Performance Modelling with Stochastic Process Algebra

Jane Hillston
LFCS, Edinburgh

Dynamic Properties of Complex Networks,
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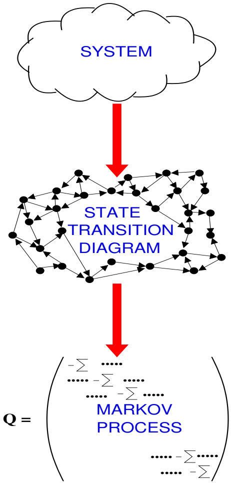
- Performance Evaluation Process Algebra (PEPA) sought to address these problems by the introduction of a suitable process algebra.
- We have sought to investigate and exploit the interplay between the process algebra and the continuous time Markov chain (CTMC)

Outline

Performance Modelling using CTMC

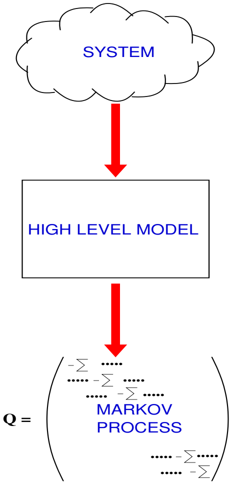
Model Construction



Performance Modelling using CTMC

Model Construction

- describing the system using a high level modelling formalism
- generating the underlying CTMC



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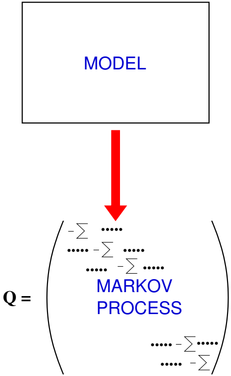
- model simplification
- model aggregation



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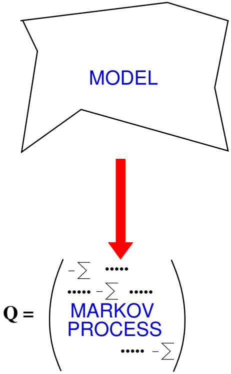

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Model Solution

- solve to find steady state or transient probability distribution
- deriving performance measures

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$$\underbrace{\alpha}_{\text{action type or name}}.\underbrace{P}_{\text{agent/ component}}$$

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$$\text{Process algebra model} \xrightarrow{\text{SOS rules}} \text{Labelled transition system}$$

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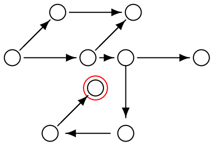
Will the system arrive in a particular state?



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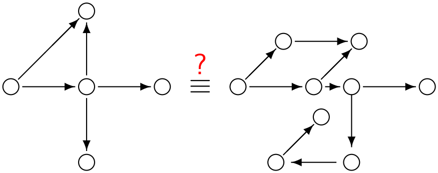

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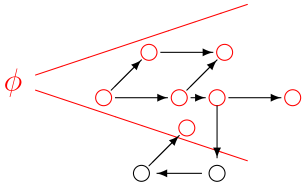

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$$P[5] \equiv (P \parallel P \parallel P \parallel P \parallel P)$$

A simple example: processors and resources

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\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
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\end{aligned}
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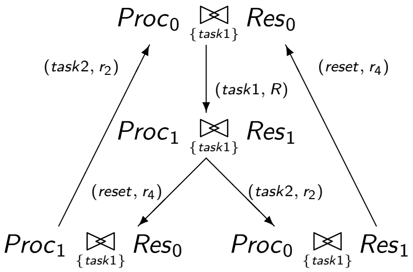

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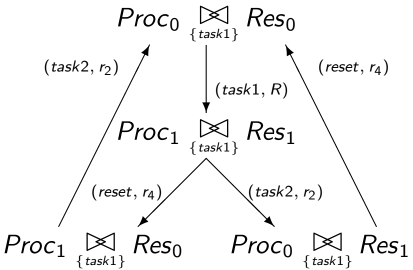

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$$
\mathbf{Q} = \begin{pmatrix}
-R & R & 0 & 0 \\
0 & -(r_2 + r_4) & r_4 & r_2 \\
r_2 & 0 & -r_2 & 0 \\
r_4 & 0 & 0 & -r_4
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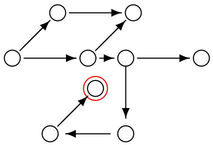
Reachability analysis

How long will it take for the system to arrive in a particular state?



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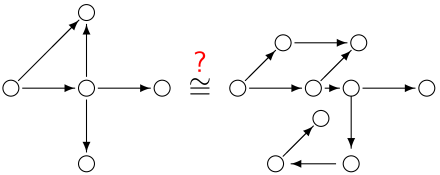
Specification matching

With what probability does system behaviour match its specification?



