Stochastic Process Algebras

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Outline

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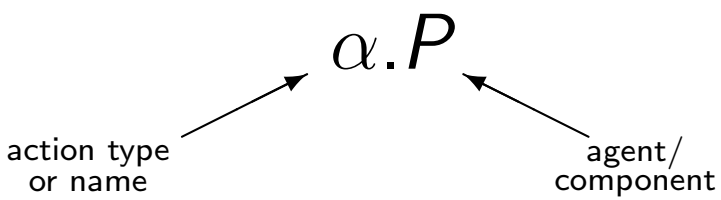
- Models consist of agents which engage in actions.



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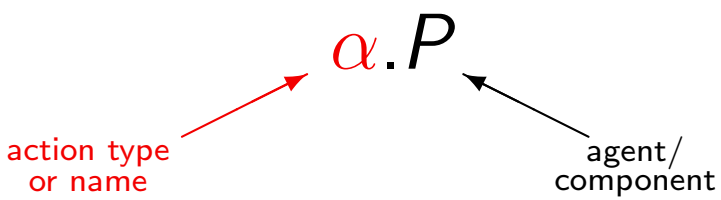

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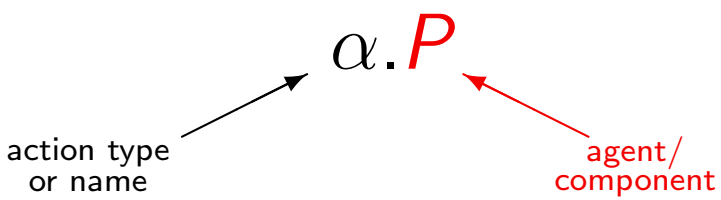

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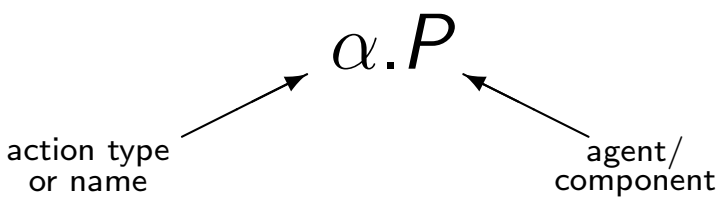

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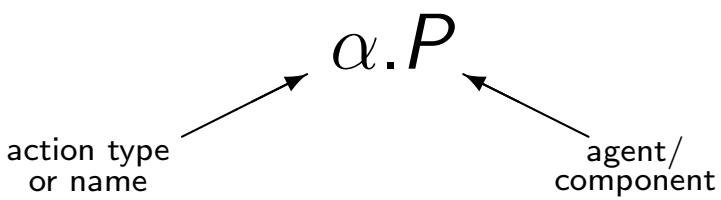

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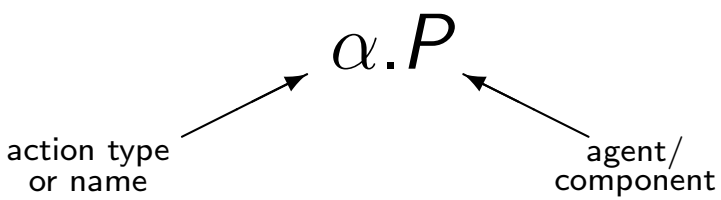

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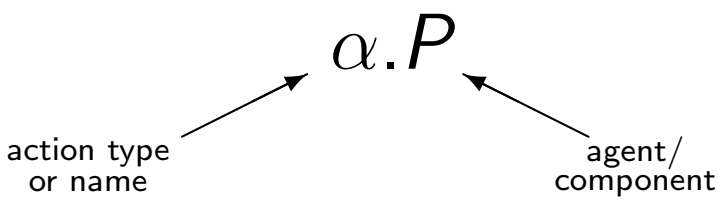

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Example

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A simple version of the Web can be considered to be the interaction of these components:

$$WEB \stackrel{def}{=} (Browser \parallel Browser) \mid Server$$

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- This can be viewed as a graph in which each node is a state of the model (comprised of the local states of each of the components) and the arcs represent the actions which can cause the move from one state to another.
- The language is also equipped with observational equivalence which can be used to compare models.

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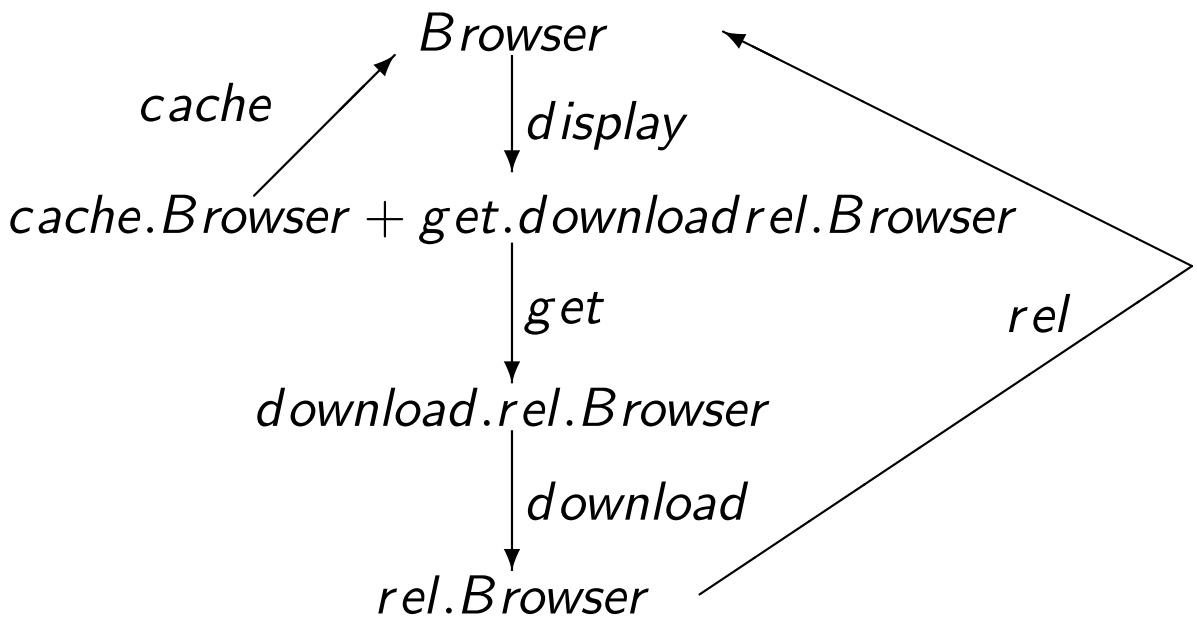

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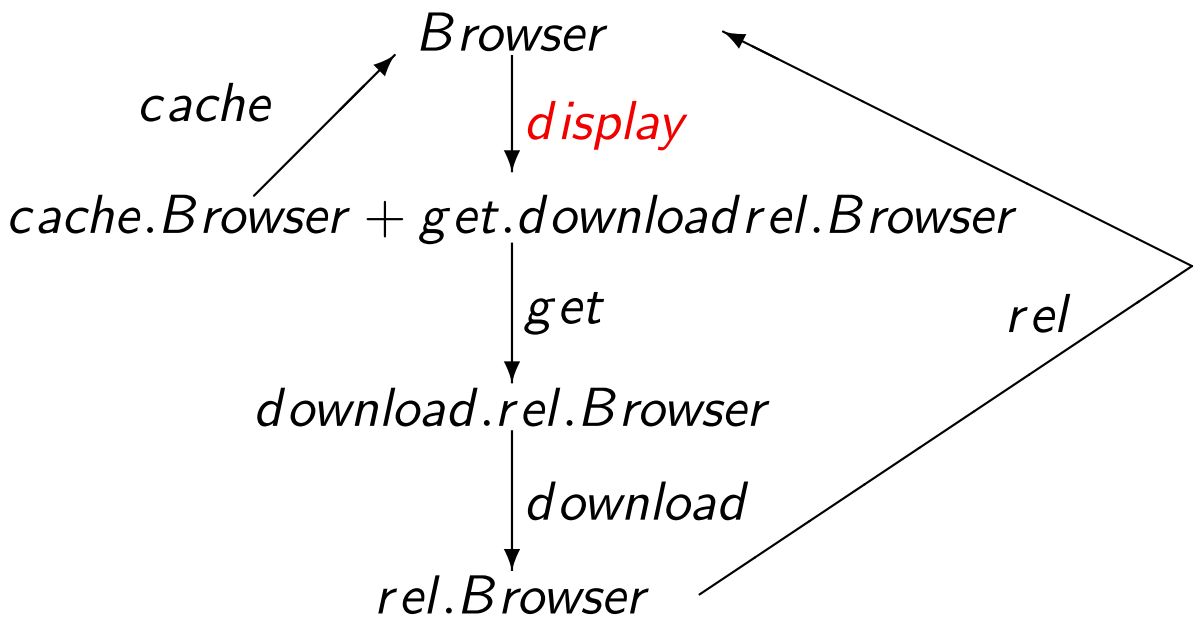

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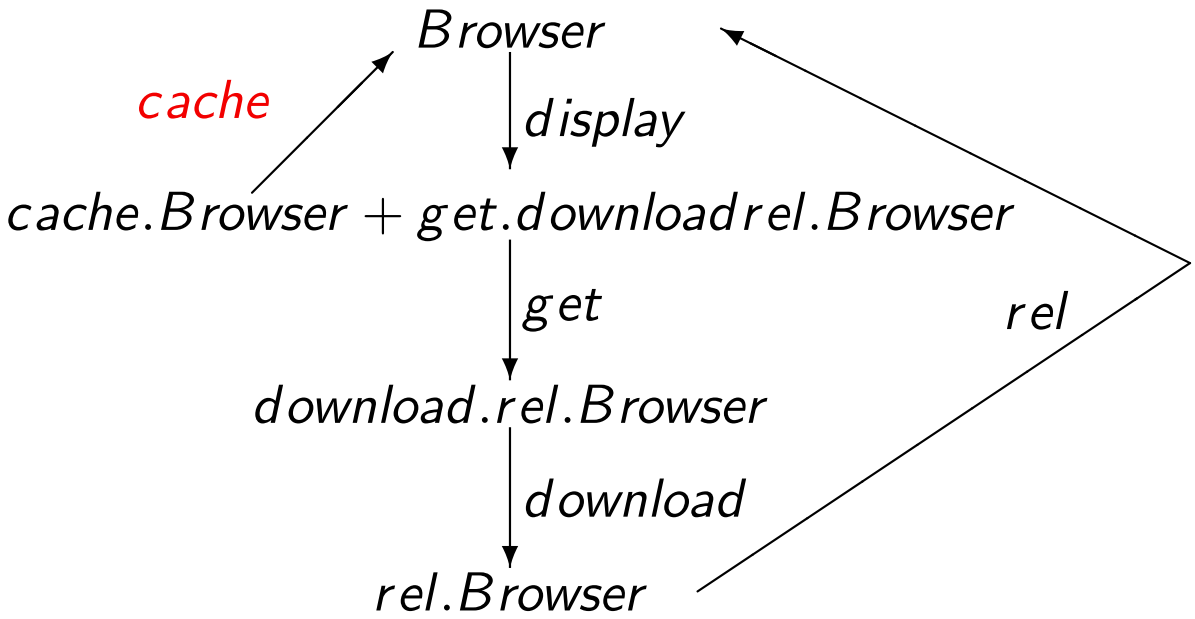

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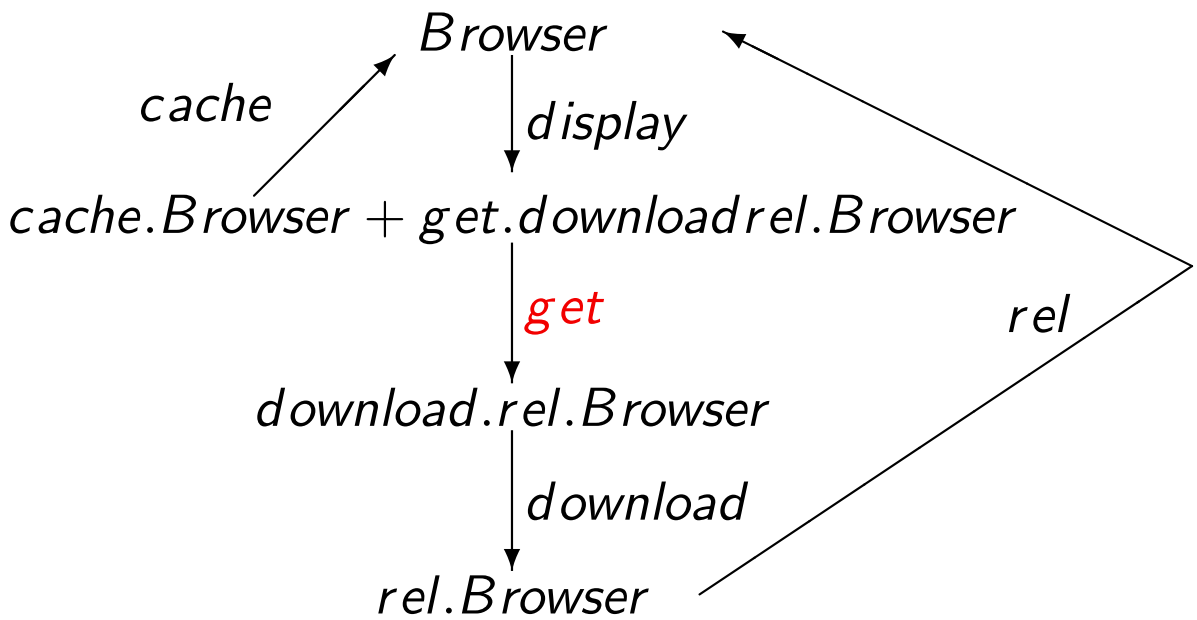

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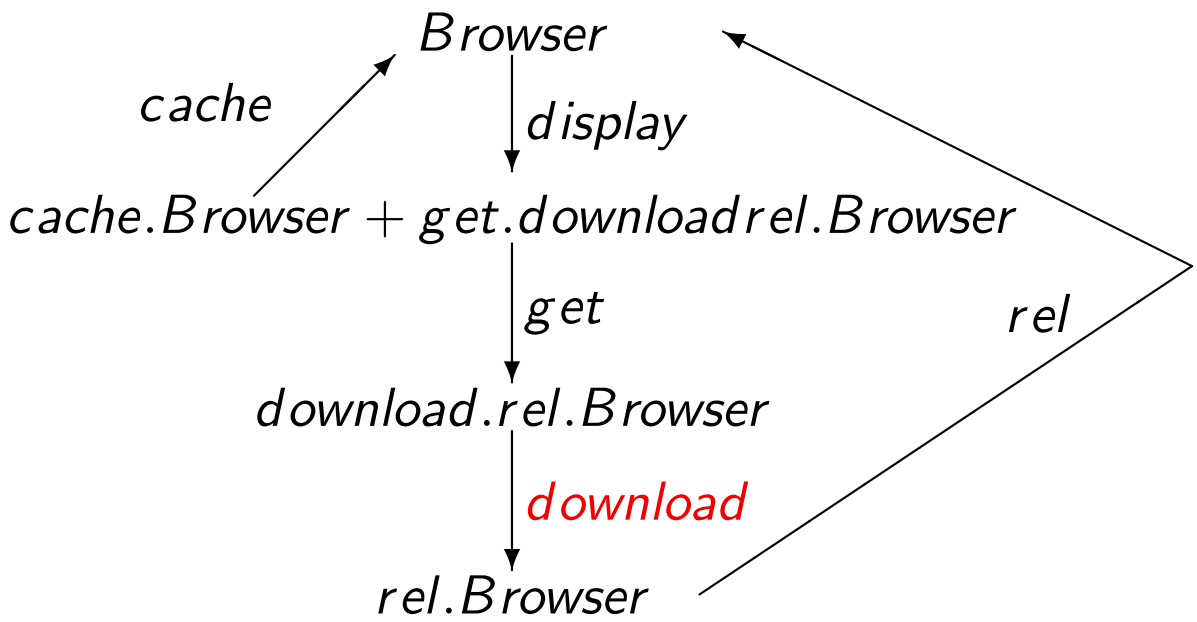

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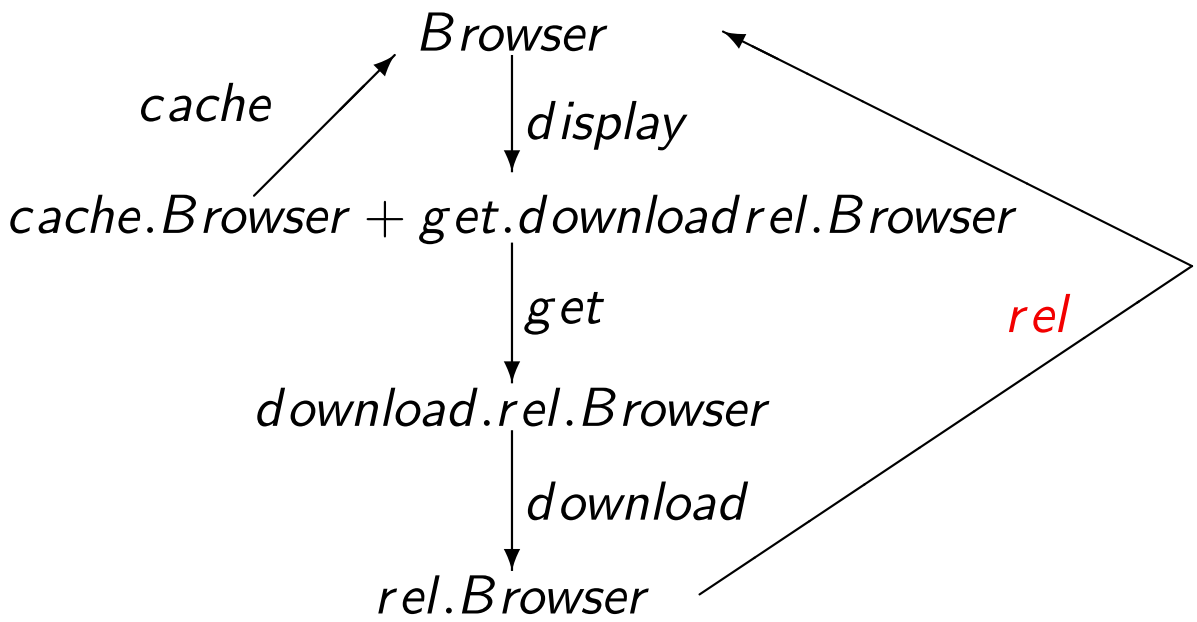

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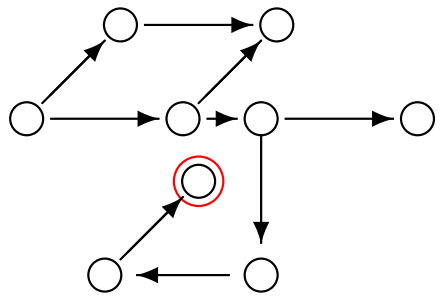

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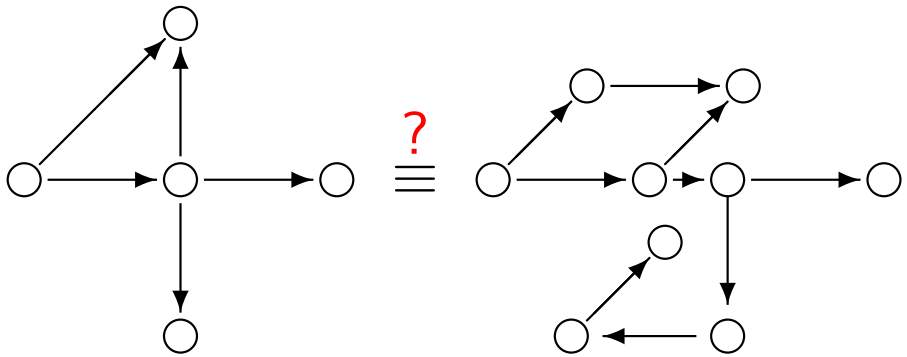
Does system behaviour match its specification?



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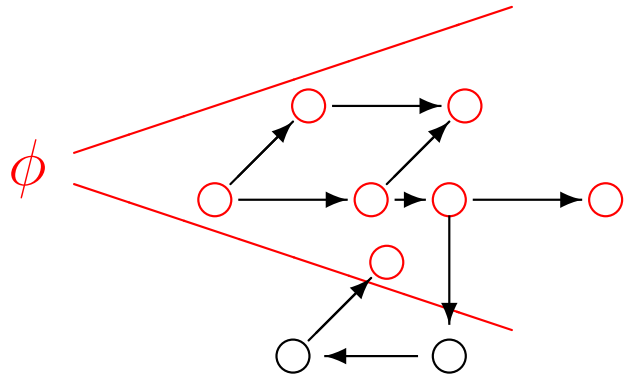
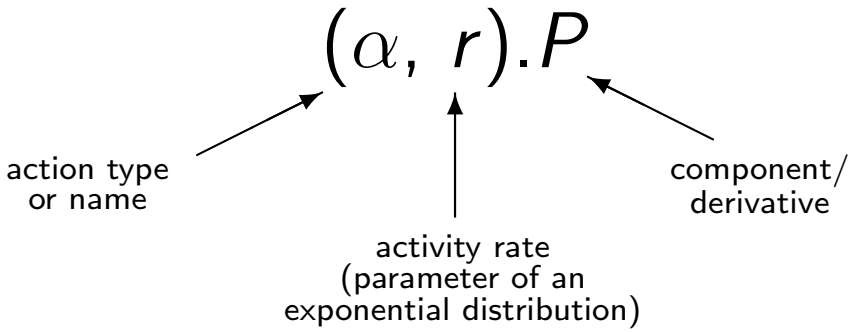
Does a given property ϕ hold within the system?



Performance Evaluation Process Algebra

- Models are constructed from components which engage in activities.



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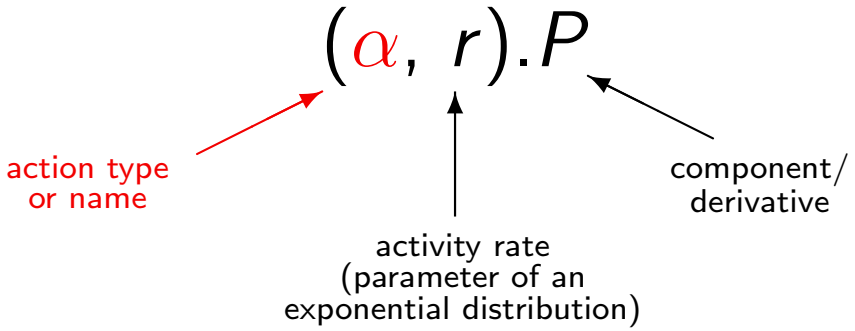

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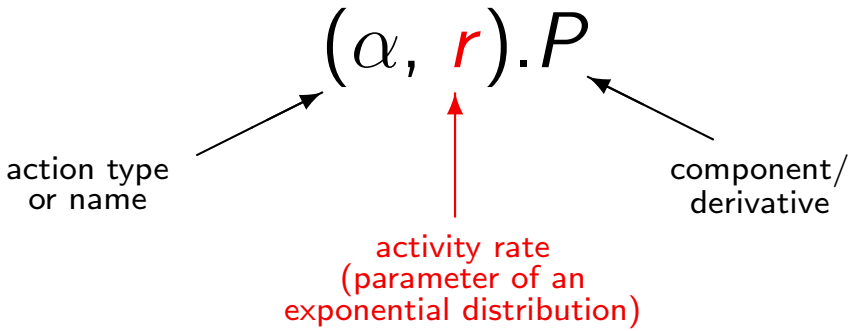

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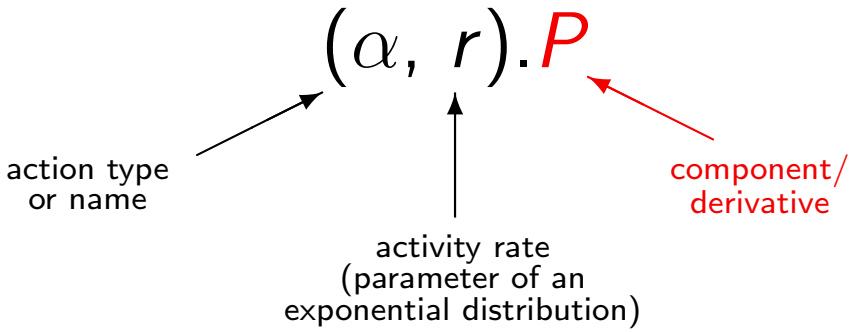

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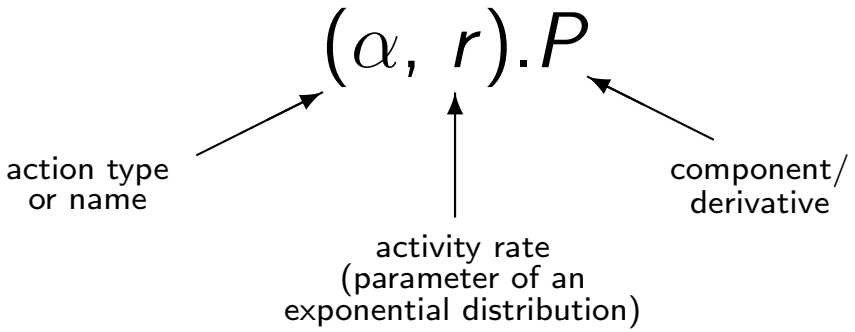

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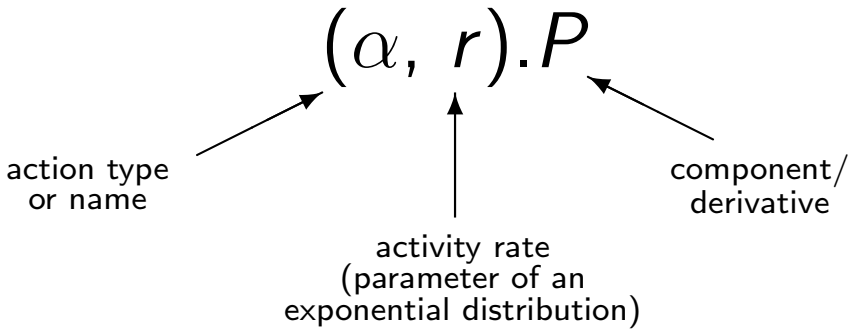

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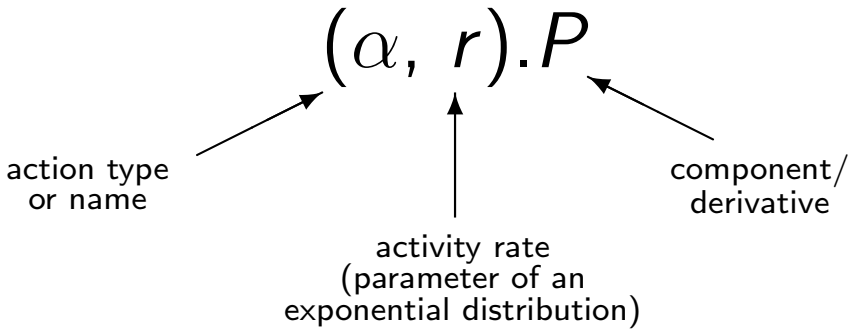

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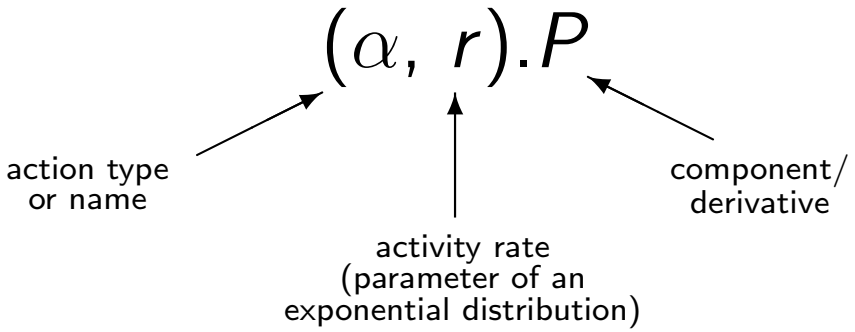

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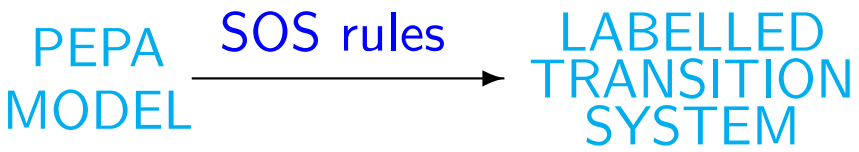

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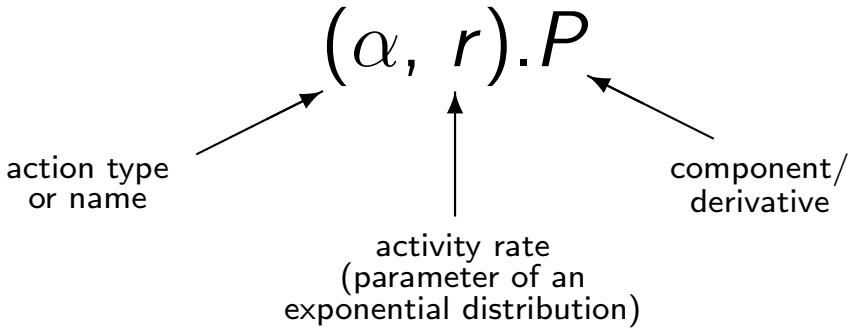

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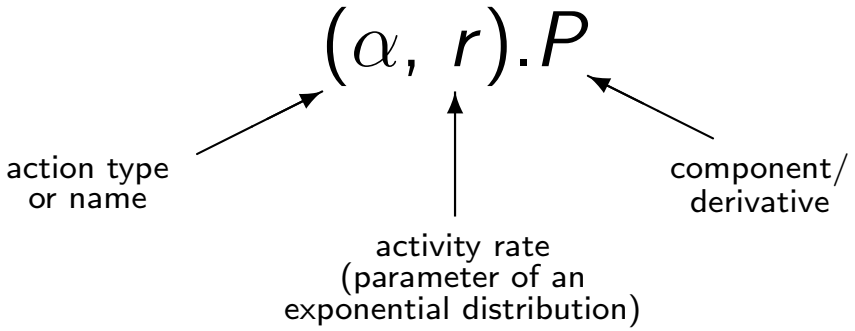

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$$L = \{get, download, rel\}$$

Integrated analysis

- Qualitative verification can now be complemented by quantitative verification:

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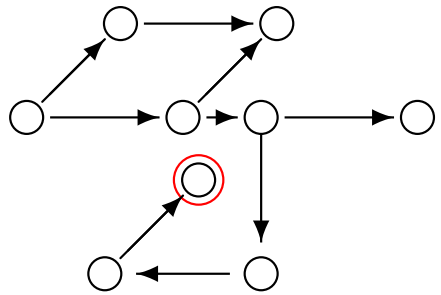
Reachability analysis

How long will it take for the system to arrive in a particular state?

