Quantitative Analysis of Collective Adaptive Systems

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LFCS, University of Edinburgh

23rd January 2018

Outline

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Collective Systems

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Collective Systems

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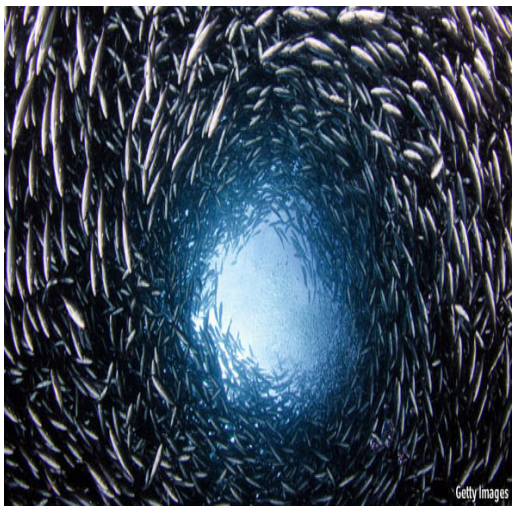
in the natural world ....



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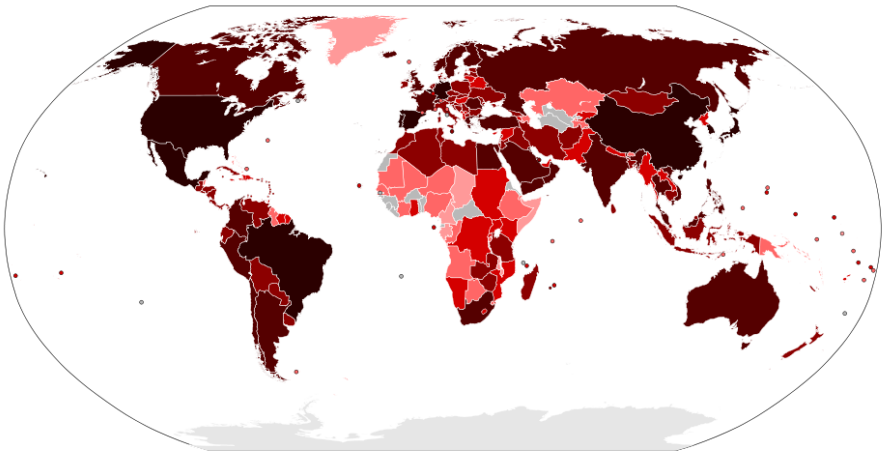

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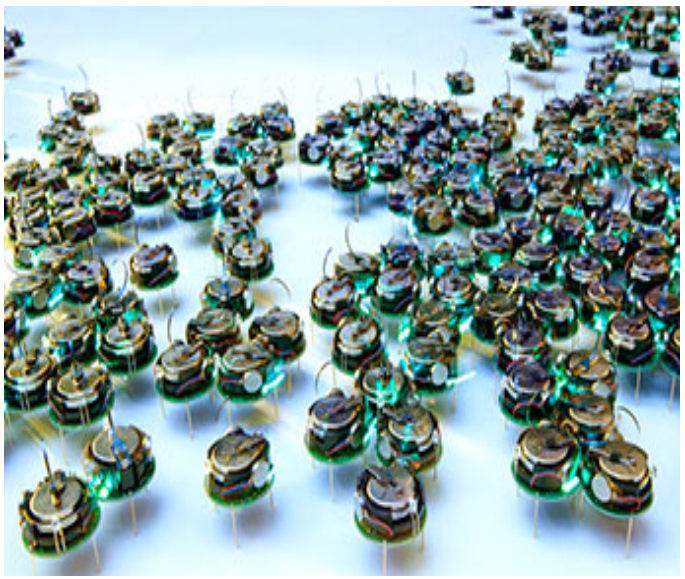

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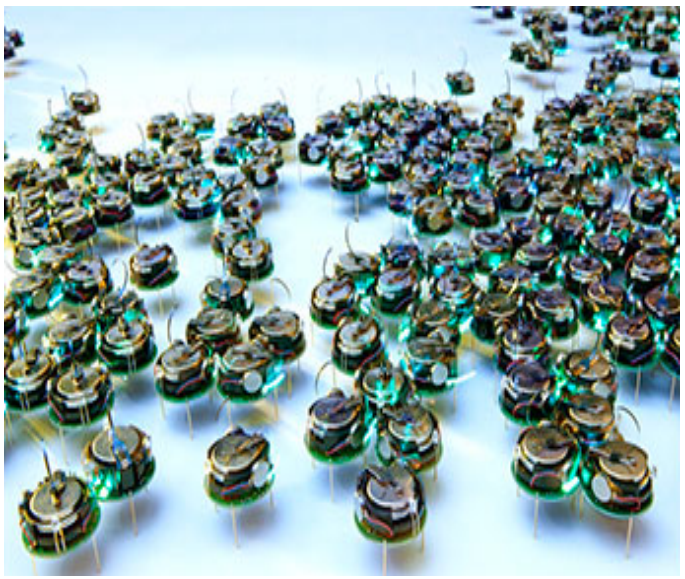

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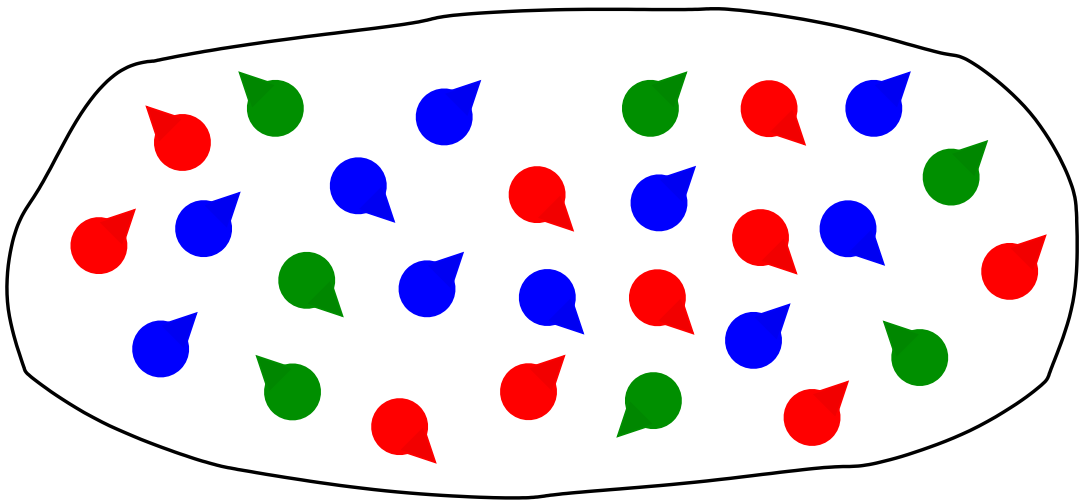
Most of these systems are also adaptive to their environment

Collective Adaptive Systems

From a computer science perspective these systems can be viewed as being made up of a large number of interacting entities.

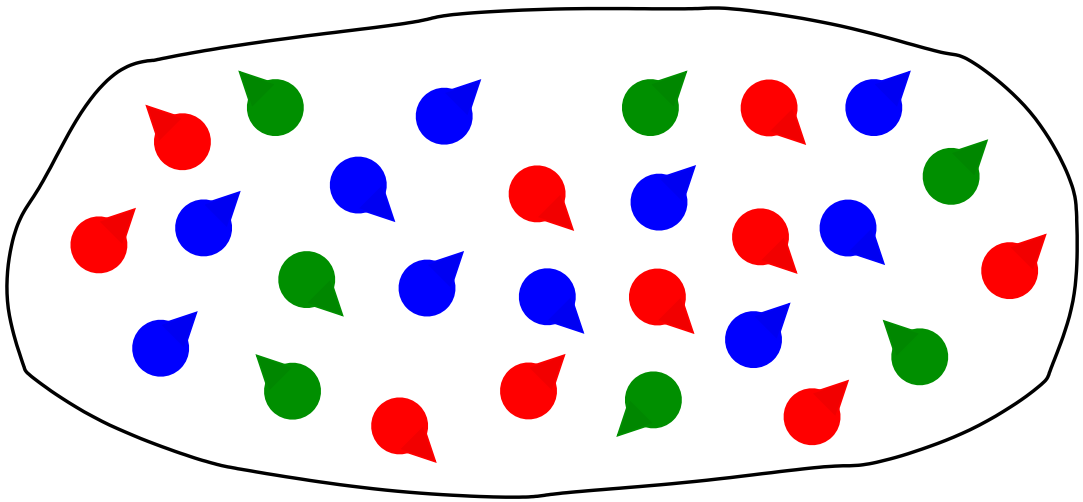


Each entity may have its own properties, objectives and actions.

At the system level these combine to create the collective behaviour.

Collective Adaptive Systems

The behaviour of the system is thus dependent on the behaviour of the individual entities.