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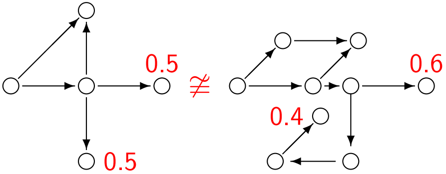
Specification matching

Does the "*frequency profile*" of the system match that of the specification?



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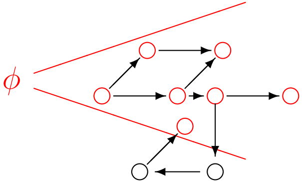
Model checking

Does a given property ϕ hold within the system with a given probability?



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Model checking

For a given starting state
how long is it until
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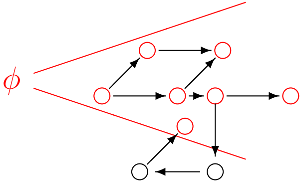


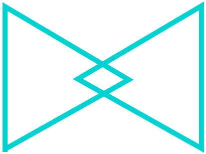
Theory

The theoretical development underpinning PEPA focused on the interaction between the process algebra and the underlying mathematical structure, the Markov process.

This work can be broadly categorised into three areas:

- Designing the language
- Manipulating models
- Solving models and deriving measures

Designing the language



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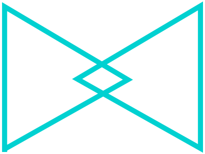

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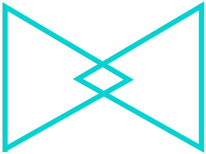

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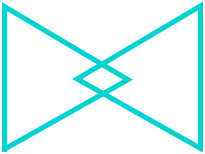

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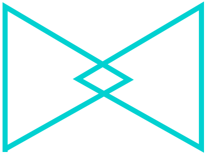

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- Thus the apparent rate of a cooperation is the minimum of the apparent rates of the co-operands.

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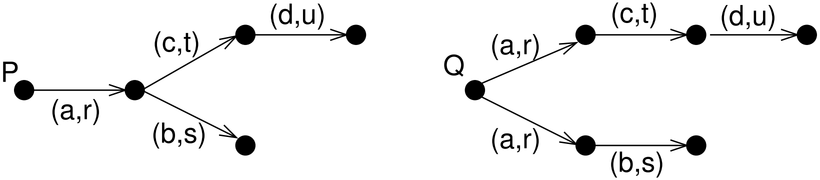

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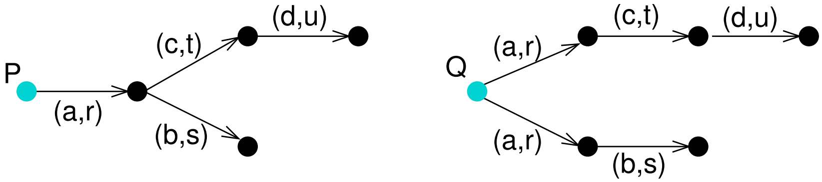

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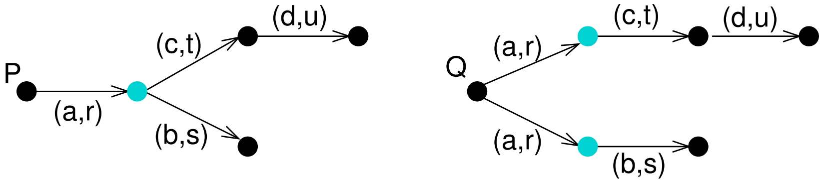

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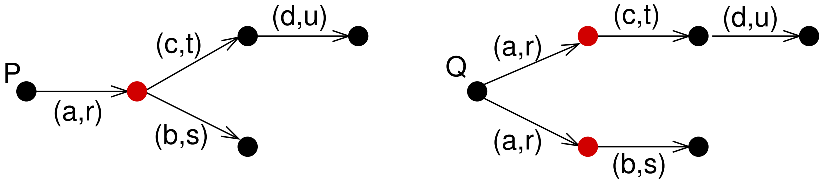

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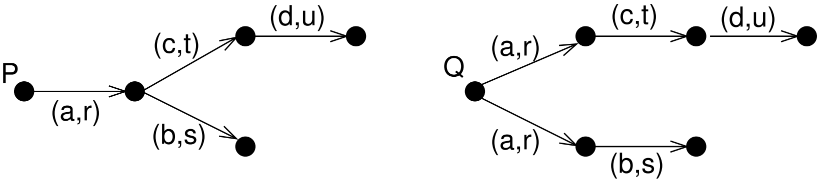

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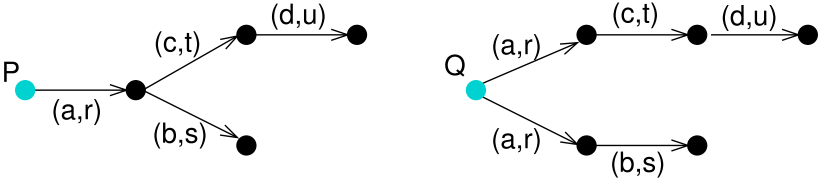

