Process algebras for quantitative analysis: Lecture 7 — Scalable Analysis of Scalable Systems

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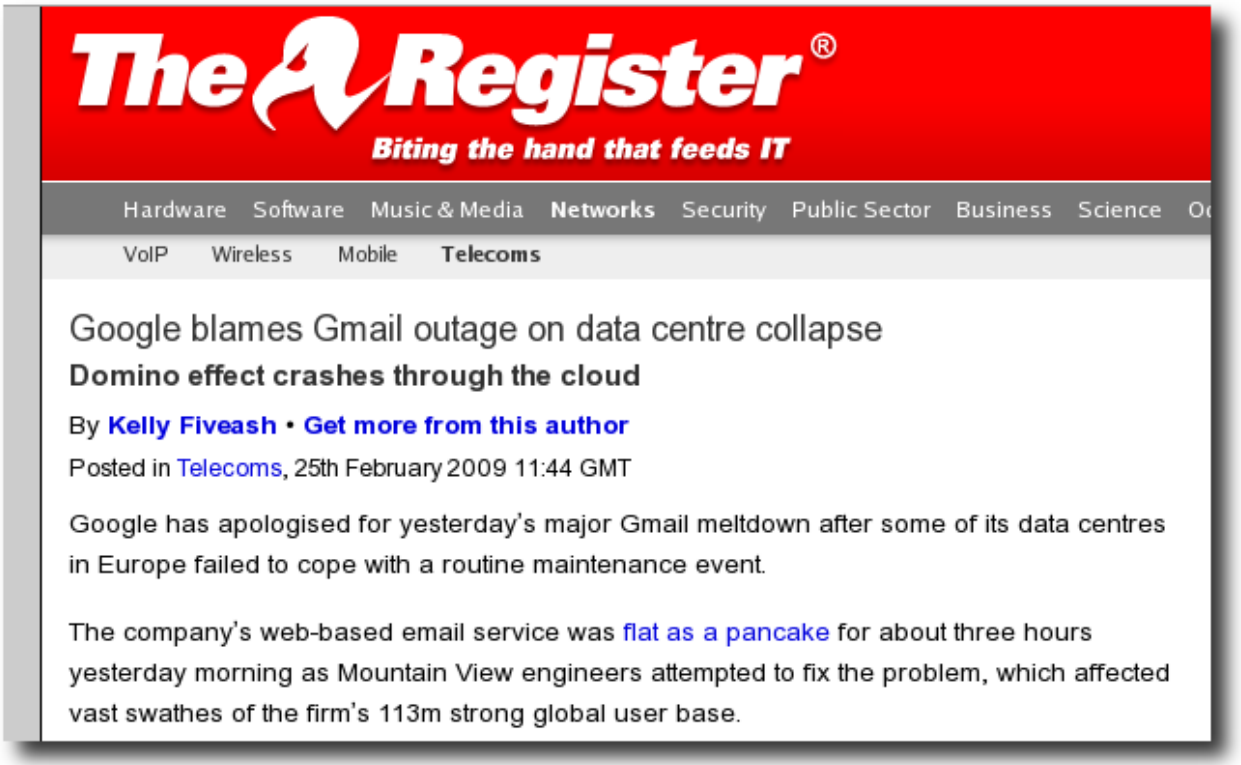
The University of Edinburgh
Scotland

Joint work with Allan Clark and Mirco Tribastone

Scalable analysis of scalable systems



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Google blames Gmail outage on data centre collapse

Domino effect crashes through the cloud

By **Kelly Fiveash** • **Get more from this author**

Posted in Telecoms, 25th February 2009 11:44 GMT

Google has apologised for yesterday's major Gmail meltdown after some of its data centres in Europe failed to cope with a routine maintenance event.

The company's web-based email service was flat as a pancake for about three hours yesterday morning as Mountain View engineers attempted to fix the problem, which affected vast swathes of the firm's 113m strong global user base.

Scalable analysis of scalable systems

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- Computer systems are currently deployed in active use with no real assurance of graceful degradation of service as the user population grows.

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- Computer systems are currently deployed in active use with no real assurance of graceful degradation of service as the user population grows.
- Because we cannot rely on them to provide service in times of greatest need, such systems are as unreliable in practice as ones which contain programming errors.