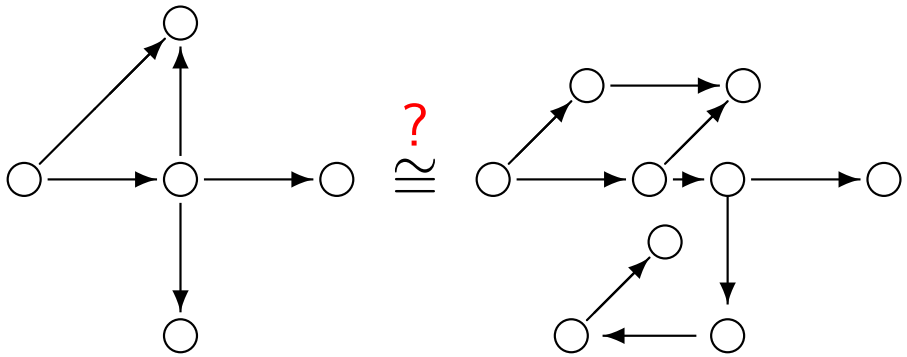


Integrated analysis

- Qualitative verification can now be complemented by quantitative verification:

Specification matching

With what probability
does system behaviour
match its specification?



Integrated analysis

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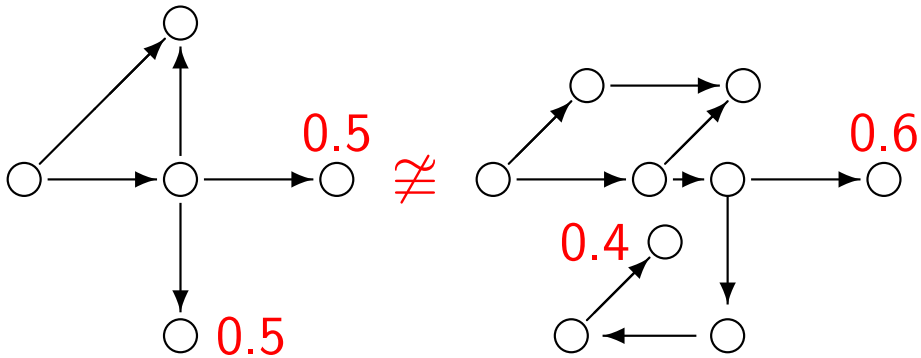
Specification matching

Does the "*frequency profile*" of the system match that of the specification?

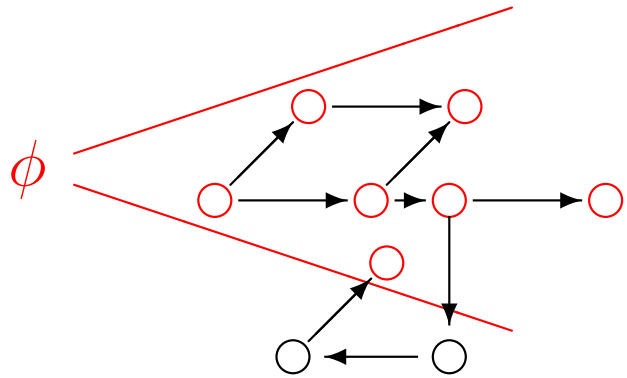


Integrated analysis

- Qualitative verification can now be complemented by quantitative verification:

Model checking

Does a given property ϕ hold within the system with a given probability?

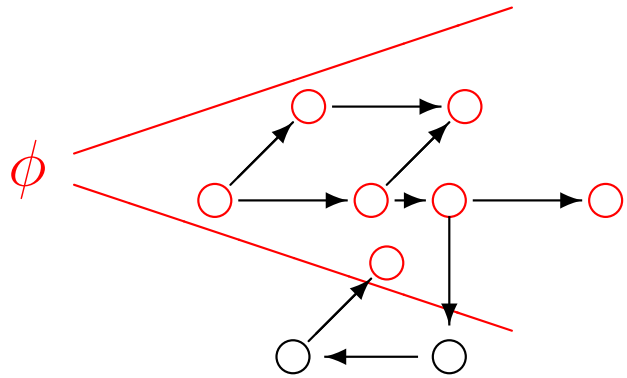


Integrated analysis

- Qualitative verification can now be complemented by quantitative verification:

Model checking

For a given starting state
how long is it until
a given property ϕ holds?



SPA Languages

SPA

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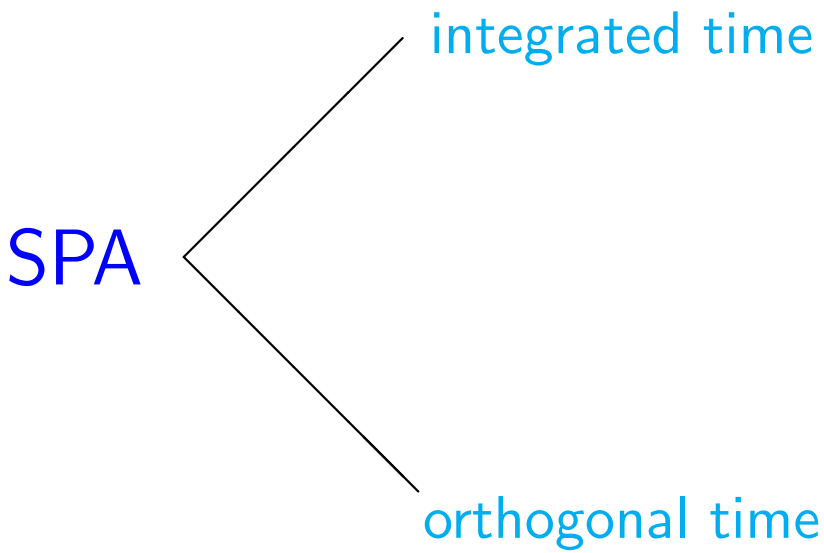

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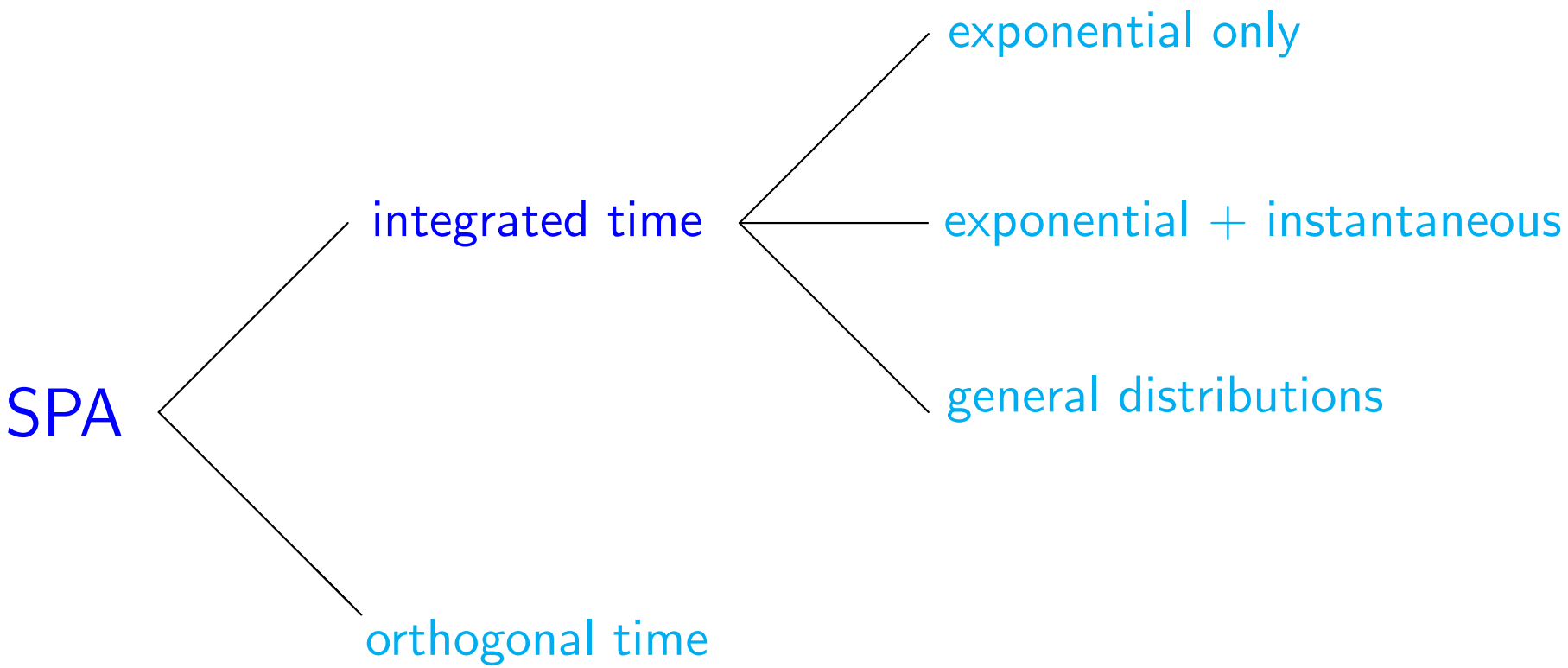

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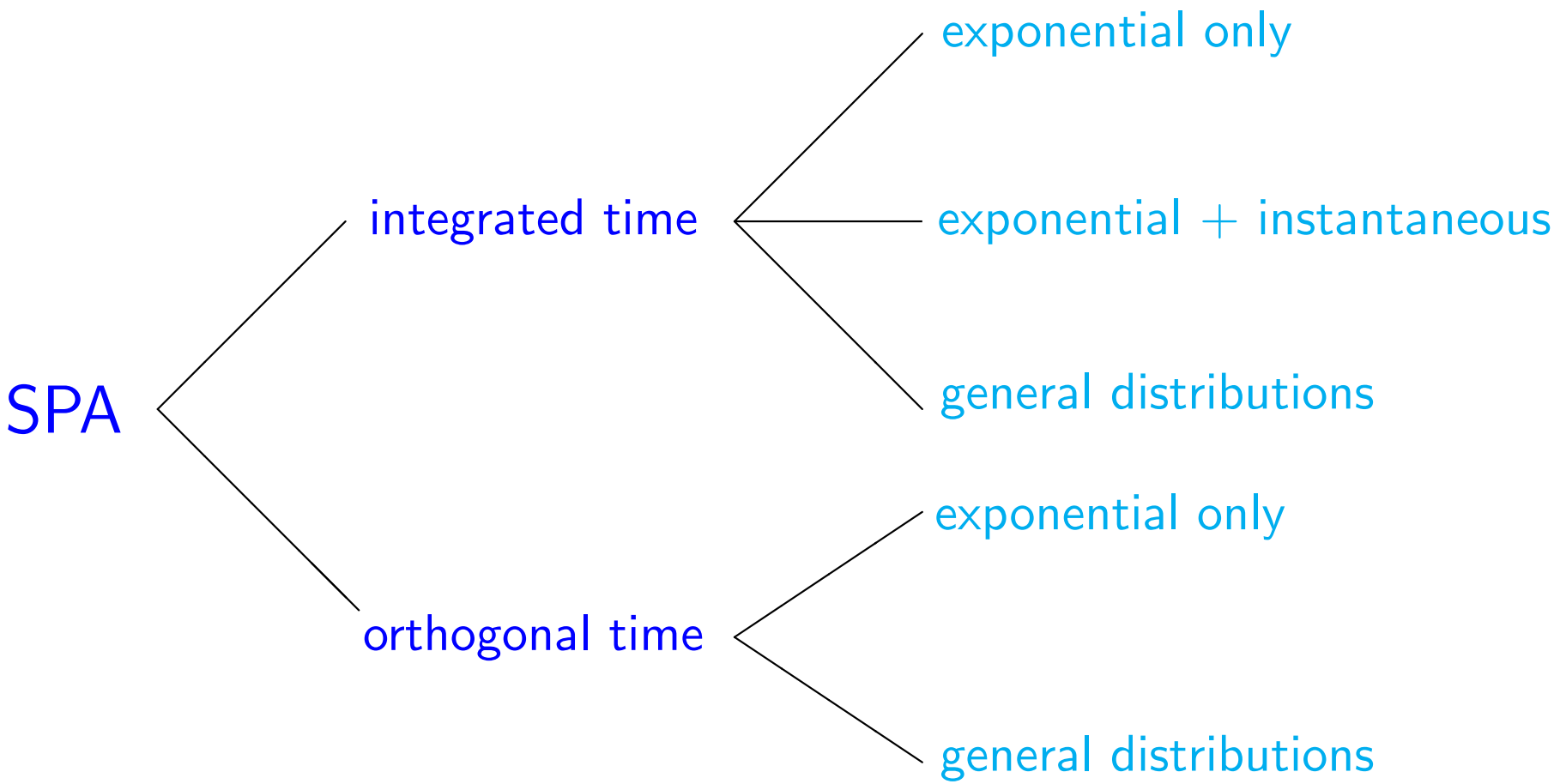

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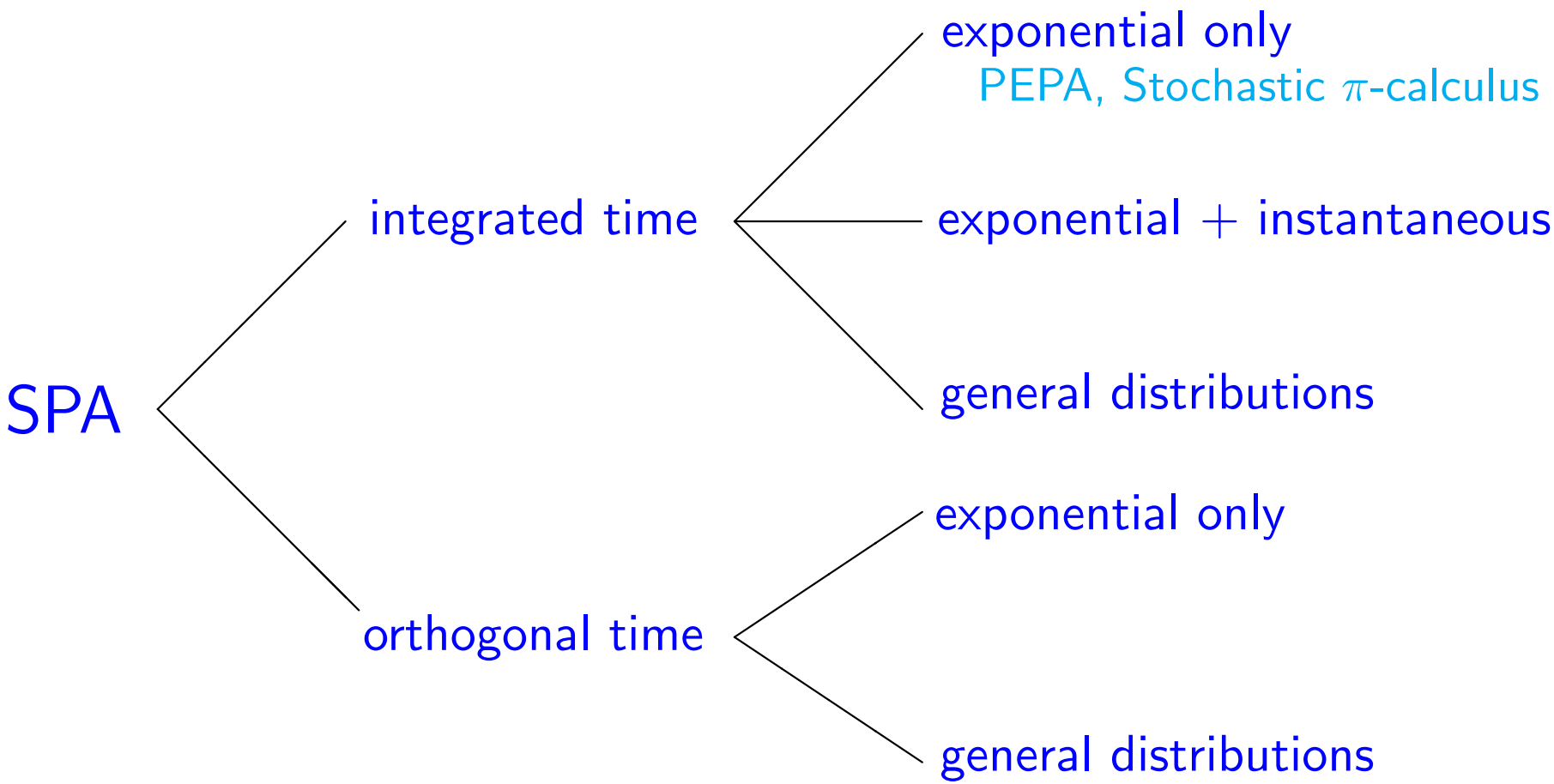

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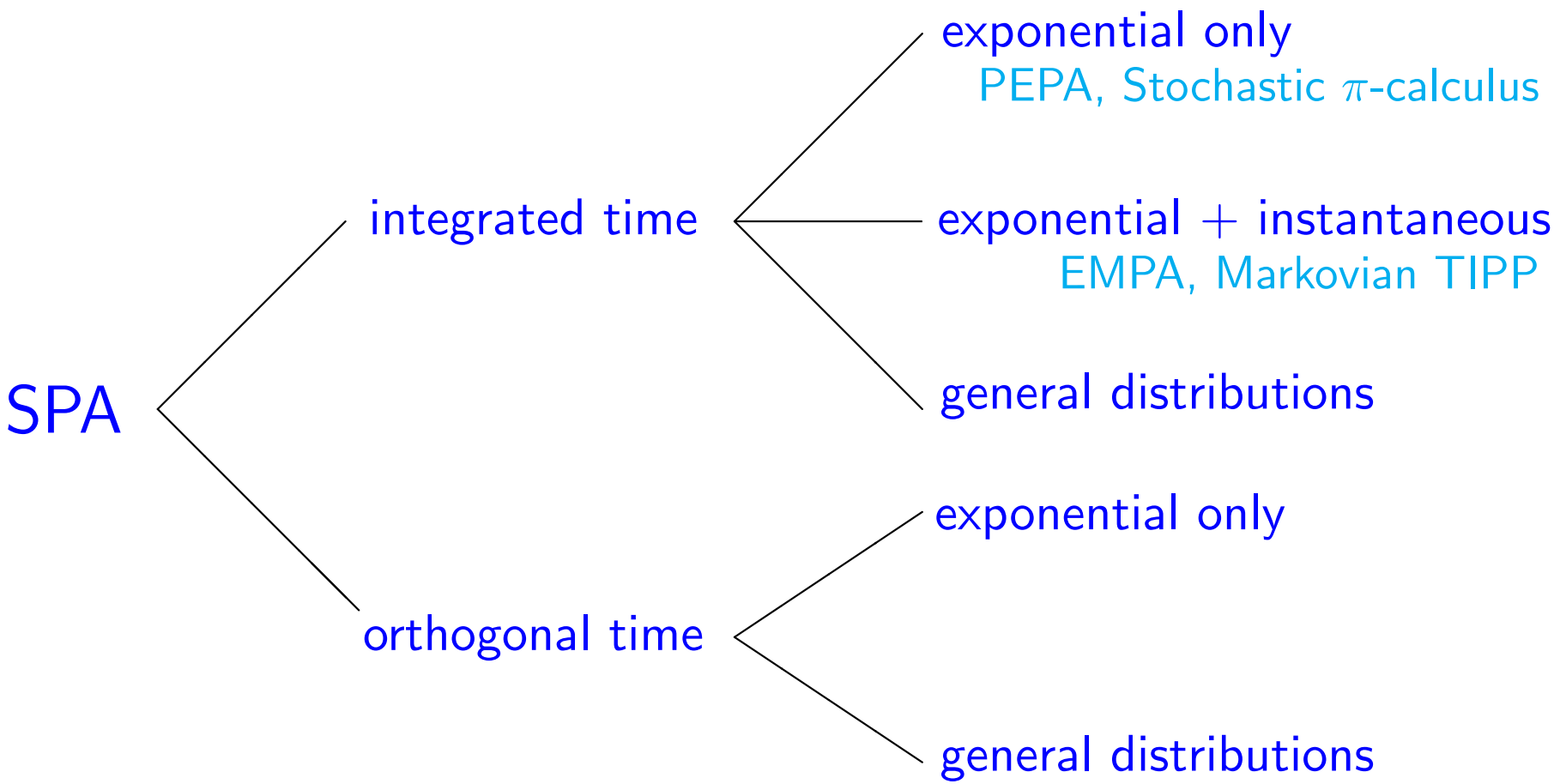

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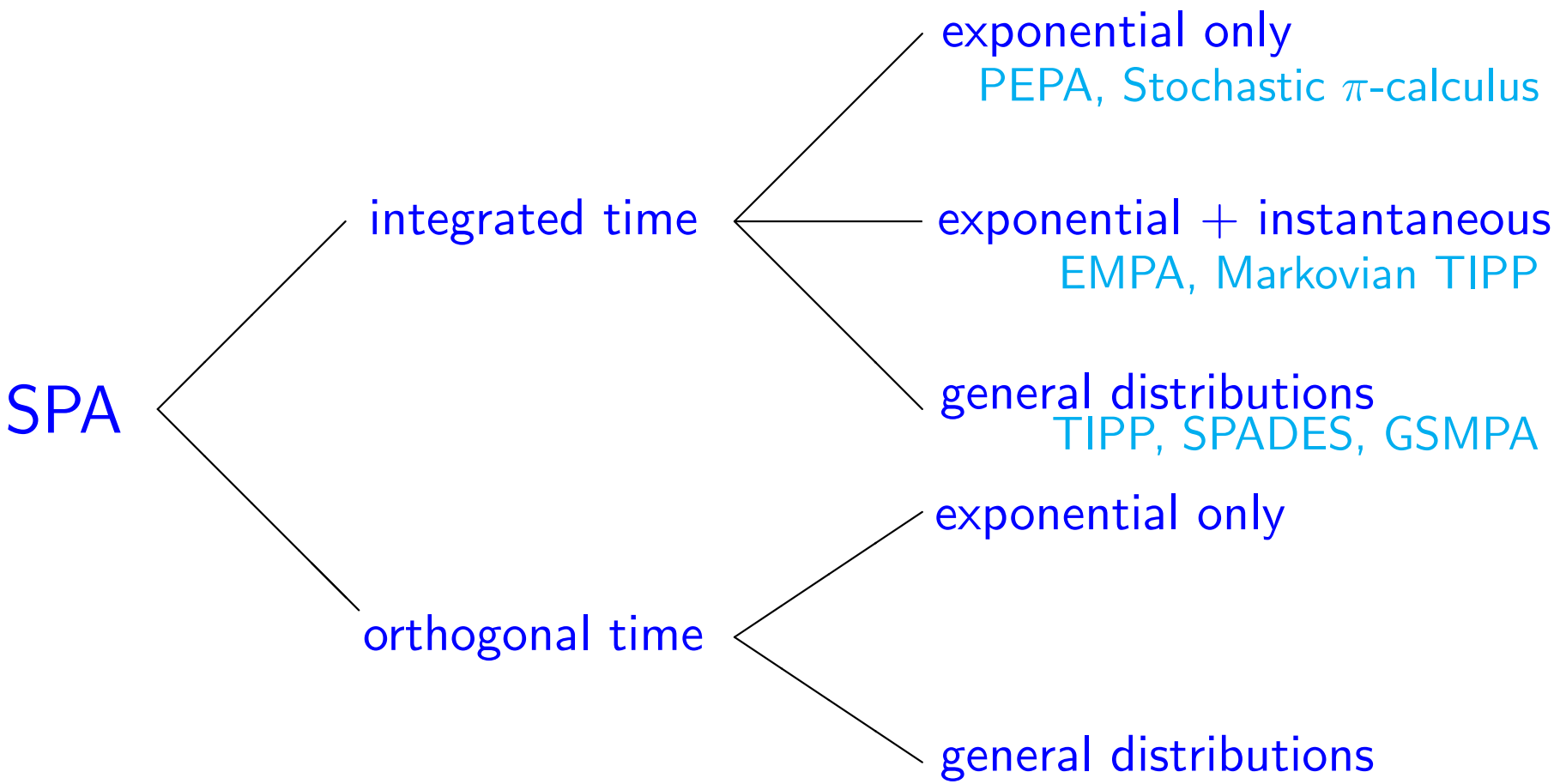

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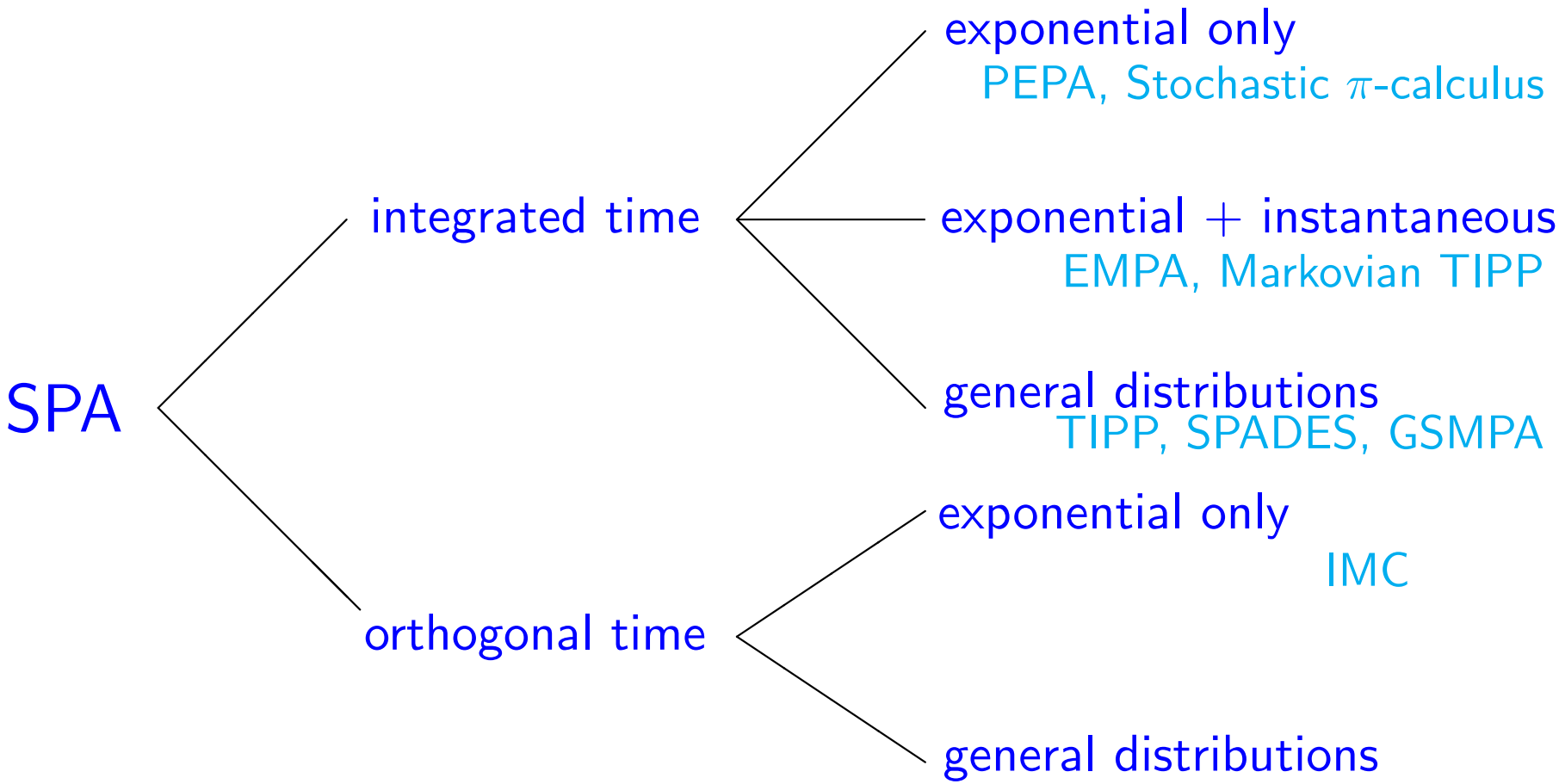

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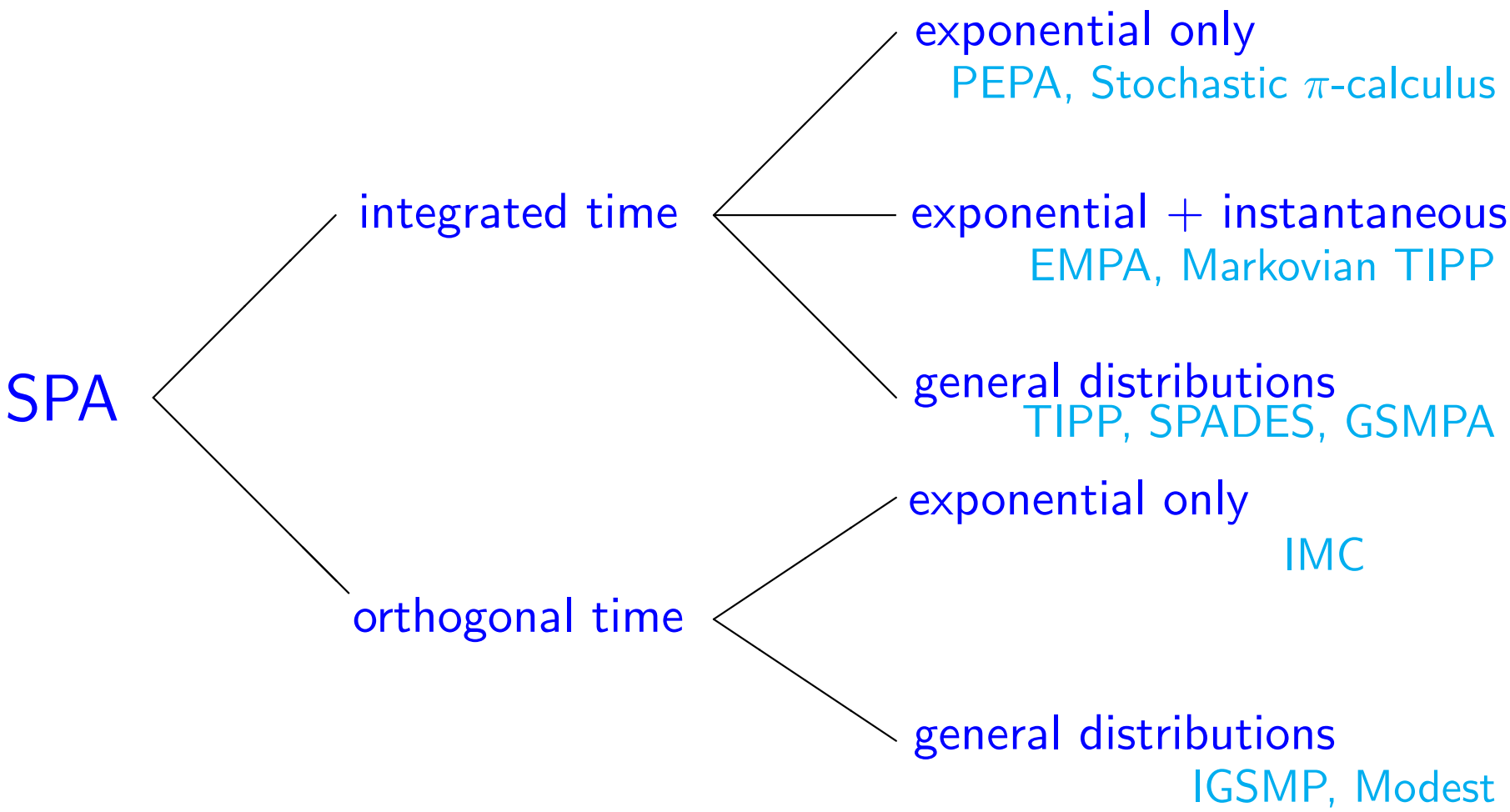