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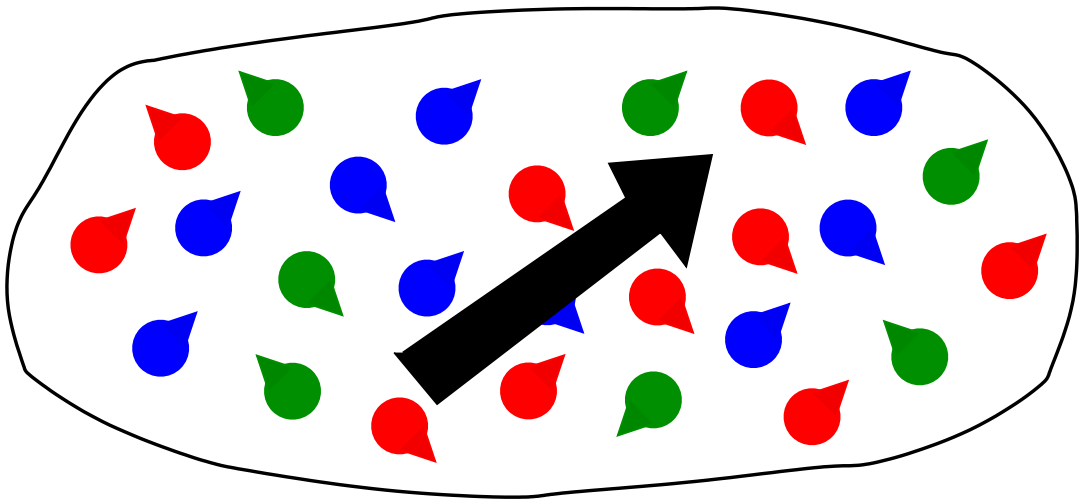

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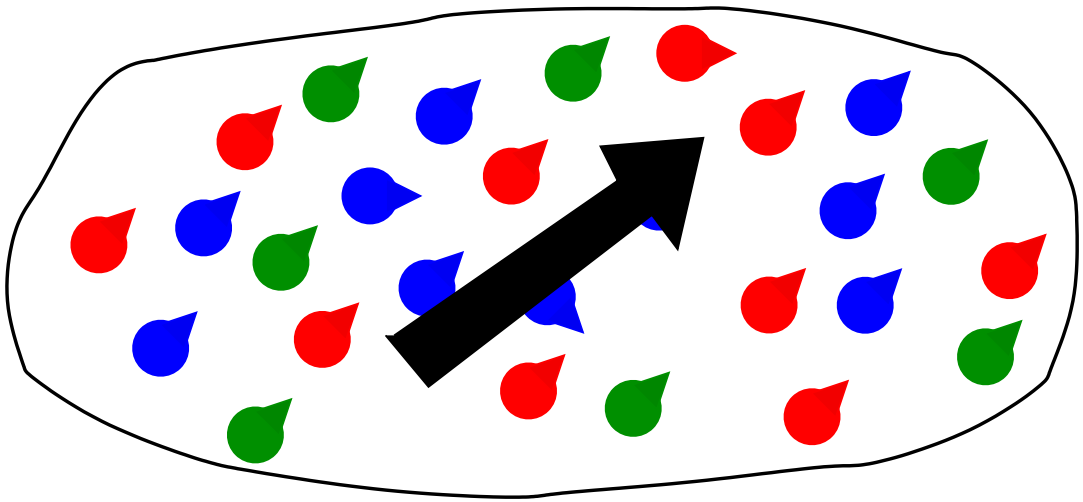
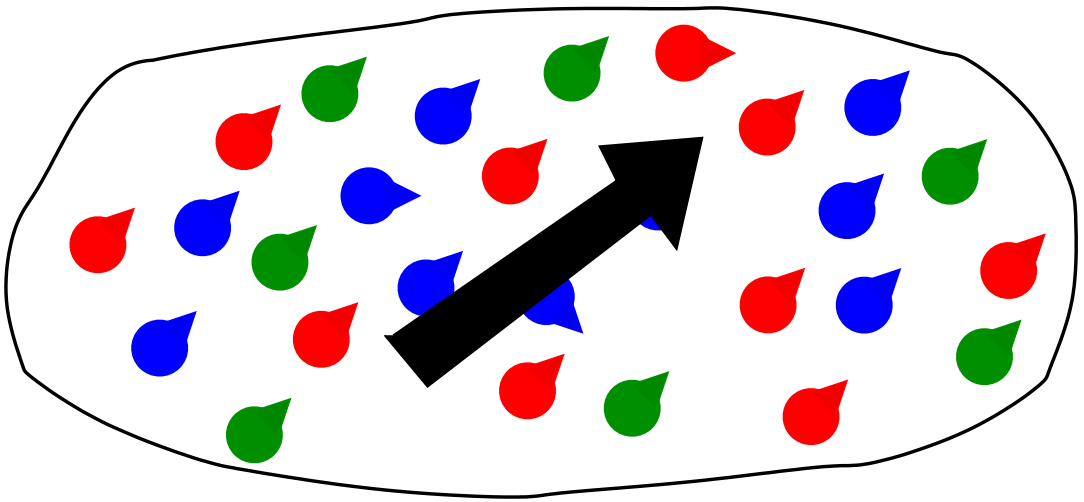


And the behaviour of the individuals will be influenced by the state of the overall system.

Collective Adaptive Systems

Such systems are often embedded in our environment and need to operate without centralised control or direction.



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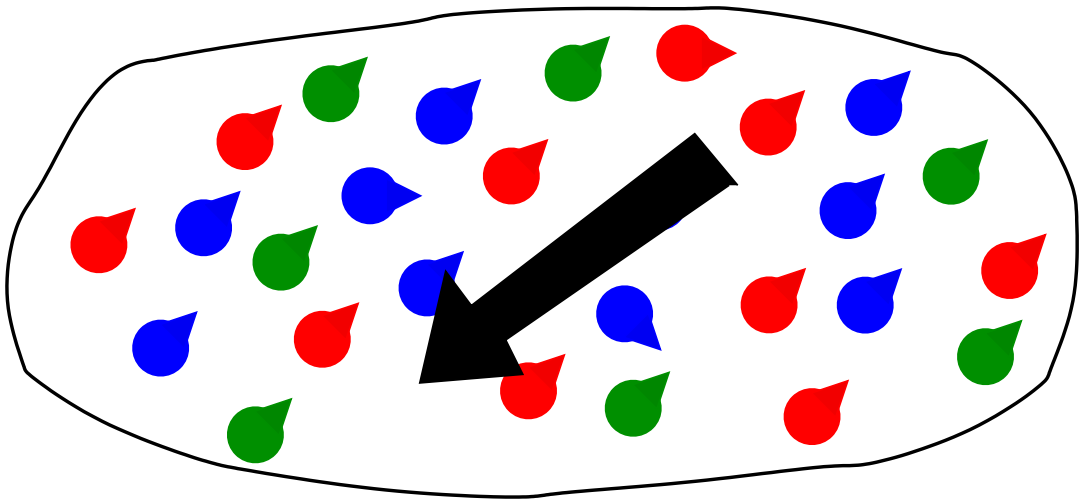

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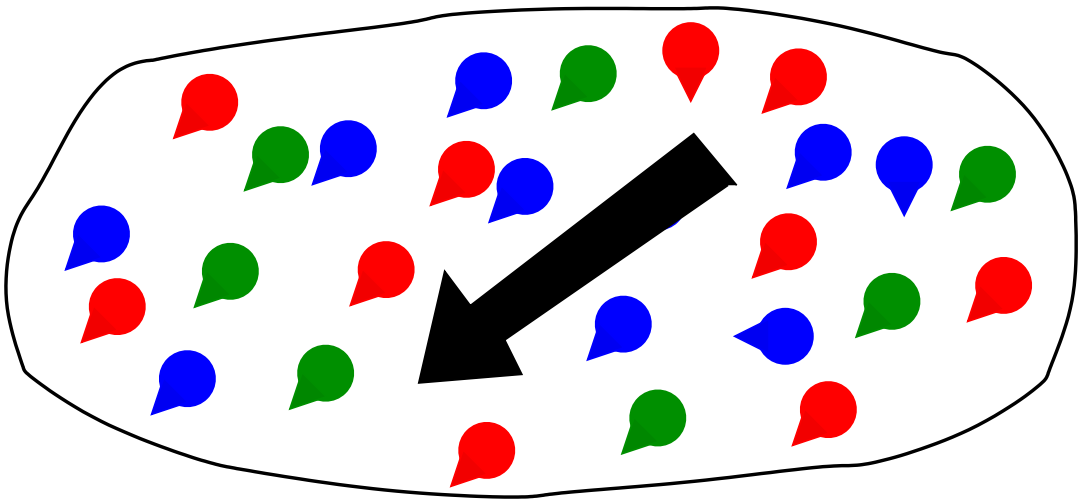

Moreover when conditions within the system change it may not be feasible to have human intervention to adjust behaviour appropriately.

Thus systems must be able to autonomously adapt.

The Informatic Environment

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For instance, may examples of such systems can be found in components of Smart Cities, such as smart urban transport and smart grid electricity generation and storage.

Performance Modelling

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This often involves a trade-off between the interests of the users, who want more resource, and the interests of system operators, who want to minimise the resource.

Performance Modelling for Smart Cities

Capacity planning

- How many buses do I need to maintain service at peak time in a smart urban transport system?