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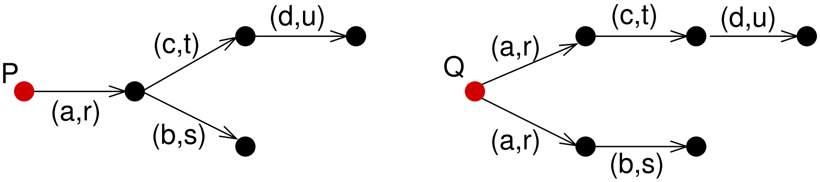

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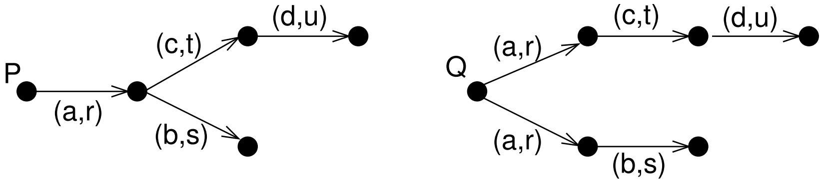

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The resulting equivalence relation is a bisimulation in the style of Larsen and Skou, and coincides with the Markov process notion of lumpability.

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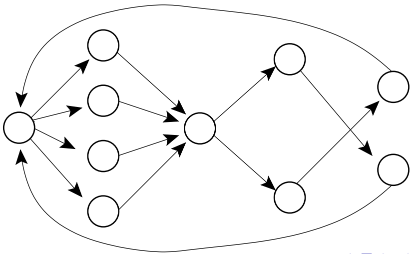

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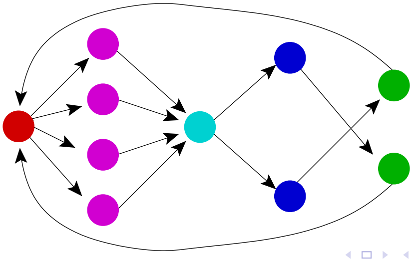

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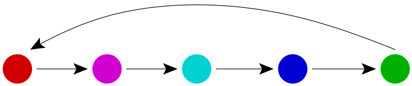
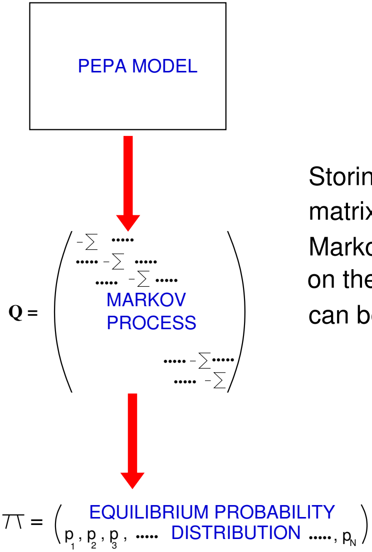


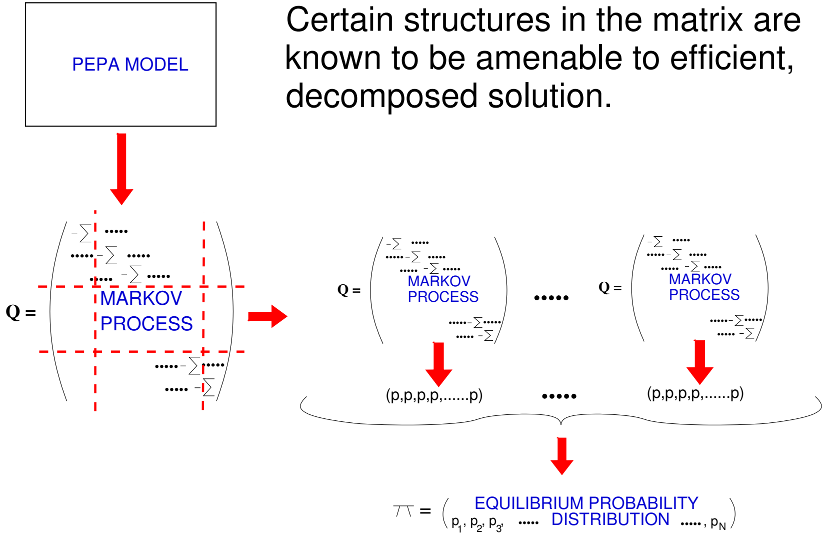
Characterising efficient solution



Storing and manipulating the matrix which represents the Markov process places limitations on the size of model which can be analysed.

Characterising efficient solution

Certain structures in the matrix are known to be amenable to efficient, decomposed solution.