Outline

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Disk model in SRMC

```
DiskA::{
  Working = (read, r).Working
          + (write, w).Working
          + (fail, f).Failed;
  Failed  = (correct, c).Working;
};
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Disk model in SRMC

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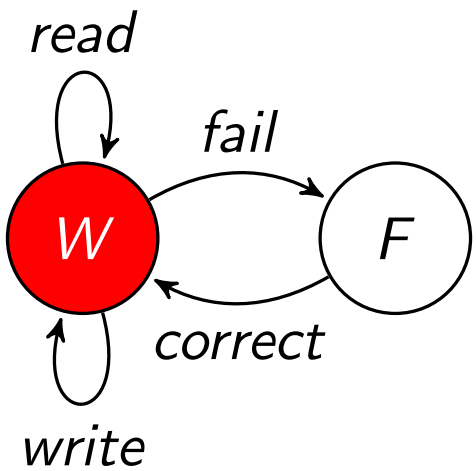

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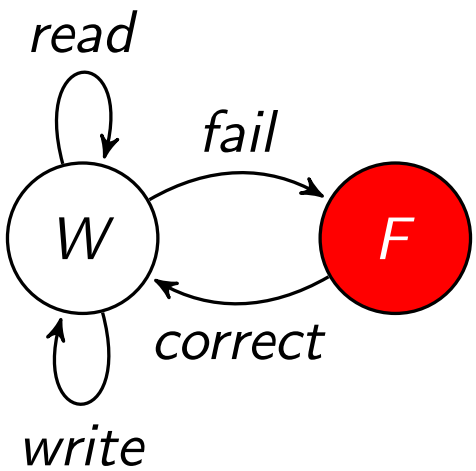

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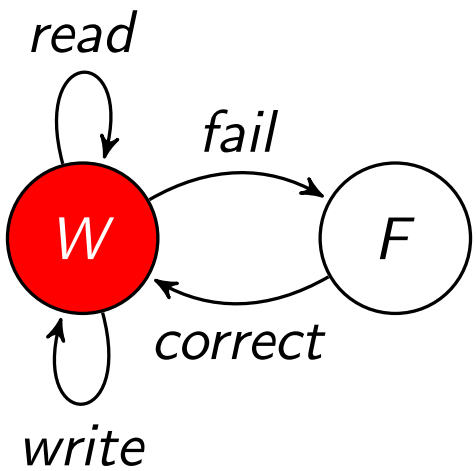