Performance Modelling for Smart Cities

System Configuration

- What capacity do I need at bike stations to minimise the movement of bikes by truck?

Performance Modelling for Smart Cities

System Tuning

- What strategy can I use to maintain supply-demand balance within a smart electricity grid?

Quantitative Modelling

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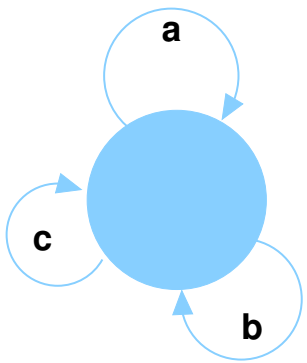

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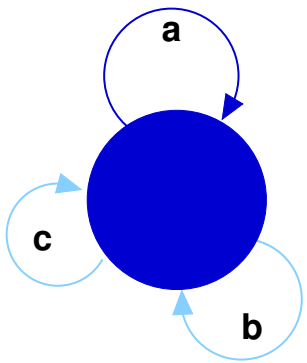

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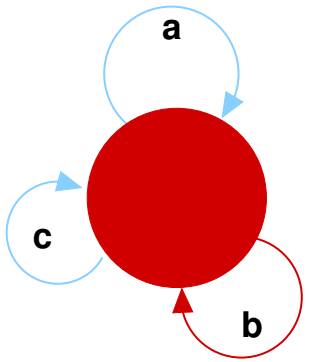

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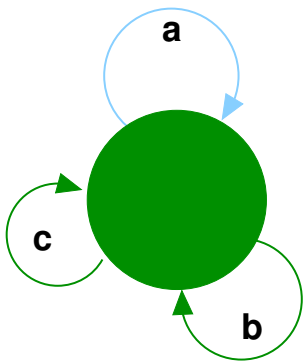

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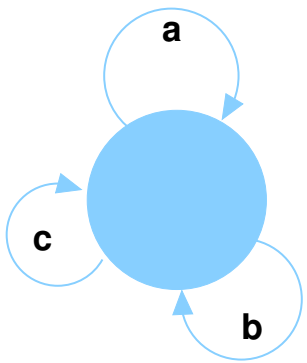

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Primary examples include:

- Stochastic Petri Nets and
- Stochastic Process Algebras.

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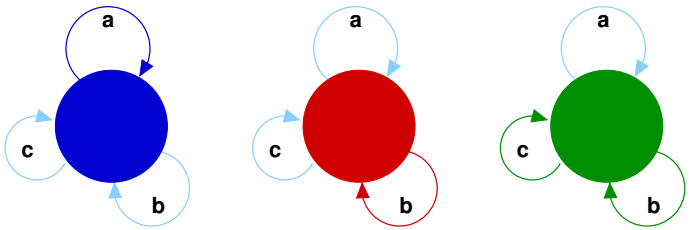
J.Hillston, A Compositional Approach to Performance Modelling, CUP, 1995

Outline

Solving discrete state models

Under the SOS semantics a SPA model is mapped to a CTMC with global states determined by the local states of all the participating components.



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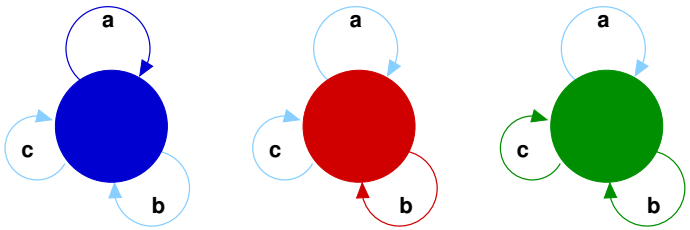

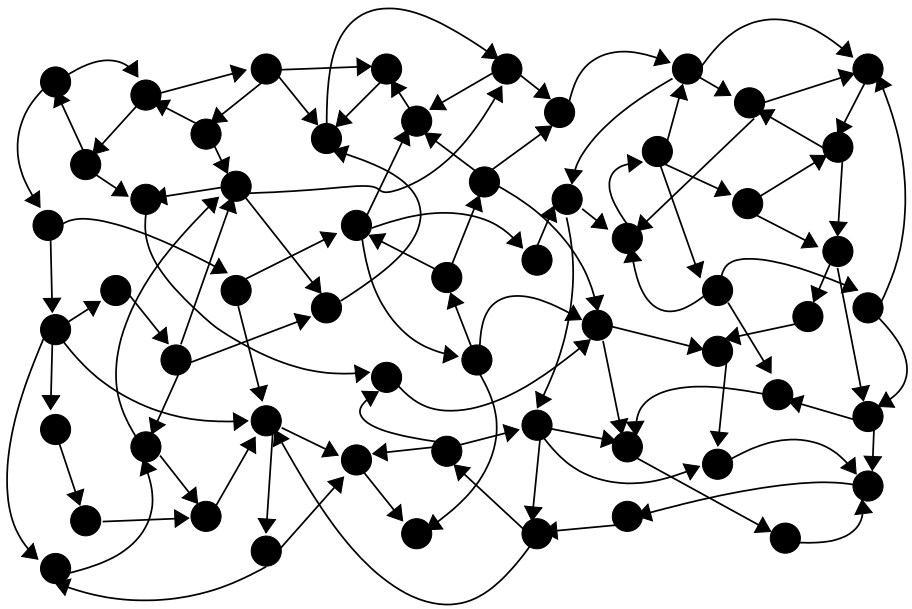

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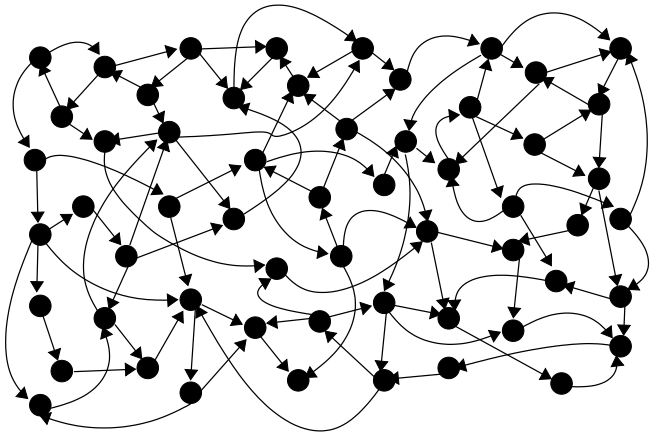

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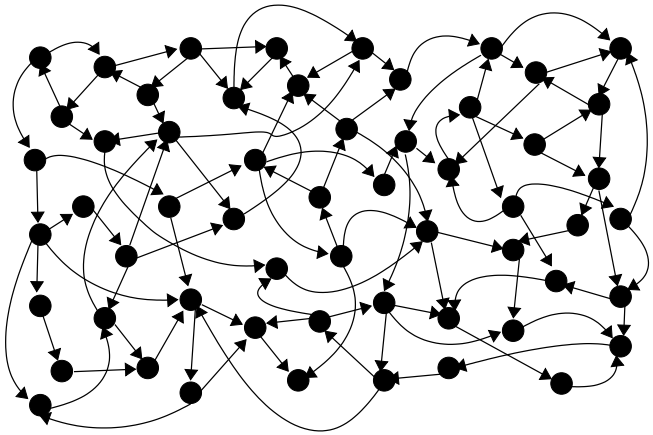

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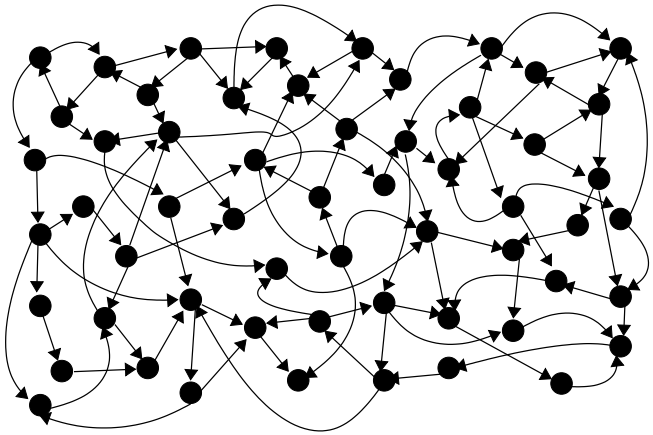

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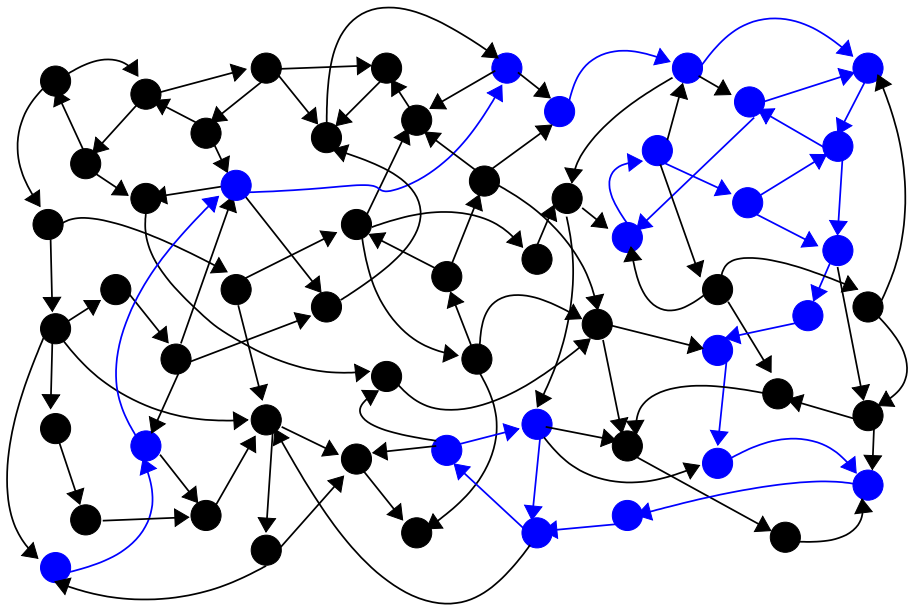
$$\pi(t) = (\pi_1(t), \pi_2(t), \ldots, \pi_N(t))$$

$$\pi(\infty)Q = 0$$

Solving discrete state models

Alternatively they may be studied using stochastic simulation. Each run generates a single trajectory through the state space. Many runs are needed in order to obtain average behaviours.



