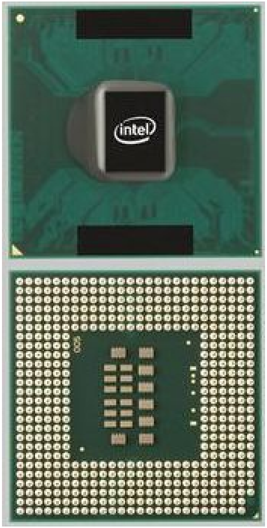


ENIAC c. 1946

Advancing Computer Technology

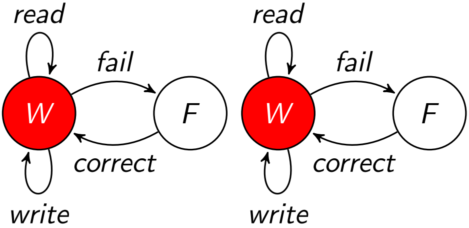
standard laptop c. 2006

Advancing Computer Technology



Intel dual core

Disks revisited



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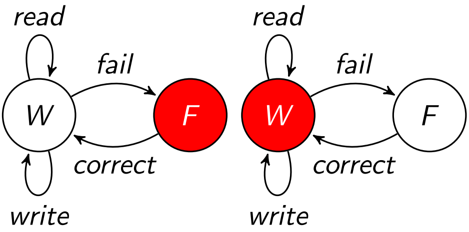

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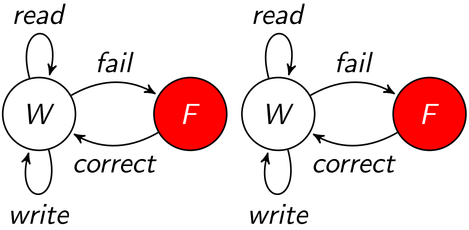

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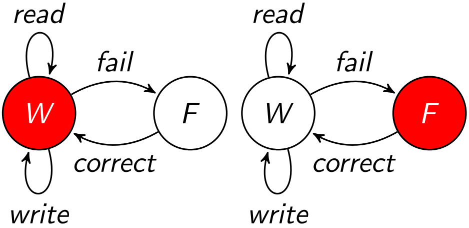

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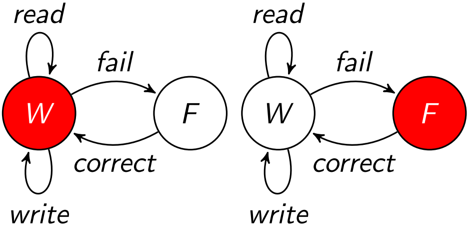

Constructing a model directly in terms of the possible states rapidly becomes a daunting prospect

Advancing Modelling Technology

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Later version of the formalisms included information about the timing and resource use associated with the represented processes: these are stochastic process algebras.

Disks revisited

Disk

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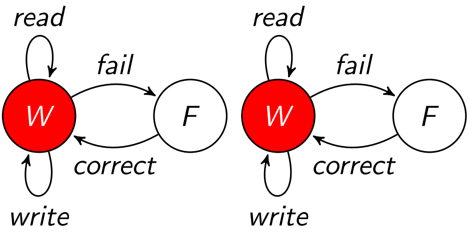
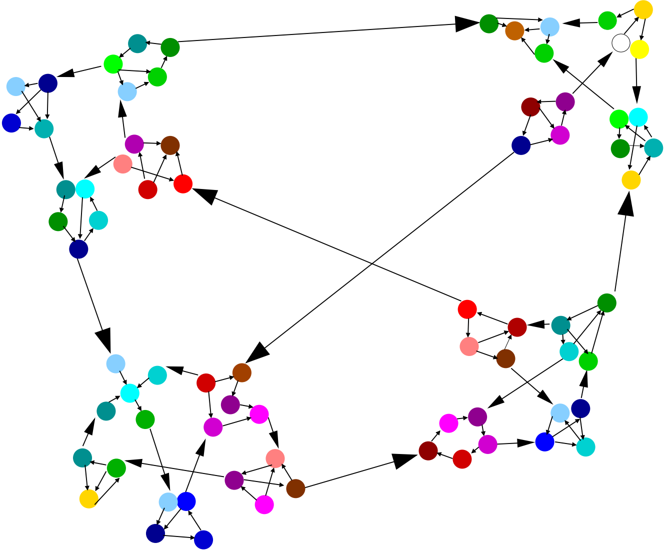


