

Tuning Systems: From Composition to Performance

Jane Hillston
LFCS, Edinburgh

BCS Roger Needham Award Lecture,
The Royal Society, London,
8th December 2004

The PEPA project

- ▶ The PEPA project started in Edinburgh in 1991.

Tuning Systems: from
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Introduction

Theory

Applications

Future Work



The PEPA project

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- ▶ **Performance Evaluation Process Algebra (PEPA)** sought to address these problems by the introduction of a suitable process algebra.

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- ▶ Process algebras offered a compositional description technique supported by apparatus for formal reasoning.
- ▶ Performance Evaluation Process Algebra (PEPA) sought to address these problems by the introduction of a suitable process algebra.
- ▶ We have sought to investigate and exploit the **interplay** between the **process algebra** and the continuous time **Markov chain** (CTMC)

Outline

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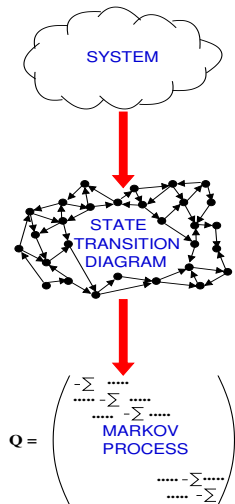
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Performance Modelling using CTMC

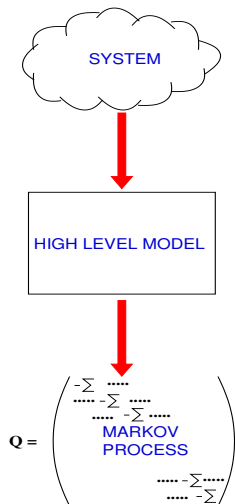
Model Construction



Performance Modelling using CTMC

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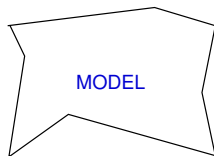
- ▶ describing the system using a high level modelling formalism
- ▶ generating the underlying CTMC



Performance Modelling using CTMC

Model Construction

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Model Manipulation

- ▶ model simplification
- ▶ model aggregation

$$Q = \begin{pmatrix} -\Sigma & \dots \\ \dots & -\Sigma & \dots \\ \dots & \dots & -\Sigma \end{pmatrix}$$

MARKOV PROCESS

Model Construction

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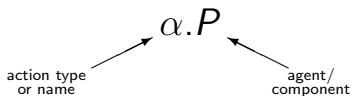
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Model Solution

- ▶ solving the CTMC to find steady state probability distribution
- ▶ deriving performance measures

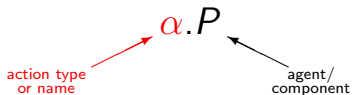
Process Algebra

- ▶ Models consist of **agents** which engage in **actions**.



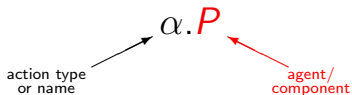
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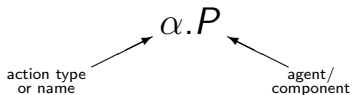
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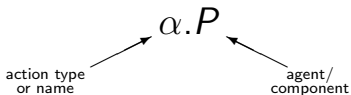


- ▶ The language is used to generate a **labelled transition system** for functional verification: reachability analysis, specification matching and model checking.

Process algebra model $\xrightarrow{\text{SOS rules}}$ Labelled transition system

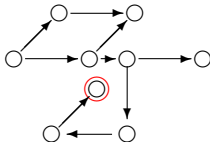
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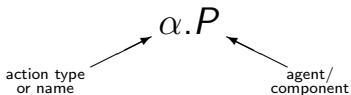
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Will the system arrive
in a particular state?



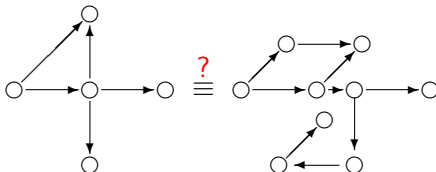
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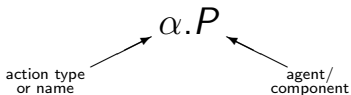
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Does system behaviour
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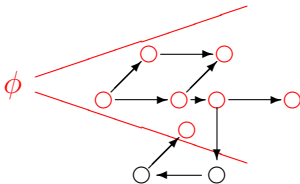
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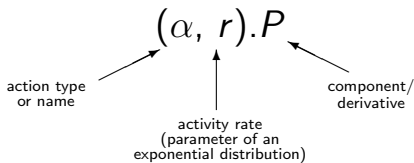
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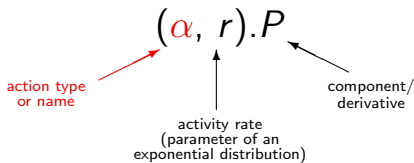
Performance Evaluation Process Algebra