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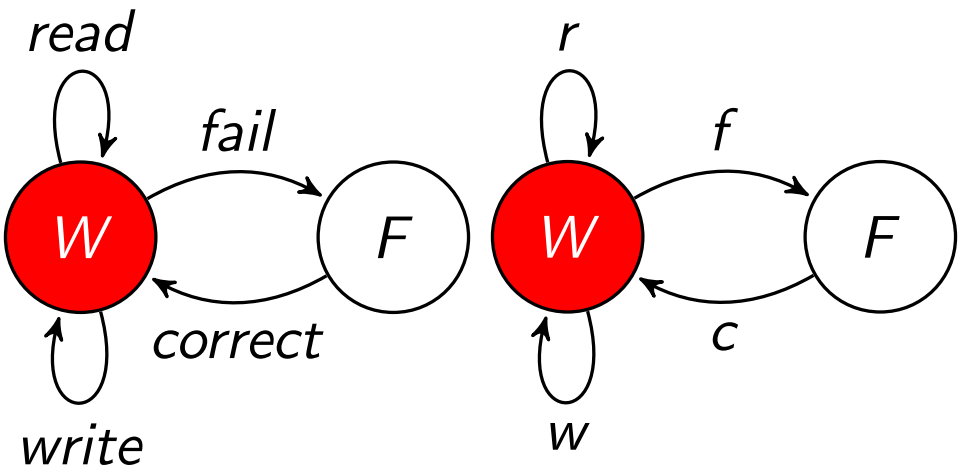

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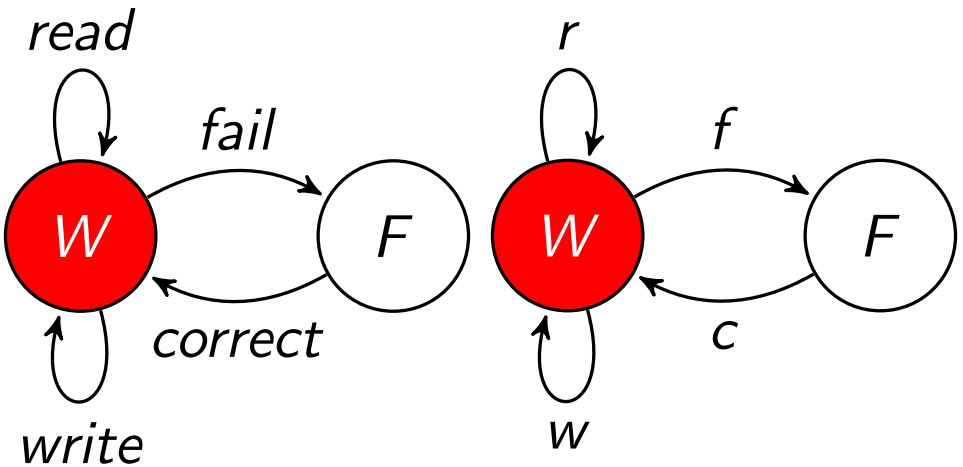

Rate matrix

$$R = \begin{pmatrix} & f \\ c & \end{pmatrix}$$

Disk model in SRMC

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