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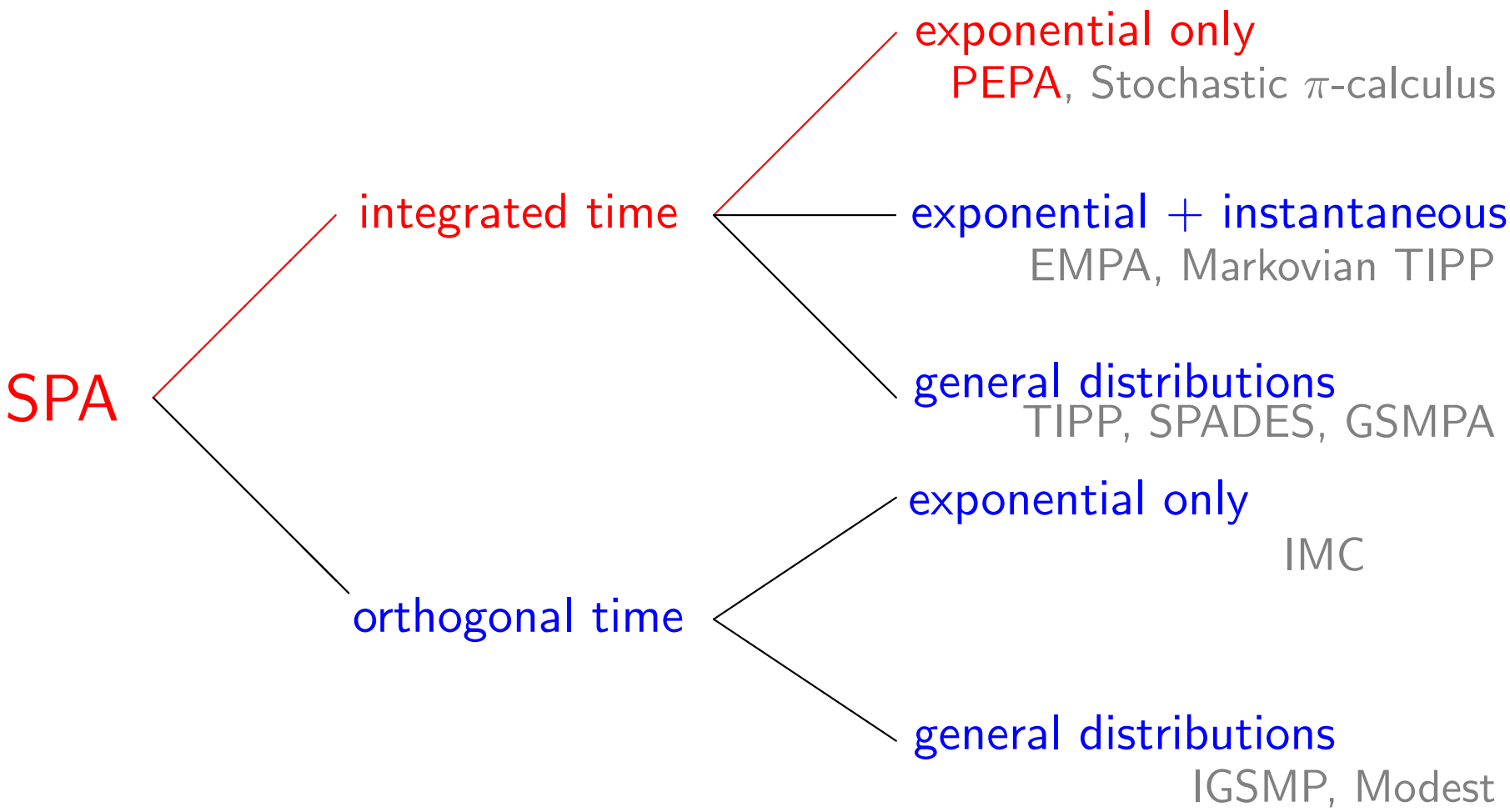
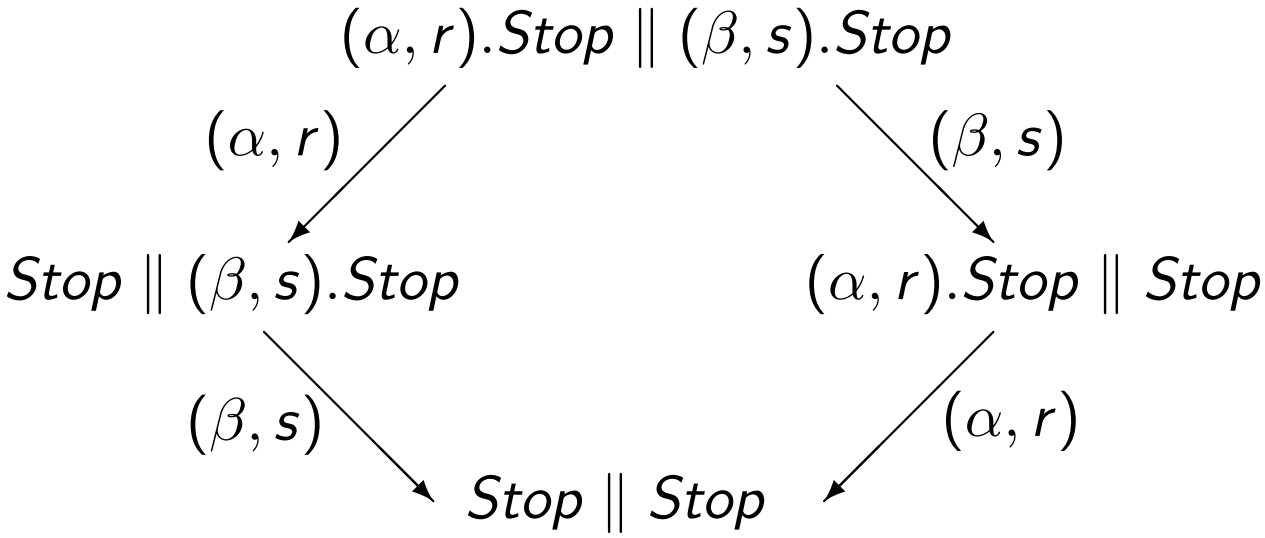


The Importance of Being Exponential



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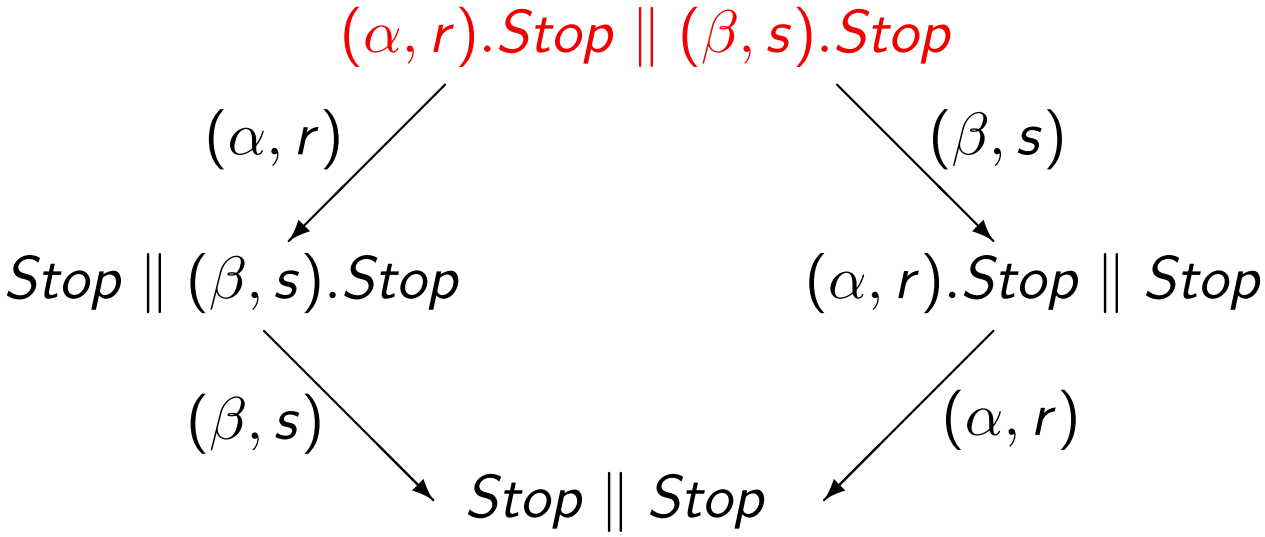

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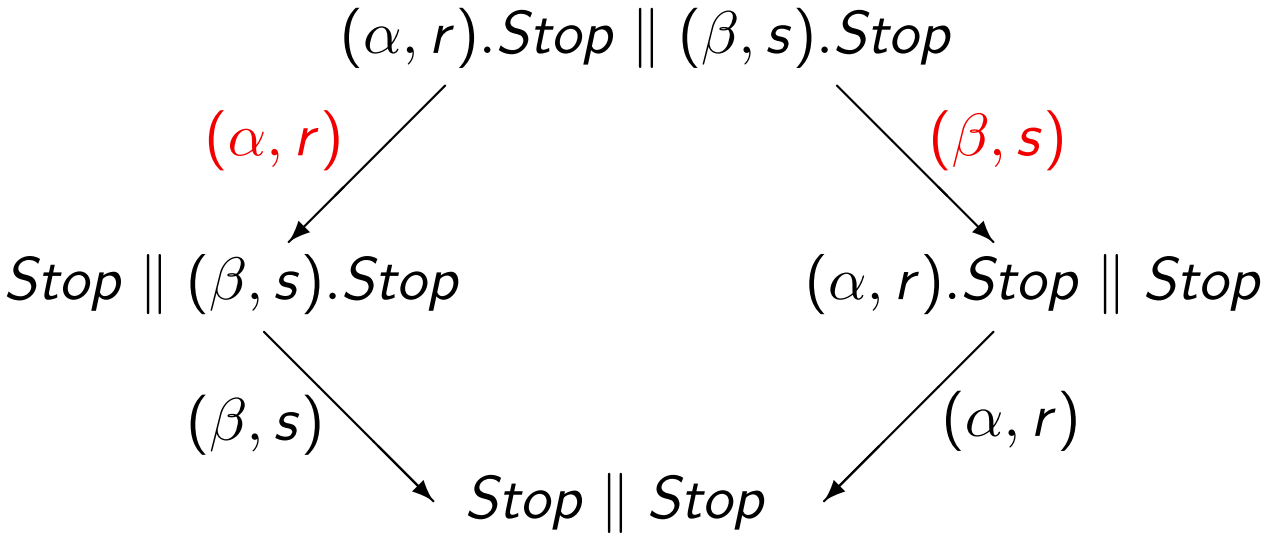

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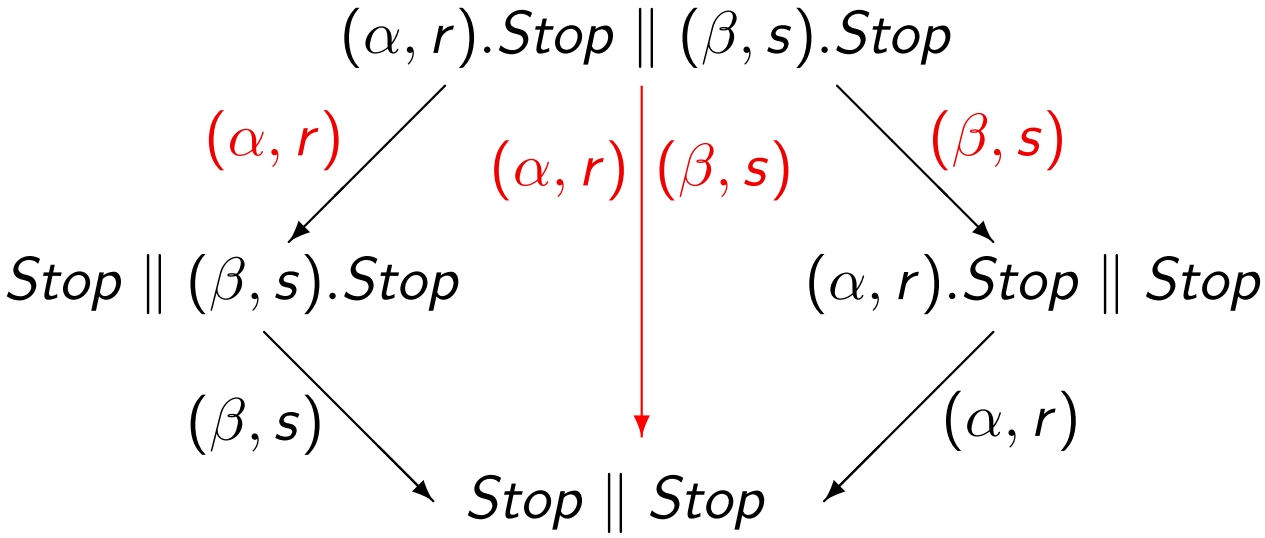

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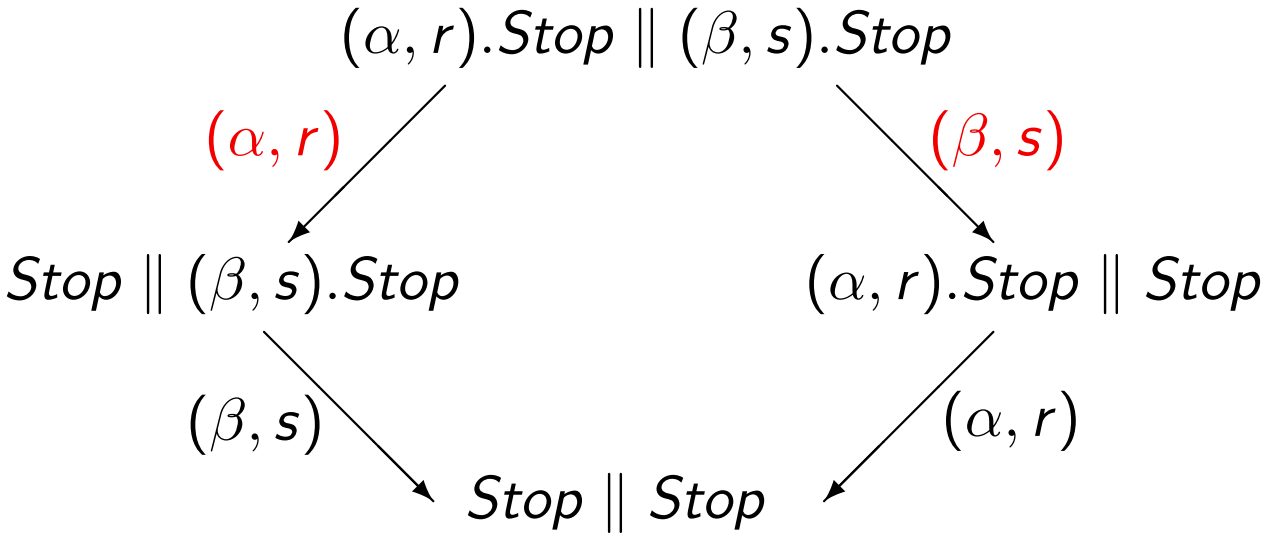

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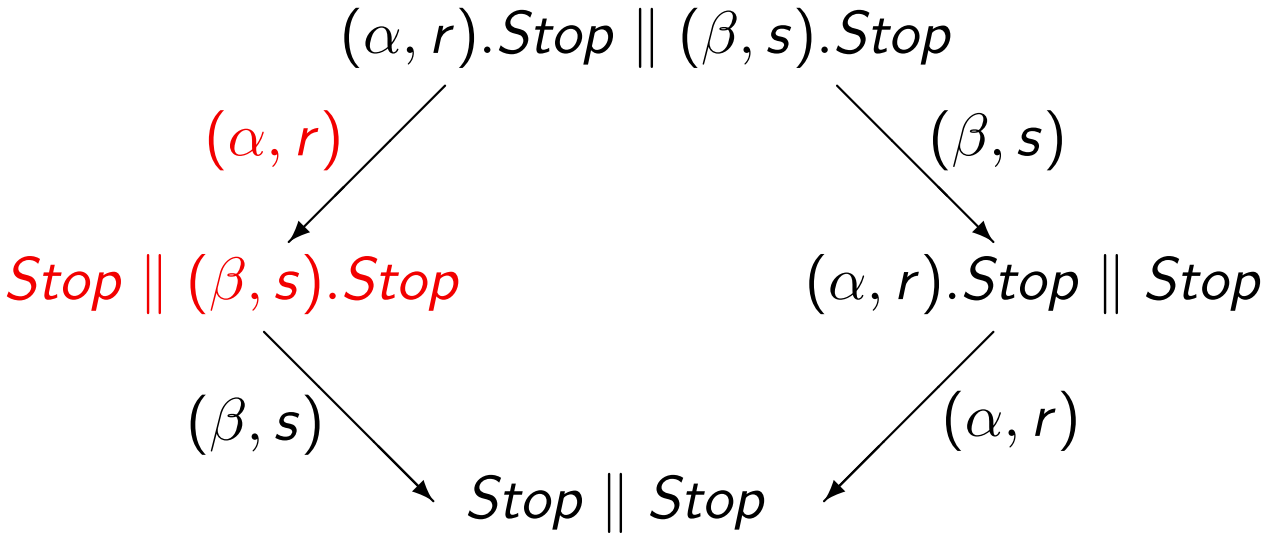

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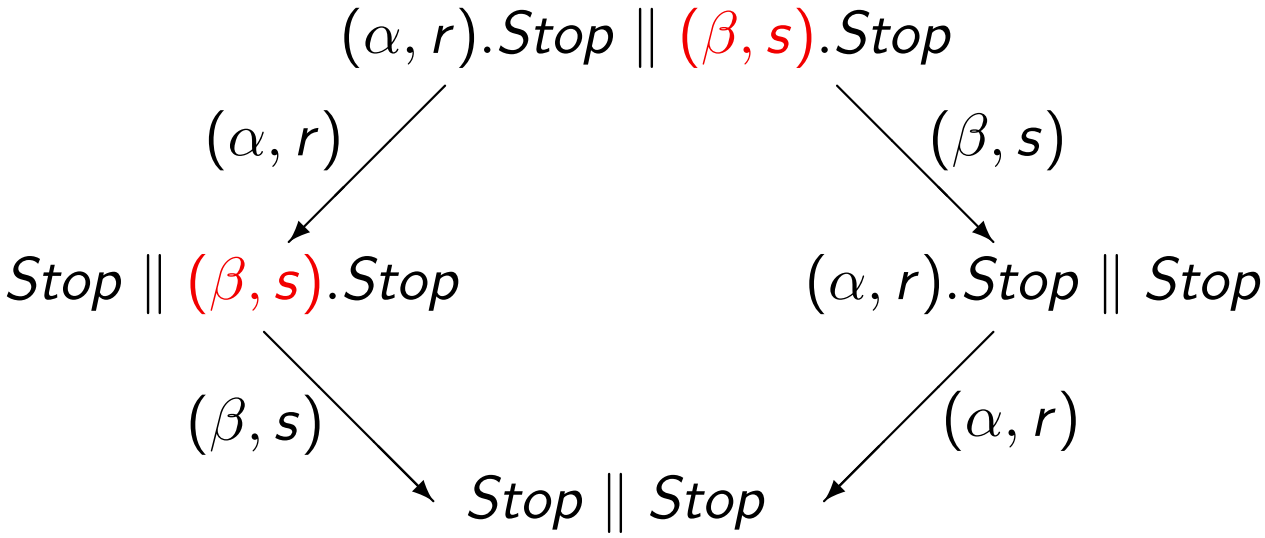

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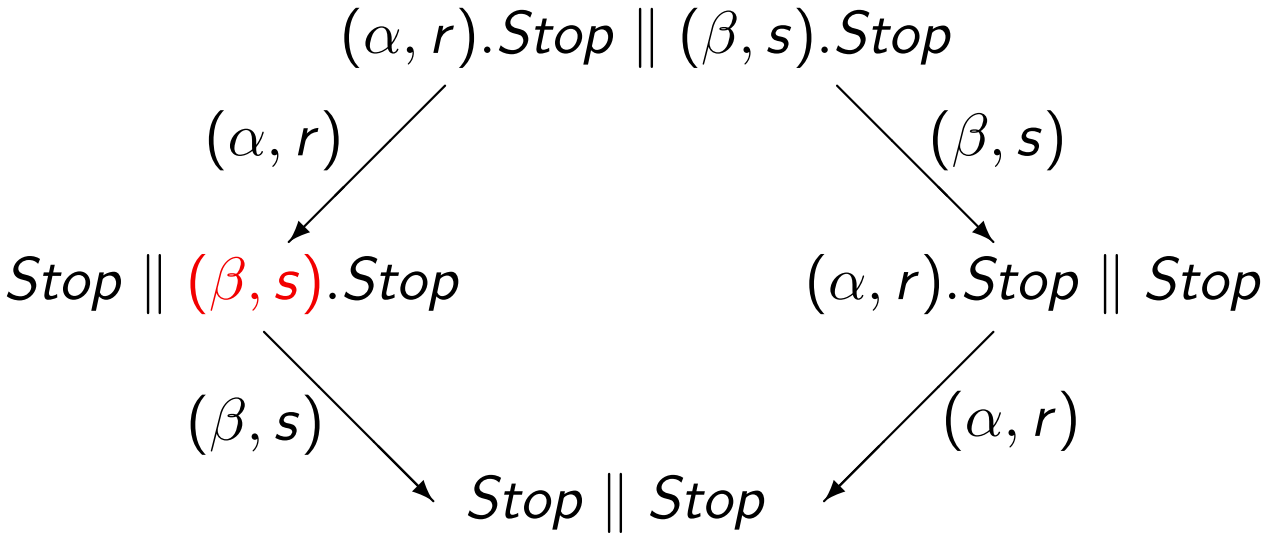
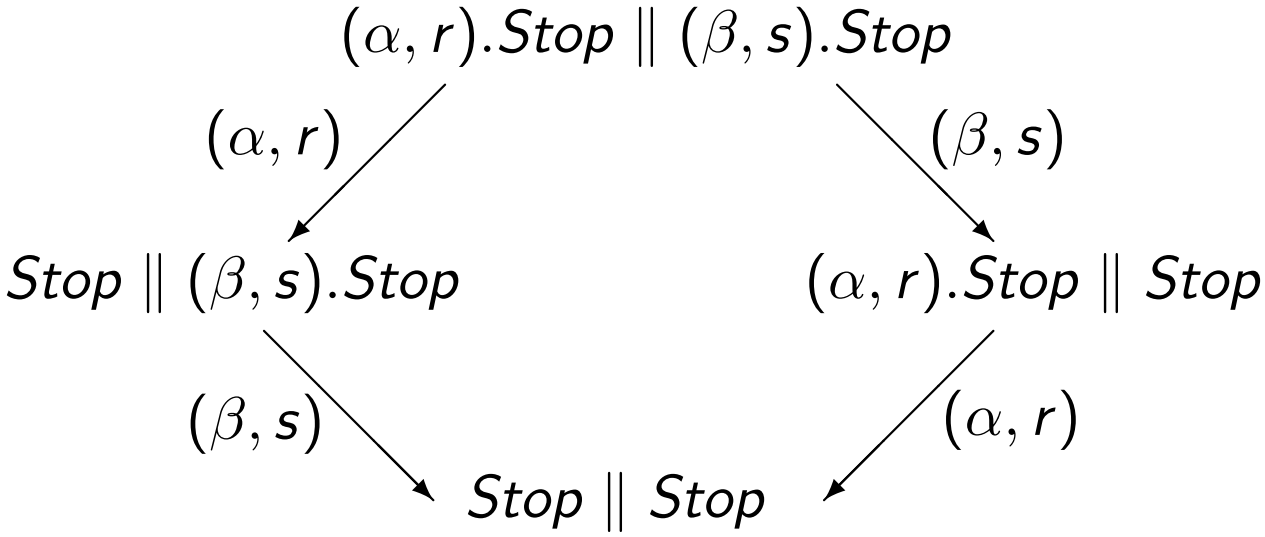


The memoryless property of the negative exponential distribution means that residual times do not need to be recorded.

The Importance of Being Exponential



We retain the expansion law of classical process algebra:

$$(\alpha, r).Stop \parallel (\beta, s).Stop =$$
$$(\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)$$

only if the negative exponential distribution is assumed.

Parallel Composition

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- In classical process algebra is it often associated with communication.
- When the activities of the process algebra have a duration the definition of parallel composition becomes more complex.

Who Synchronises...?

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CCS-style

- Actions are partitioned into input and output pairs.
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Most stochastic process algebras adopt CSP-style synchronisation.

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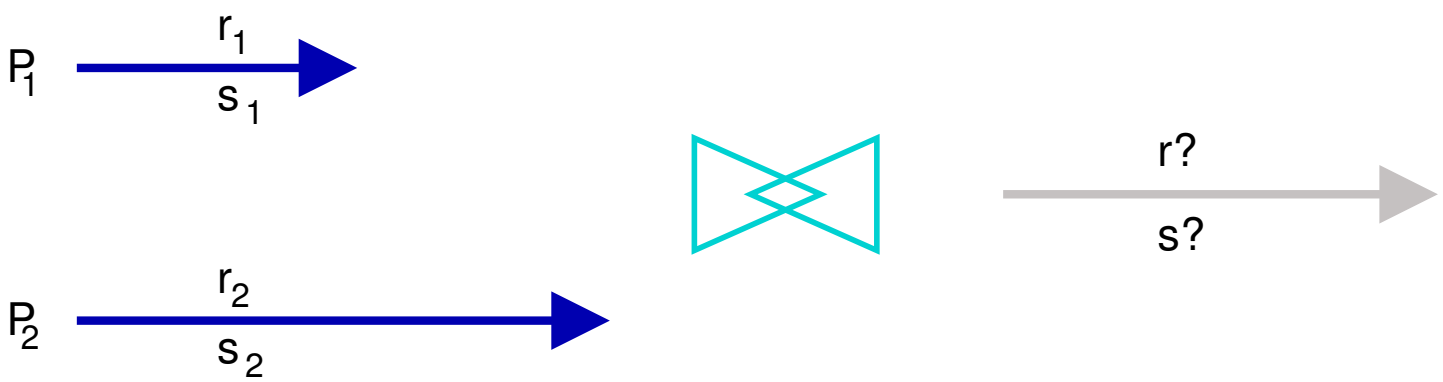

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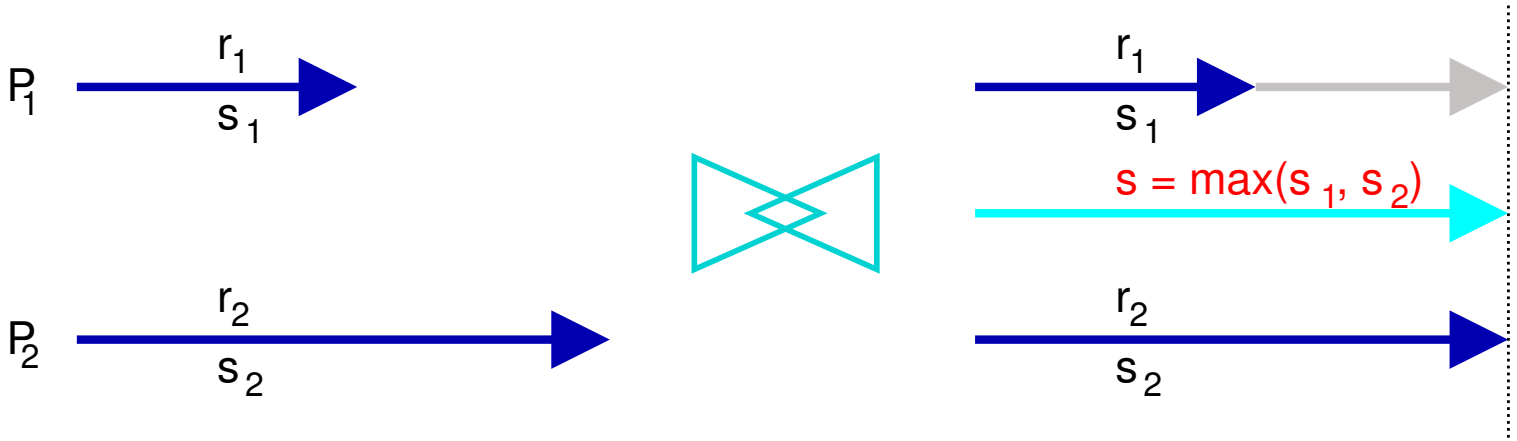
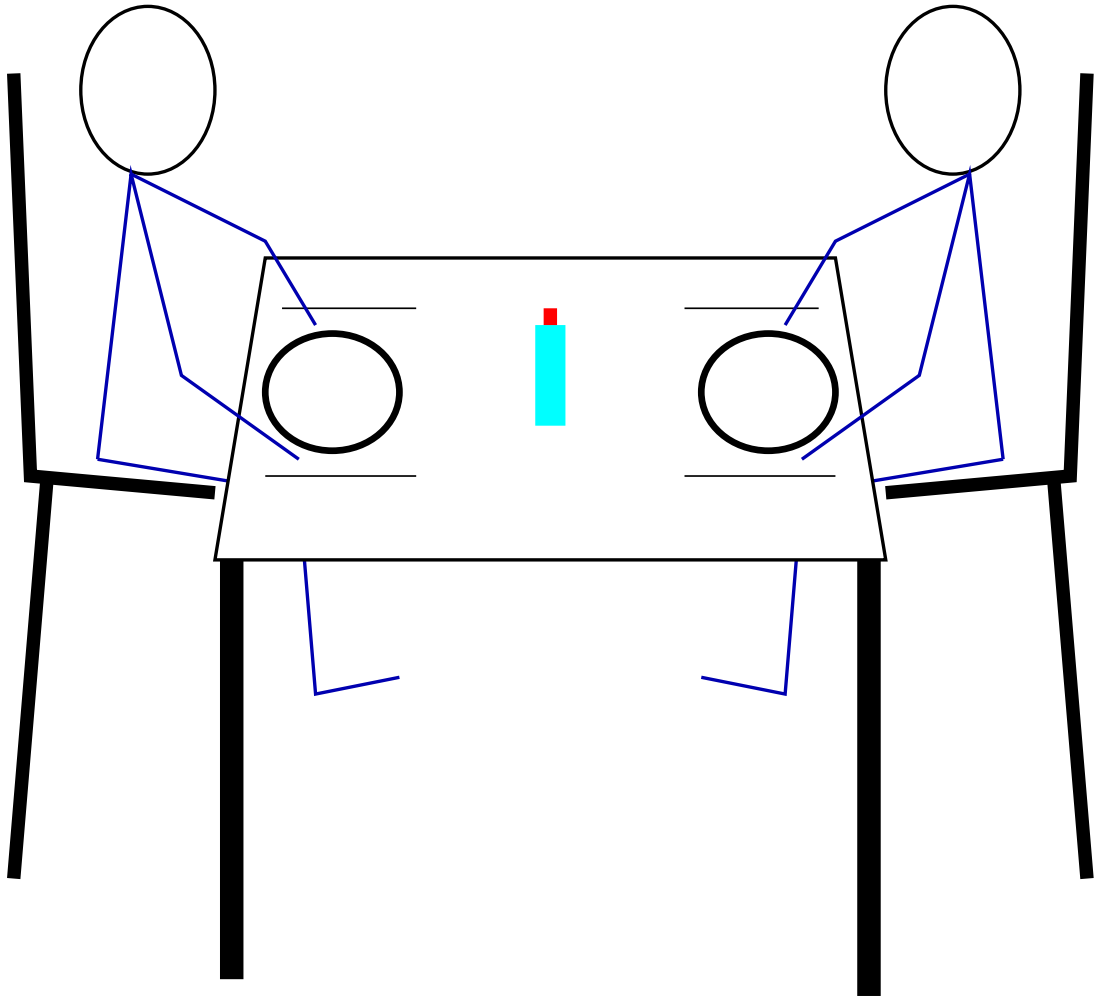