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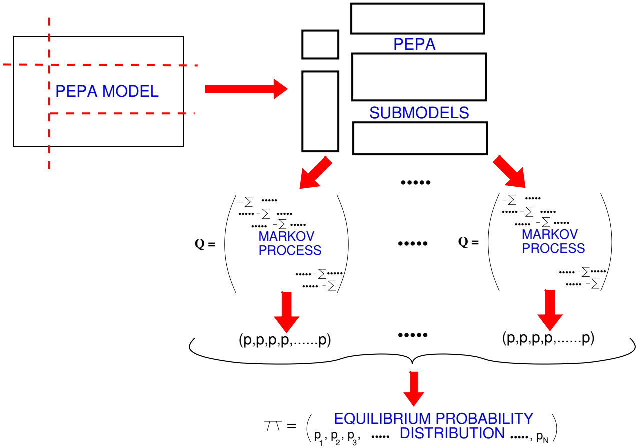


Finding the corresponding structures in the process algebra means that these techniques can be applied automatically, before the monolithic matrix is formed.

Applications

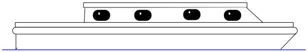
- Developing models of real applications has always been an integral part of the PEPA project.
- This allows us to demonstrate to ourselves and others that the theory we have developed is useful.
- It is also a valuable source of inspiration for new theory and future directions.

PEPA Case Studies (1)

- Protocols for fault-tolerant systems (Clark, Gilmore, Hillston and Ribaudo, Edinburgh and Turin)
- Multimedia traffic characteristics (Bowman et al, Kent)
- Database systems (The STEADY group, Heriot-Watt University)
- Software Architectures (Pooley, Bradley and Thomas, Heriot-Watt and Durham)
- Switch behaviour in active networks (Hillston, Kloul and Mokhtari, Edinburgh and Versailles)
- Mobility and QOS protocols in wireless networks Hillston, Laurenson and Wang, Edinburgh

PEPA Case Studies (2)

- Locks and movable bridges in inland shipping in Belgium (Knapen, Hasselt)



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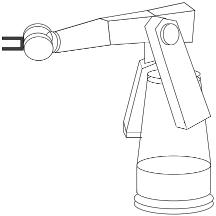

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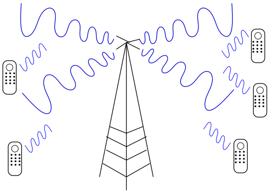

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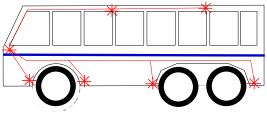
- Automotive diagnostic expert systems (Console, Picardi and Ribaudo, Turin)



Tool Support

PEPA Eclipse Plug-in
(Edinburgh University)

Möbius modelling platform
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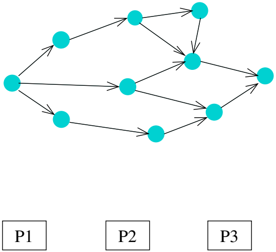

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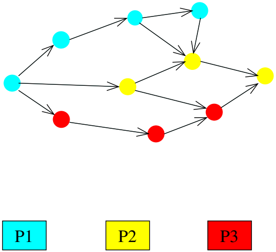

A schedule maps tasks to processors

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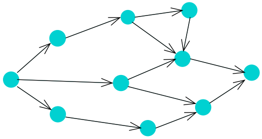

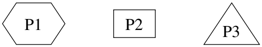

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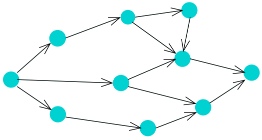

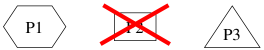

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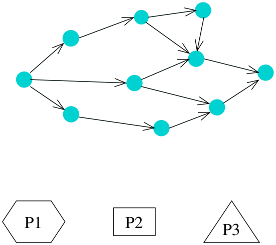

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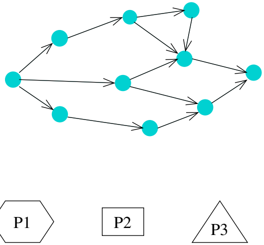

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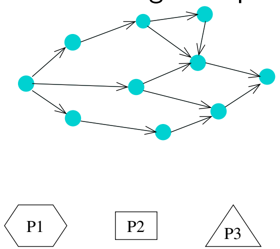

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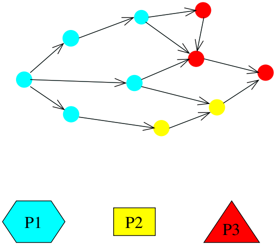

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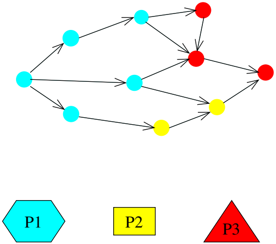

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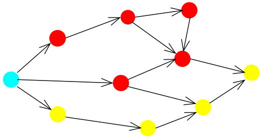

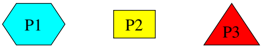

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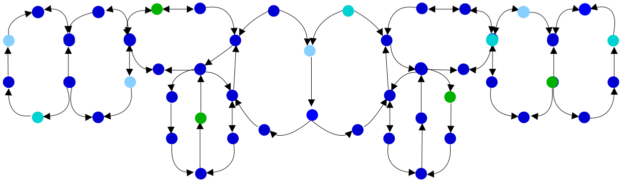
Collective Dynamics

The behaviour of many systems can be interpreted as the result of the collective behaviour of a large number of interacting entities.