Compositional View of Behaviour



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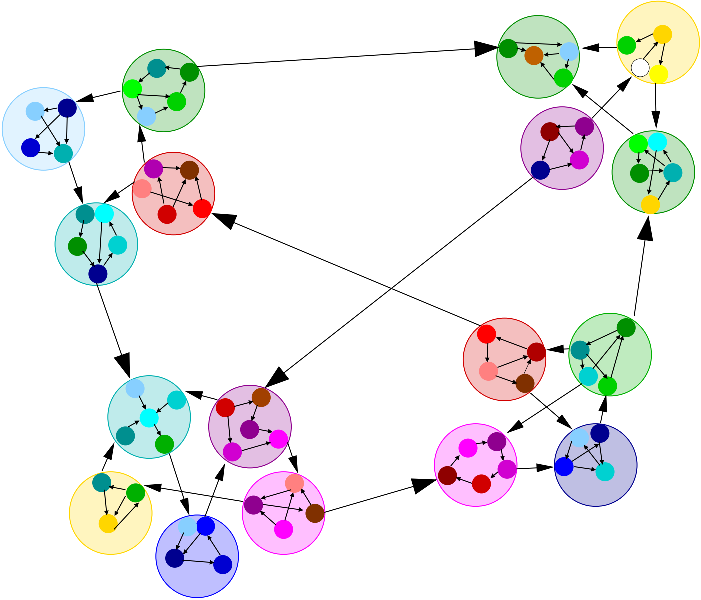

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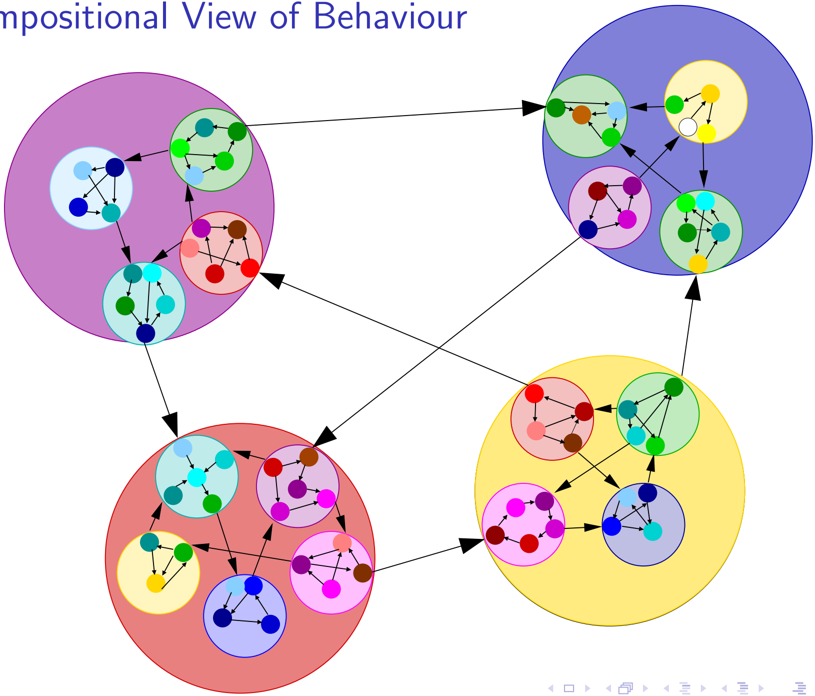

Constructing models from simple components

Client

$$Client_{idle} \stackrel{def}{=} (request, \lambda).Client_{waiting}$$
$$Client_{waiting} \stackrel{def}{=} (response, \top).Client_{idle}$$

Server

$$Server_{idle} \stackrel{def}{=} (request, \top).Server_{computing}$$
$$Server_{computing} \stackrel{def}{=} (compute, \pi).Server_{responding}$$
$$Server_{responding} \stackrel{def}{=} (response, \rho).Server_{idle}$$

System

$$System \stackrel{def}{=} Client_{idle}[3] \underset{\mathcal{L}}{\bowtie} Server_{idle}[2]$$
$$\text{where } \mathcal{L} = \{request, response\}$$

Other applications

The compositional modelling style of process algebras has also been applied to modelling other things as well as computers, most notably biochemical pathways in systems biology.