Challenges for modelling CAS

The compositional framework provided by stochastic process algebras are well suited to modelling collective behaviour but leave a number of challenges:

- Richer forms of interaction
- The influence of space on behaviour
- Capturing adaptivity

Richer forms of interaction

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Languages like SCEL offer these richer communication patterns, with components which include a knowledge store which can be manipulated by other components and attribute-based communication.

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For quantitative modelling there is a tension between keeping state spaces tractable and capturing local knowledge in agents.

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It would be preferable to model space explicitly but this poses significant challenges both for model expression and model solution.

Capturing adaptivity

- Existing process algebras, tend to work with a fixed set of actions for each entity type.
- Some stochastic process algebras allow the rate of activity to be dependent on the state of the system.
- But for truly adaptive systems there should also be some way to identify the goal or objective of entity in addition to its behaviour.

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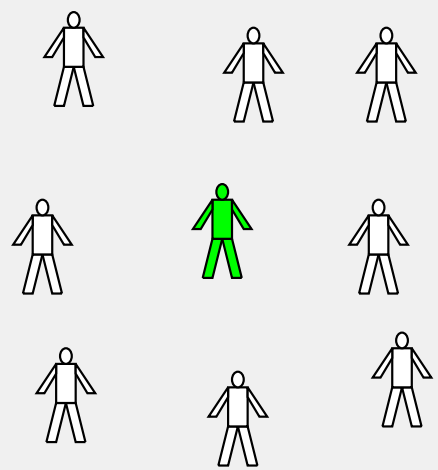

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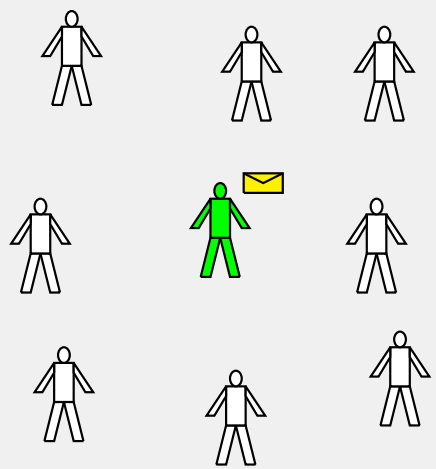

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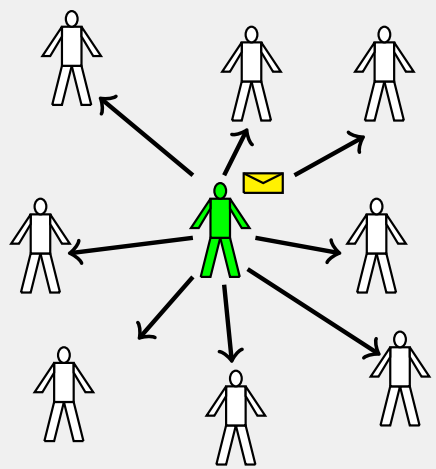

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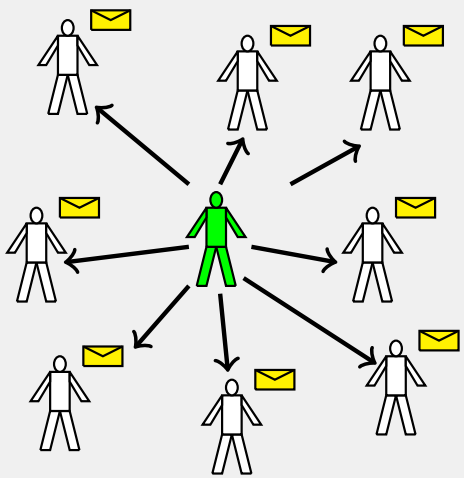

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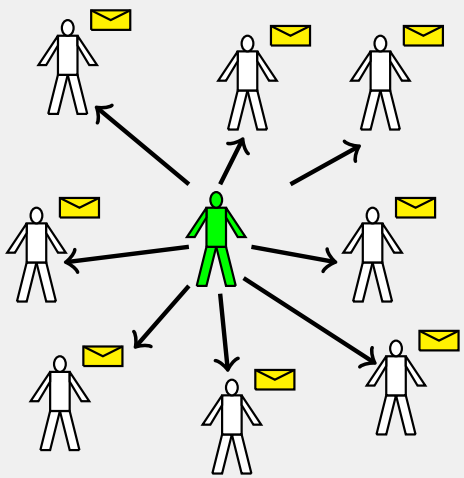
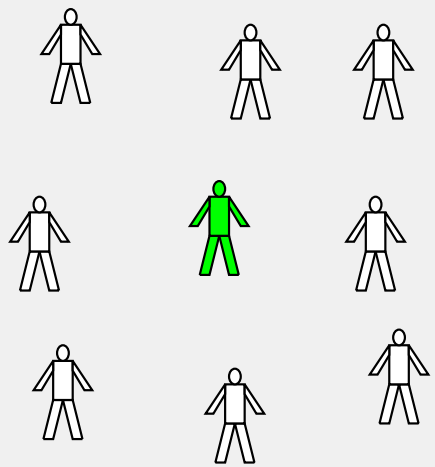


Collecting: 1-to-1



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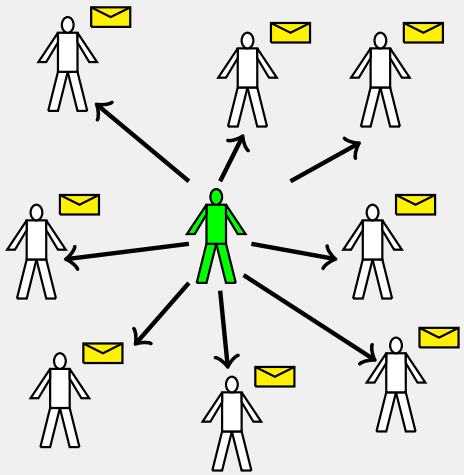

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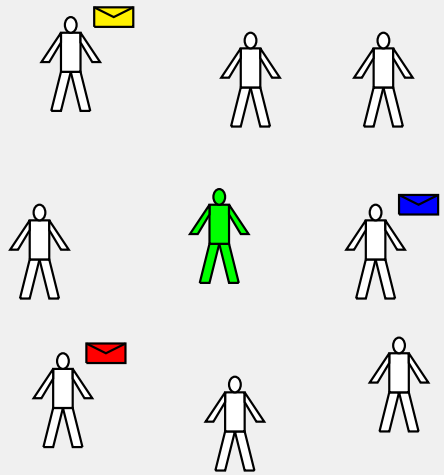
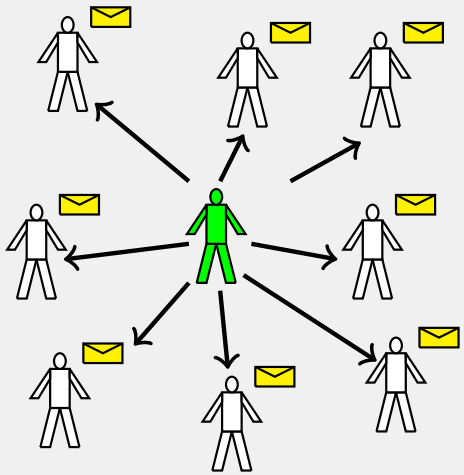


Interaction patterns in CAS

Typically, CAS exhibit two kinds of interaction pattern:

1. **Spreading**: one agent spreads relevant information to a given group of other agents
2. **Collecting**: one agent changes its behaviour according to data collected from one agent belonging to a given group of agents.