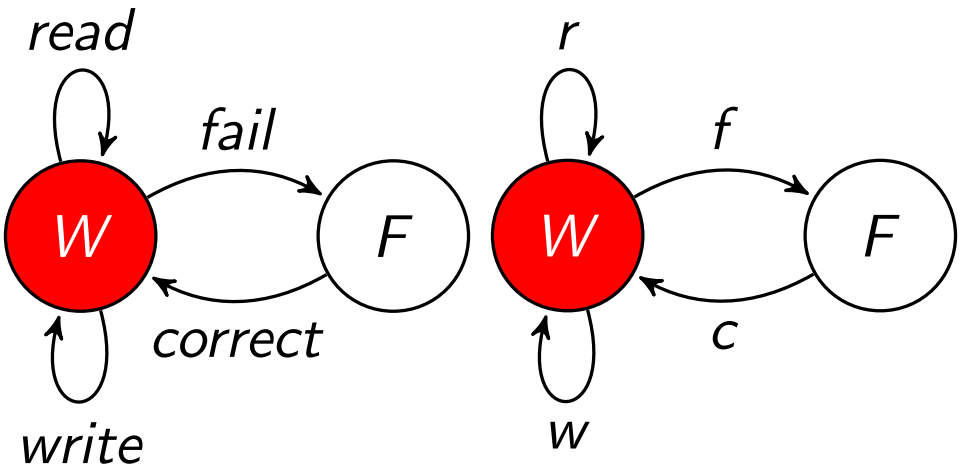

Generator matrix

$$Q = \begin{pmatrix} -f & f \\ c & -c \end{pmatrix}$$

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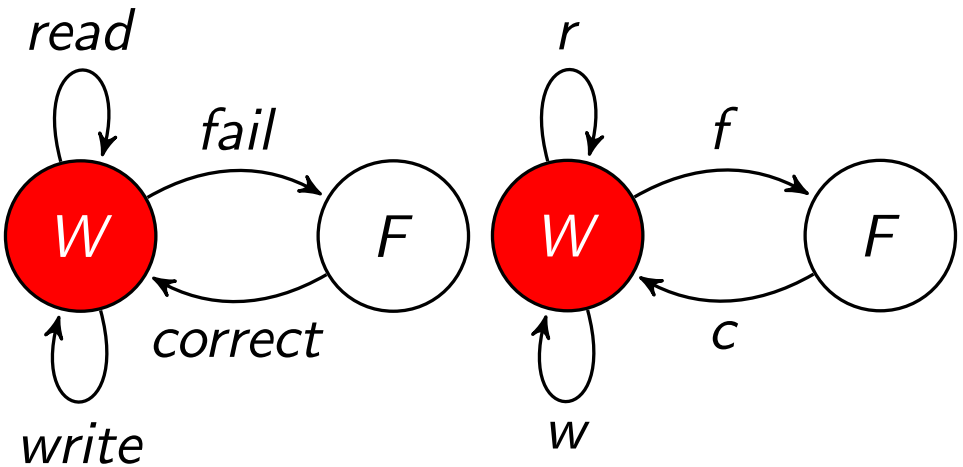

Global balance equation

$$\begin{pmatrix} \pi_W & \pi_F \end{pmatrix} \begin{pmatrix} -f & f \\ c & -c \end{pmatrix} = 0$$