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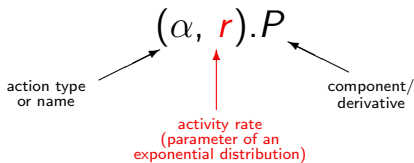
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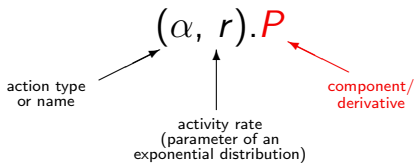
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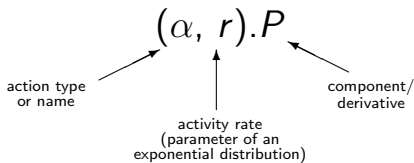
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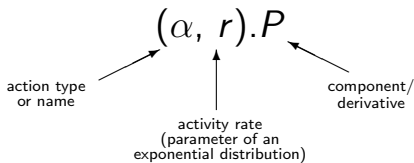
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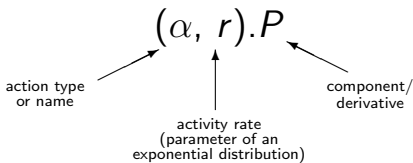


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PEPA
MODEL

Performance Evaluation Process Algebra

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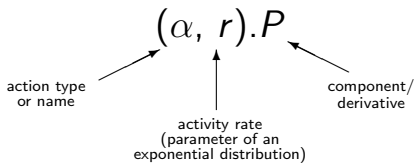


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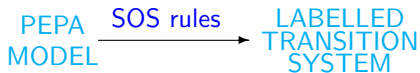
PEPA MODEL $\xrightarrow{\text{SOS rules}}$

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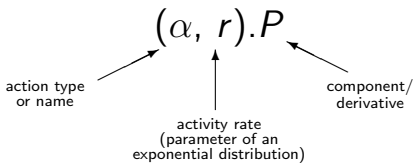


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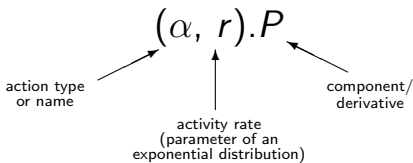


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Benefits of Quantification

- ▶ Each PEPA expression has an underlying CTMC which can be derived **automatically**.

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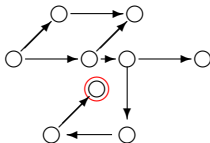
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Benefits of Quantification

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Reachability analysis

How long will it take for the system to arrive in a particular state?

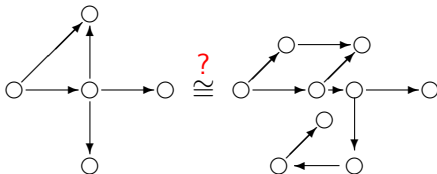


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Specification matching

With what probability
does system behaviour
match its specification?

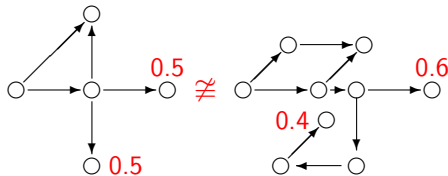


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Specification matching

Does the “*frequency profile*” of the system match that of the specification?

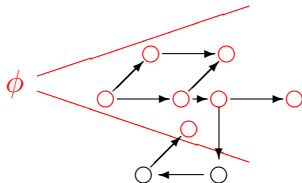


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Model checking

Does a given property ϕ
hold within the system
with a given probability?

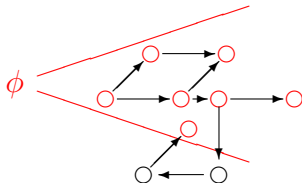


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Model checking

For a given starting state
how long is it until
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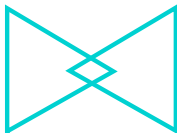


The theoretical development underpinning PEPA has focused on the interaction between the process algebra and the underlying mathematical structure, the Markov process.