Spreading: 1-to-many



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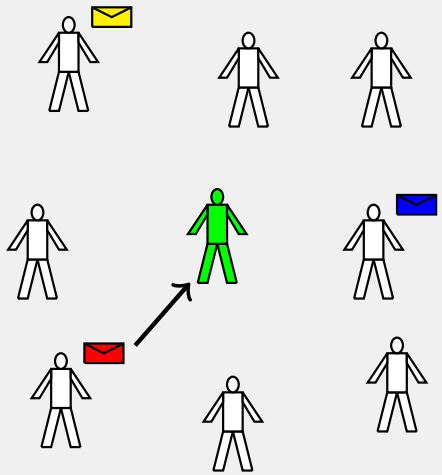

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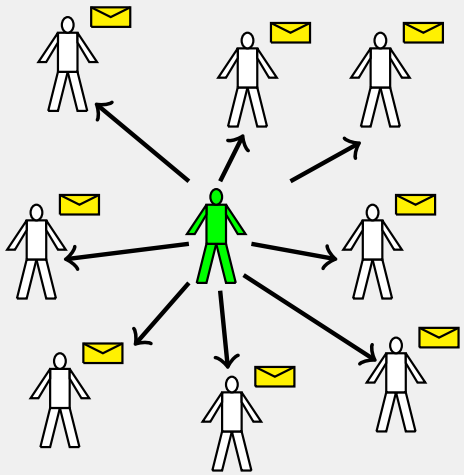

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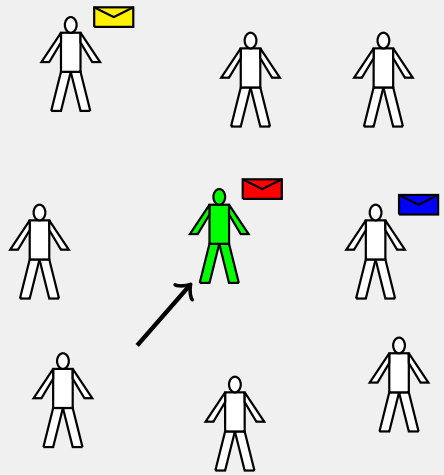

CAS: CARMA perspective

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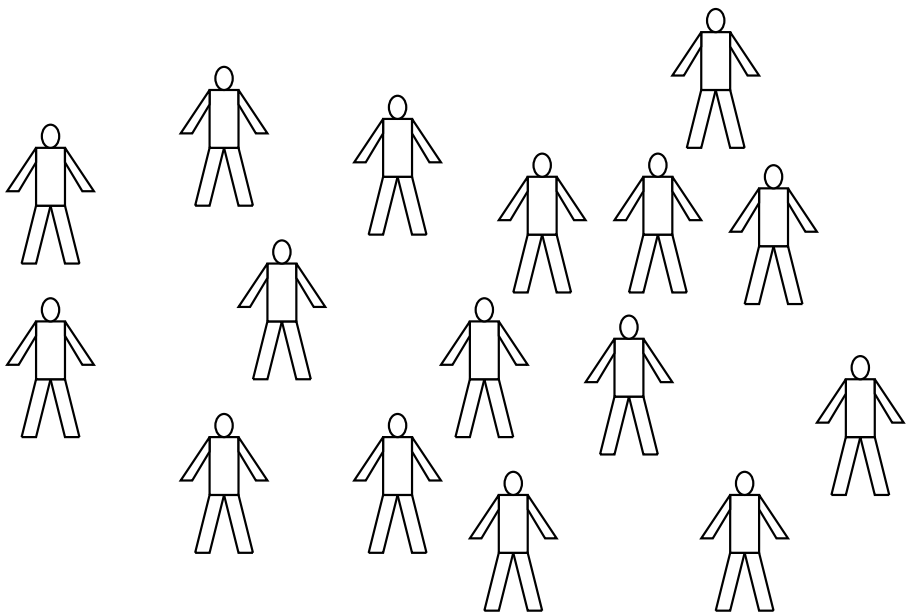

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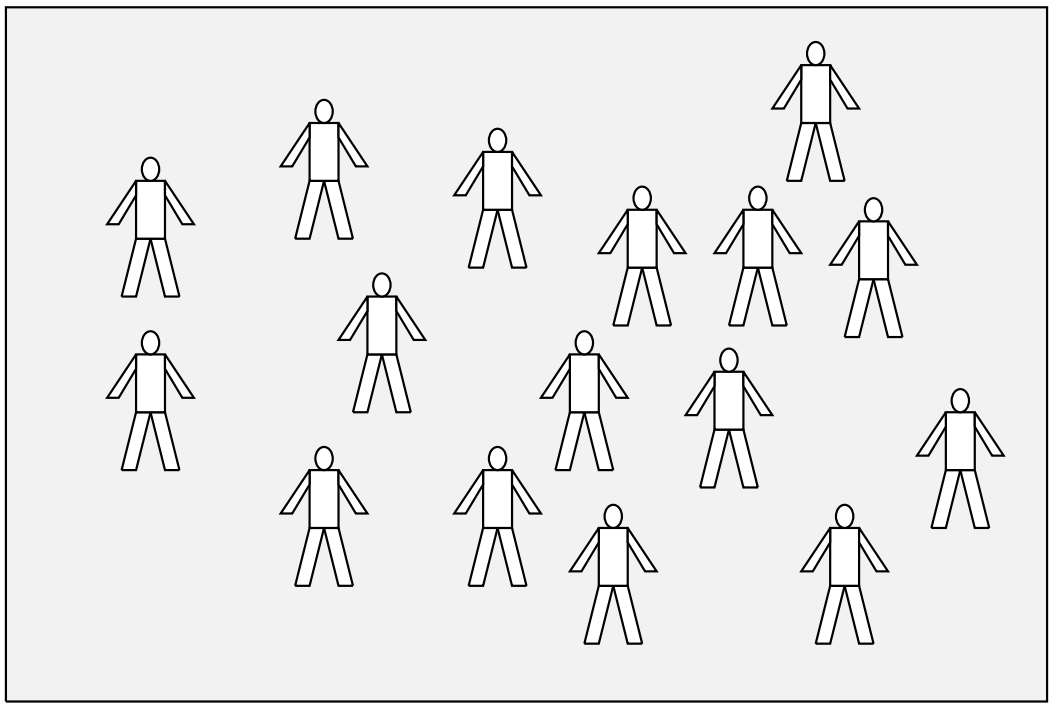
Collective **Environment**



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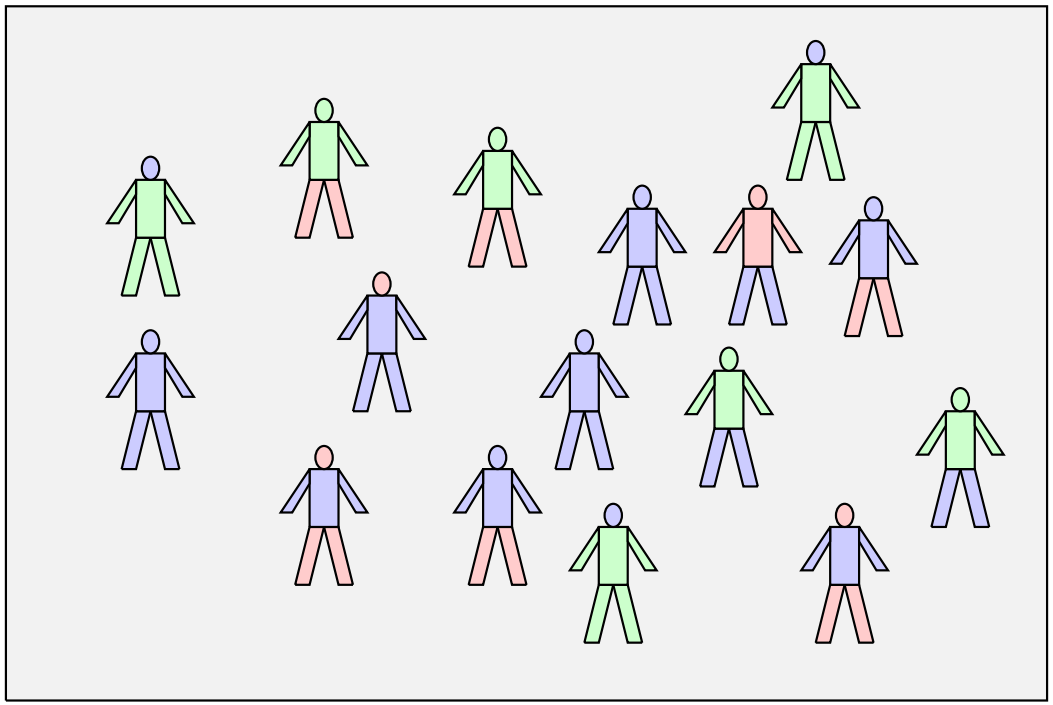