Disk model in SRMC

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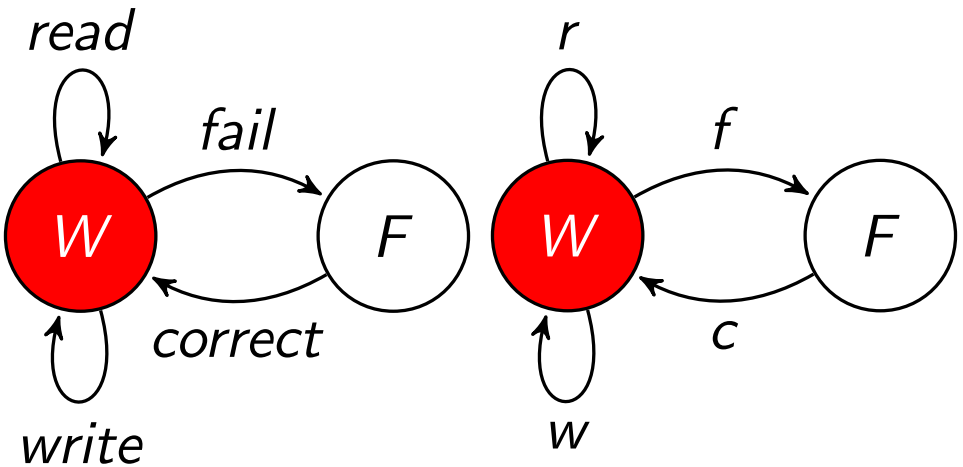

Normalisation

$$\pi_W + \pi_F = 1$$

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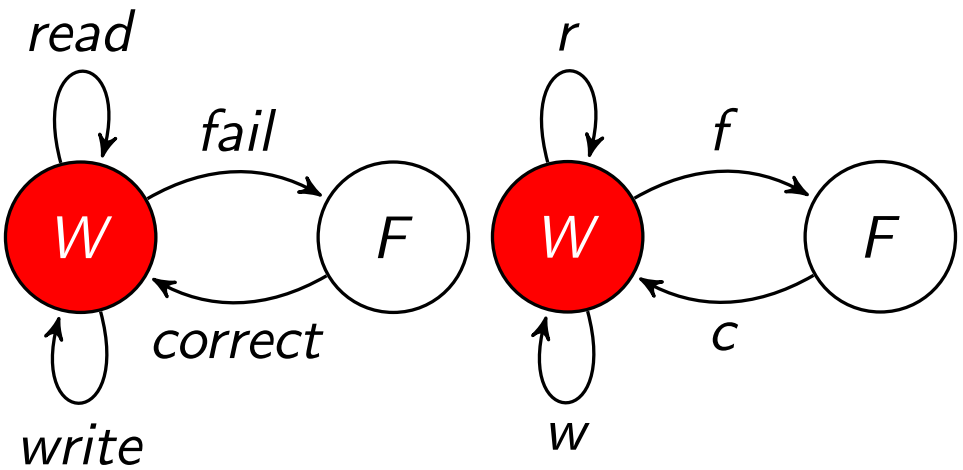