This work can be broadly categorised into three areas:

- ▶ Designing the language
- ▶ Manipulating models
- ▶ Solving models and deriving measures

Designing the language



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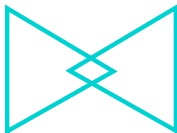
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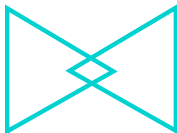


Designing the language



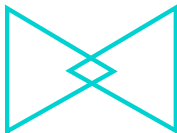
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Designing the language



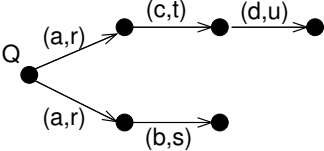
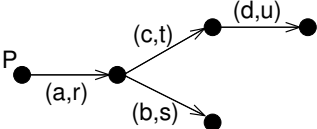
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- ▶ In PEPA each component has a bounded capacity to carry out activities of any particular type, determined by the apparent rate for that type.
- ▶ Synchronisation, or cooperation cannot make a component exceed its bounded capacity.
- ▶ Thus the apparent rate of a cooperation is the **minimum** of the apparent rates of the co-operands.

Equivalence Relations

In process algebra equivalence relations are defined based on the notion of **observability**:

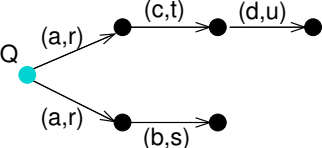
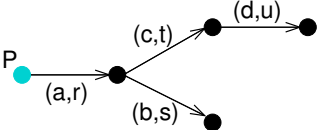
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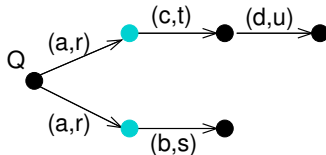
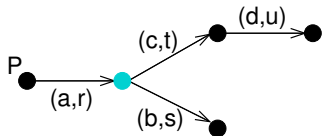
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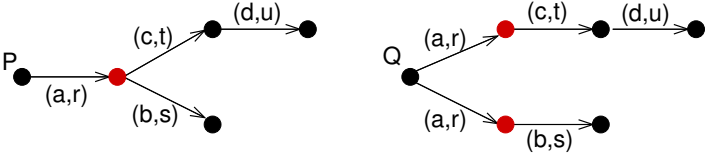
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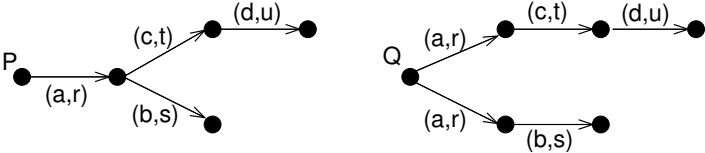
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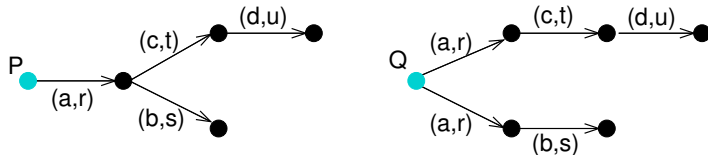
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In PEPA **observation** is assumed to include the ability to record **timing** information over a number of runs.

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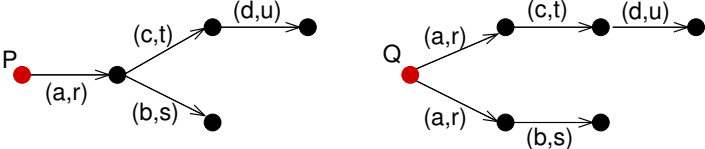
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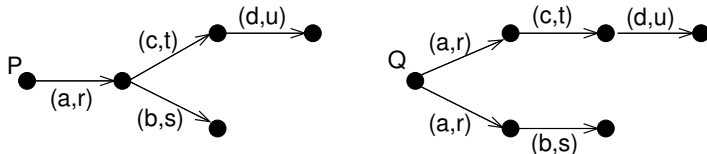
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Equivalence Relations

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In PEPA observation is assumed to include the ability to record timing information over a number of runs.

The resulting equivalence relation is a **bisimulation** in the style of Larsen and Skou, and coincides with the Markov process notion of **lumpability**.

Model Manipulation

Model simplification: use a **model-model** equivalence to substitute one model by another which is more attractive from a solution point of view, e.g. smaller state space, special class of model, etc.