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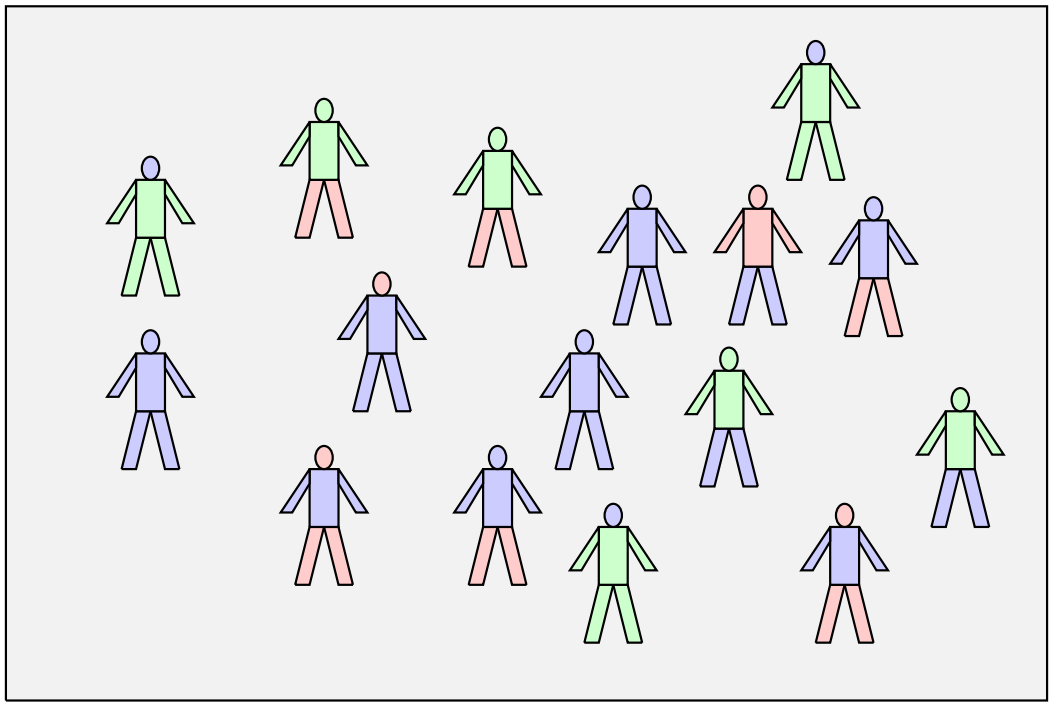

Processes are referenced via their attributes!

Collective Adaptive Resource-sharing Markovian Agents

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- models the behavioural part of a system

Environment...

- models the rules intrinsic to the context where agents operate;
- mediates and regulates agent interactions.

Components

Agents in CARMA are defined as components C of the form (P, γ) where...

- P is a process, representing agent behaviour;
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The participants of an interaction are identified via predicates...

- the counterpart of a communication is selected according its properties

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The execution of an action takes an exponentially distributed time; the rate of each action is determined by the environment.

Interaction primitives

Syntax

$$
\begin{array}{rcll}
act & ::= & \alpha^{\star}[\pi]\langle\vec{e}\rangle\sigma & \text{Broadcast output} \\
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- α is an action type;
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- σ is the effect of the action on the store.

Updating the store

After the execution of an action, a process can update the component store:

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Remark:

- Processes running in the same component can implicitly interact via the local store;
- Updates are instantaneous.

More on synchronisation

Predicates regulating broadcast/unicast inputs can refer also to the received values.

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Example:

A value greater than 0 is expected from a component with a *trust_level* less than 3:

$$\alpha^{\star}[(x > 0) \wedge (\mathit{trust\_level} < 3)](x)\sigma.P$$

Examples of interactions...

Broadcast synchronisation:

$$
\begin{aligned}
&(\ \mathsf{stop}^{\star}[\mathsf{bl} < 5\%]\langle v\rangle\sigma_1.P\ ,\{\mathit{role} = \text{“}\mathit{master}\text{”}\}) \parallel \\
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$$\Downarrow$$

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  - an evolution rule ρ that regulates component interactions (receiving probabilities, action rates,...).

Example: Smart Taxi System

System description:

- We consider a set of taxis operating in a city, providing service to users;
- Both taxis and users are modelled as components.
- The city is subdivided into a number of patches arranged in a grid over the geography of the city.
- The users arrive randomly in different patches, at a rate that depends on the specific time of day.
- After arrival, a user makes a call for a taxi and then waits in that patch until they successfully engage a taxi and move to another randomly chosen patch.
- Unengaged taxis move about the city, influenced by the calls made by users.

J.Hillston and M.Loreti. Specification and analysis of open-ended systems with CARMA. In LNCS 9068, 2015.

Taxis and Users: stores

Both kinds of component use the local store to publish the relevant data that will be used to represent the state of the agent.

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User processes

Users

process $User =$

$$\begin{aligned} & Wait : \mathsf{call}^{\star}[\top]\langle \mathsf{my.loc}.x, \mathsf{my.loc}.y\rangle.Wait \\ & \quad + \\ & \quad \mathsf{take}[\mathsf{loc}.x == \mathsf{my.loc}.x \ \wedge \ \mathsf{loc}.y == \mathsf{my.loc}.y] \\ & \qquad \langle \mathsf{my.dest}.x, \mathsf{my.dest}.y\rangle.\mathbf{kill} \end{aligned}$$

endprocess

Taxi processes

Taxis

process $Taxi =$

$F : \mathsf{call}^{\star}[(\mathsf{my.loc.x} \neq \mathsf{posx}) \wedge \mathsf{my.loc.y} \neq \mathsf{posy}](posx, posy)$
$\{\mathsf{dest} := [x := posx, y := posy]\}.G$

$+$

$\mathsf{take}[\top](posx, posy)$
$\{\mathsf{dest} := [x := posx, y := posy], \mathsf{occupancy} := 1\}.G$

$G : \mathsf{move}^{\star}[\bot]\langle\circ\rangle$
$\{\mathsf{loc} := \mathsf{dest}, \mathsf{dest} := [x := 3, y := 3], \mathsf{occupancy} := 0\}.F$

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Modelling arrivals

The Arrivals process has a single attribute loc.

Arrivals process for users

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The precise role of this process will be clear when the environment is described.

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The environment manages these aspects of system behaviour, and others in the evolution rule.

The evolution rule ρ

ρ is a function, dependent on current time, the global store and the current state of the collective, returns a tuple of functions $\varepsilon = \langle \mu_p, \mu_w, \mu_r, \mu_u \rangle$ known as the evaluation context

- $\mu_p(\gamma_s, \gamma_r, \alpha)$: the probability that a component with store γ_r can receive a broadcast message α from a component with store γ_s;
- $\mu_w(\gamma_s, \gamma_r, \alpha)$: the weight to be used to compute the probability that a component with store γ_r can receive a unicast message α from a component with store γ_s;
- $\mu_r(\gamma_s, \alpha)$ computes the execution rate of action α executed at a component with store γ_s;
- $\mu_u(\gamma_s, \alpha)$ determines the updates on the environment (global store and collective) induced by the execution of action α at a component with store γ_s.

Evolution rule: μ_p

Defining the probabilities of broadcast actions

$$\begin{array}{l} \mathsf{prob}\{ \\ \quad \top, \mathsf{call}^\star : \mathsf{global.p_{lost}} \\ \quad \mathsf{default}\ \ 1 \\ \} \end{array}$$

- $\mathsf{call}^\star$ can be missed with a probability p_{lost} defined in the global store.
- All the other interactions occur with probability 1.

Evolution rule: μ_w

Defining the weights of unicast actions

$$
\begin{aligned}
&\mathsf{prob}\{\\
&\quad \top, \mathsf{take} : \mathit{Takeprob}(\mathit{real}(\#\{\mathit{Taxi}[F] \mid \\
&\qquad\qquad (\mathsf{my.loc.x} == \mathsf{sender.loc.x}) \wedge \\
&\qquad\qquad (\mathsf{my.loc.y} == \mathsf{sender.loc.y})\}));\\
&\}
\end{aligned}
$$

- Each taxi receives a user request (take) with a weight that depends on the number of taxis in the patch.

Evolution rule: μ_r

Defining the rates of actions

$$
\begin{aligned}
&\text{rate}\{\\
&\quad \top, \text{take} : \text{global.r}_\text{t}\\
&\quad \top, \text{call}^\star : \text{global.r}_\text{c}\\
&\quad \top, \text{move}^\star : \mathit{Mtime}(\text{now}, \text{sender.loc}, \text{sender.dest}, 6)\\
&\quad \top, \text{arrival}^\star : \mathit{Atime}(\text{now}, \text{sender.loc}, 1)\\
&\quad \text{default } 0\\
&\}
\end{aligned}
$$

While take and call have constant rates, the rates of the actions move and arrival are functions that depend on time, reflecting shifting traffic patterns within the city over the course of a day.

Evolution rule: μ_u

In the taxi example, the arrival of a new user is achieved via the update rule:

Update rule

$$\begin{array}{l}\text{update}\{\\ \quad \top, \text{arrival}^{\star} : \textbf{new}\ \text{User}(\text{sender.loc}, \text{DestLoc}(\text{now}, \text{sender.loc}), \text{Wait})\\ \}\end{array}$$

Measures

To extract data from a system, a CARMA specification also contains a set of measures.

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The number of waiting users at a location

$$\textbf{measure}\ \mathsf{WaitingUser}_{00}[i := 0] = \#\{\mathsf{User}[\mathsf{Wait}] \mid \mathsf{my.loc.x} == 0 \ \wedge\ \mathsf{my.loc.y} == 0\};$$

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The number of taxis relocating

$$\textbf{measure}\ \mathsf{Taxi\_Relocating}[i := 1] = \#\{\mathsf{Taxi}[G] \mid \mathsf{my.occupancy} == 0\};$$

Two Scenarios

We consider a grid of 3×3 patches, i.e., a set of locations (i, j) where $0 \leq i, j \leq 2$, and two different scenarios:

Scenario 1: Users arrive in all the patches at the same rate;

Scenario 2: At the beginning users arrive with a higher probability to the patches at the border of the grid; subsequently, users arrive with higher probability in the centre of the grid.