Barrier Synchronisation

Timed Synchronisation



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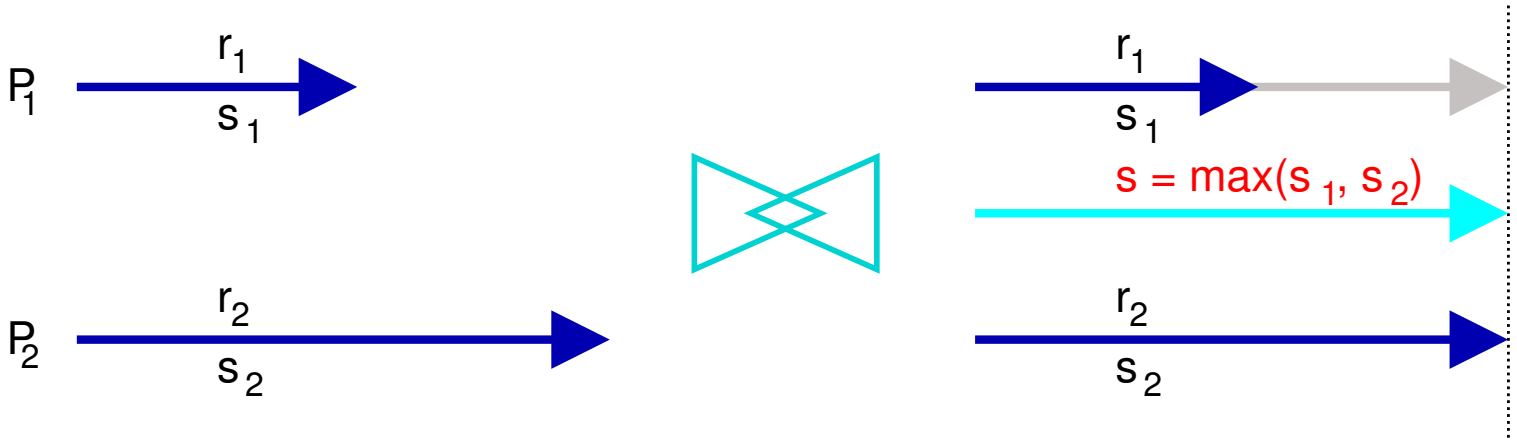

s is no longer exponentially distributed

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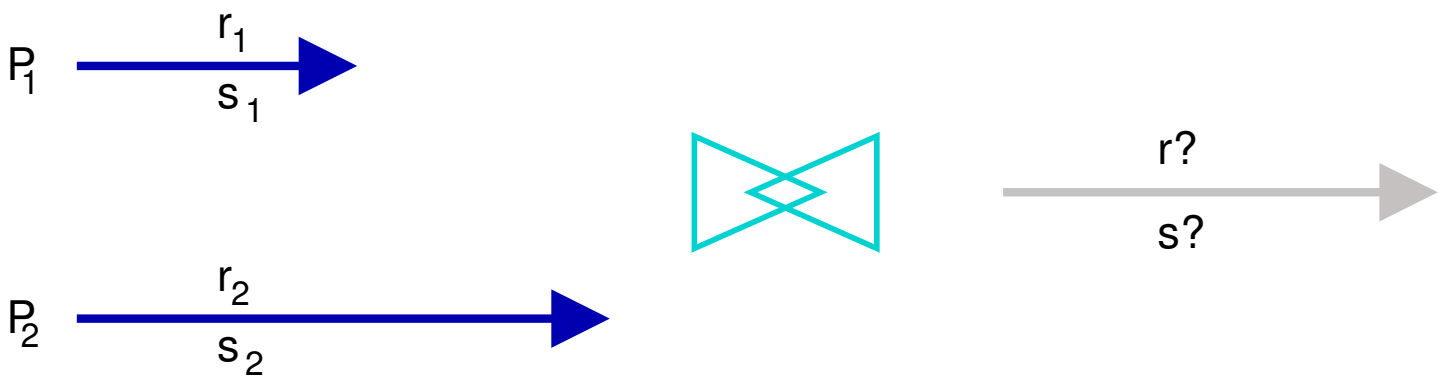

algebraic considerations limit choices

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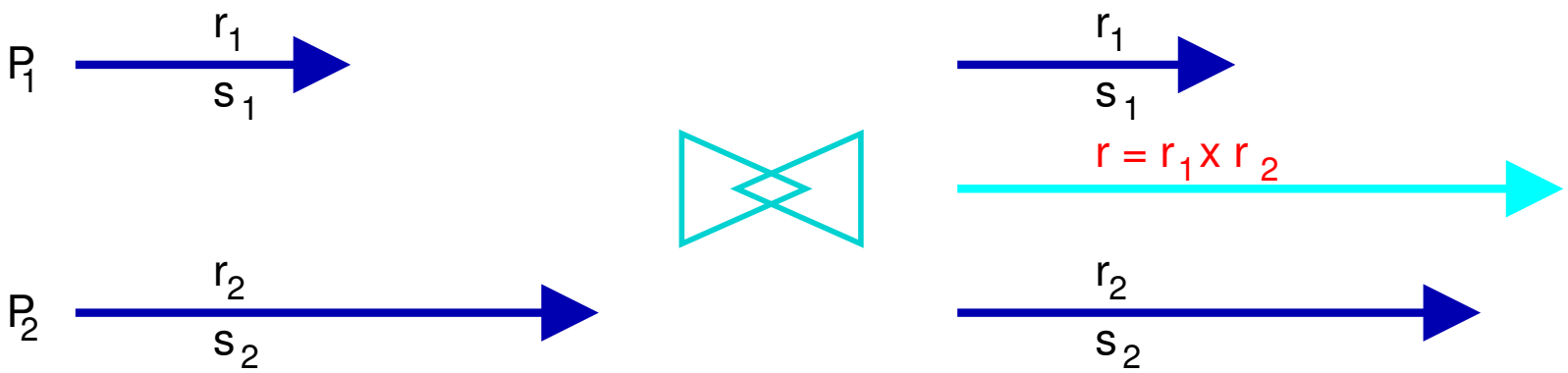
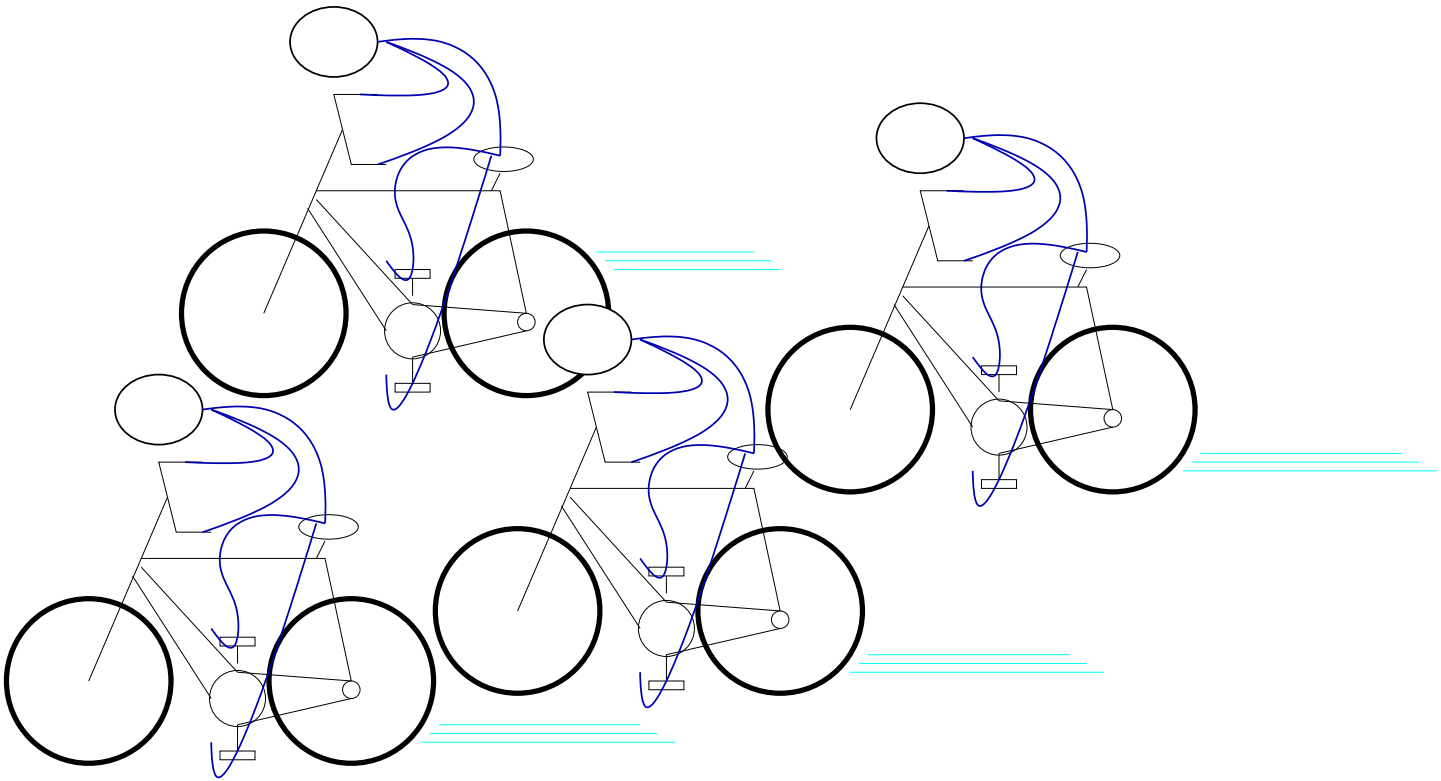


TIPP: new rate is product of individual rates

Timed Synchronisation



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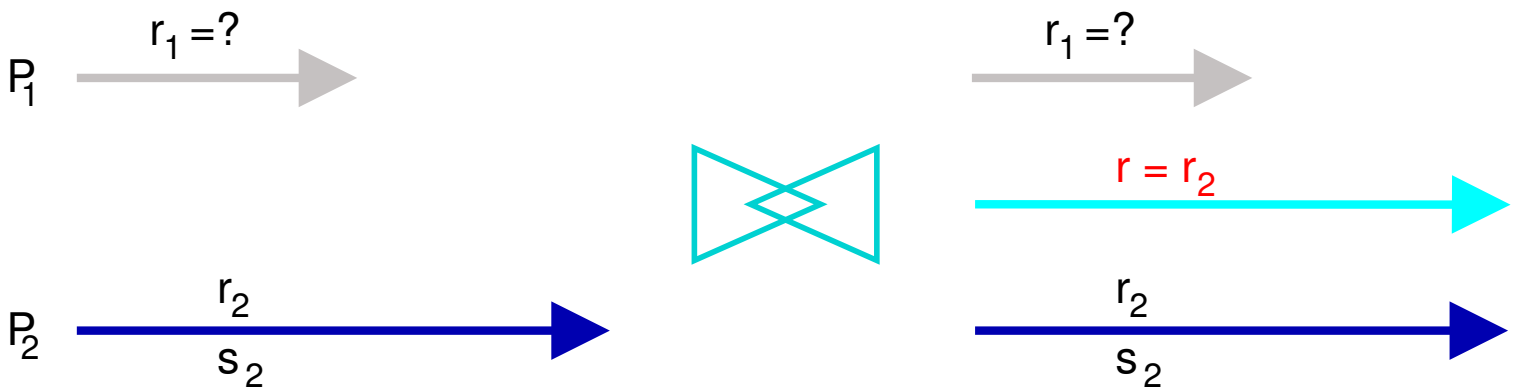
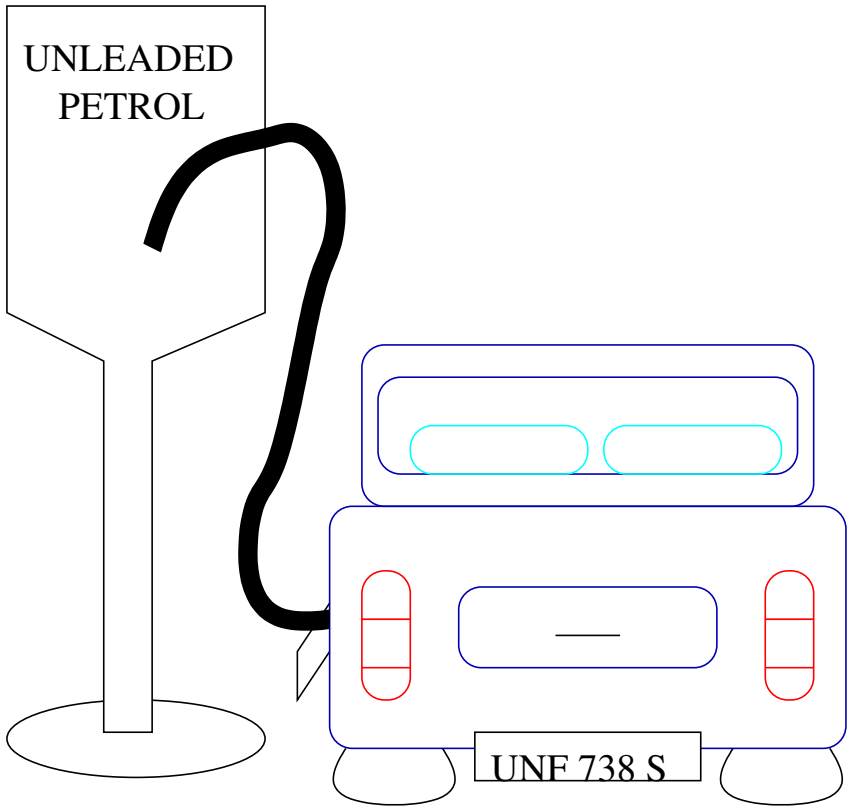


EMPA: one participant is passive

Timed Synchronisation



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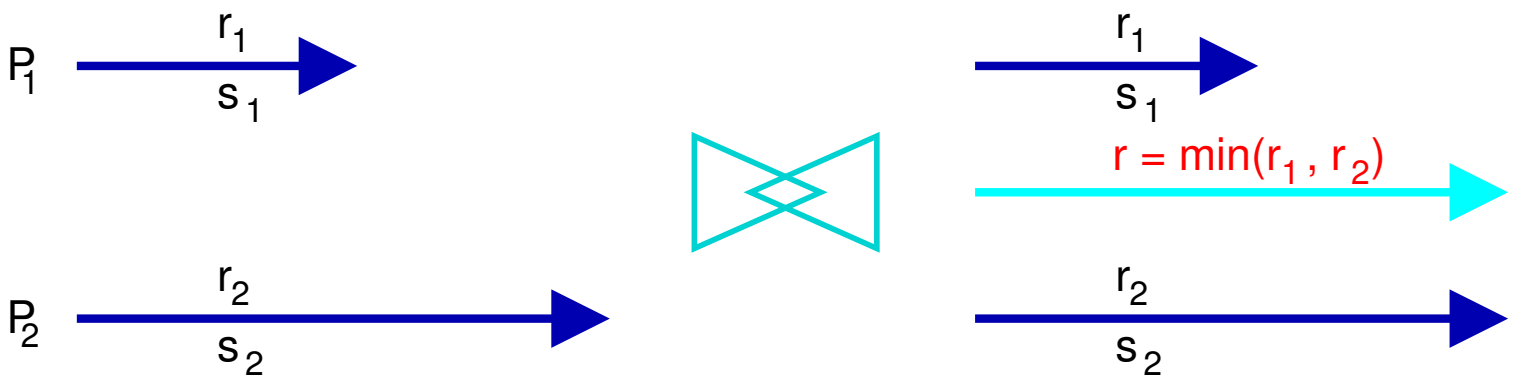

bounded capacity: new rate is the minimum of the rates

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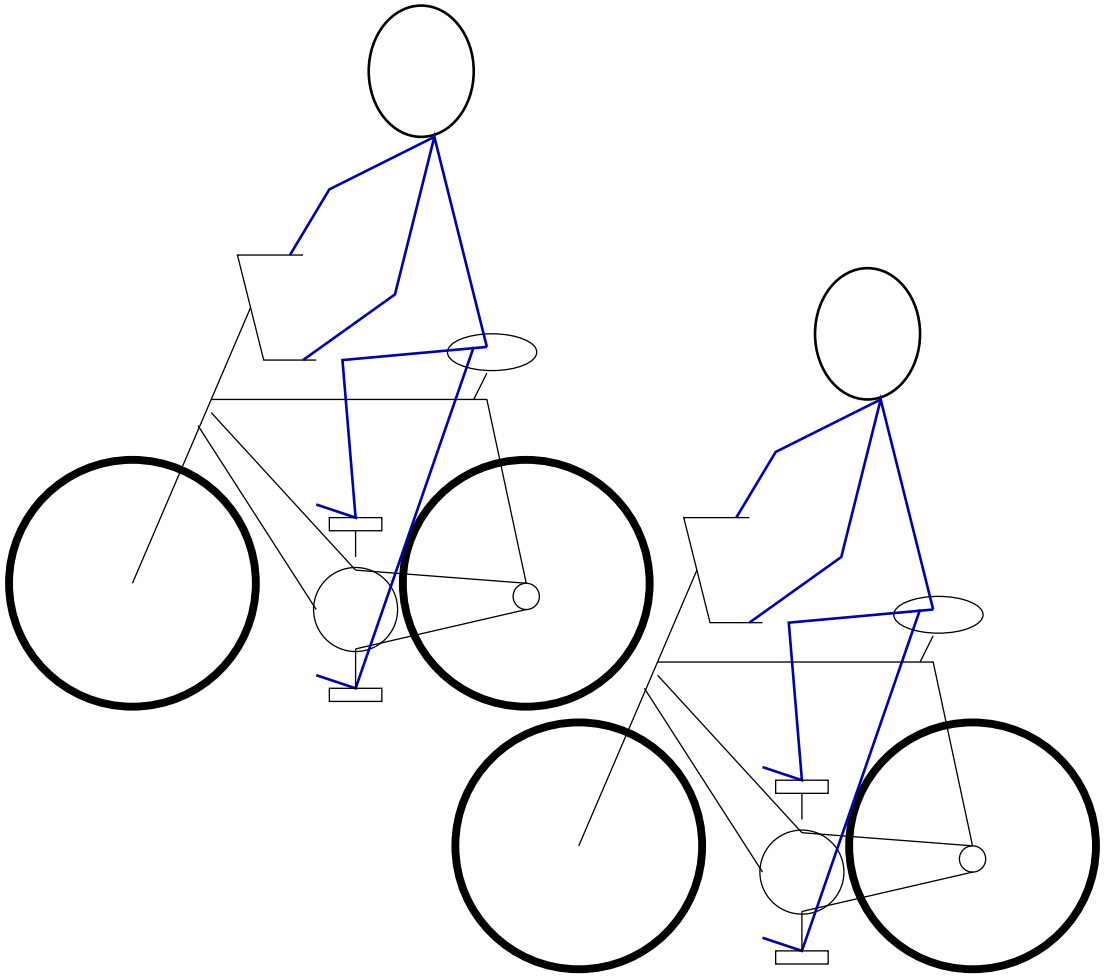

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- Synchronisation, or cooperation cannot make a component exceed its bounded capacity.
- Thus the apparent rate of a cooperation is the minimum of the apparent rates of the co-operands.

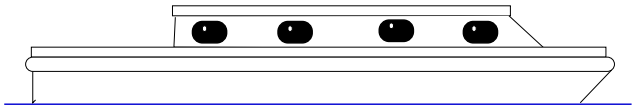
Outline

PEPA Case Studies (1)

- Multiprocessor access-contention protocols (Gilmore, Hillston and Ribaudo, Edinburgh and Turin)
- Protocols for fault-tolerant systems (Clark, Gilmore, Hillston and Ribaudo, Edinburgh and Turin)
- Multimedia traffic characteristics (Bowman et al, Kent)
- Database systems (The STEADY group, Heriot-Watt University)
- Software Architectures (Pooley, Bradley and Thomas, Heriot-Watt and Durham)
- Switch behaviour in active networks (Hillston, Kloul and Mokhtari, Edinburgh and Versailles)

PEPA Case Studies (2)

- Locks and movable bridges in inland shipping in Belgium (Knapen, Hasselt)



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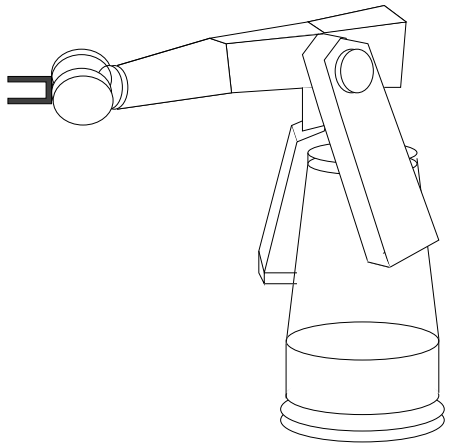

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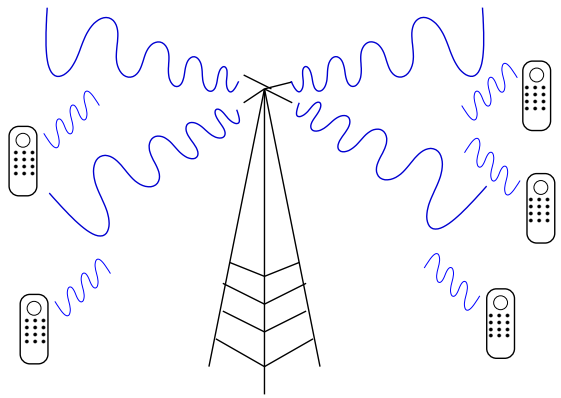

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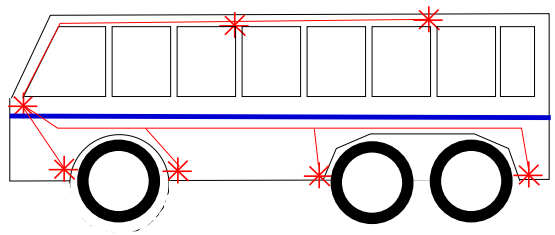
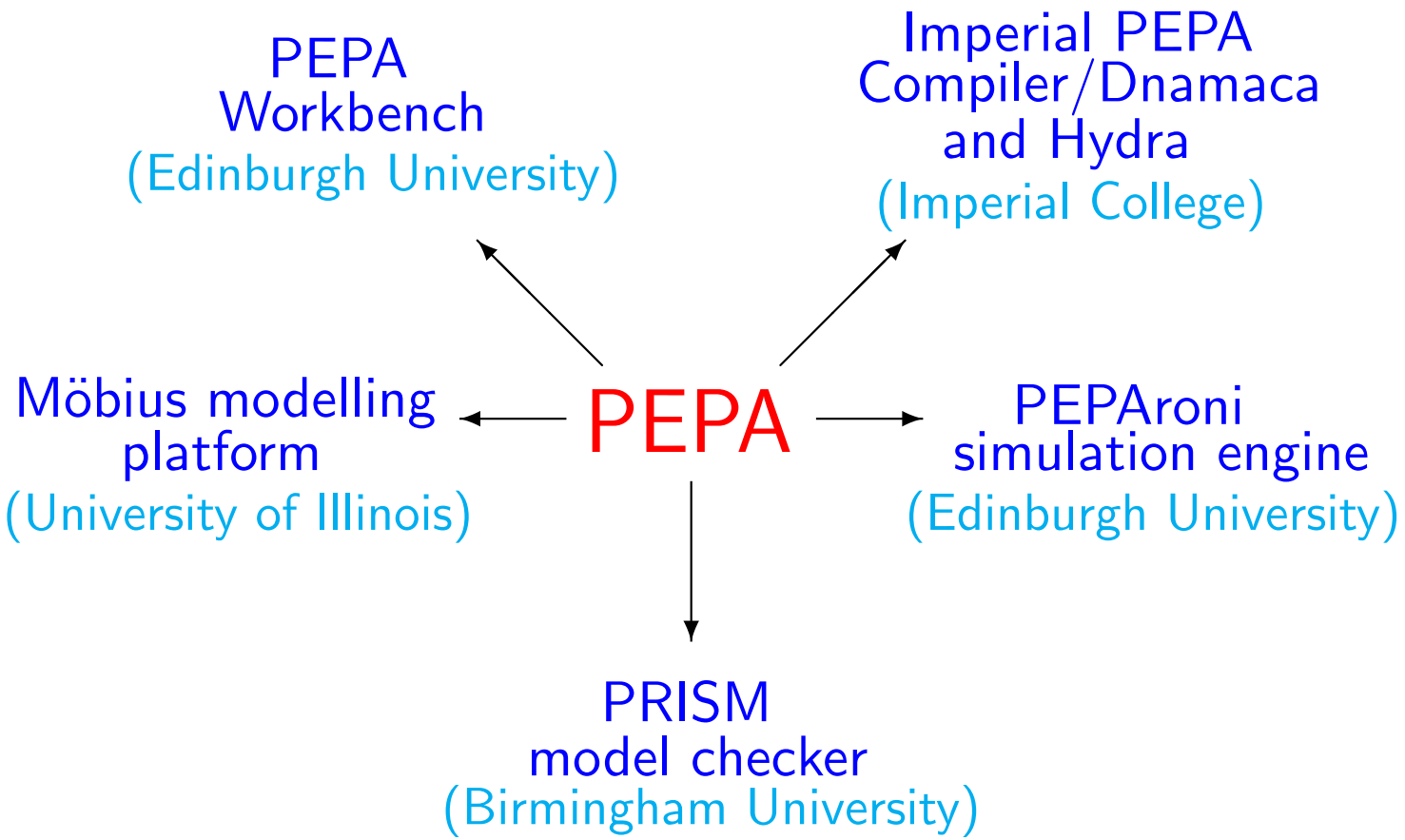
- Cellular telephone networks (Kloul, Fourneau and Valois, Versailles)
- Automotive diagnostic expert systems (Console, Picardi and Ribaudo, Turin)



Tool Support



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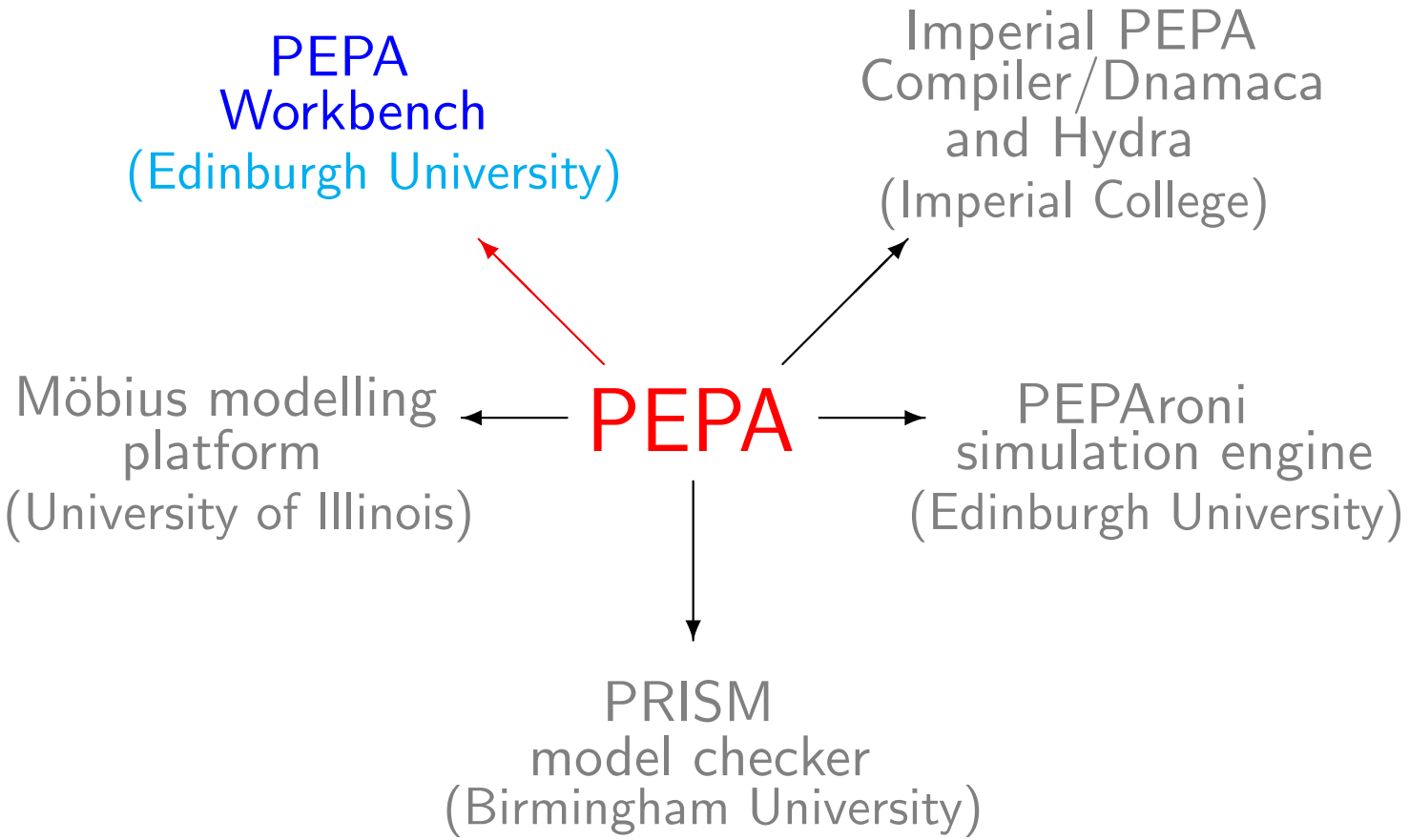