Model Manipulation

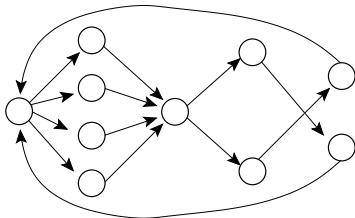
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Model aggregation: use a **state-state** equivalence to establish a partition of the state space of a model, and replace each set of states by one **macro-state**, i.e. take a different stochastic representation of the same model.

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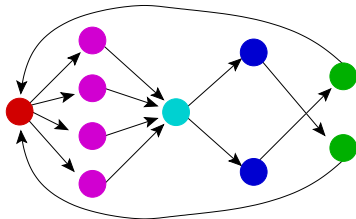
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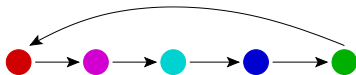
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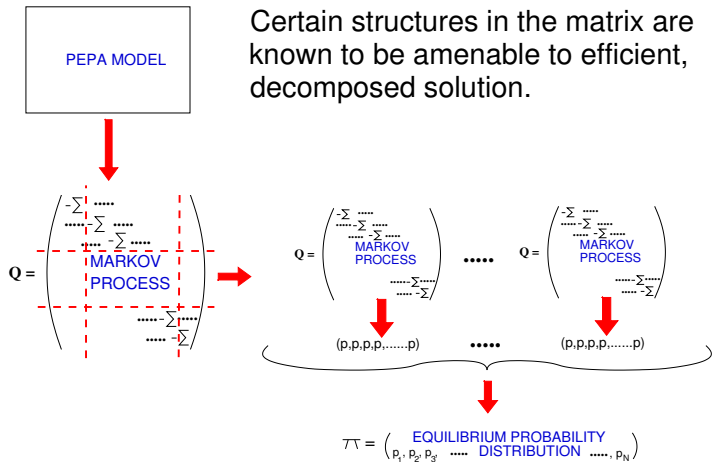
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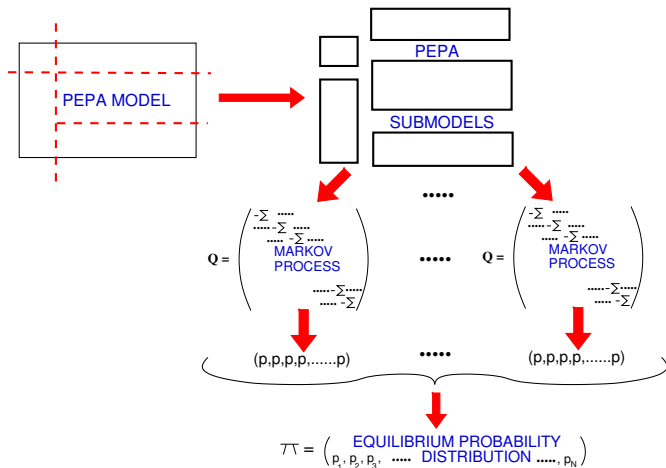
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Characterising efficient solution



Characterising efficient solution



Finding the corresponding structures in the process algebra means that these techniques can be applied automatically, before the monolithic matrix is formed.

Applications

- ▶ Developing models of real applications has always been an integral part of the PEPA project.
- ▶ This allows us to demonstrate to ourselves and others that the theory we have developed is useful.
- ▶ It is also a valuable source of inspiration for new theory and future directions.

PEPA Case Studies (1)

- ▶ Multiprocessor access-contention protocols ([Gilmore, Hillston and Ribaldo, Edinburgh and Turin](#))
- ▶ Protocols for fault-tolerant systems ([Clark, Gilmore, Hillston and Ribaldo, Edinburgh and Turin](#))
- ▶ Multimedia traffic characteristics ([Bowman et al, Kent](#))
- ▶ Database systems ([The STEADY group, Heriot-Watt University](#))
- ▶ Software Architectures ([Pooley, Bradley and Thomas, Heriot-Watt and Durham](#))
- ▶ Switch behaviour in active networks ([Hillston, Kloul and Mokhtari, Edinburgh and Versailles](#))

PEPA Case Studies (2)

- ▶ Locks and movable bridges in inland shipping in Belgium ([Knapen](#), [Hasselt](#))