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These are investigated by placing the same collective in different environments.

Smart Taxi System Collective

collective {
    new : Arrival(0 : 2, 0 : 2);
    new Taxi(0 : 2, 0 : 2, 3, 3, 0, F);
}

Quantitative Analysis

The semantics of CARMA gives rise to a Continuous Time Markov Chain (CTMC).

This can be analysed by

- by numerical analysis of the CTMC for small systems;
- by stochastic simulation of the CTMC;
- by fluid approximation of the CTMC under certain restrictions (particularly on the environment).

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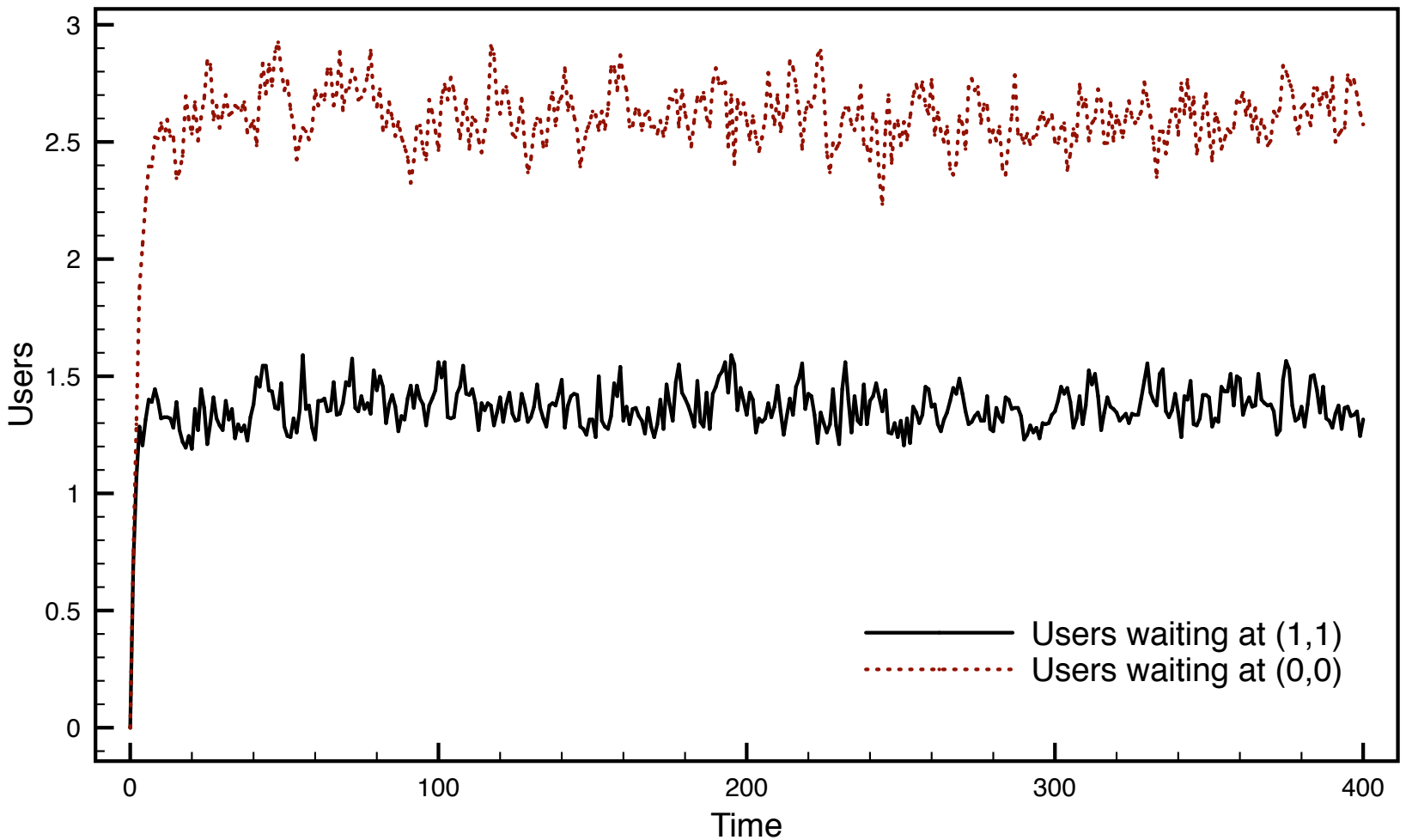
Here we show the results of stochastic simulation.