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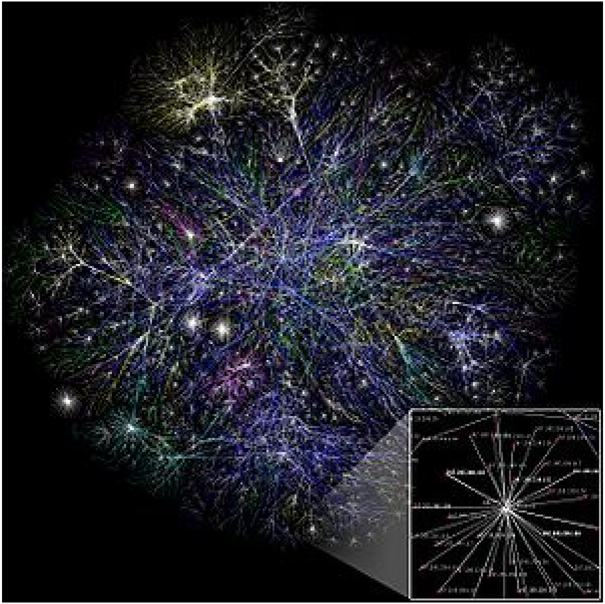
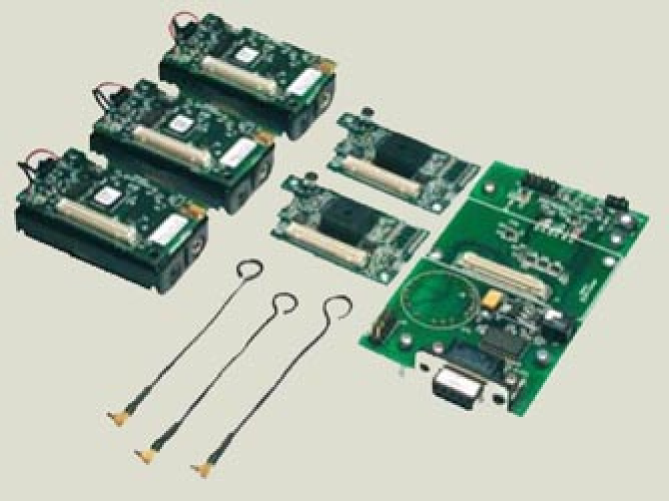
In general stochastic process algebras provide a good framework for modelling systems in which the collective behaviour emerges as the result of a large number of individuals, acting and interacting in a predefined, but stochastic, manner.

Internet scale



Mote scale



Smaller and smaller devices create the possibily of a network around each person.

Challenges of modelling ubiquitous systems

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Mark Weiser 1994

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Unfortunately the scale of the systems is such that our existing modelling techniques are severely challenged by ubiquitous systems.

State-space explosion

Disks	States
1	2

State-space explosion

Disks	States
1	2
2	4

State-space explosion

Disks	States
1	2
2	4
6	64

State-space explosion

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1	2
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10	1024

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Disks	States
1	2
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State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

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2^{150} states

State-space explosion

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2^{150} states $= 2^{152}$ bytes

State-space explosion

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2^{150} states $= 2^{152}$ bytes $= 2^{82} \times 2^{70}$ bytes

State-space explosion

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$$2^{150} \text{ states} = 2^{152} \text{ bytes} = 2^{82} \times 2^{70} \text{ bytes} = 2^{82} \text{ zettabytes}$$

Compositionality and Abstraction

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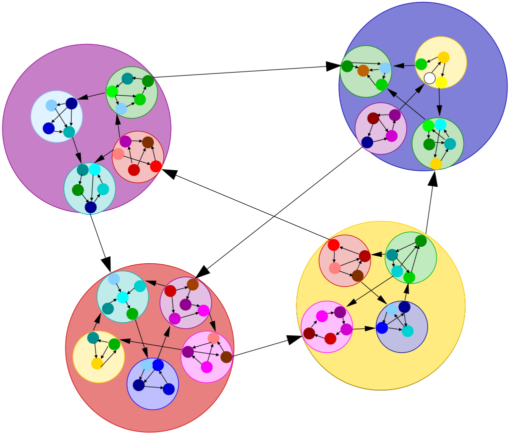