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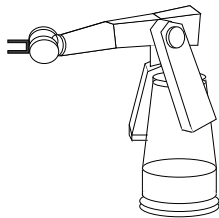
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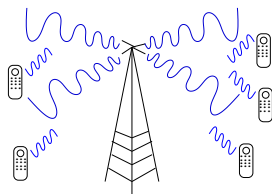
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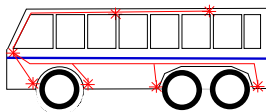
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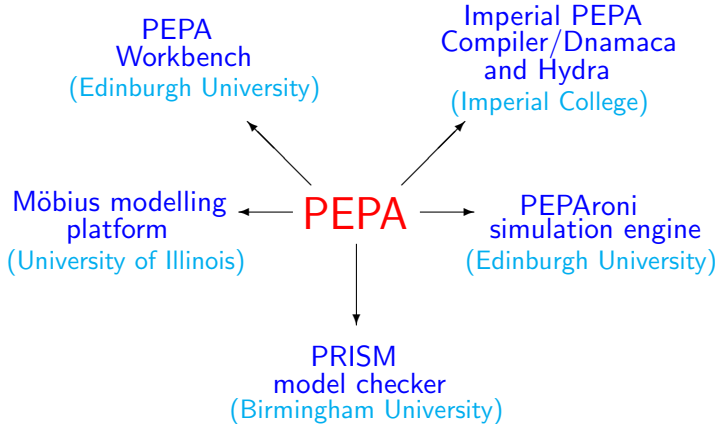


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- ▶ Automotive diagnostic expert systems ([Console, Picardi and Ribaldo, Turin](#))



Tool Support



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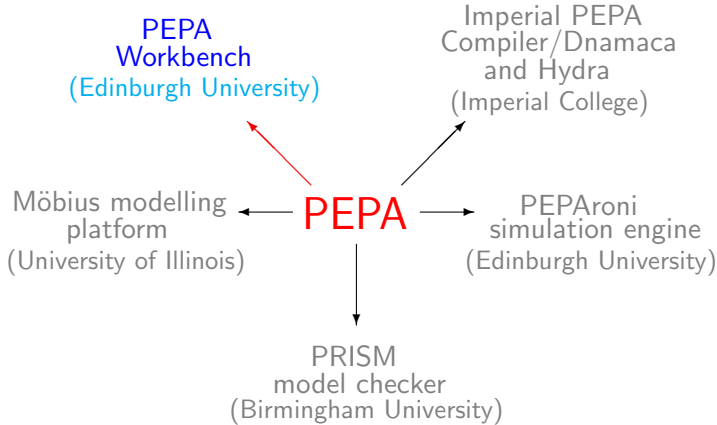
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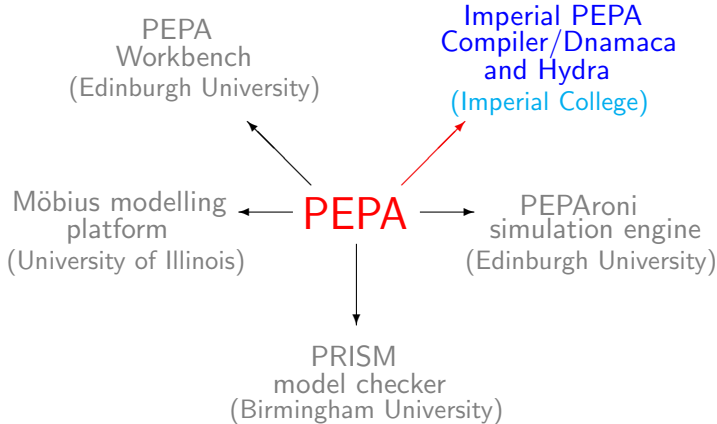
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Future Work

Tool Support



Tool Support



Tuning Systems: from
Composition to
Performance

Jane Hillston. LFCS,
University of Edinburgh.