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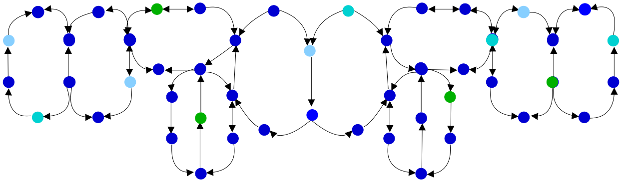

For such systems we are often as interested in the population level behaviour as we are in the behaviour of the individual entities.

Process Algebra and Collective Dynamics

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- Stochastic extensions, such as PEPA, enable quantified behaviour of the dynamics of systems.

In the CODA project we are developing stochastic process algebras and associated theory, tailored to the construction and evaluation of the collective dynamics of large systems of interacting entities.

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Large scale software systems
Issues of scalability are important for user satisfaction and resource efficiency but such issues are difficult to investigate using discrete state models.

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Biochemical signalling pathways
Understanding these pathways has the potential to improve the quality of life through enhanced drug treatment and better drug design.

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Epidemiological systems

Improved modelling of these systems could lead to improved disease prevention and treatment in nature and better security in computer systems.

Solving discrete state models

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As the size of the state space becomes large it becomes infeasible to carry out numerical solution and extremely time-consuming to conduct stochastic simulation.

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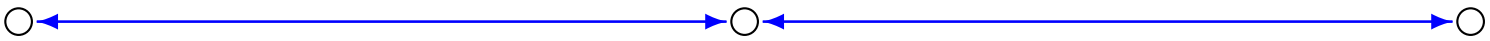

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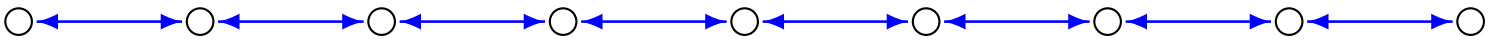

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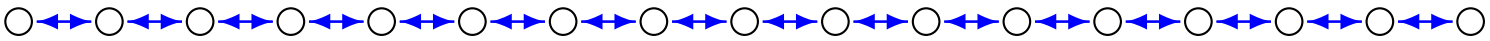

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Use continuous state variables to approximate the discrete state space.

Use ordinary differential equations to represent the evolution of those variables over time.

New mathematical structures: differential equations

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Appropriate for models in which there are large numbers of components of the same type, i.e. models of populations and situations of collective dynamics.

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- Models are constructed from components which engage in activities.

$$(\alpha,\ r).P$$

action type or name

activity rate (parameter of an exponential distribution)

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Nevertheless we are able to define a structured operational semantics which defines the possible transitions of an abitrary abstract state and from this derive the ODEs.

Capturing behaviour in the Generator Function

$$
\begin{aligned}
Proc_0 &\stackrel{\text{def}}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{\text{def}}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{\text{def}}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{\text{def}}{=} (reset, r_4).Res_0 \\
System &\stackrel{\text{def}}{=} Proc_0[N_P] \underset{\{transfer\}}{\bowtie} Res_0[N_R]
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Numerical Vector Form

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4) \in \mathbb{N}^4, \quad \xi_1 + \xi_2 = N_P \;\text{ and }\; \xi_3 + \xi_4 = N_R$$

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Generator Function

$$
f(\xi, l, \alpha): \quad
\begin{aligned}
f(\xi, (-1, 1, -1, 1), task1) &= \min(r_1\xi_1, r_3\xi_3) \\
f(\xi, (1, -1, 0, 0), task2) &= r_2\xi_2 \\
f(\xi, (0, 0, 1, -1), reset) &= r_4\xi_4
\end{aligned}
$$

Extraction of the ODE from f

Generator Function

$$
\begin{aligned}
f(\xi, (-1,1,-1,1), \mathit{task1}) &= \min(r_1\xi_1, r_3\xi_3) \\
f(\xi, (1,-1,0,0), \mathit{task2}) &= r_2\xi_2 \\
f(\xi, (0,0,1,-1), \mathit{reset}) &= r_4\xi_4
\end{aligned}
$$