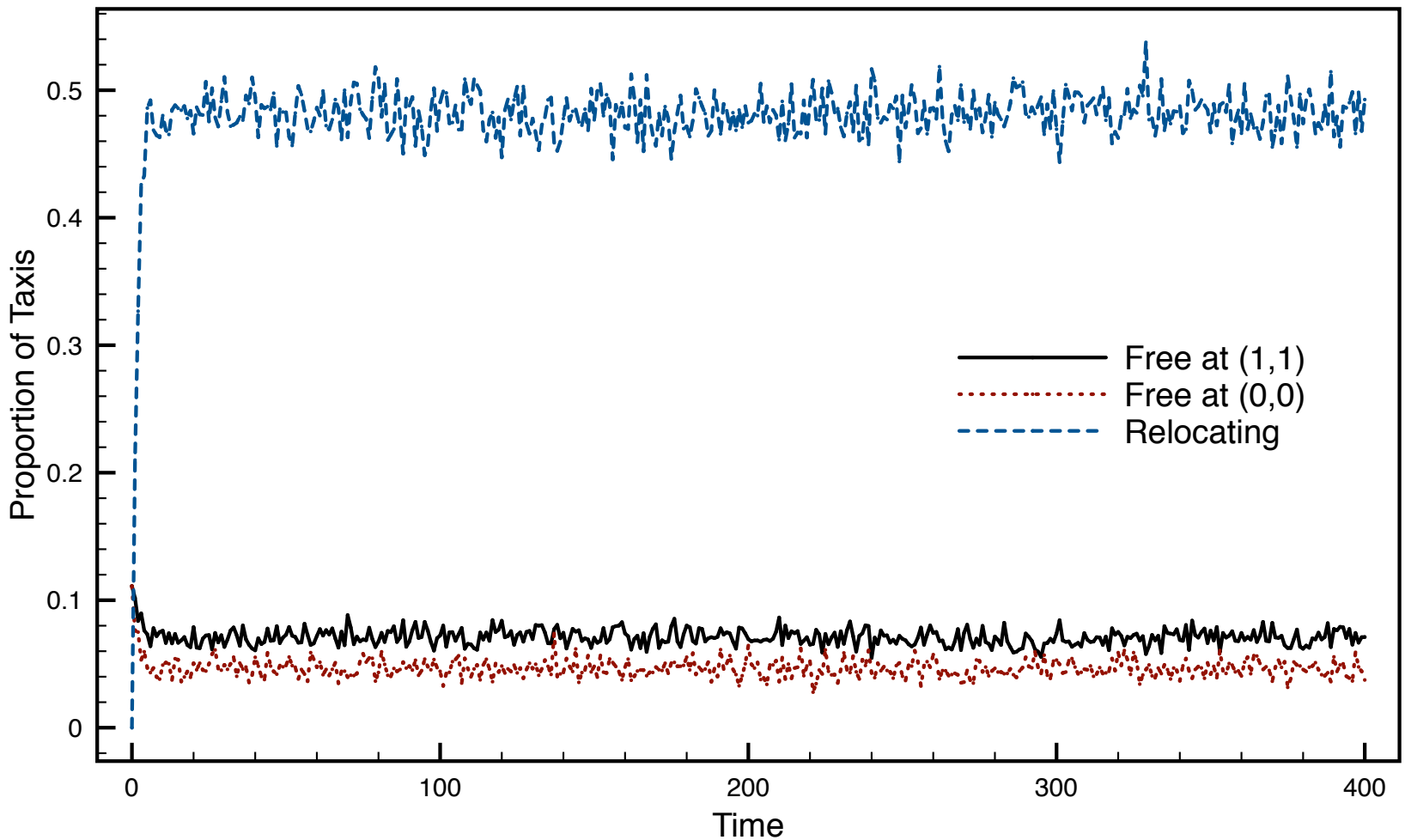
Scenario 1 results

Average number of users waiting at $(1,1)$ and $(0,0)$



Scenario 1 results

Proportion of free taxis at $(1,1)$ and $(0,0)$ and in transit

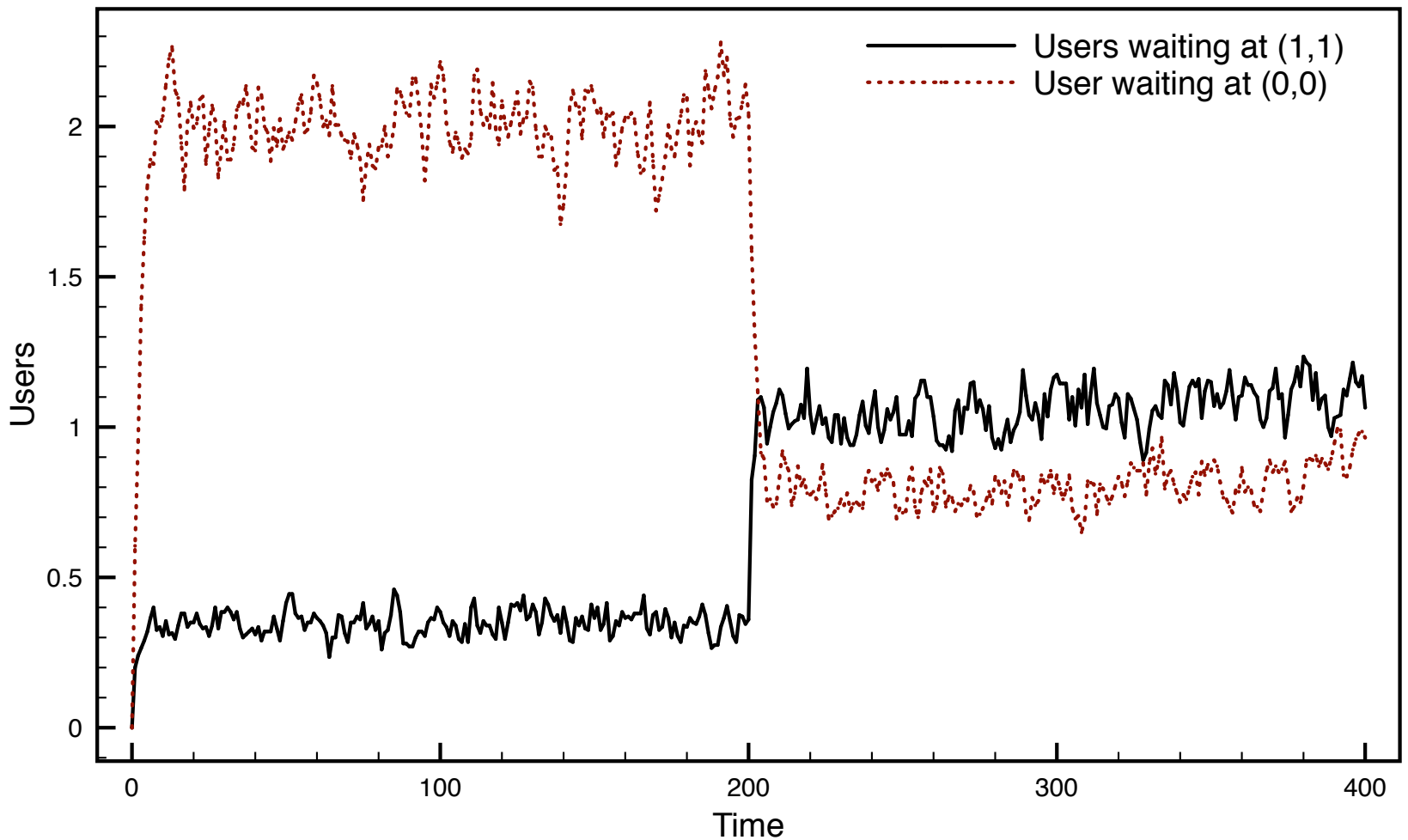


Comments: Scenario 1

- In Scenario 1 after an initial startup period, around 2.5 users are waiting for a taxi in the peripheral location while only 1.5 users are waiting for a taxi in location $(1, 1)$.
- In this scenario a larger fraction of users are delivered to location $(1, 1)$ so soon a larger fraction of taxis are available to collect users at the centre.
- A large fraction of taxis (around 50%) are continually moving between the different patches.

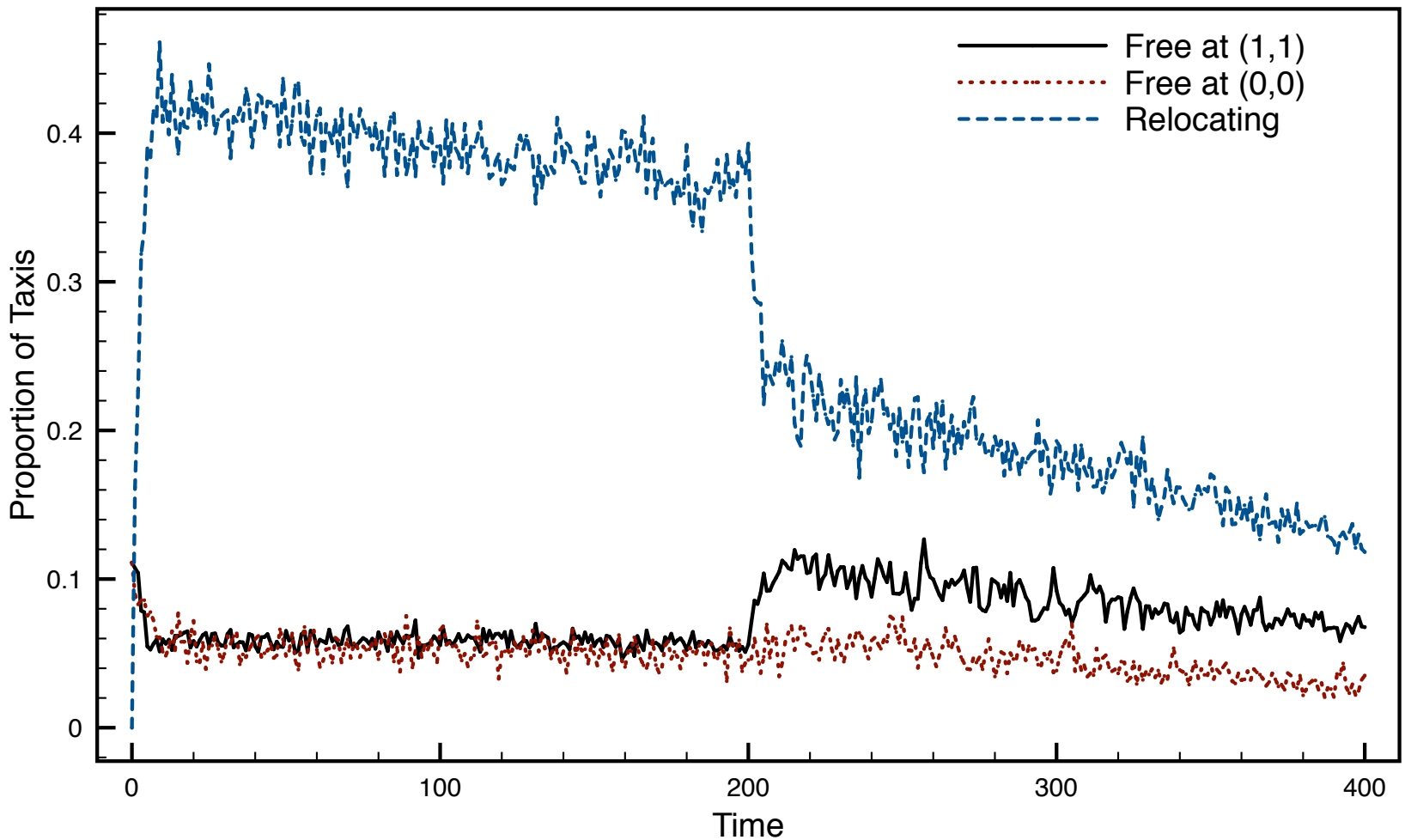
Scenario 2 results

Average number of users waiting at $(1,1)$ and $(0,0)$



Scenario 2 results

Proportion of free taxis at $(1,1)$ and $(0,0)$ and in transit



Comments: Scenario 2

- In Scenario 2 the location of new arrivals depends on the current time:

  $[0, 200)$: 3/4 of users arrive on the border and only 1/4 in the centre;

  $[200, 400)$: 1/4 of users arrive on the border and 3/4 in the centre.

- Results in the first phase are similar to Scenario 1.
- After time 200, the number of users waiting for a taxi in the border decreases below 1 whilst the average waiting for a taxi in the centre increases to just over 1 and the fraction of taxis continually moving is reduced to 20%.

Outline

Concluding remarks

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- Their role within infrastructure, such as within smart cities, make it essential that quantitive aspects of behaviour is taken into consideration, as well as functional correctness.
- The complexity of these systems poses challenges both for model construction and model analysis.
- CARMA aims to address many of these challenges, supporting rich forms of interaction, using attributes to capture explicit locations and the environment to allow adaptivity.

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- Their role within infrastructure, such as within smart cities, make it essential that quantitive aspects of behaviour is taken into consideration, as well as functional correctness.
- The complexity of these systems poses challenges both for model construction and model analysis.
- CARMA aims to address many of these challenges, supporting rich forms of interaction, using attributes to capture explicit locations and the environment to allow adaptivity.
- Fluid approximation based analysis offers hope for scalable quantitative analysis techniques, but this is yet to included in the tool.

Thanks

Thanks

Thanks to my collaborators and colleagues on the QUANTICOL project, especially Michele Loreti.

This work has been funded by the CEC through the FET-Proactive QUANTICOL project

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