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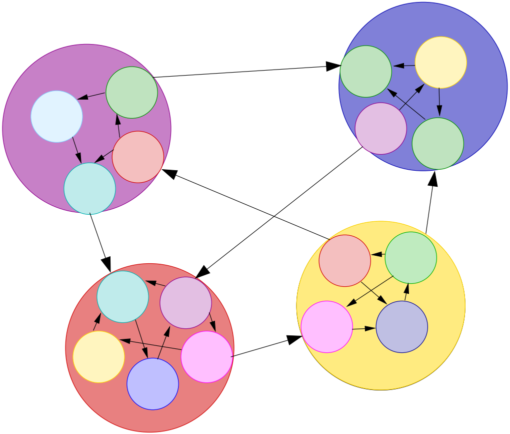

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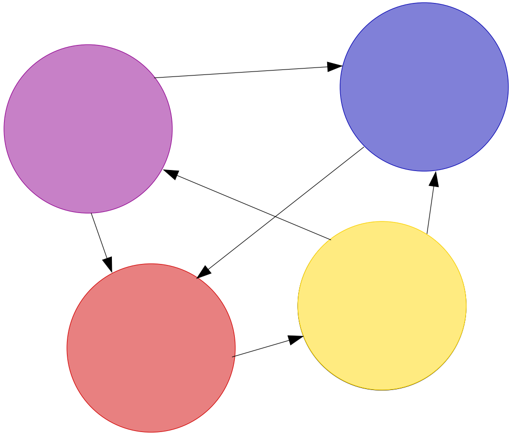

Identity and Individuality

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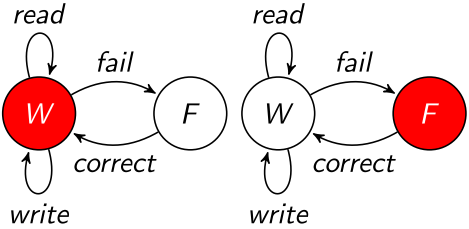

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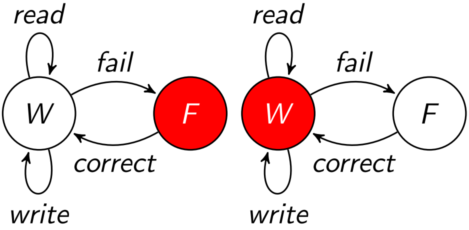

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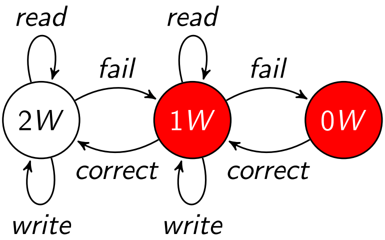

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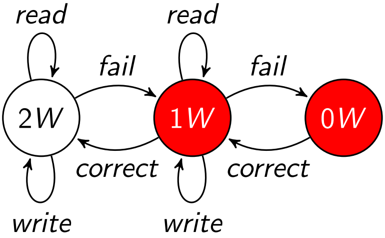

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Even better reductions can be achieved when we no longer regard the components as individuals.

Collective Behaviour

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Performance as an emergent behaviour

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Example Service Level Agreement

90% of requests receive a response within 3 seconds.

Disk model in PEPA

Disk

$$\begin{aligned} \mathit{Working} &\stackrel{\mathit{def}}{=} (\mathit{read}, r).\mathit{Working} \\ &+ (\mathit{write}, w).\mathit{Working} \\ &+ (\mathit{fail}, f).\mathit{Failed} \\ \mathit{Failed} &\stackrel{\mathit{def}}{=} (\mathit{correct}, c).\mathit{Working} \end{aligned}$$

- We have W working disks and F failed ($W + F = N$).

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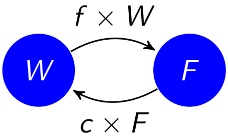

$$
\begin{aligned}
\mathrm{d}W/\mathrm{d}t &= -f \times W + c \times F \\
\mathrm{d}F/\mathrm{d}t &= f \times W - c \times F
\end{aligned}
$$

Conclusions

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The compositional style supports representation of large systems.

The shift to the collective dynamic view supports analysis of very large systems.

Nevertheless there are significant challenges ahead as ubiquitous systems become a reality.

Acknowledgements

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