Differential Equation

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= F_{\mathcal{M}}(x) = \sum_{l \in \mathbb{Z}^d} l \sum_{\alpha \in \mathcal{A}} f(x, l, \alpha) \\
&= (-1,1,-1,1)\min(r_1x_1, r_3x_3) + (1,-1,0,0)r_2x_2 \\
&\qquad + (0,0,1,-1)r_4x_4
\end{aligned}
$$

Extraction of the ODE from f

Generator Function

$$
\begin{aligned}
f(\xi, (-1,1,-1,1), \mathit{task1}) &= \min(r_1\xi_1, r_3\xi_3) \\
f(\xi, (1,-1,0,0), \mathit{task2}) &= r_2\xi_2 \\
f(\xi, (0,0,1,-1), \mathit{reset}) &= r_4\xi_4
\end{aligned}
$$

Differential Equation

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\min(r_1x_1, r_3x_3) + r_2x_2 \\
\frac{dx_2}{dt} &= \min(r_1x_1, r_3x_3) - r_2x_2 \\
\frac{dx_3}{dt} &= -\min(r_1x_1, r_3x_3) + r_4x_4 \\
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- Worms like Nimbda, Slammer, Code Red, Sasser and Code Red 2 have caused the Internet to become unusable for many hours at a time until security patches could be applied and routers fixed.
- The estimated cost of computer worms and related activities is about $50 billion a year.

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- We wish to study the emergent behaviour of Internet worms as they spread to thousands and then hundreds-of-thousands of hosts.
- Explicit state-based methods for calculating steady-state, transient or passage-time measures are limited to state-spaces of the order of 10^9.
- By transforming our stochastic process algebra model into a set of ODEs, we can obtain a plot of model behaviour against time for models with global state spaces in excess of 10^{10000} states.

Susceptible-Infective-Removed over a network

- This is our most basic infection model and is used to verify that we get recognisable qualitative results.

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- This state is termed removed and is an absorbing state for that component in the system.

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- Additionally, an attempted network connection can fail or timeout as indicated by the *fail* action.
- This might be due to network contention or the lack of availability of a susceptible machine to infect.
- As large scale worm infections tend not to waste time determining whether a given host is already infected or not, we assume that a certain number of infections will attempt to reinfect hosts; in this instance, the host is unaffected.

Susceptible-Infective-Removed over a network

$$
\begin{aligned}
S &\stackrel{\mathit{def}}{=} (\mathit{infectS}, \top).I \\
I &\stackrel{\mathit{def}}{=} (\mathit{infectI}, \beta).I + (\mathit{infectS}, \top).I + (\mathit{patch}, \gamma).R \\
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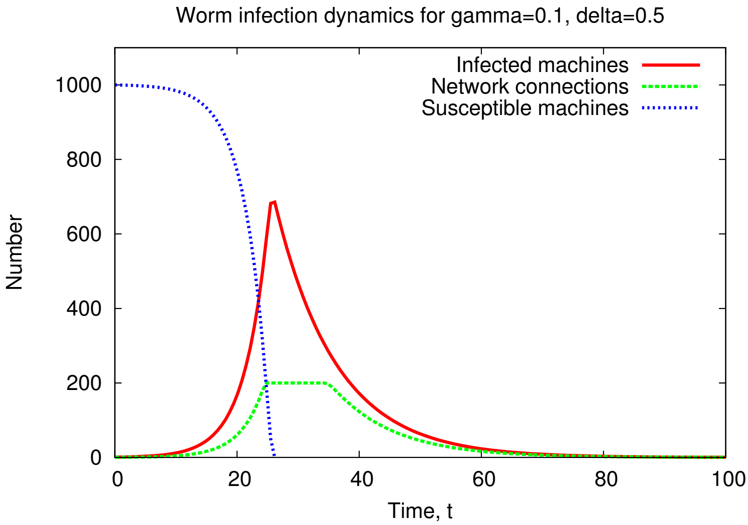
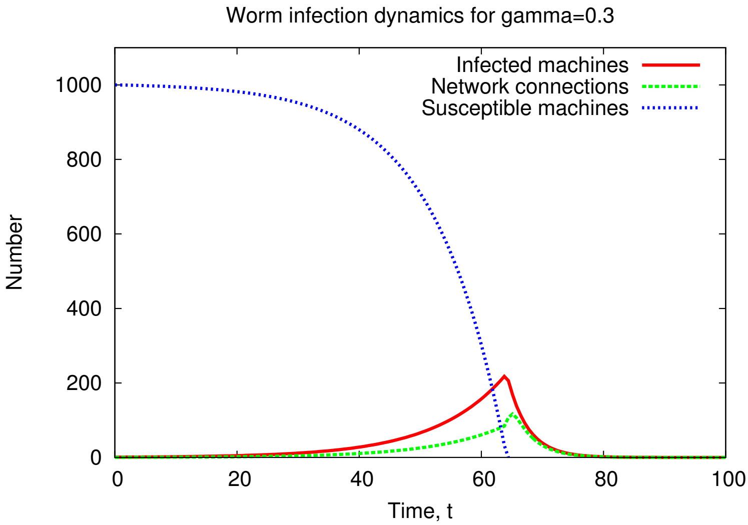
\mathit{Sys} \stackrel{\mathit{def}}{=} (S[N] \parallel I) \underset{L}{\bowtie} \mathit{Net}[M] \quad \text{where } L = \{\, \mathit{infectI}, \mathit{infectS} \,\}
$$