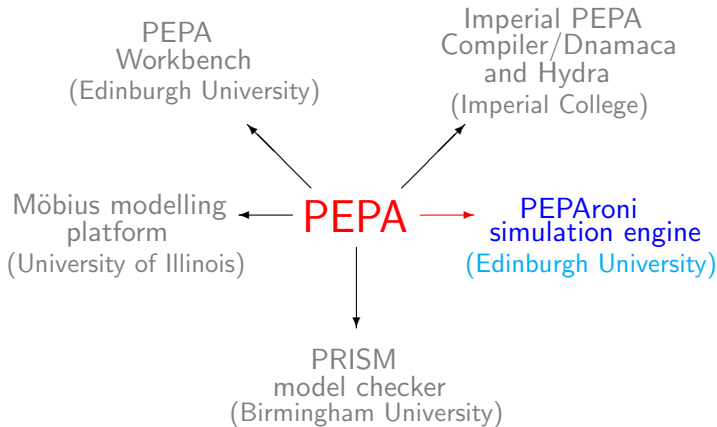
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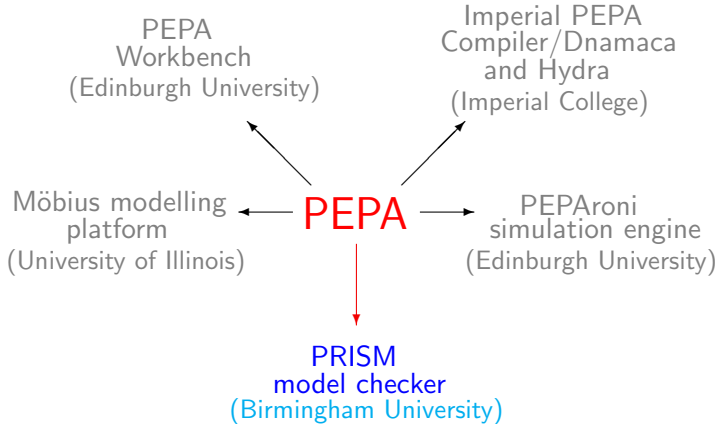
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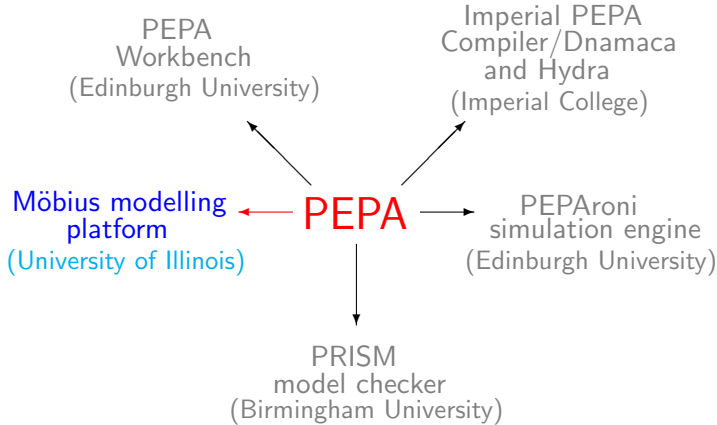
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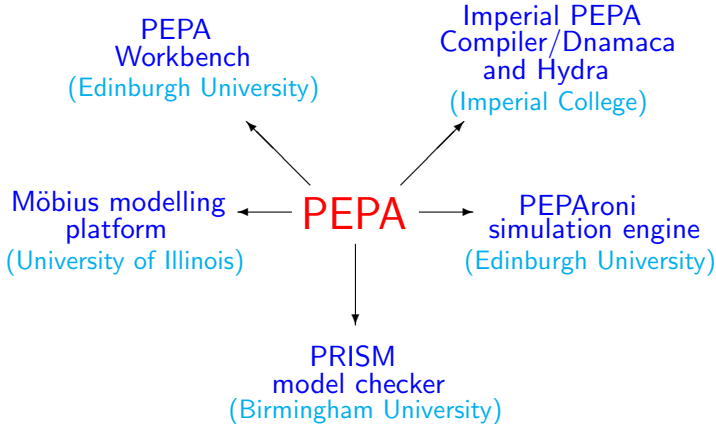
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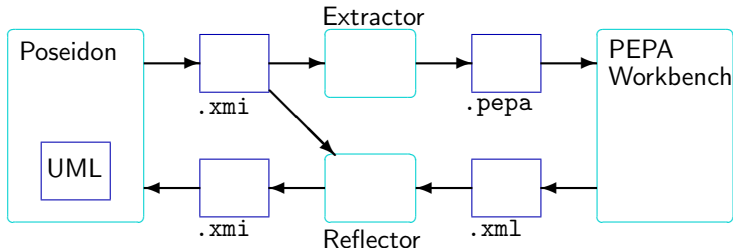
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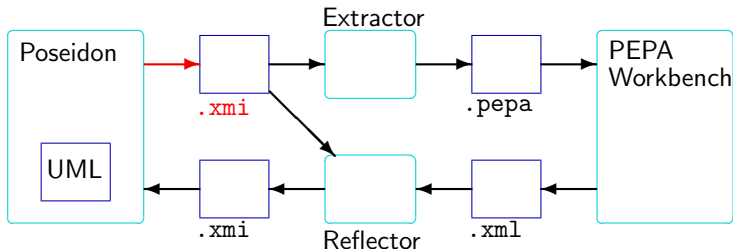
PEPA via UML

In the European-funded **DEGAS** research project we have been investigating ways to make performance modelling using PEPA more accessible to software designers.