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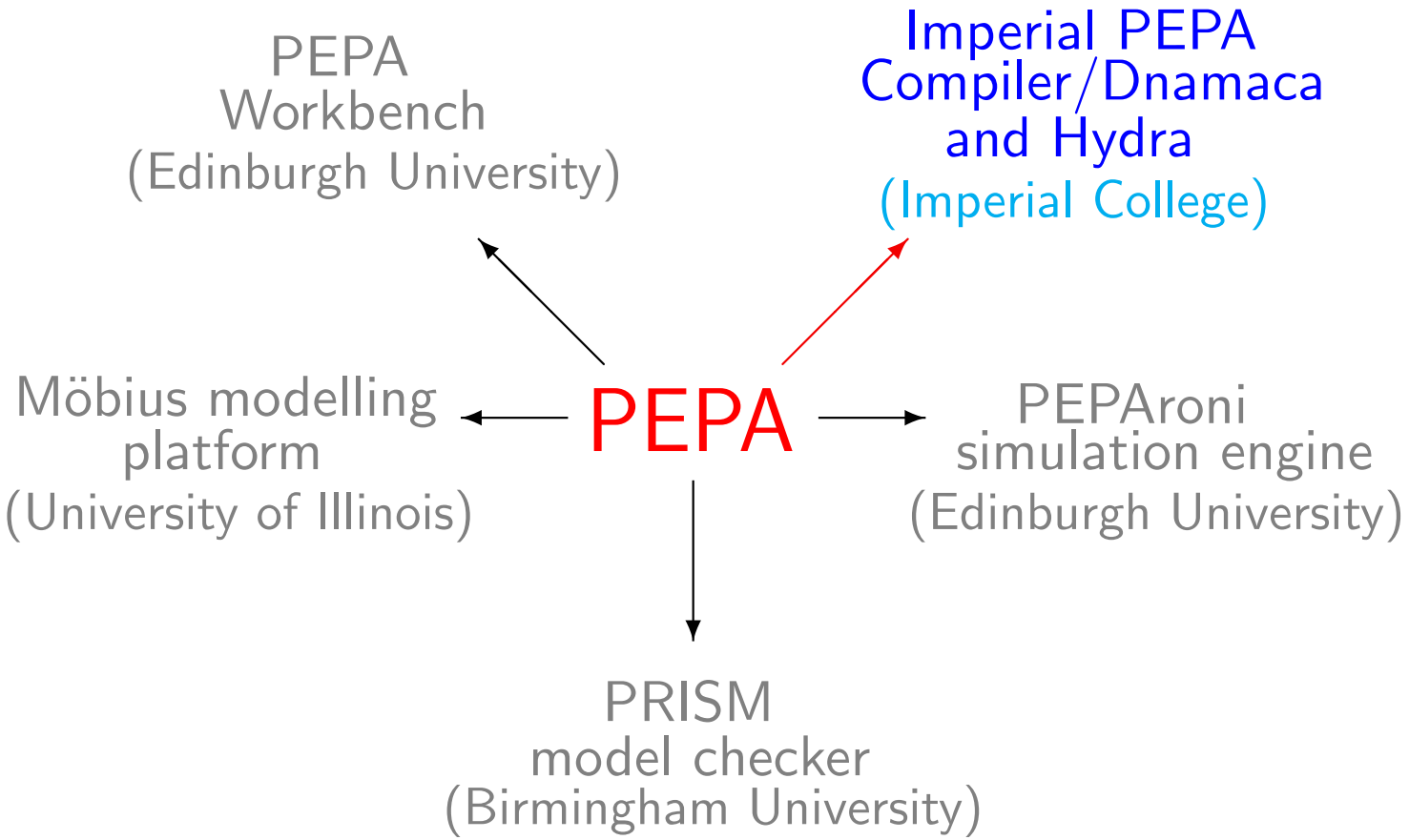

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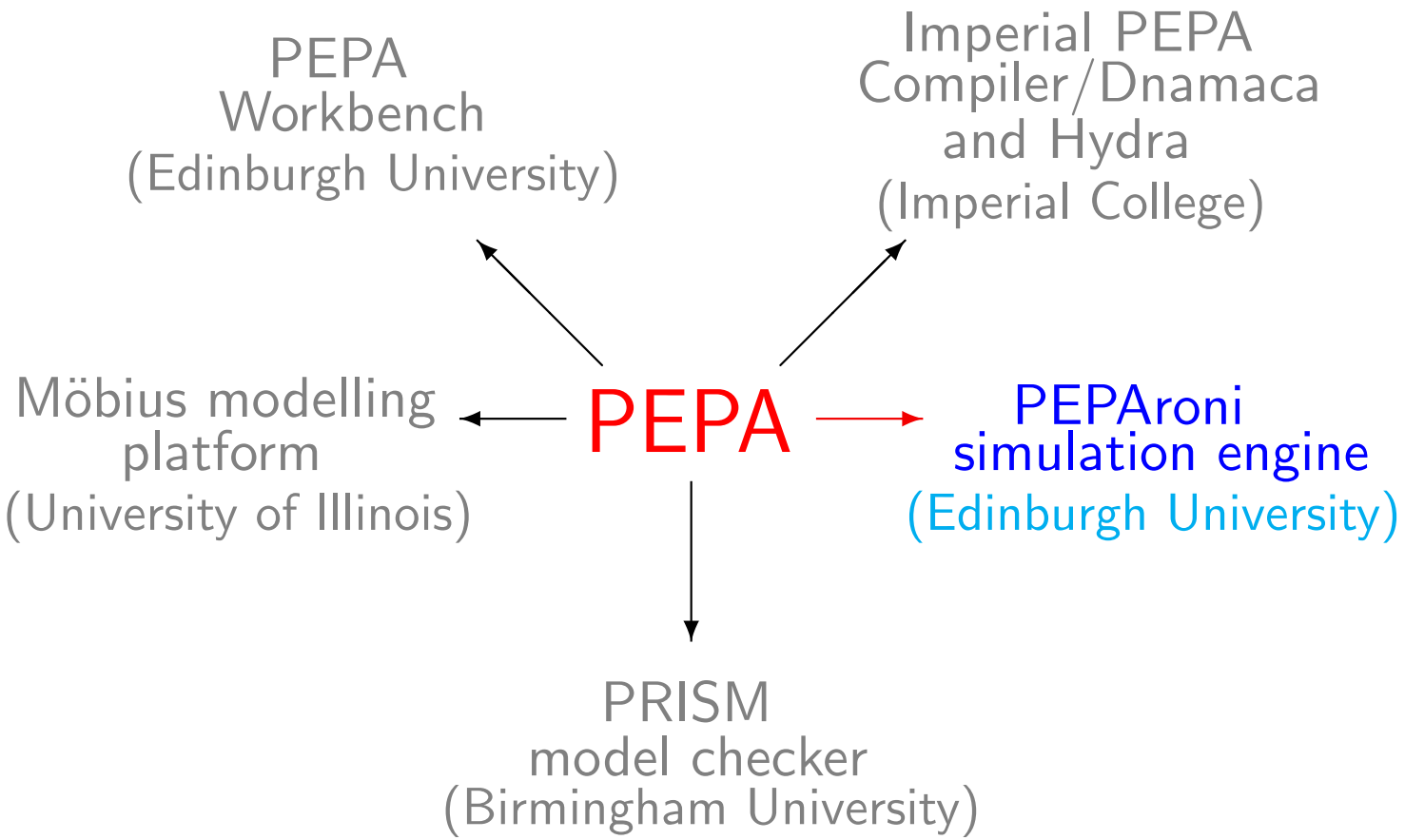

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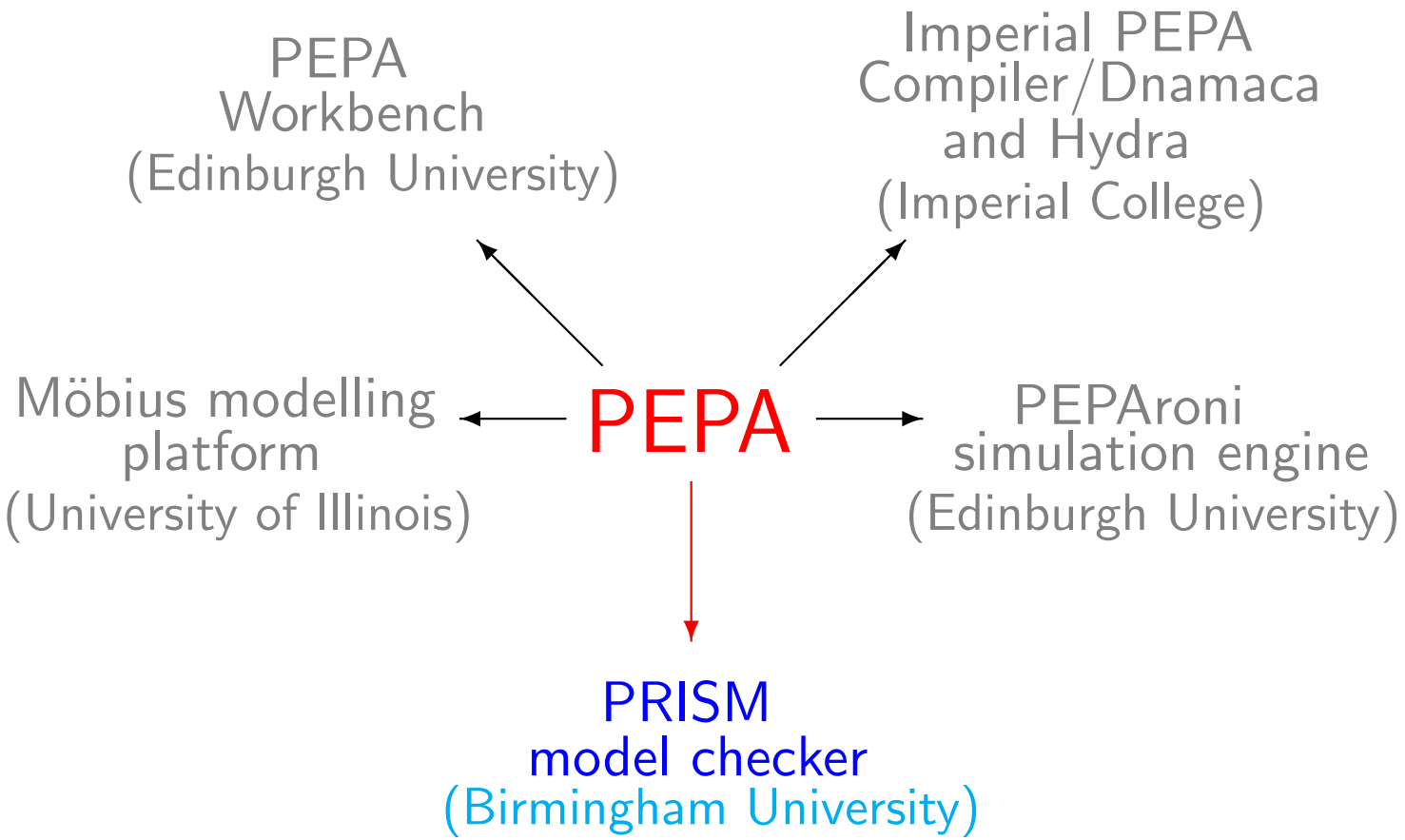

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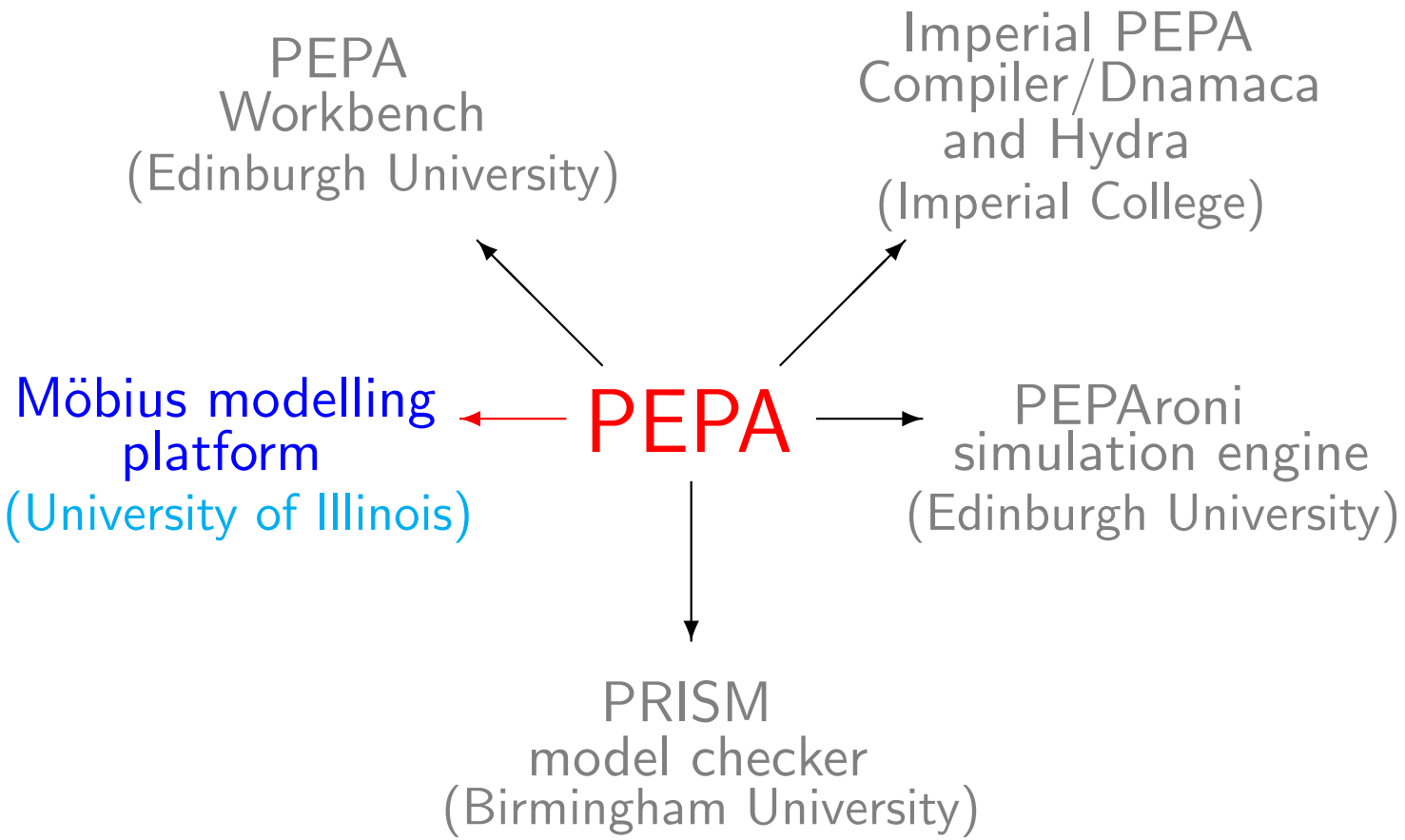

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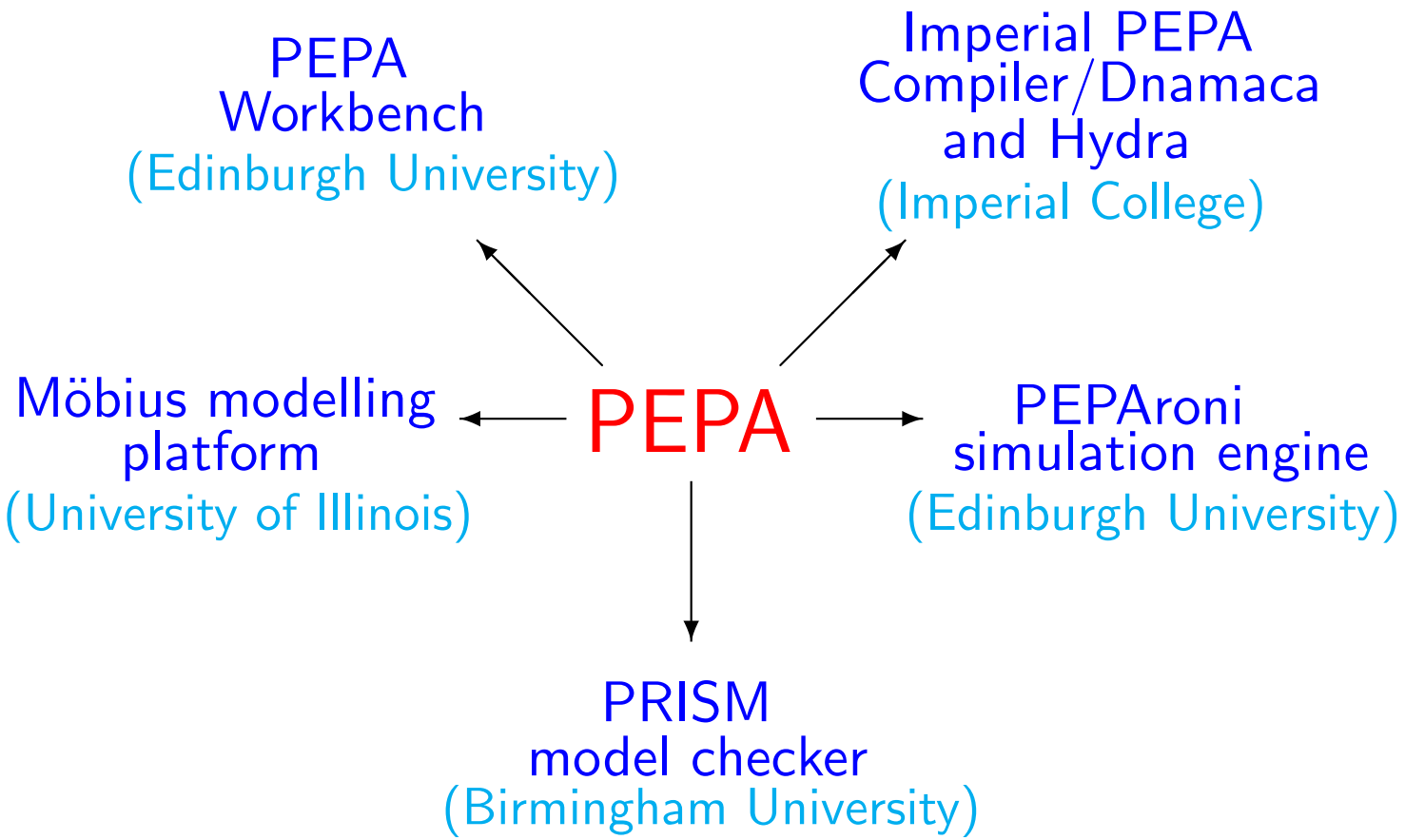

Roland the Gunslinger

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  - illustrate the main features of the language,
  - give you some experience of how models are constructed, and
  - demonstrate a variety of solution techniques.

Roland alone

In the first scenario we consider Roland alone, with the single activity of firing his gun which is a six-shooter. When his gun is empty Roland will reload the gun and then continue shooting.

$$
\begin{aligned}
Roland_6 &\stackrel{def}{=} (fire, r_{fire}).Roland_5 \\
Roland_5 &\stackrel{def}{=} (fire, r_{fire}).Roland_4 \\
Roland_4 &\stackrel{def}{=} (fire, r_{fire}).Roland_3 \\
Roland_3 &\stackrel{def}{=} (fire, r_{fire}).Roland_2 \\
Roland_2 &\stackrel{def}{=} (fire, r_{fire}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (fire, r_{fire}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_6
\end{aligned}
$$

Roland with two guns

All self-respecting gun-slingers have one gun in each hand. A simplistic way to model this is two instances of *Roland* in parallel:

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$$Roland_6 \underset{\{reload\}}{\bowtie} Roland_6$$

From now on we restrict Roland to his shotgun, which has two bullets and requires both hands for firing.

Roland meets an Enemy

- Upon his travels Roland encounters some enemies and when he does so he must fight them.
- Roland is the wildest gunslinger in the west so we assume that no enemy has the skill to seriously harm Roland.
- Each time Roland fires he might miss or hit his target.
- But with nothing to stop him he will keep firing until he successfully hits (and kills) the enemy.
- We assume that some sense of cowboy honour prevents any enemy attacking Roland if he is already involved in a gun fight.

The model

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, r_{attack}).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\quad + (miss, r_{miss}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\quad + (miss, r_{miss}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2
\end{aligned}
$$

Parameter settings for the $Roland_2$ model

parameter	*value*	*explanation*
r_{fire}	1.0	Roland can fire the gun once per-second
$p_{hit\text{-}success}$	0.8	Roland has an 80% success rate
r_{hit}	0.8	$r_{fire} \times p_{hit\text{-}success}$
r_{miss}	0.2	$r_{fire} \times (1 - p_{hit\text{-}success})$
r_{reload}	0.3	It takes Roland about 3 seconds to reload
r_{attack}	0.01	Roland is attacked once every 100 seconds

Steady state analysis

We can calculate the probability that at arbitrary time Roland is involved in a battle based on the steady state probability that *Roland* is in any of the states in which a battle is on-going, i.e. $Roland_2$, $Roland_1$ and $Roland_{empty}$.