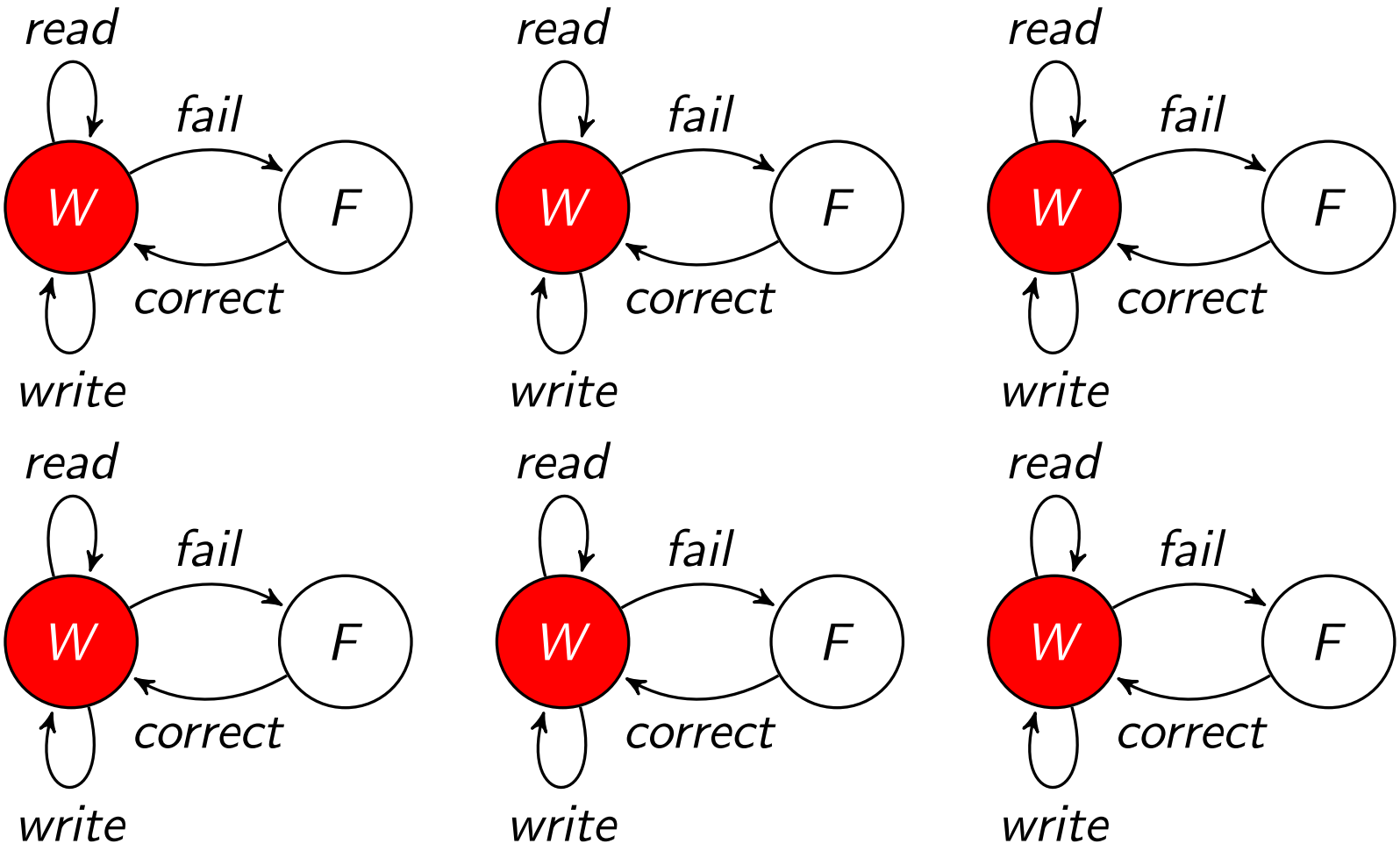


Global balance solution

$$\begin{pmatrix} \pi_W \\ \pi_F \end{pmatrix} = \begin{pmatrix} \frac{c}{f+c} \\ \frac{f}{f+c} \end{pmatrix}$$

Six disks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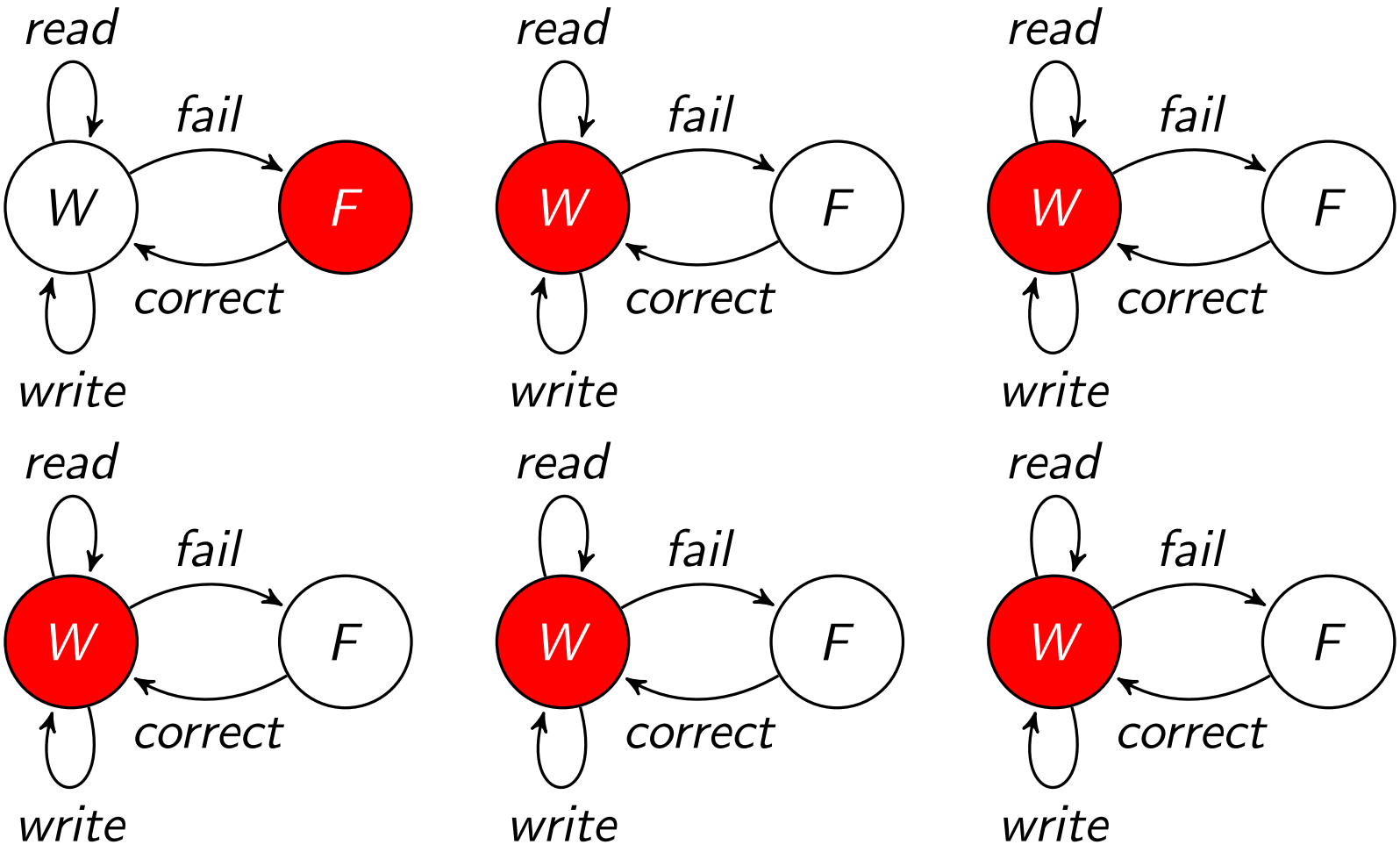