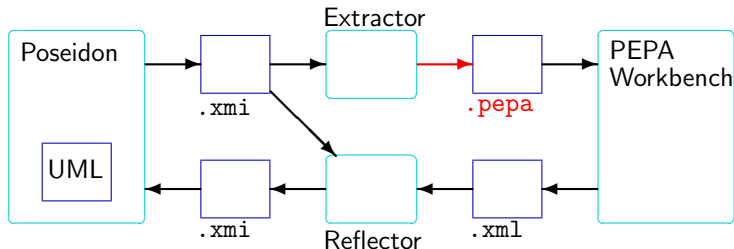
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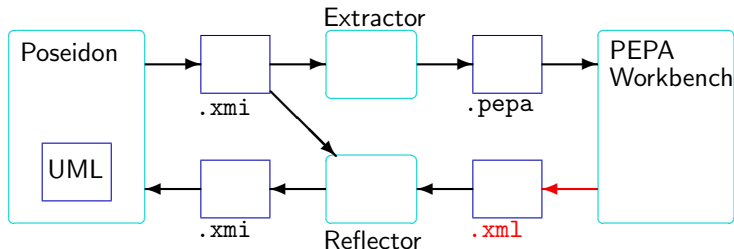
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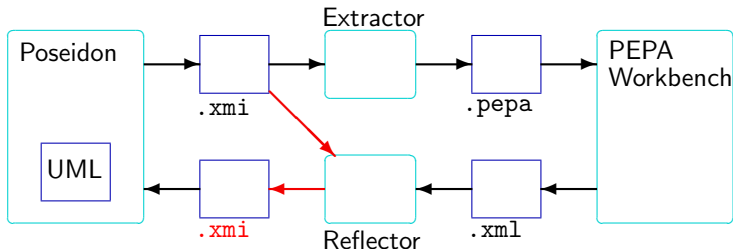
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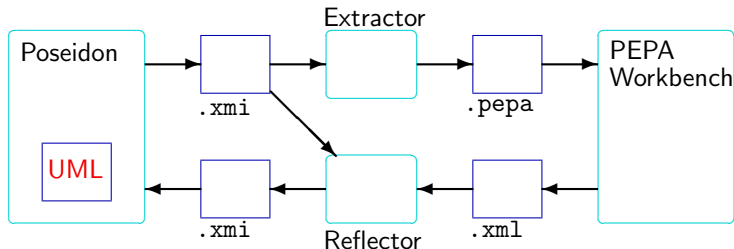
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It is essential that results are reported in terms which make sense to the software designer, i.e. in terms of the original UML model.

Grid Scheduling

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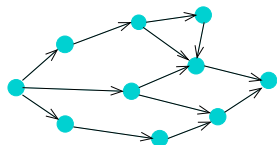
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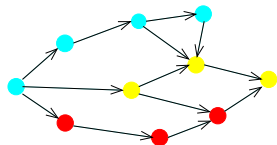
P1

P2

P3

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P1

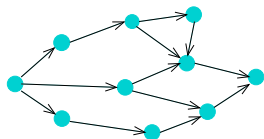
P2

P3

A **schedule** maps tasks to processors

Grid Scheduling

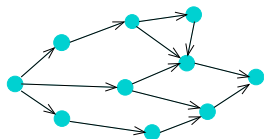
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In a grid processors are **heterogeneous**...

Grid Scheduling

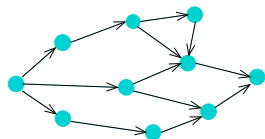
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In a grid processors are heterogeneous and **dynamic**.

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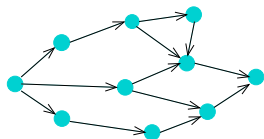


- ▶ Current performance parameters obtained from the **Network Weather Service**.



Grid Scheduling

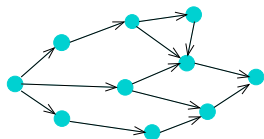
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- ▶ Current performance parameters obtained from the Network Weather Service.
- ▶ Highly **abstract model components** configured to represent different scheduling possibilities.