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```
 State Measure 'roland peaceful'
mean                9.5490716180e-01
State Measure 'roland in battle'
mean                0.0450928382e-01
```

Passage-Time Analysis

Passage-time analysis allows us to calculate measures such as the probability that Roland has killed his enemy at a given time after he is attacked.

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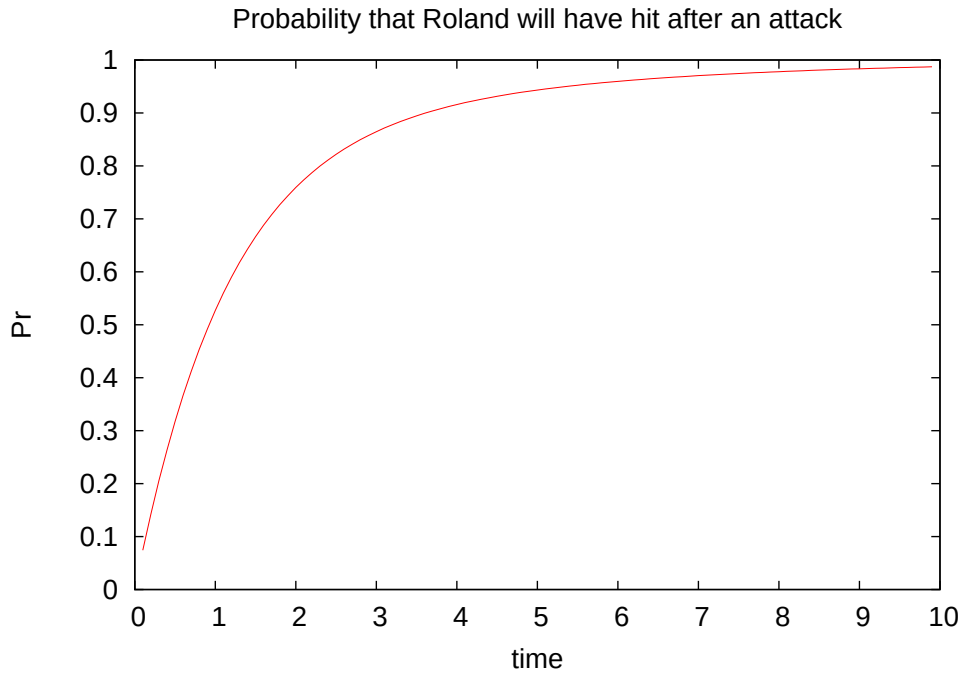
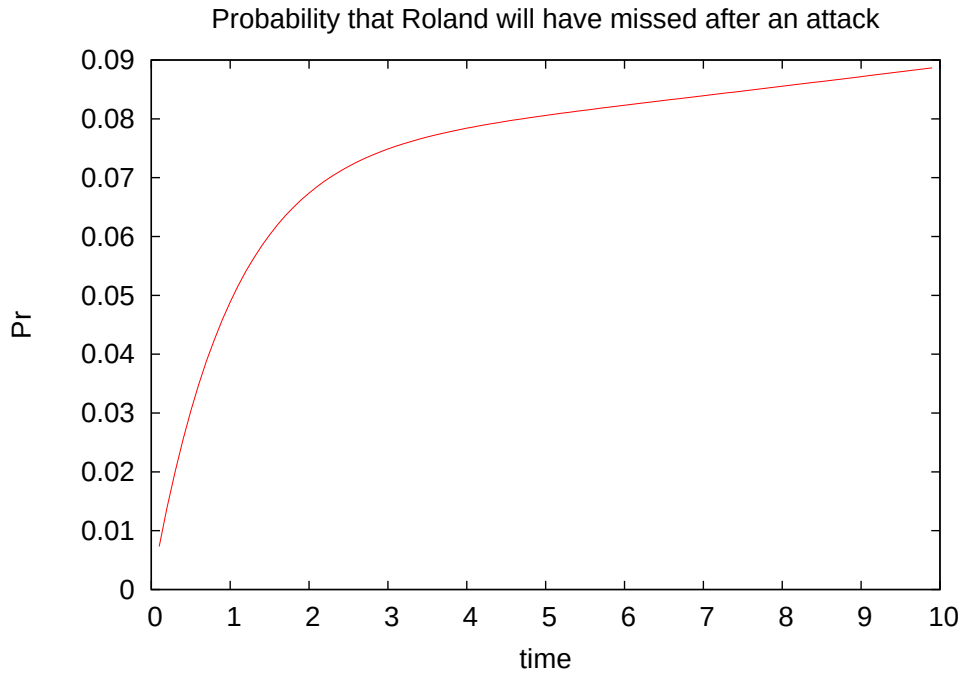
This would involve calculating the probability that the model performs a *hit* action within the given time after performing an *attack* action.

Passage-Time Analysis results



The probability that Roland will successfully perform a *hit* action a given time after an *attack*.

Passage-Time Analysis results



The probability that Roland has performed a *miss* action a given time after an *attack* action.

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- We now consider a model in which the enemies appear explicitly and allow them to fight back.
- However for now we still assume that there are rather ineffectual and so they never seriously injure Roland.
- This model can be used to calculate properties such as the likelihood that an enemy will manage to fire one shot before they are killed by Roland.

Revised Model

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\qquad + (miss, r_{miss}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\qquad + (miss, r_{miss}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2 \\
\\
Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\
Enemies_{attack} &\stackrel{def}{=} (fire, r_{e\text{-}miss}).Enemies_{attack} \\
&\qquad + (hit, \top).Enemies_{idle} \\
\\
Roland_2 &\underset{\{hit\}}{\bowtie} Enemies_{idle}
\end{aligned}
$$

Additional parameters

parameter	*value*	*explanation*
r_{attack}	0.01	Roland is attacked once every 100 seconds
$r_{e\text{-}miss}$	0.3	Enemies can fire only once every 3 seconds

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- This model can be thought of as an approximation to a more complicated component similar to the one which models Roland.
- Here the rate at which the enemy fires encompasses all of the actions, including the reloading of an empty gun.
- We may choose to model a component in such an abstract way when the focus of our modelling is really elsewhere in the model.

Model Validation

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This should never occur and hence the probability should be zero.

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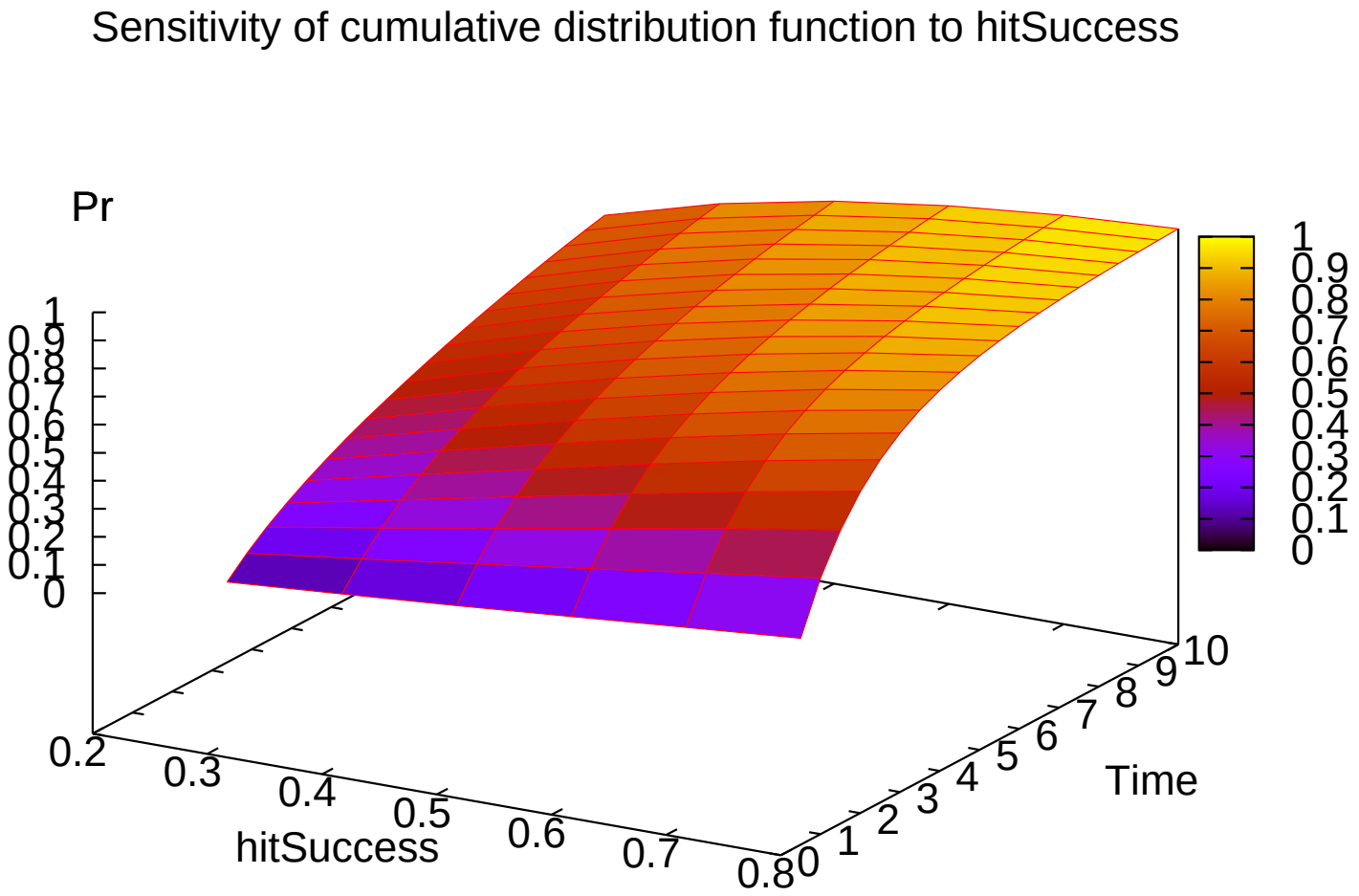
- Sensitivity analysis is performed by solving the model many times while varying the rates slightly.
- For this model we chose to vary three of the rates involved and measured the passage time between an *attack* and a *hit* activity, for each combination of rates.

Sensitivity Analysis: Results



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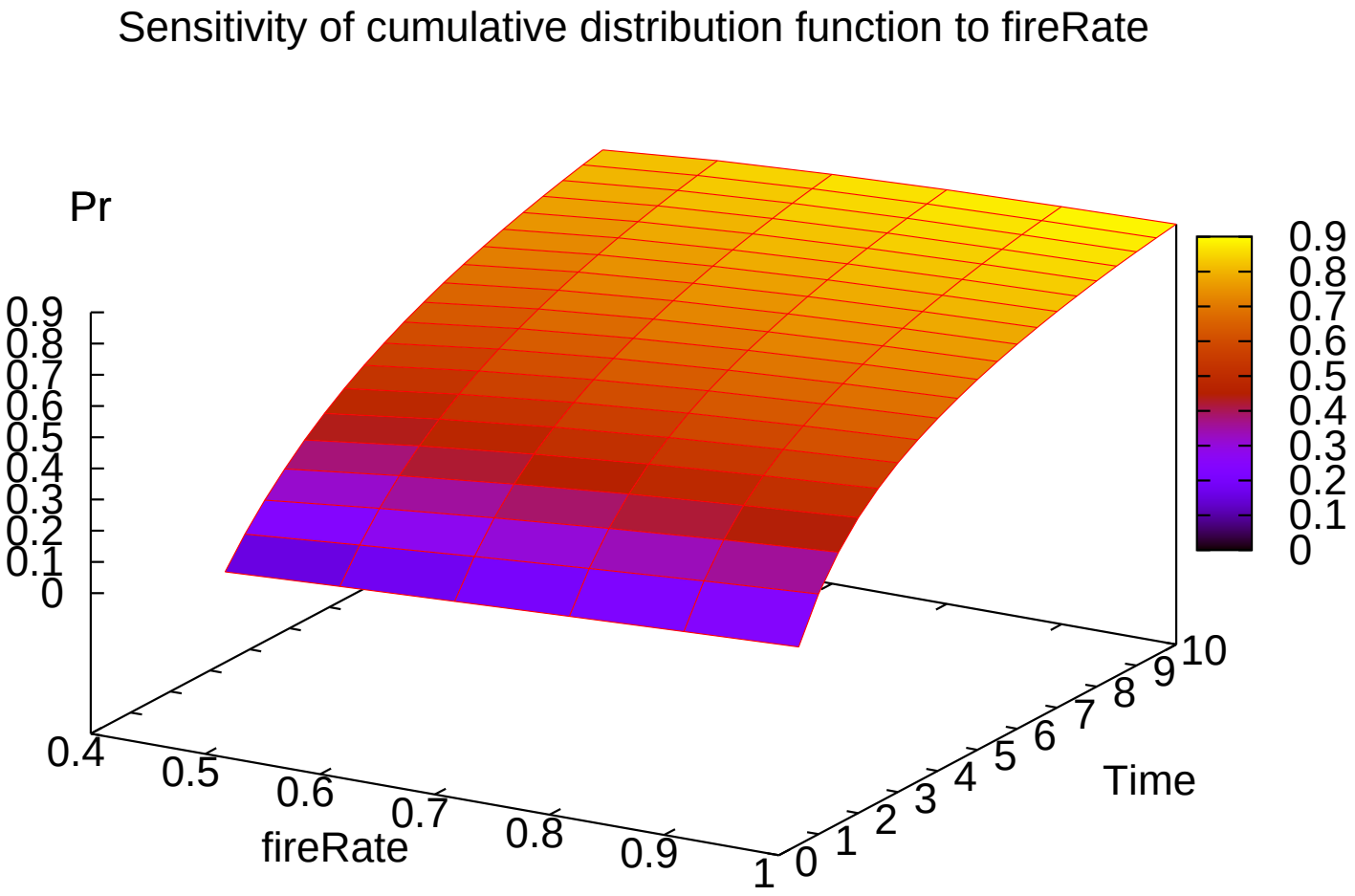

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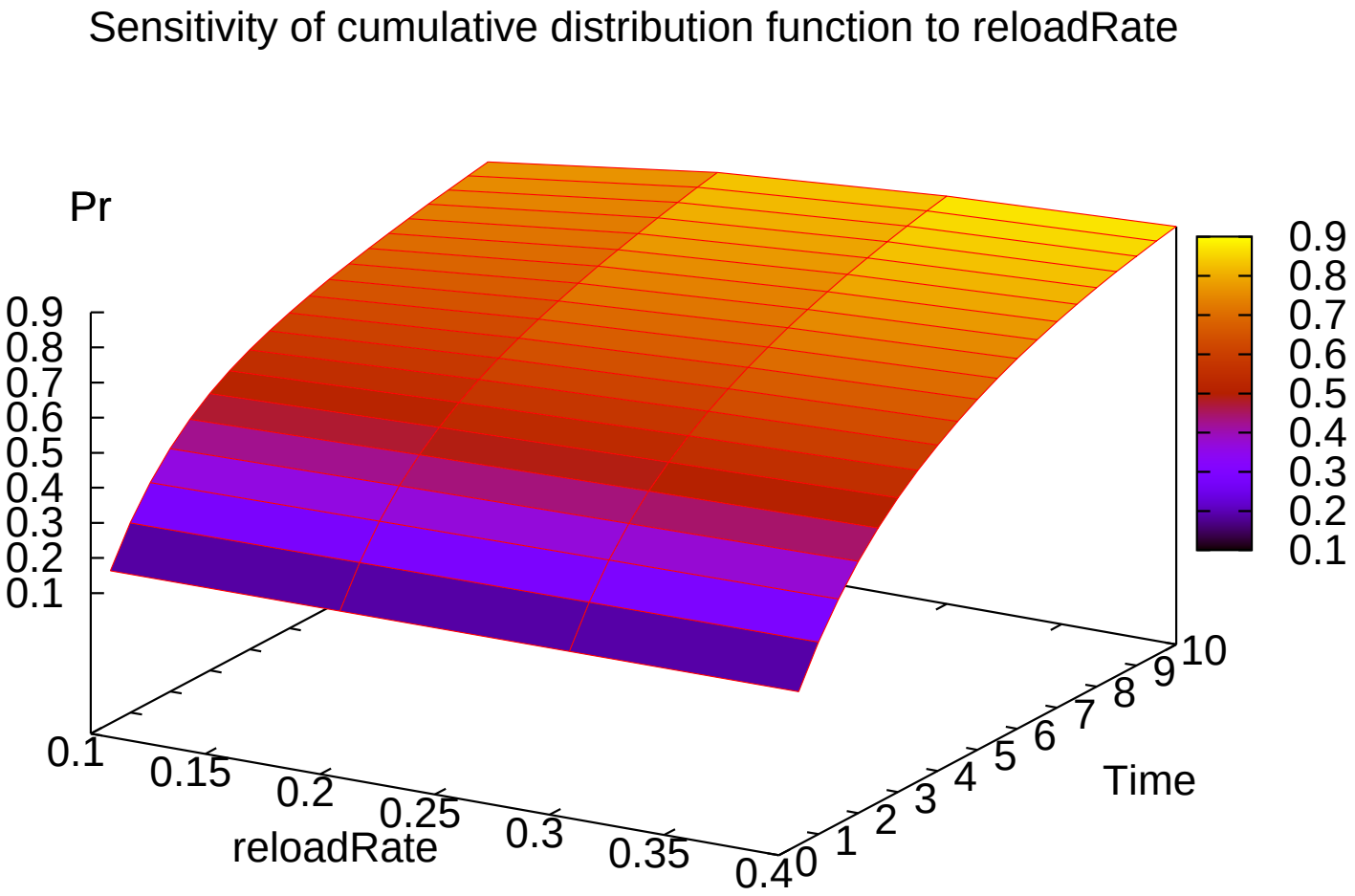

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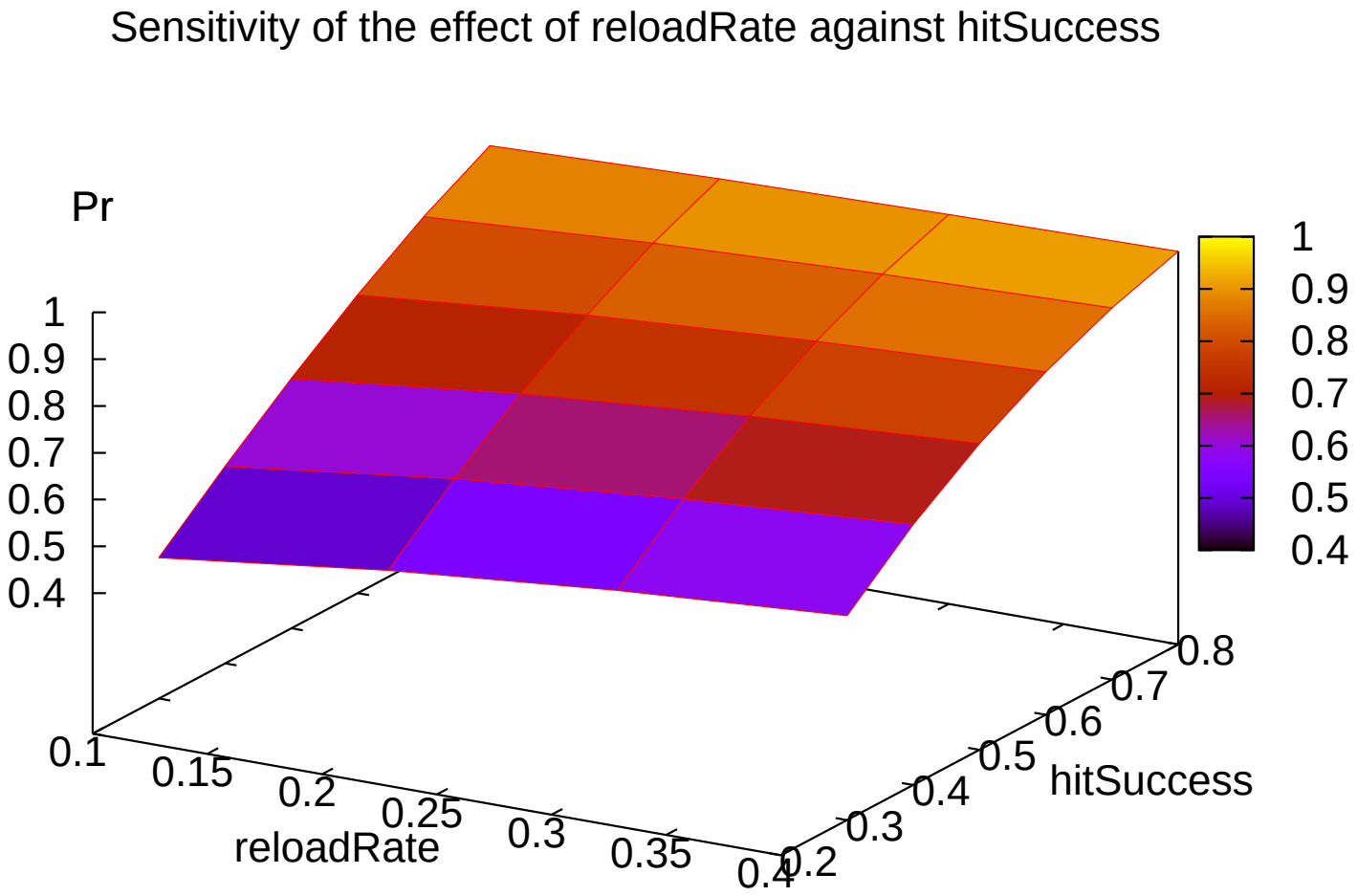

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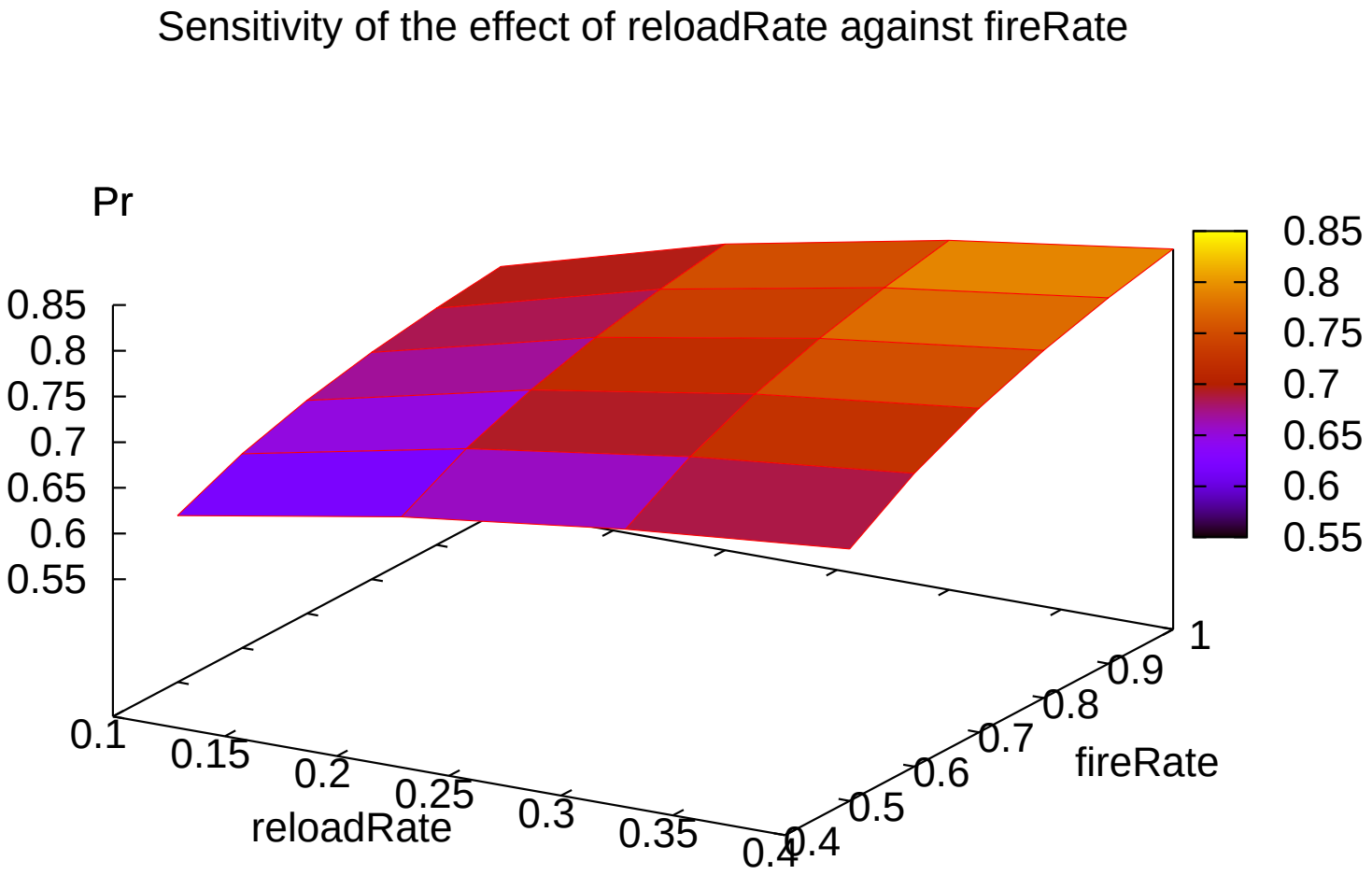

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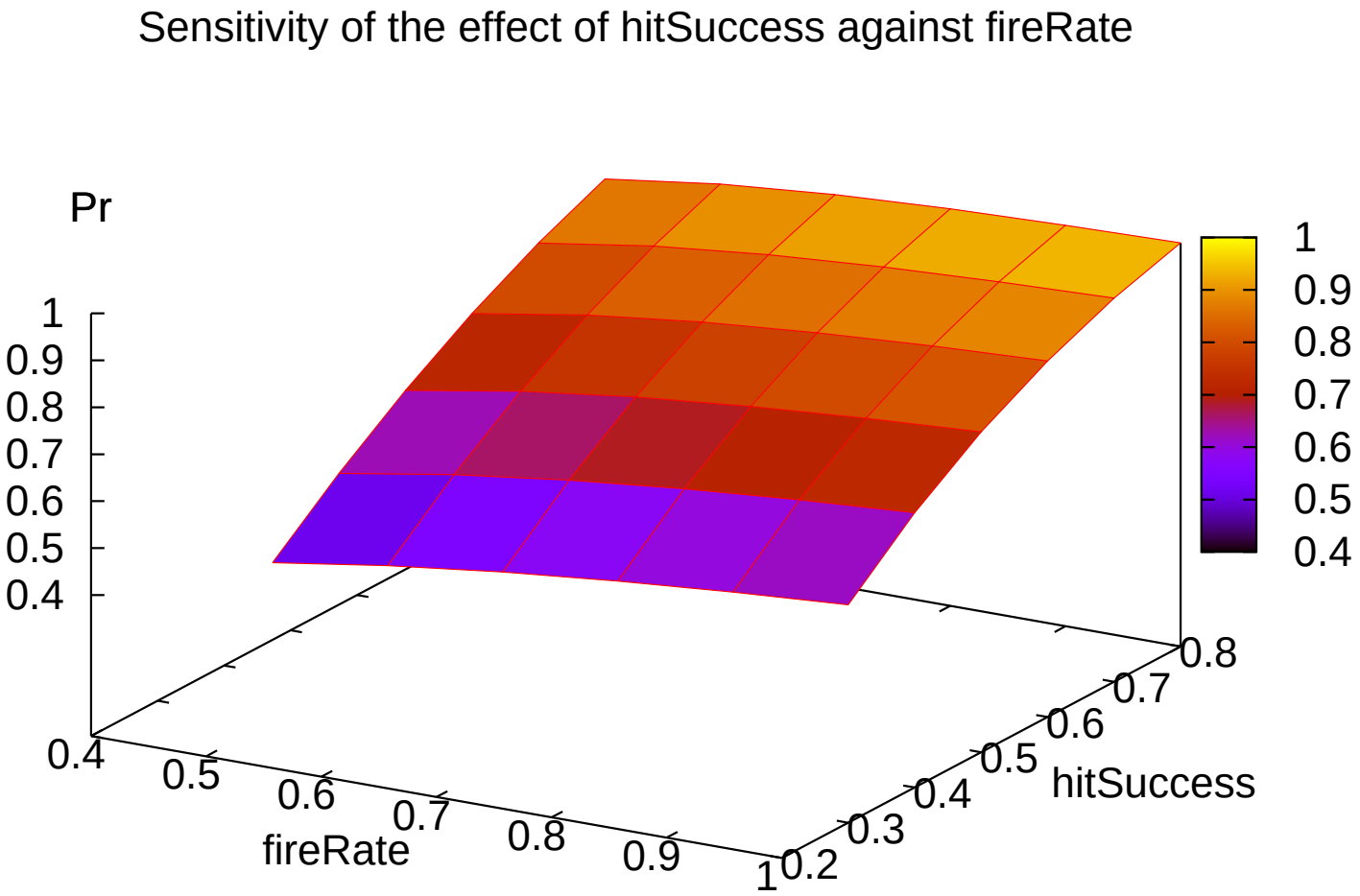

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- We assume that the enemies can only hit Roland once every 50 seconds. This rate approximates the rate of a more detailed model in which we would assign a process to the enemies which is much like that of the process which describes Roland.
- The only new parameter is $r_{e\text{-}hit}$ which is assigned a value 0.02 to reflect this assumption.

New Roland

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&+ (miss, r_{miss}).Roland_1 \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&+ (miss, r_{miss}).Roland_{empty} \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2 \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_{dead} &\stackrel{def}{=} Stop
\end{aligned}
$$

New Enemy

$$
\begin{aligned}
Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\
Enemies_{attack} &\stackrel{def}{=} (e\text{-}hit, r_{e\text{-}hit}).Enemies_{idle} \\
&\quad + \quad (hit, \top).Enemies_{idle}
\end{aligned}
$$

$$
Roland_{idle} \underset{\{hit, attack, e\text{-}hit\}}{\bowtie} Enemies_{idle}
$$

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Transient Analysis Transient analysis on this model can be used to calculate the probability that Roland is dead after a given amount of time. As time increases this should tend towards probability 1.

Passage-Time Analysis Passage-time analysis could be used to calculate the probability of a given event happening at a given time after another given event, e.g. from an attack on Roland until he dies or wins the gun fight.

Roland makes a friend

In the next revision of the model we introduce an accomplice who is befriended by Roland and who, when Roland is attacked, fights alongside him.

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Whenever either Roland or the accomplice kills the enemy the other must witness this action, so as to stop firing at a dead opponent (it would be a waste of ammunition!).

A new component for Roland

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
&+ (befriend, r_{befriend}).Roland_{idle} \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).Roland_{hit} + (miss, r_{miss}).Roland_1 \\
&+ (a\text{-}hit, \top).Roland_{idle} \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).Roland_{hit} \\
&+ (miss, r_{miss}).Roland_{empty} \\
&+ (a\text{-}hit, \top).Roland_{idle} \\
Roland_{hit} &\stackrel{def}{=} (reload, r_{reload}).Roland_{idle} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2 \\
&+ (a\text{-}hit, \top).Roland_{hit}
\end{aligned}
$$

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$$\begin{aligned} Acmpl_{abs} &\stackrel{def}{=} (befriend, r_{befriend}).Acmpl_{idle} \\ &+ (hit, \top).Acmpl_{abs} + (attack, \top).Acmpl_{abs} \end{aligned}$$

Component for the Accomplice

$$
\begin{aligned}
Acmpl_{idle} &\stackrel{def}{=} (attack, \top).Acmpl_2 \\
Acmpl_2 &\stackrel{def}{=} (a\text{-}hit, r_{a\text{-}hit}).Acmpl_{hit} + (hit, \top).Acmpl_{idle} \\
&\quad + (miss, r_{miss}).Acmpl_1 + (enemy\text{-}hit, \top).Acmpl_{abs} \\
Acmpl_1 &\stackrel{def}{=} (a\text{-}hit, r_{a\text{-}hit}).Acmpl_{hit} + (hit, \top).Acmpl_{hit} \\
&\quad + (miss, r_{miss}).Acmpl_{empty} + (enemy\text{-}hit, \top).Acmpl_{abs} \\
Acmpl_{hit} &\stackrel{def}{=} (reload, r_{a\text{-}reload}).Acmpl_{idle} \\
Acmpl_{empty} &\stackrel{def}{=} (reload, r_{a\text{-}reload}).Acmpl_2 + (hit, \top).Acmpl_{hit}
\end{aligned}
$$

Parameter Settings for the Accomplice

parameter	*value*	*explanation*
$r_{befriend}$	0.001	Roland befriends a stranger once every 1000 seconds
$r_{a\text{-}fire}$	1.0	the accomplice can also fire once per second
$p_{a\text{-}hit\text{-}success}$	0.6	the accomplice has a 60 percent accuracy
$r_{a\text{-}hit}$	0.6	$r_{fire} \times p_{hit\text{-}success}$
$r_{a\text{-}miss}$	0.4	$r_{fire} \times (1.0 - p_{hit\text{-}success})$
$r_{a\text{-}reload}$	0.25	it takes the accomplice 4 seconds to reload

Component for the Enemy

The component representing the enemy is similar to before.

$$
\begin{aligned}
Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\
Enemies_{attack} &\stackrel{def}{=} (enemy\text{-}hit, r_{e\text{-}hit}).Enemies_{attack} \\
&+ (hit, \top).(enemy\text{-}die, r_{e\text{-}die}).Enemies_{idle} \\
&+ (a\text{-}hit, \top).(enemy\text{-}die, r_{e\text{-}die}).Enemies_{idle}
\end{aligned}
$$

The system equation is as follows:

$$
(Roland_{idle} \underset{\{attack, hit, a\text{-}hit, befriend\}}{\bowtie} Acmpl_{abs}) \underset{\{attack, hit, a\text{-}hit, enemy\text{-}hit\}}{\bowtie} Enemies_{idle}
$$

Model Analysis

Steady-State Analysis

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Steady-State Analysis

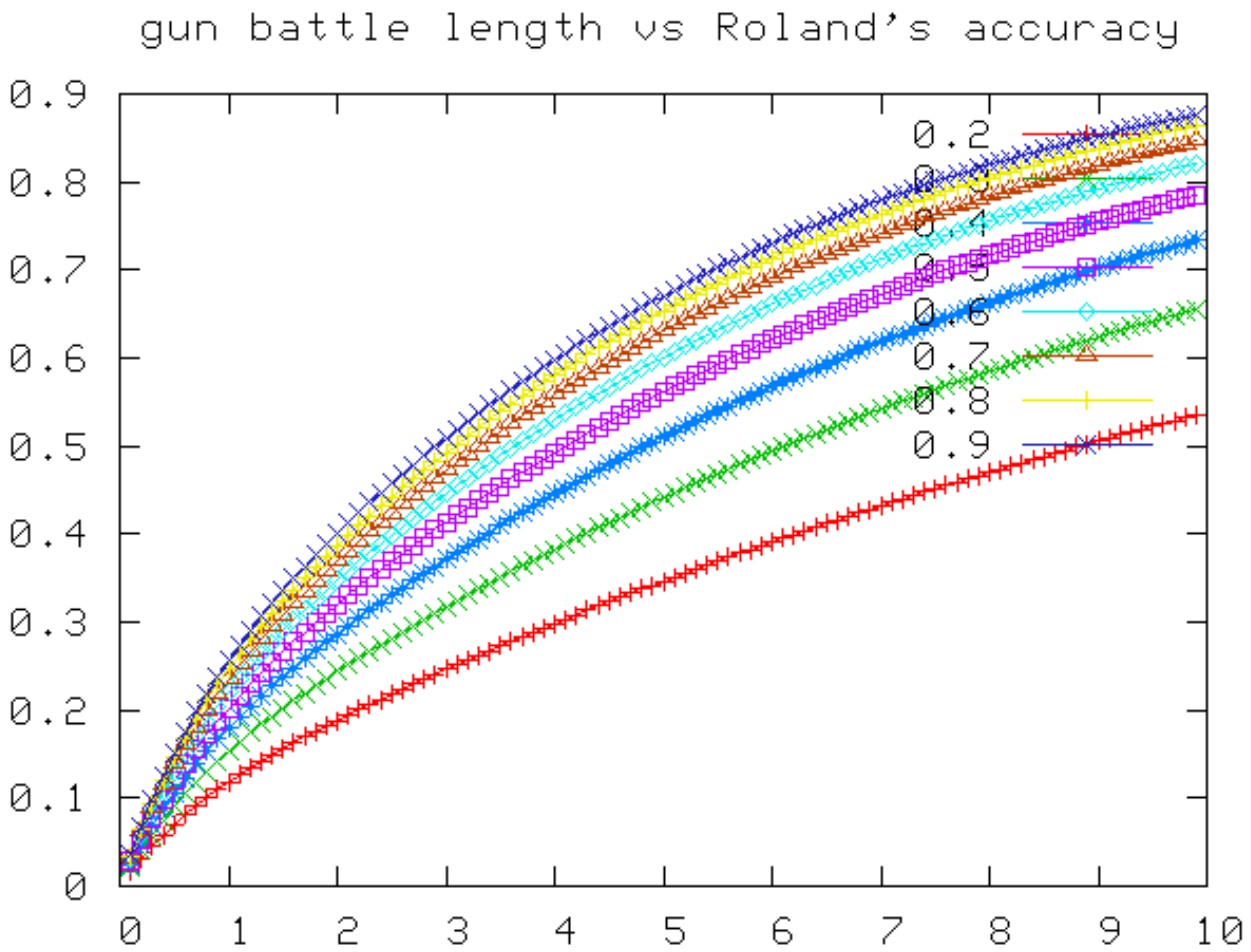
- As before we can determine the probability that Roland is involved in a gun battle at an arbitrary time.
- We could also determine the likelihood that Roland has an accomplice at an arbitrary time.
- Since Roland cannot perform a befriending action while currently involved in a battle, the probabilty that Roland is in such a battle clearly affects the probability that he is alone in his quest.
- For example, if Roland's success rate is reduced, gun battles will take longer to resolve and Roland will be involved in a gun battle more often. Consequently he will befriend fewer accomplices.

Model Analysis

Transient Analysis

An example transient analysis would be to determine the expected time after Roland has set off before he meets his first accomplice.

Model Analysis



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Passage-Time Analysis

- An example analysis would be to calculate the passage-time from an *attack* action until the death of the enemy or of the accomplice.
- Since all gun battles now end in the enemy being killed stopping the analysis there would give us the expected duration of any one gun battle.
- There is also the possibility to start the analysis from the *befriend* action and stop it with the death of the accomplice.

Hiding

- Currently there is nothing in the model to stop an enemy from disrupting the interaction between Roland and his accomplice, e.g. by performing a *befriend* action.

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- Currently there is nothing in the model to stop an enemy from disrupting the interaction between Roland and his accomplice, e.g. by performing a *befriend* action.
- One way to avoid this is to 'hide' those actions only Roland and the accomplice should cooperate on.
- To do this for our model we can simply change the system equation:

$$((Roland_{idle} \underset{L_1}{\bowtie} Acmpl)/\{befriend\}) \underset{L_2}{\bowtie} Enemies_{idle}$$

where $L_1 = \{attackhit, a\text{-}hit, befriend\}$ and $L_2 = \{attack, hit, a\text{-}hit, enemy\text{-}hit\}$.

Outline

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- PEPA has been integrated into the Möbius multi-paradigm modelling tool.
- The PEPA Plug-in Project permits Markovian steady-state analysis and stochastic simulation.

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- The examples discussed throughout this presentation are available at the PEPA Home Page:
  `http://www.dcs.ed.ac.uk/pepa/`

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- Eclipse Core makes PEPA tools available within the Eclipse framework.
- Eclipse UI implements a rich graphical user interface including an editor for PEPA descriptions and views for performance evaluation.

Services Available in PEPAto

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The output of this tool is a list of messages grouped into two categories:

- Error messages prevent further model analysis (e.g. state space derivation)
- Warning messages are less severe and allow further processing

Basic Analysis

An Error message is reported for:

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- Rate not declared

Basic Analysis

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- Rate not declared
- Process not defined

Basic Analysis

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Local Deadlock

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Q2 &\stackrel{def}{=} (\epsilon, v).Q1 \\
P1 &\underset{\{\alpha\}}{\bowtie} Q1
\end{aligned}
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Complete Action Type Set

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- Hiding $P\backslash\{L\}$:
  $\mathcal{A}\big(P\backslash\{L\}\big) = \mathcal{A}(P) - L$

Example

$$
\begin{array}{lcl}
P1 & \stackrel{def}{=} & (\alpha, r).P2 \\
P2 & \stackrel{def}{=} & (\gamma, t).P1 \\
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$$

Local Deadlock Detection

A cooperation $P \underset{L}{\bowtie} Q$ gives rise to a local deadlock if

$$\exists\, \alpha \in L : \alpha \notin \mathcal{A}(P) \cap \mathcal{A}(Q), \alpha \in \mathcal{A}(P) \cup \mathcal{A}(Q)$$

Redundant Action

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Redundant Action Detection

A cooperation $P \underset{L}{\bowtie} Q$ has a redundant action type α if

$$\exists\, \alpha \in L : \alpha \notin \mathcal{A}(P) \cup \mathcal{A}(Q)$$

Non-guarded Recursive Definition

Consider the following model snippet:

$$
\begin{array}{lcl}
\ldots & & \\
P1 & \stackrel{def}{=} & P2 \parallel P3 \\
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The derivation graph of component $P1$ gives rise to infinite-depth left recursion:

$$\begin{array}{lcl} P1 & \stackrel{(\gamma,t)}{\longrightarrow} & P2 \parallel P3 \parallel P3 \\ & \stackrel{(\gamma,t)}{\longrightarrow} & P2 \parallel P3 \parallel P3 \parallel P3 \\ & \stackrel{(\gamma,t)}{\longrightarrow} & P2 \parallel P3 \parallel P3 \parallel P3 \parallel P3 \\ & \stackrel{(\gamma,t)}{\longrightarrow} & \ldots \end{array}$$

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\ldots & &
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Non-guarded Definition Detection

For each process definition $A \stackrel{def}{=} P$ compute $\mathcal{U}(P)$.
A has infinite recursion if it defines a cooperation and $A \in \mathcal{U}(P)$

Example

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$$\mathcal{U}(P2 \parallel P3) = \{P1, P2, P3, \ldots\}$$

Example

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Example

$$
\begin{array}{lcl}
\ldots & & \\
P1 & \stackrel{def}{=} & P2 \parallel P3 \\
P2 & \stackrel{def}{=} & (\gamma, t).P1 \\
\ldots & &
\end{array}
$$

Non-guarded Definition Detection

For each process definition $A \stackrel{def}{=} P$ compute $\mathcal{U}(P)$.
A has infinite recursion if it defines a cooperation and $A \in \mathcal{U}(P)$

$$\mathcal{U}(P2 \parallel P3) = \{P1, P2, P3, \ldots\}$$
$$\mathcal{U}((\gamma, t).P1) = \{P1, P2, P3, \ldots\}$$

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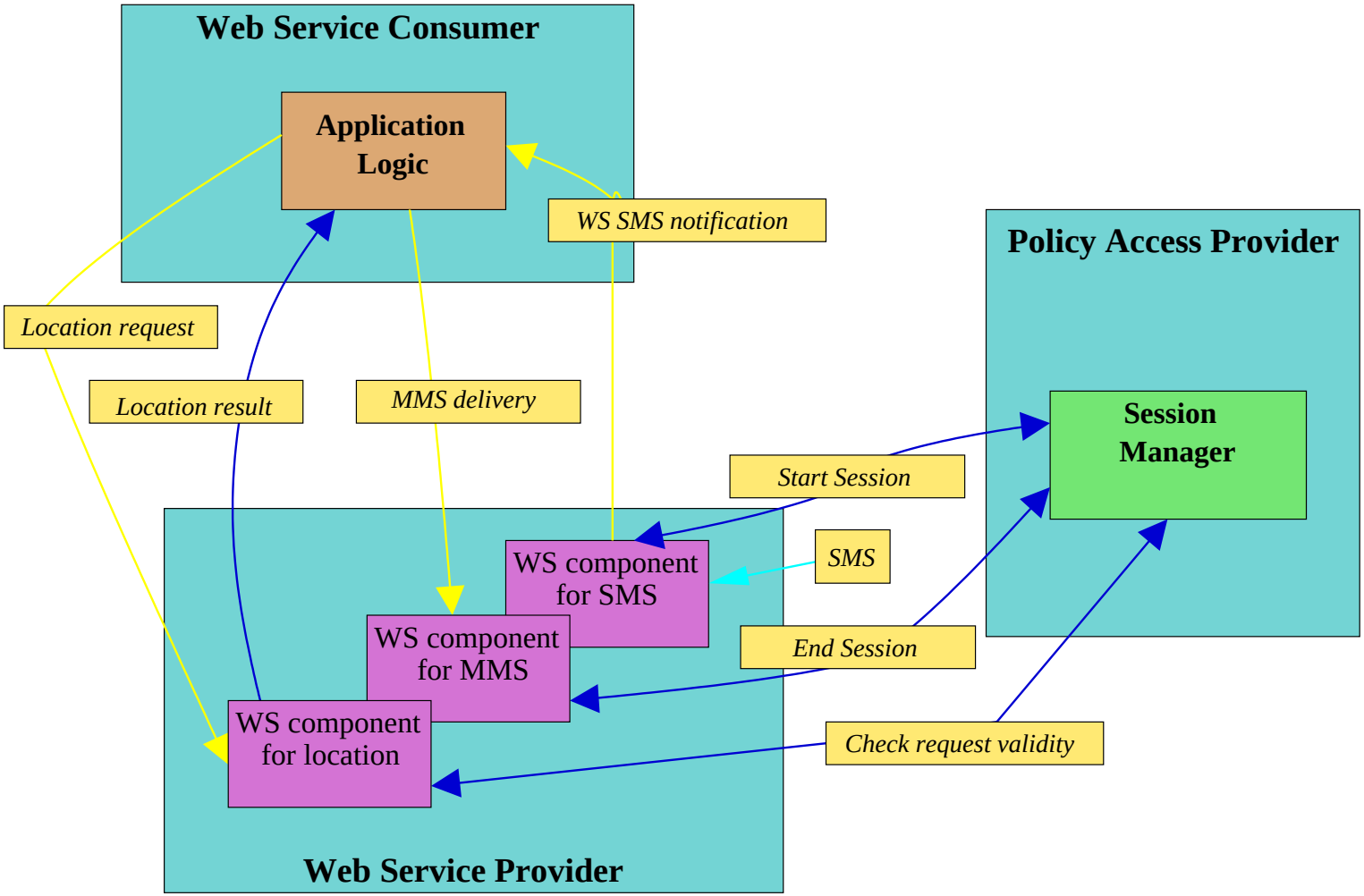
Moreover the service provider imposes a restriction that only one request may be handled for each SMS message received.

Schematic view



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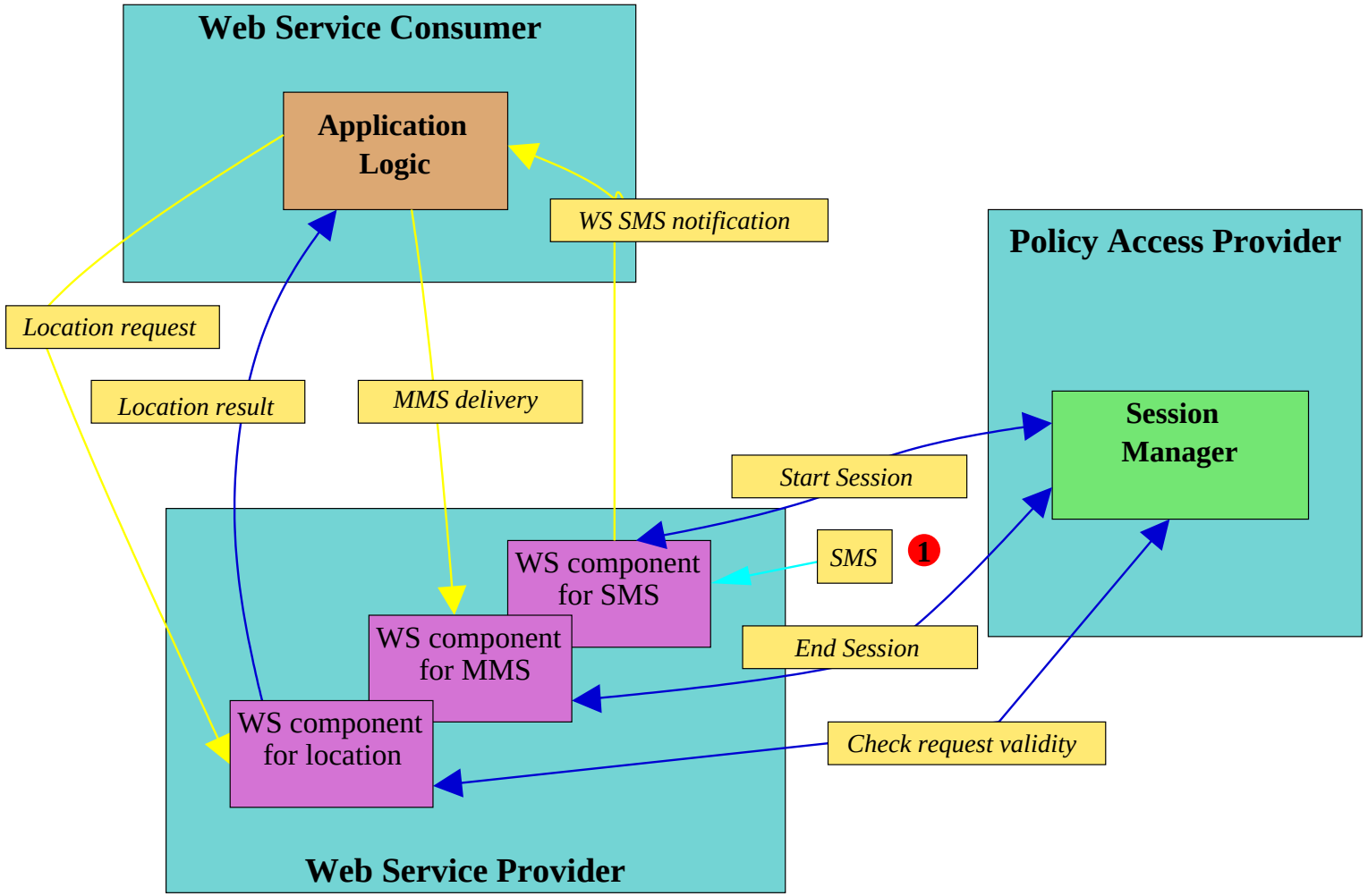

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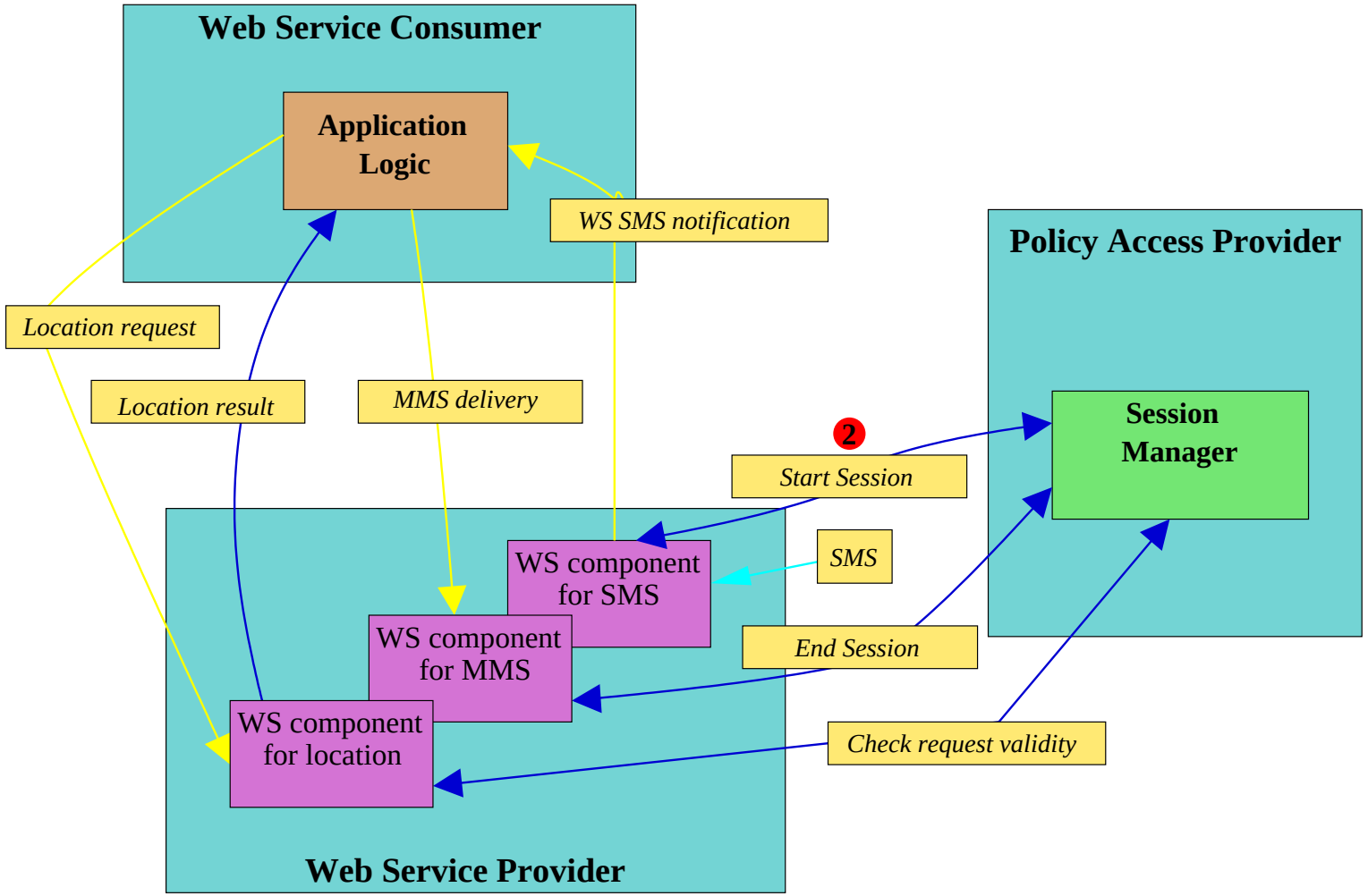

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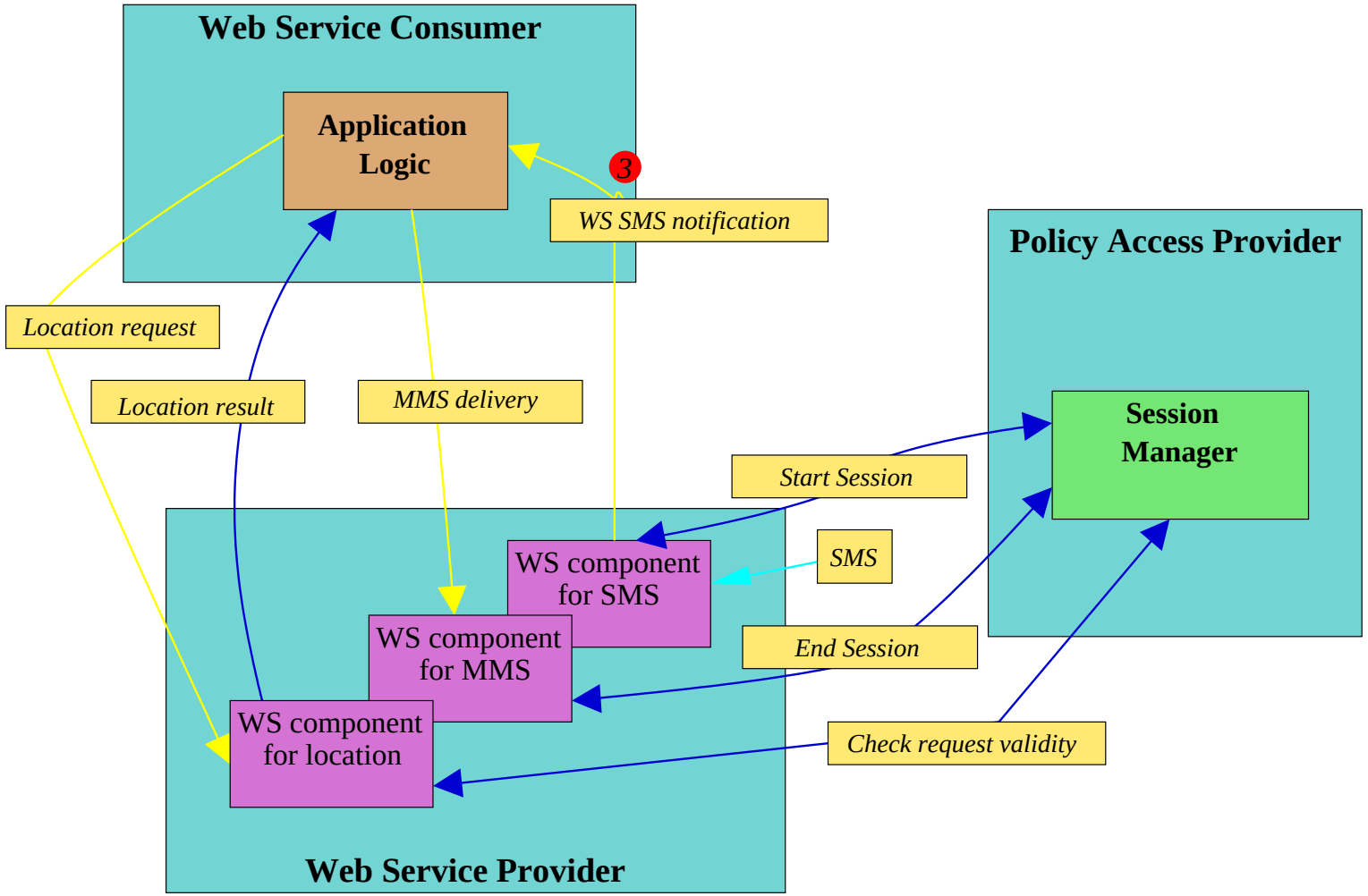

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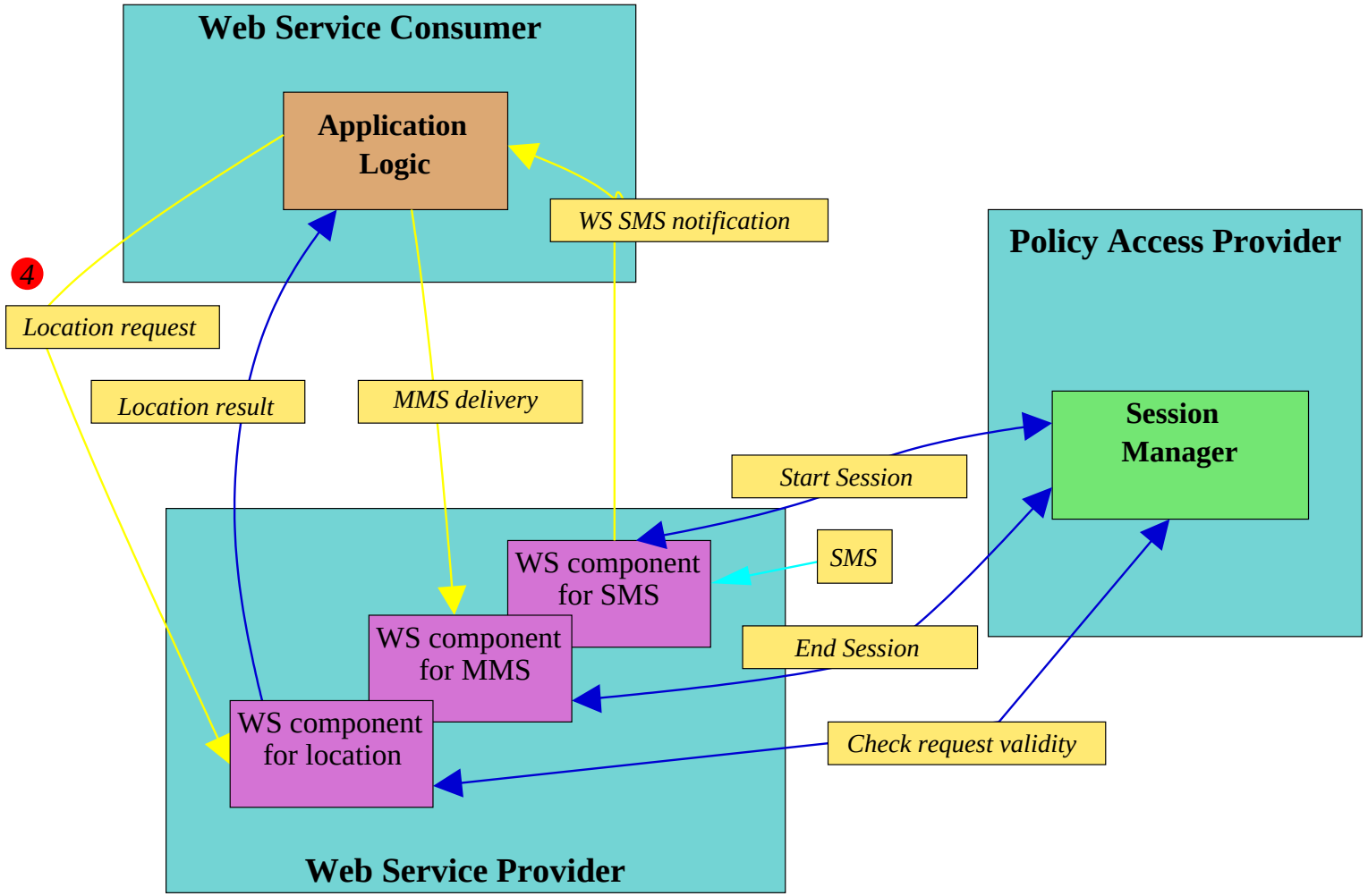

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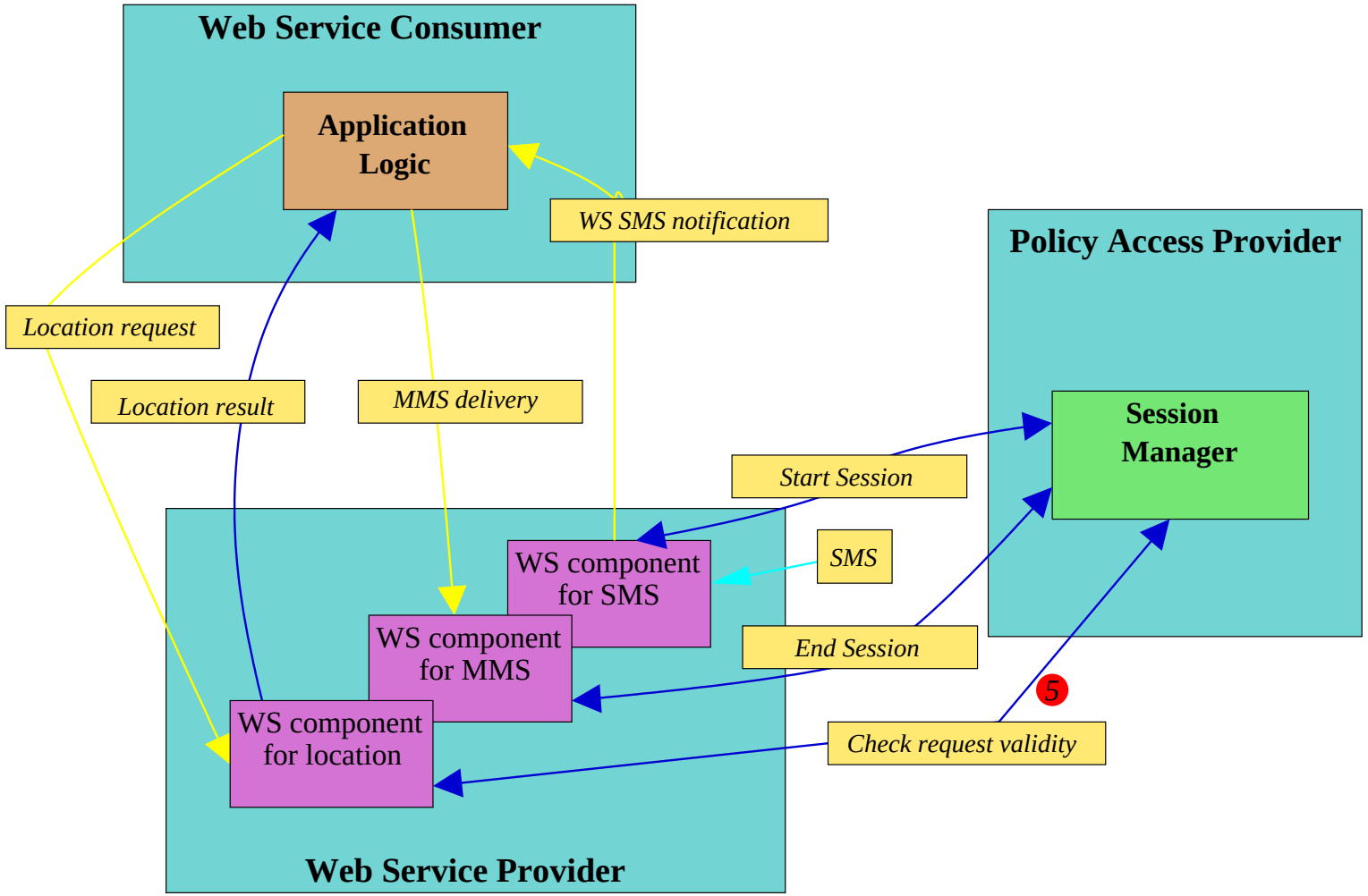

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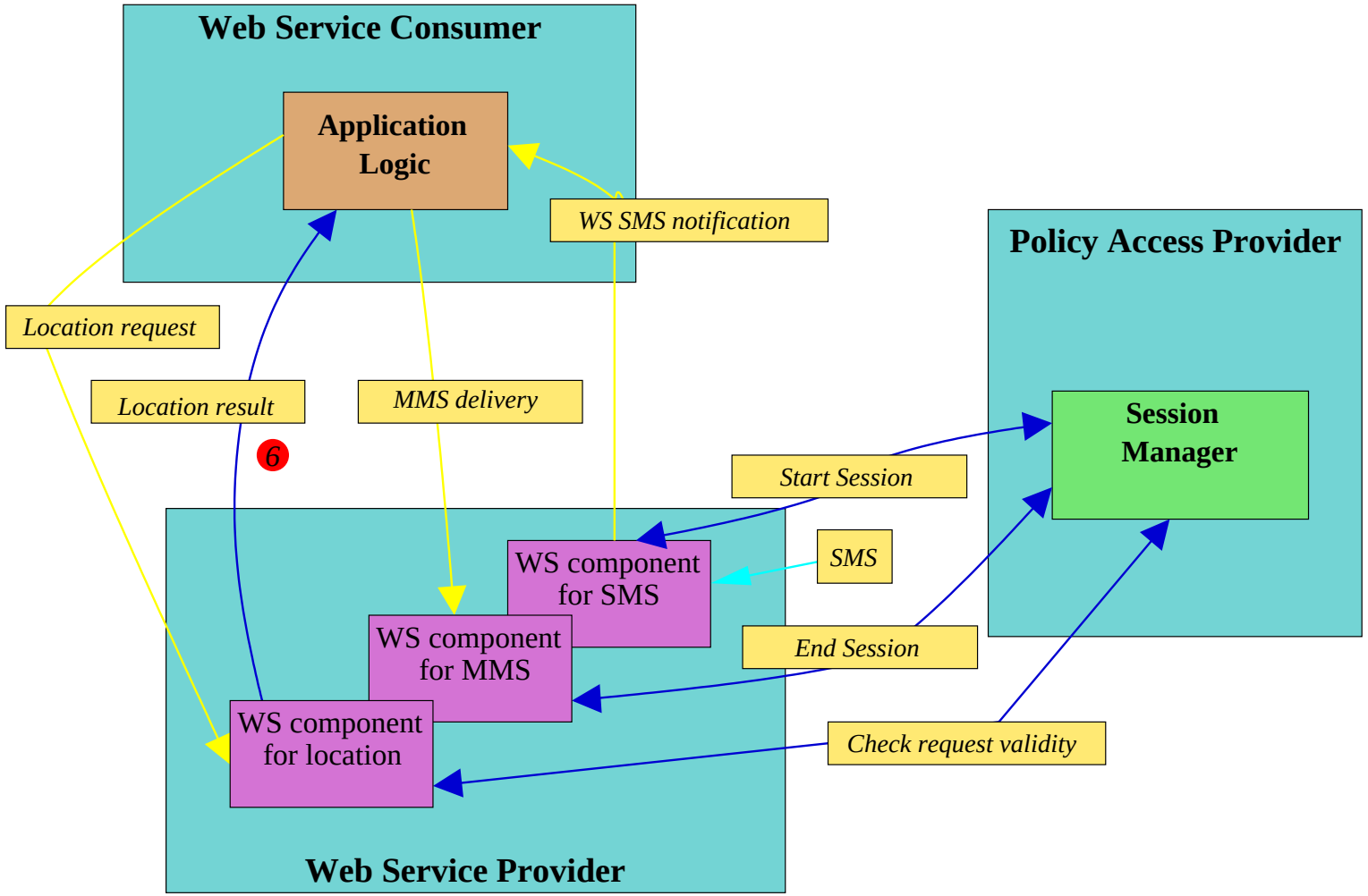

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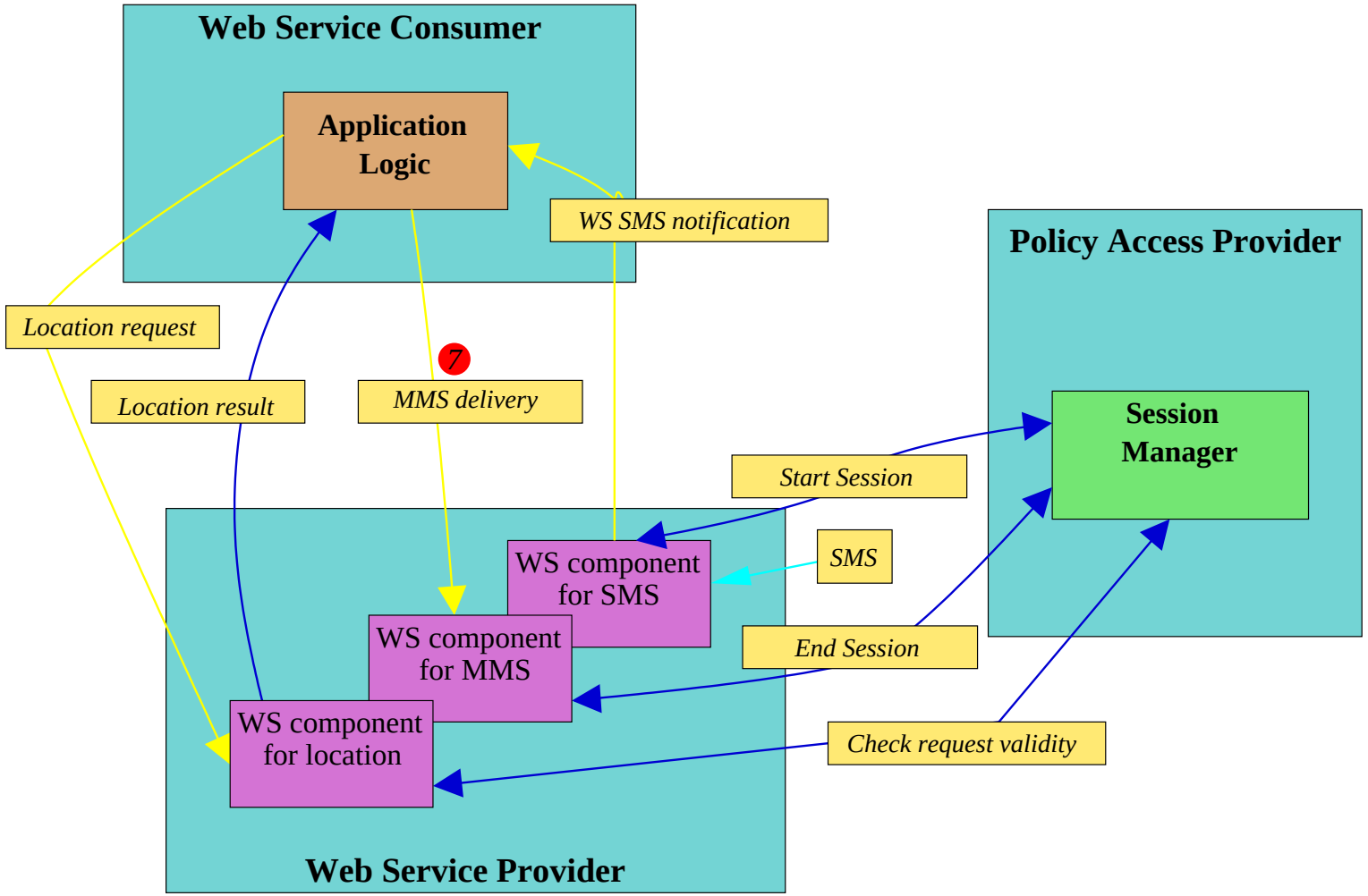

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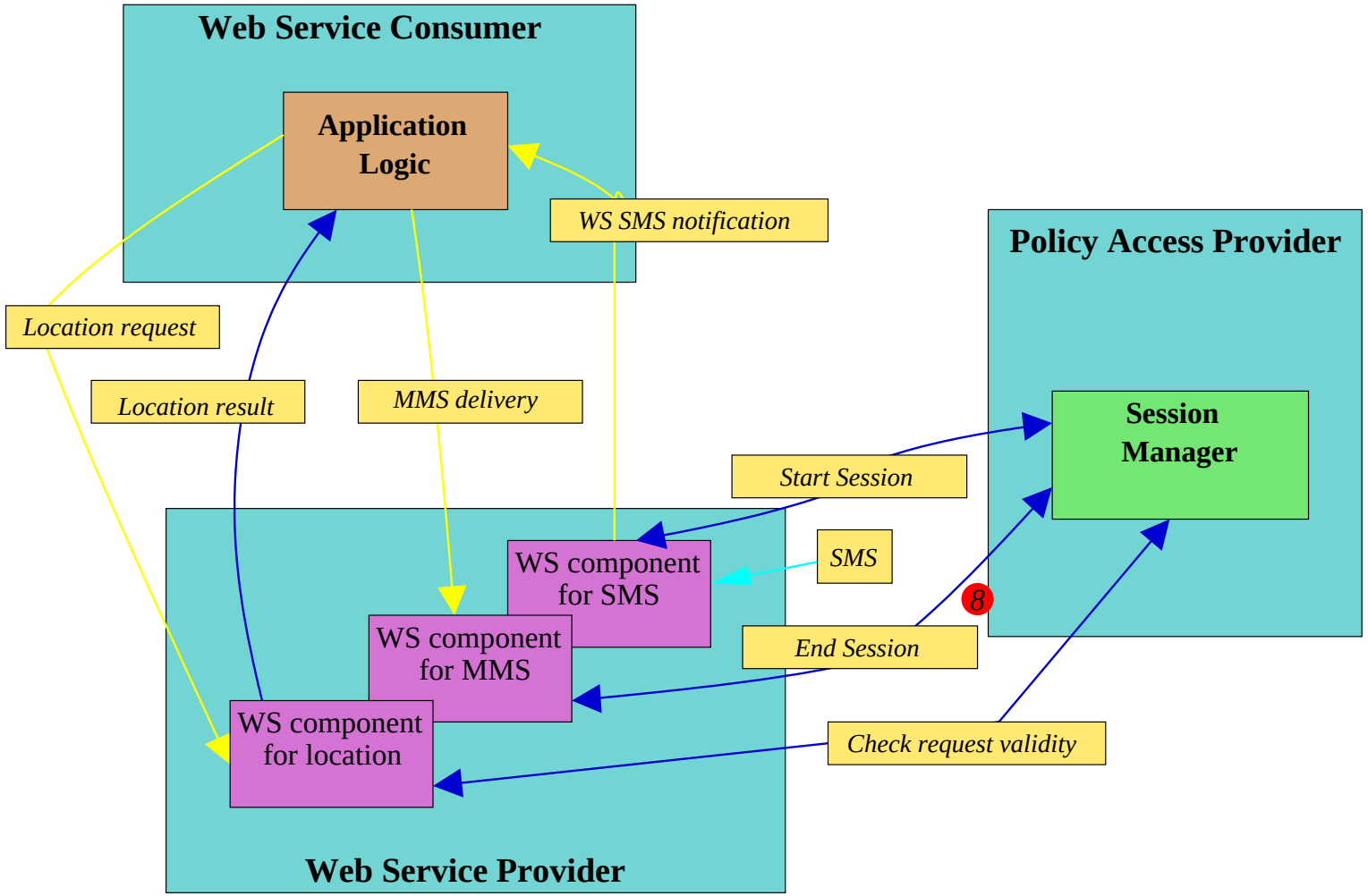

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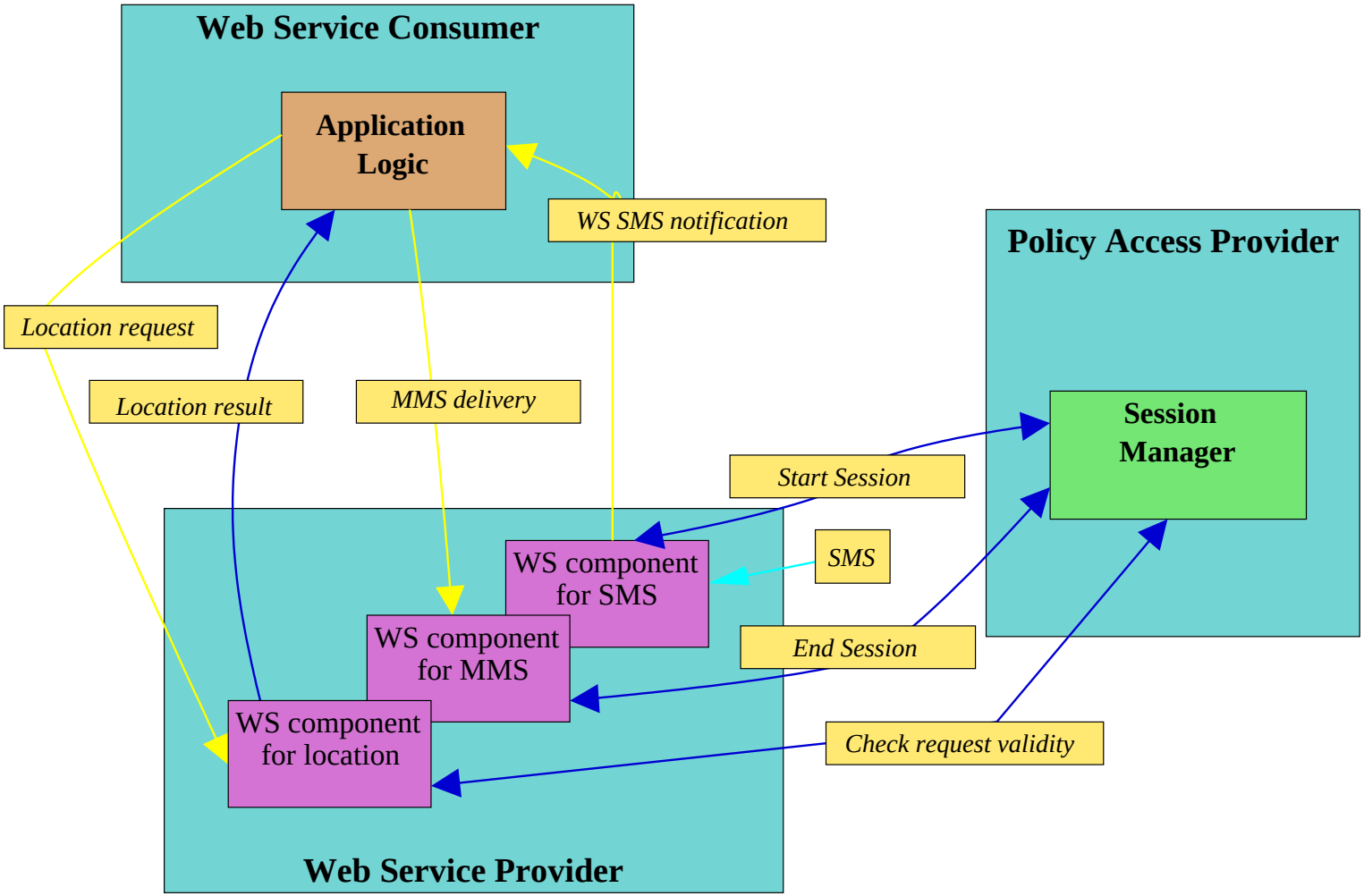

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Concurrency is introduced into the model by allowing multiple sessions rather than by representing the constituent web services separately.

Component *Customer*

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We associate the user-perceived system performance with the throughput of the *getMap* action which can be calculated directly from the steady state probability distribution of the underlying Markov chain.