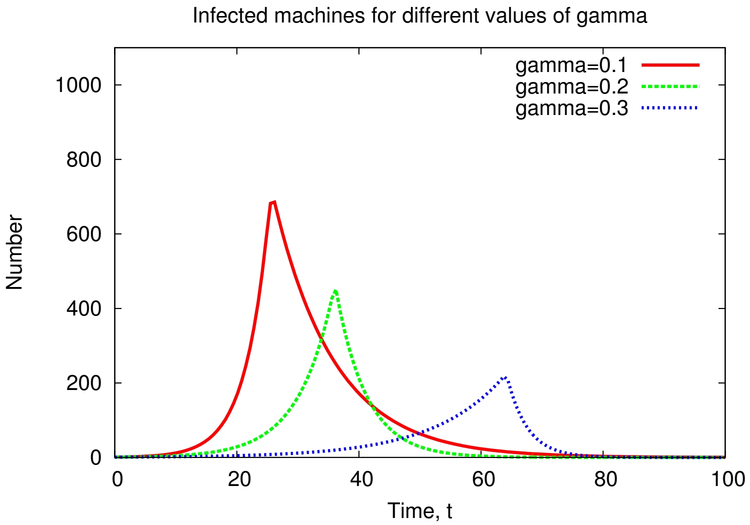
Patch rate $\gamma = 0.1$. Connection failure rate $\delta = 0.5$



Patch rate $\gamma = 0.3$. Connection failure rate $\delta = 0.5$



Increasing machine patch rate γ from 0.1 to 0.3



Susceptible-Infective-Removed-Reinfection (SIRR) model

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- A small modification in the process model of infection allows for removed computers to become susceptible again after a delay.
- We use this to model a faulty or incomplete security upgrade or the mistaken removal of security patches which had previously defended the machine against attack.

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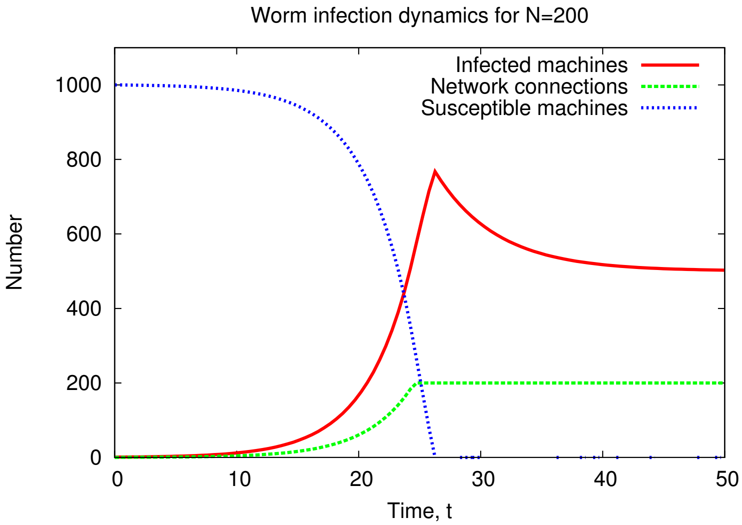
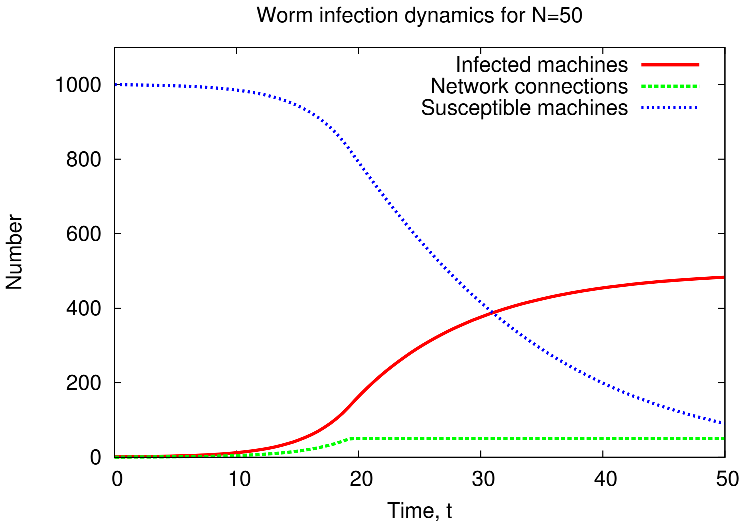
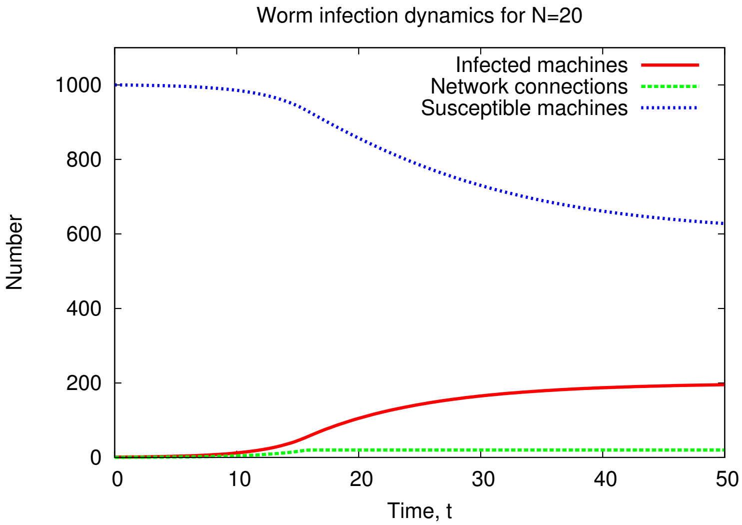