Grid Scheduling

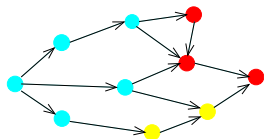
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- ▶ Fast evaluation and comparison of alternatives.

Grid Scheduling

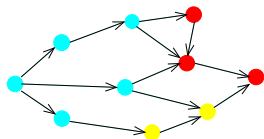
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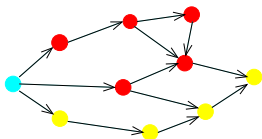


The dynamic nature may mean that tasks have to be re-scheduled during the course of the computation.

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- ▶ Current performance parameters obtained from the Network Weather Service.
- ▶ Highly abstract model components configured to represent different scheduling possibilities.
- ▶ Fast evaluation and comparison of alternatives.

Future Work

- ▶ The state space explosion problem still remains a major challenge.
- ▶ Extending the range of applicability of the modelling language for new application areas.
- ▶ Improving the analysis capabilities of the modelling tools.

New directions

New theory

- ▶ Support of mobility

PEPA nets

New application domains

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New directions

New theory

- ▶ Support of mobility
- ▶ Focus on performance measures

Use of more expressive distributions and logics

New application domains

New directions

New theory

- ▶ Support of mobility
- ▶ Focus on performance measures
- ▶ New mathematical structures

Ordinary, random and stochastic differential equations

New application domains

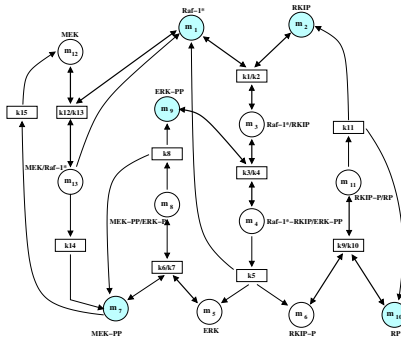
New directions

New theory

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- ▶ Focus on performance measures
- ▶ New mathematical structures

New application domains

- ▶ Biochemical signalling pathways



ERK-MAPK signalling pathway

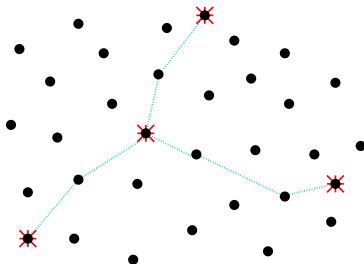
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New application domains

- ▶ Biochemical signalling pathways
- ▶ Wireless sensor networks
- ▶ Security and timing attacks



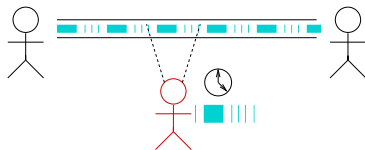
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Acknowledgements

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Coauthors:

- ▶ Anne Benoit, Linda Brodo, Muffy Calder, Catherine Canevet, Murray Cole, Graham Clark, Pierpaolo Degano, Susanna Donatelli, Stephen Gilmore, Zully Grant-Duff, Valentin Haenel, Peter Harrison, Robert Holton, Leïla Kloul, Vassilis Mertsiotakis, Amdjed Mokhtari, Corrado Priami, Matthew Prowse, Laura Recalde, Michael Rettelbach, Marina Ribaudó, Manuel Silva, Perdita Stevens, Joanna Tomasik, Nigel Thomas.

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Tool developers:

- ▶ Ashok Argent-Katwala, Jeremy Bradley, Graham Clark, Nick Dingle, Ross Duncan, Stephen Gilmore, Valentin Haenel, Jon Hunter, Will Knottenbelt, Dave Parker, Matthew Prowse, Fotis Stathopoulos, Feng Wan.

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- ▶ Ashok Argent-Katwala, Howard Bowman, Jeremy Bradley, Jeremy Bryans, Aleksiy Burdakov, Luca Console, Lia Deotto, Nick Dingle, Kerstin Eder, James Edwards, Amani El-Rayes, Peter Harrison, Leïla Kloul, Luk Knapen, Will Knottenbelt, Marta Kwiatkowska, Ting Ting Lee, Steve Minton, Pete Mitton, Friedger Müffke, Gethin Norman, Claudia Picardi, Rob Pooley, Kris Powell, Marina Ribaudó, Nigel Thomas, David Thornley, Fabrice Valois.

Thank you

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