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$$
\begin{aligned}
WSConsumer &\stackrel{def}{=} (notify, \top).WSConsumer_2 \\
WSConsumer_2 &\stackrel{def}{=} (locReq, r_4).WSConsumer_3 \\
WSConsumer_3 &\stackrel{def}{=} (locRes, \top).WSConsumer_4 \\
&\quad + \quad (locErr, \top).WSConsumer \\
WSConsumer_4 &\stackrel{def}{=} (compute, r_7).WSConsumer_5 \\
WSConsumer_5 &\stackrel{def}{=} (sendMMS, r_9).WSConsumer
\end{aligned}
$$

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- If the check is successful the location must be returned to the Web Service Consumer in the form of a map (*getMap*).
- If the check revealed an invalid request (*locErr*) then an error must be returned to the Web Service Consumer (*get404*) and the session terminated (*stopSession*).

Component *WSProvider*

$$
\begin{aligned}
WSProvider &\stackrel{def}{=} (getSMS, \top).WSProvider_2 \\
WSProvider_2 &\stackrel{def}{=} (startSession, r_2).WSProvider_3 \\
WSProvider_3 &\stackrel{def}{=} (notify, r_3).WSProvider_4 \\
WSProvider_4 &\stackrel{def}{=} (locReq, \top).WSProvider_5 \\
WSProvider_5 &\stackrel{def}{=} (checkValid, 99 \cdot \top).WSProvider_6 \\
&\quad + \quad (checkValid, \top).WSProvider_{10}
\end{aligned}
$$

Component *WSProvider* cont.

$$
\begin{aligned}
WSProvider_6 &\stackrel{def}{=} (locRes, r_6).WSProvider_7 \\
WSProvider_7 &\stackrel{def}{=} (sendMMS, \top).WSProvider_8 \\
WSProvider_8 &\stackrel{def}{=} (getMap, r_8).WSProvider_9 \\
WSProvider_9 &\stackrel{def}{=} (stopSession, r_2).WSProvider \\
WSProvider_{10} &\stackrel{def}{=} (locErr, r_6).WSProvider_{11} \\
WSProvider_{11} &\stackrel{def}{=} (get404, r_8).WSProvider_9
\end{aligned}
$$

Component *PAProvider*

We consider a stateless implementation of the policy access provider.

$$\begin{aligned} PAProvider &\stackrel{def}{=} (startSession, \top).PAProvider \\ &+ (checkValid, r_5).PAProvider \\ &+ (stopSession, \top).PAProvider \end{aligned}$$

Model Component *WSComp*

The complete system is composed of some number of instances of the components interacting on their shared activities:

$$\begin{aligned} WSComp \stackrel{def}{=} & \big((Customer[N_C] \underset{L_1}{\bowtie} WSProvider[N_{WSP}]) \\ & \underset{L_2}{\bowtie} WSConsumer[N_{WSC}]\big) \\ & \underset{L_3}{\bowtie} PAProvider[N_{PAP}] \end{aligned}$$

where the cooperation sets are

$$\begin{aligned} L_1 &= \{getSMS, getMap, get404\} \\ L_2 &= \{notify, locReq, locRes, locErr, sendMMS\} \\ L_3 &= \{startSession, checkValid, stopSession\} \end{aligned}$$

Parameter Values

param	*value*	*explanation*
r_1	0.0010	rate customers request maps
r_2	0.5	rate session can be started
r_3	0.1	notification exchange between consumer and provider
r_4	0.1	rate requests for location can be satisfied
r_5	0.05	rate the provider can check the validity of the request
r_6	0.1	rate location information can be returned to consumer
r_7	0.05	rate maps can be generated
r_8	0.02	rate MMS messages can be sent from provider to customer
r_9	$10.0 * r_8$	rate MMS messages can be sent via the Web Service

Steady State Analysis for System Tuning

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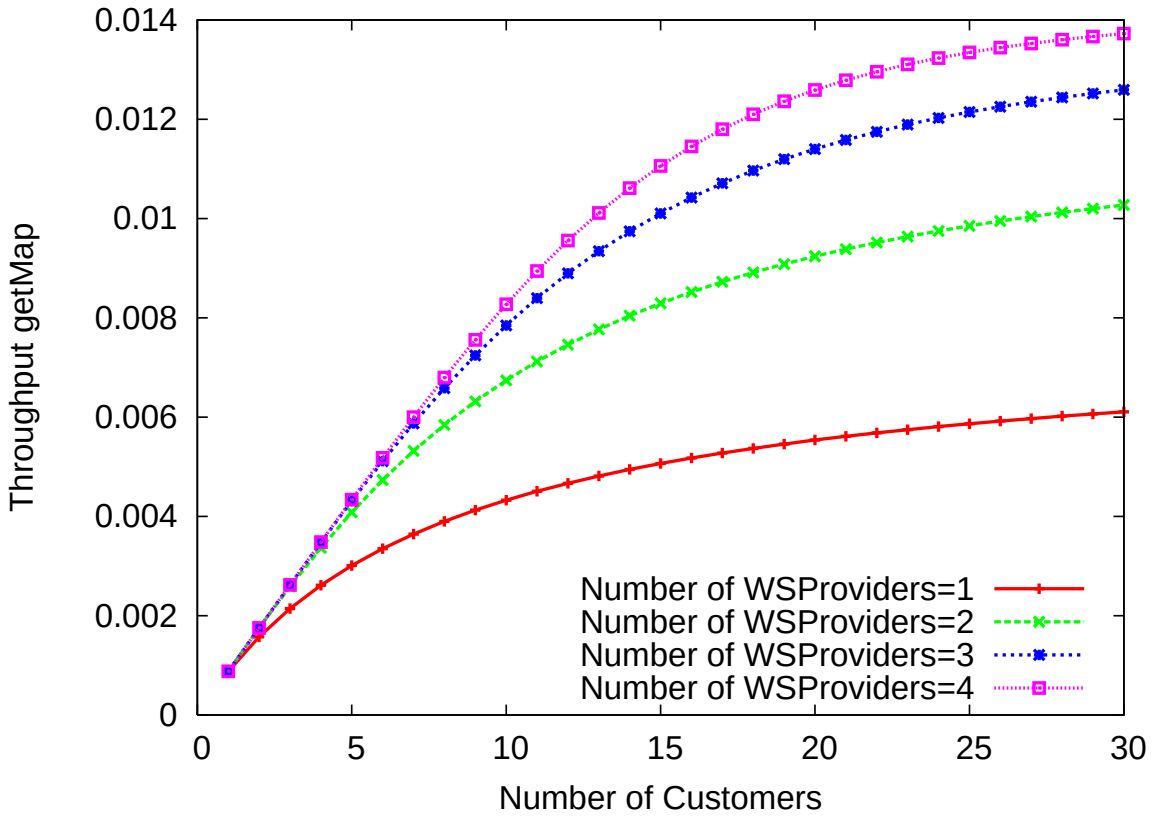
- However, there are a number of degrees of freedom which let her vary, for example, the number of threads of control of the components of the system.
- The aim of the analysis is to deliver a satisfactory service in a cost-effective way.
- The simplest example of a cost function may be a linearly dependency on the number of copies of a component or the rate at which an activity is performed.

Throughput of the *getMap* action

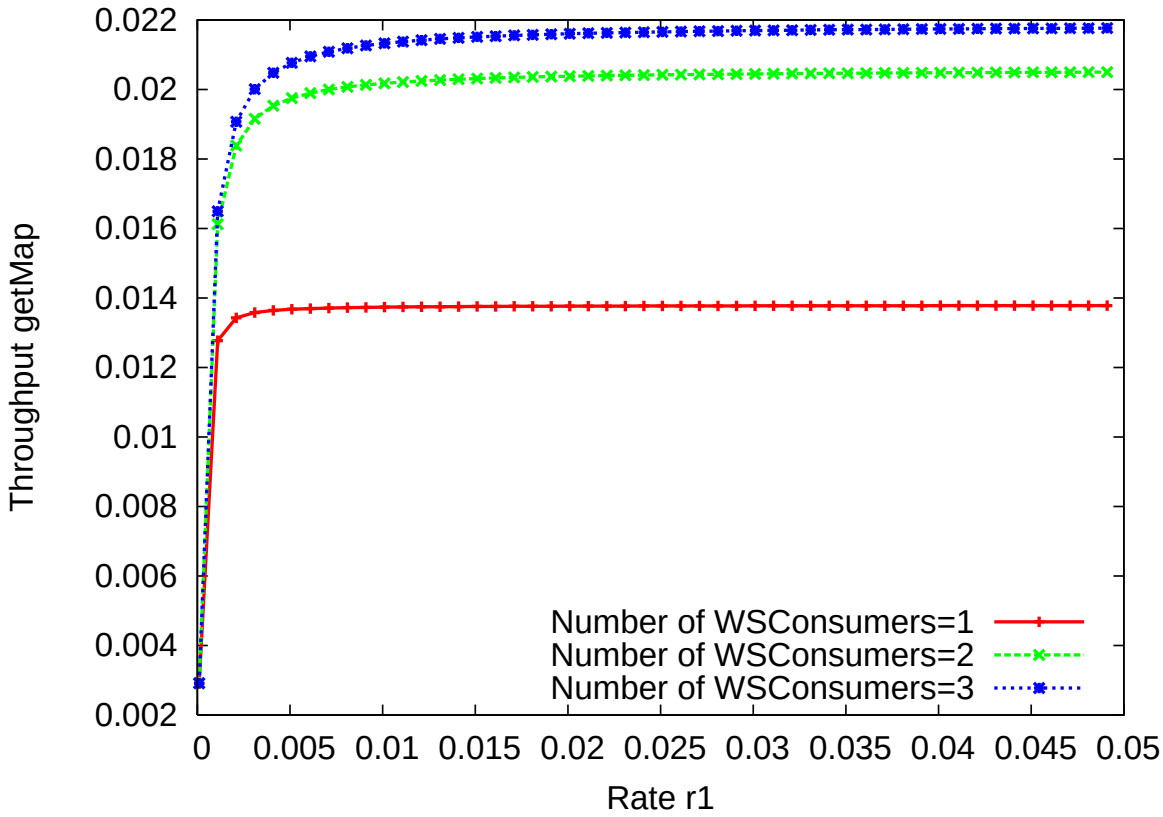


as the number of customers varies between 1 and 30 for various numbers of copies of the *WSProvider* component.

Throughput of the *getMap* action

- Under heavy load increasing the number of providers initially leads to a sharp increase in the throughput. However the gain deteriorates so that the system with four copies is just 8.7% faster than the system with three.
- In the following we settle on three copies of *WSProvider*.

Throughput of *getMap* action



as the request arrival rate (r_1) varies for differing numbers of *WSConsumer*.

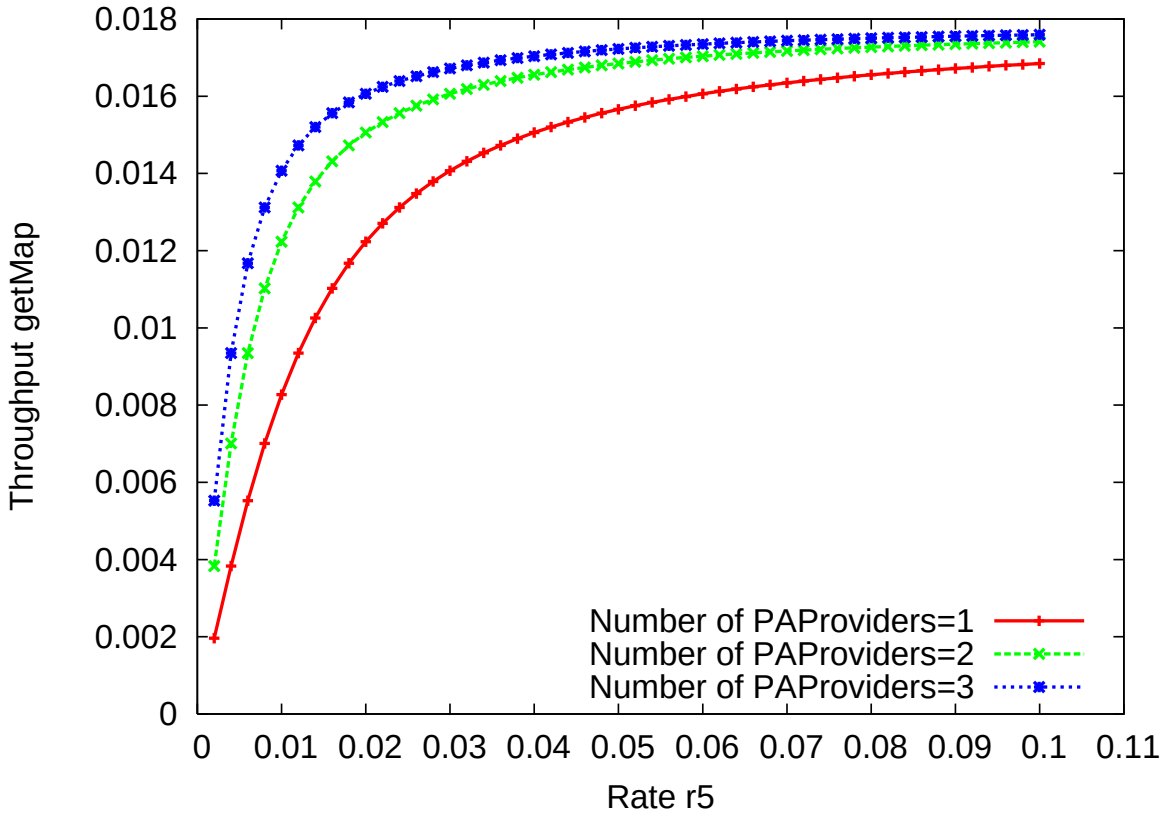
Throughput of *getMap* action

- Every line starts to plateau at approximately $r_1 = 0.010$ following an initial sharp increase. This suggests that the user is the bottle next in the system when the arrival rate is lower. Conversely, at high rates the system becomes congested.
- Whilst having two copies of *WSConsumer*, corresponding to two operating threads of control, improves performance significantly, the subsequent increase with three copies is less pronounced.
- So we set the number of copies of *WSConsumer* to 2.

Optimising the number of copies of *PAProvider*

- Here we are particularly interested in the overall impact of the rate at which the validity check is performed.
- Slower rates may mean more computationally expensive validation.
- Faster rates may involve less accuracy and lower security of the system.

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider*.

Throughput of *getMap* action

- A sharp increase followed by a constant levelling off suggests that optimal rate values lie on the left of the plateau, as faster rates do not improve the system considerably.
- As for the optimal number of copies of *PAProvider*, deploying two copies rather than one can increase the quality of service of the overall system.
- With a similar approach as previously discussed, the modeller may want to consider the trade-off between the cost of adding a third copy and the throughput increase.

An alternative design for *PAProvider*

- The original design of *PAProvider* is stateless.
- Any of its services can be called at any point, the correctness of the system being guaranteed by implementation-specific constrainsts such as session identifiers being uniquely assigned to the clients and passed as parameters of the method calls.

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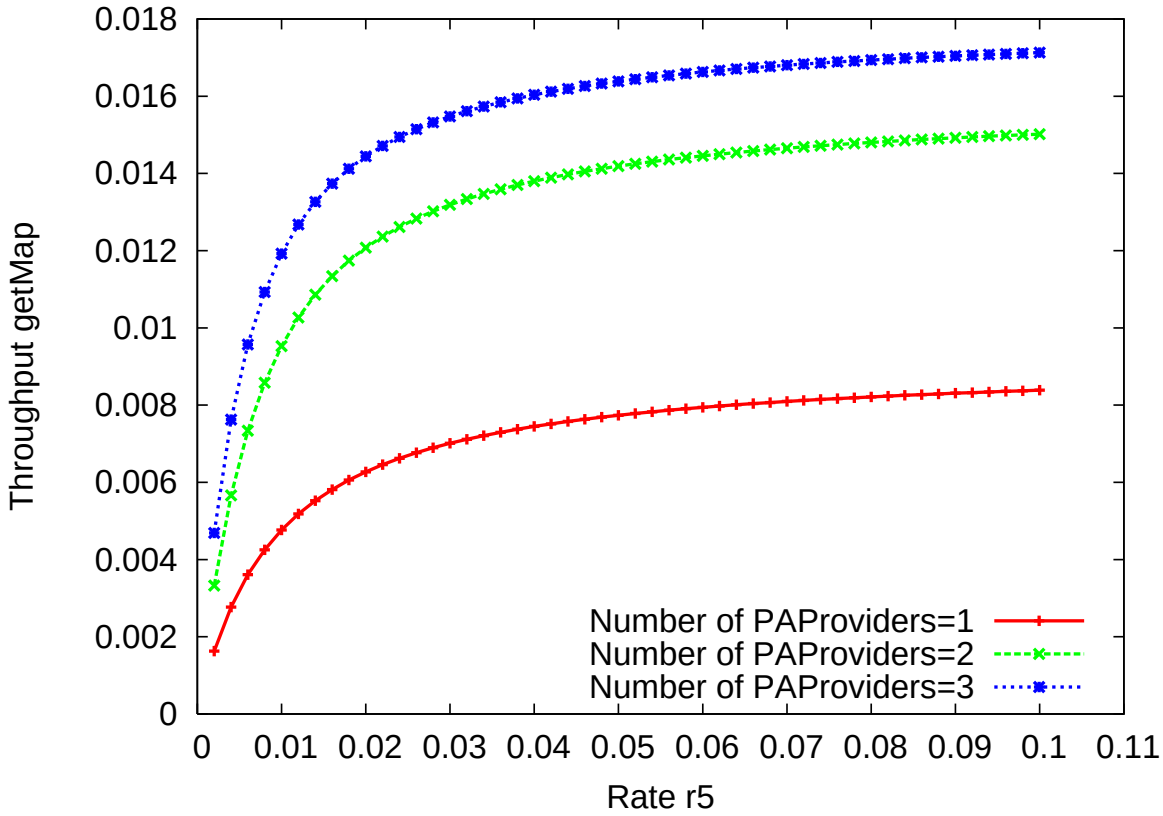
- Alternatively we may consider a stateful implementation, modelled as a sequential component with three local states.
- This implementation has the consequence that there can never be more than N_{PAP} *WSProvider* which have started a session with a *PAProvider*

Component *PAProvider* — Stateful Version

It maintains a thread for each session and carries out the validity check on behalf of the Web Service Provider.

$$
\begin{aligned}
PAProvider &\stackrel{def}{=} (startSession, \top).PAProvider_2 \\
PAProvider_2 &\stackrel{def}{=} (checkValid, r_5).PAProvider_3 \\
PAProvider_3 &\stackrel{def}{=} (stopSession, \top).PAProvider
\end{aligned}
$$

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider* (stateful version).

Throughput of *getMap* action

- In this case the incremental gain in adding more copies has become more marked.
- However, the modeller may want to prefer the original version, as three copies of the stateful provider deliver about as much as the throughput of only one copy of the stateless implementation.

Outline

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- For performance modelling the rigour of the formal description technique has had benefits for both practice and theory, and lead to enhanced analysis capabilities.

Acknowledgements

The PEPA project has been funded by SERC, EPSRC and the CEC. In particular Jane Hillston and Mirco Tribastone are supported by the SENSORIA project

We would like to thank our co-authors, Allan Clark and Stephen Gilmore for help with these slides.

Thank you