Unsecured SIR model (50 network channels)



Unsecured SIR model (20 network channels)



Example Conclusions

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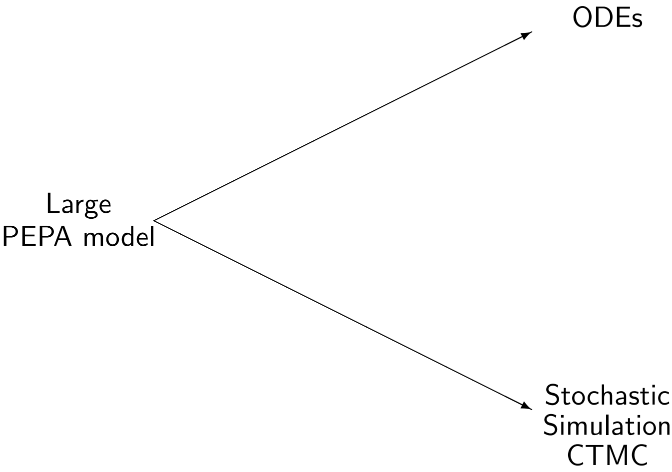
- Process algebra modelling allows the details of interactions to be recorded on the individual level but then abstracted away into appropriate population-based representations.
- The scale of problems which can be modelled in this way vastly exceeds those which are founded on explicit state representations.
- We believe the modelling methods exemplified here to be generally useful for analysing the behaviour of populations of interacting processes with complex dynamics.

Alternative Representations



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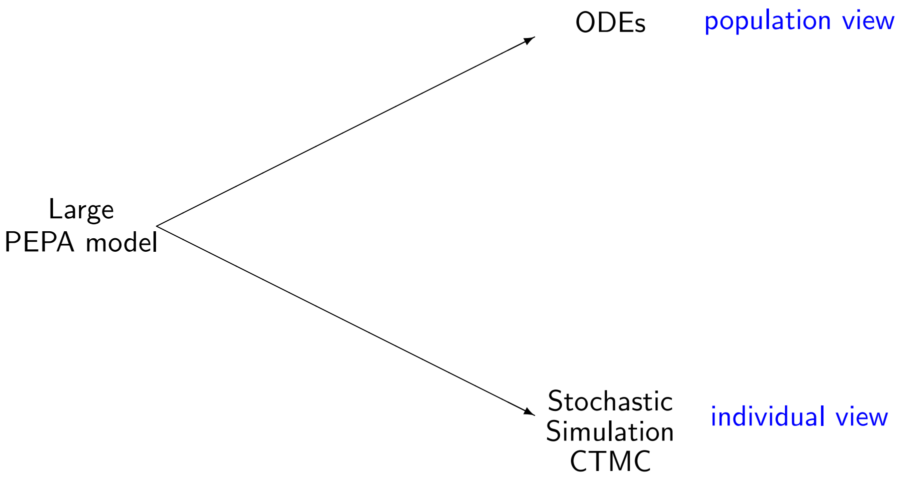

Consistency results

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- We can prove this using Kurtz's theorem: *Solutions of Ordinary Differential Equations as Limits of Pure Jump Markov Processes*, T.G. Kurtz, J. Appl. Prob. (1970).
- Lipschitz continuity of the vector field guarantees existence and uniqueness of the solution to the initial value problem.

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- This alternative view of systems has opened up many and exciting new research directions.

Thanks!

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More information:
http://www.dcs.ed.ac.uk/pepa