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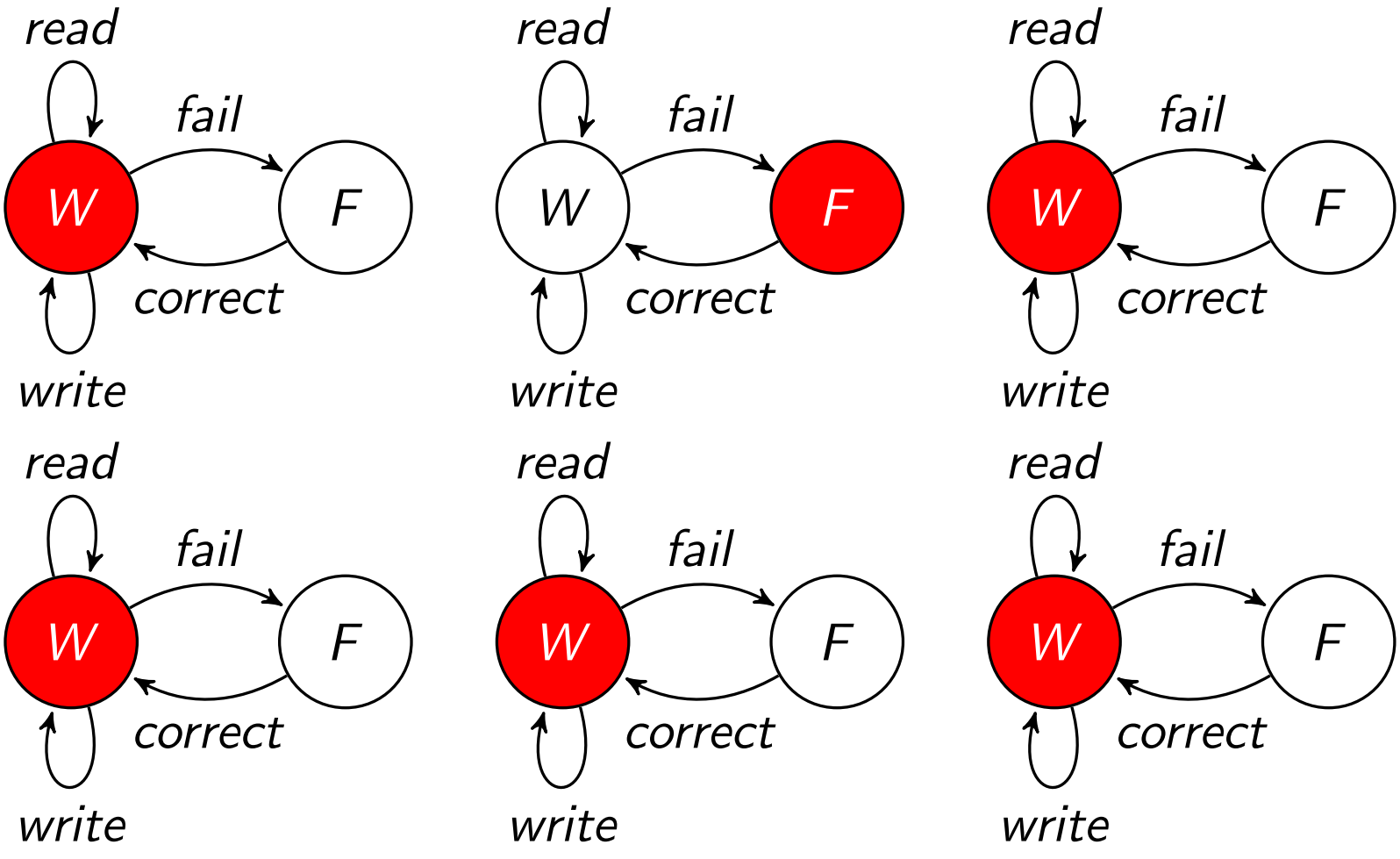
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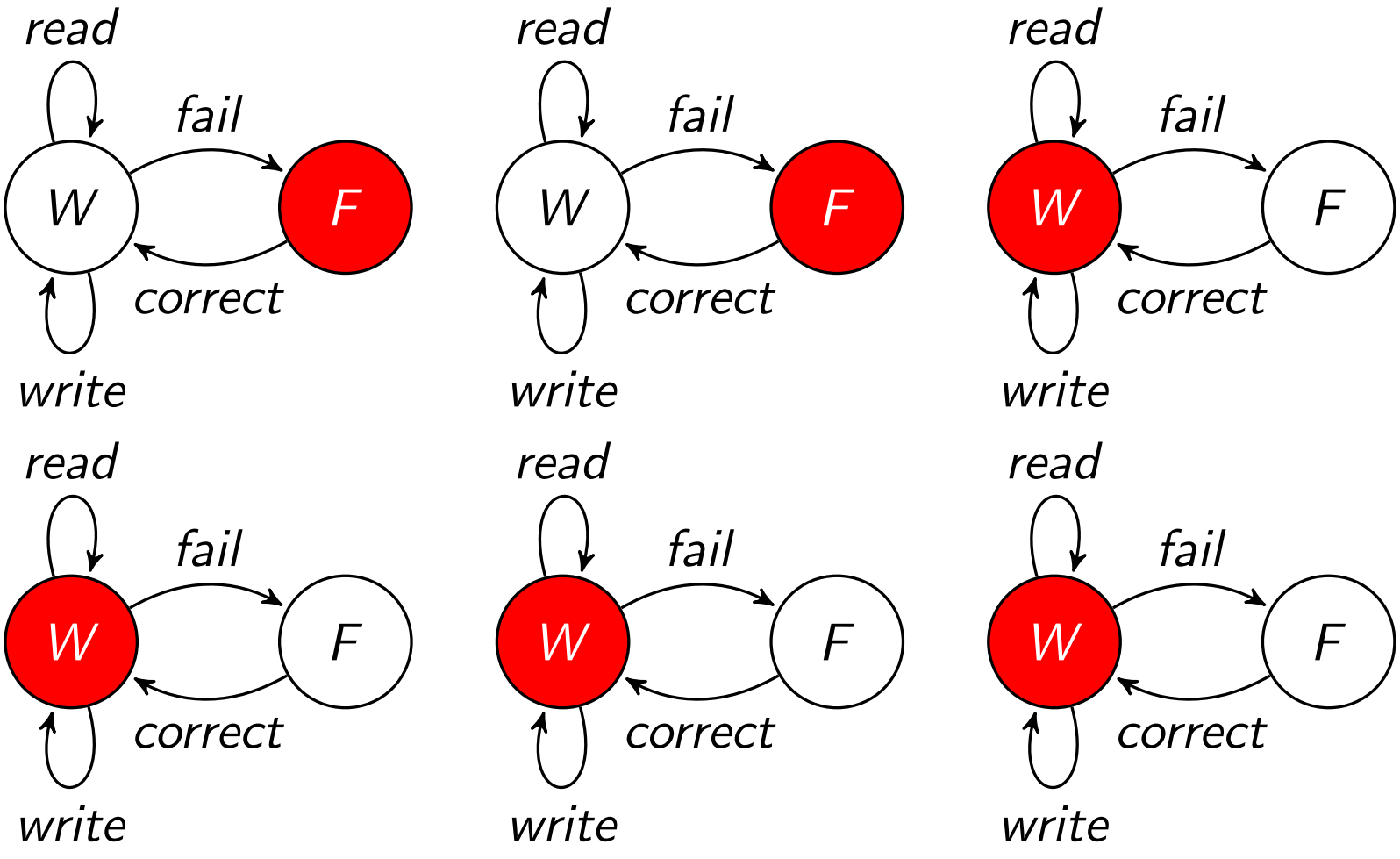
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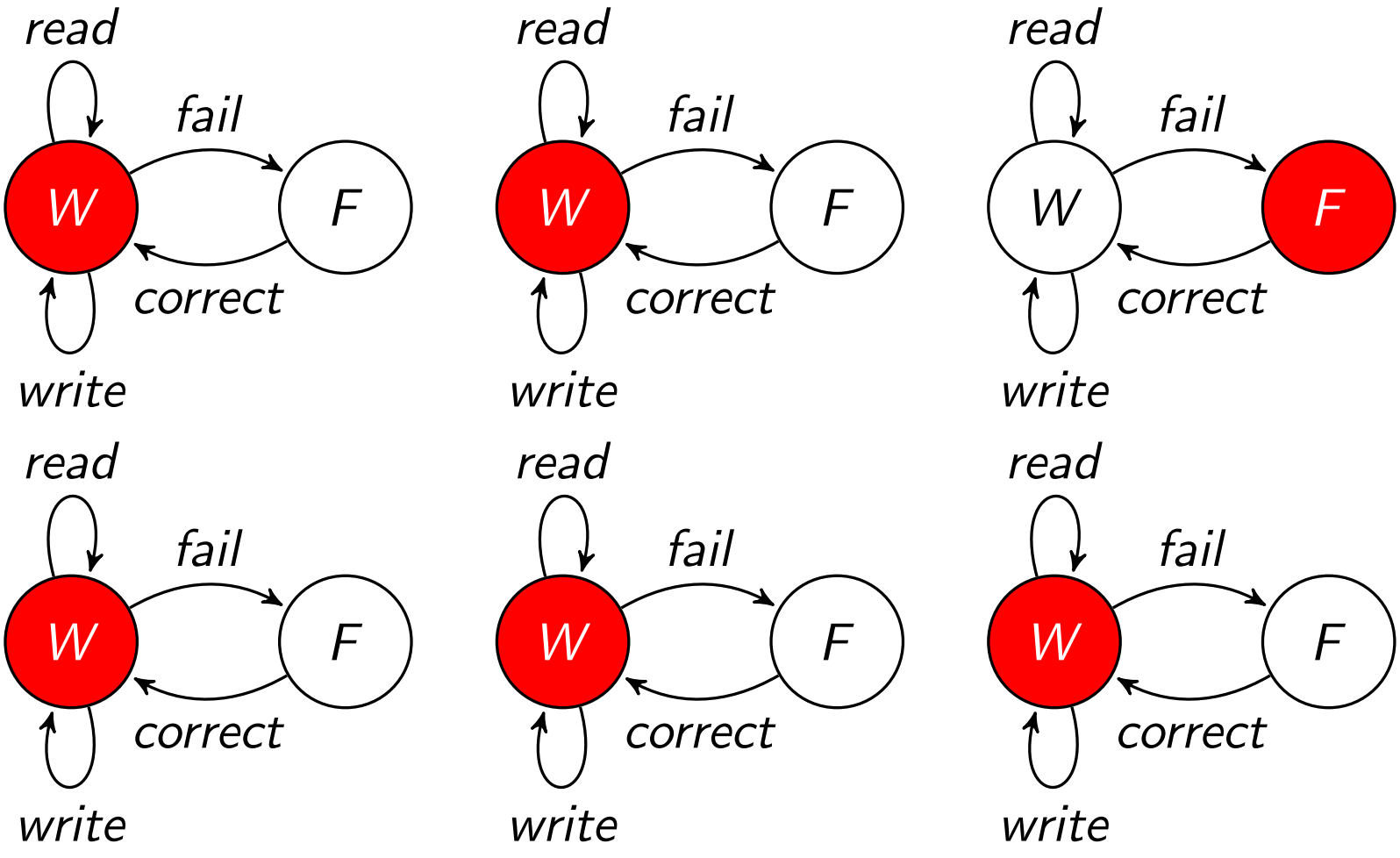
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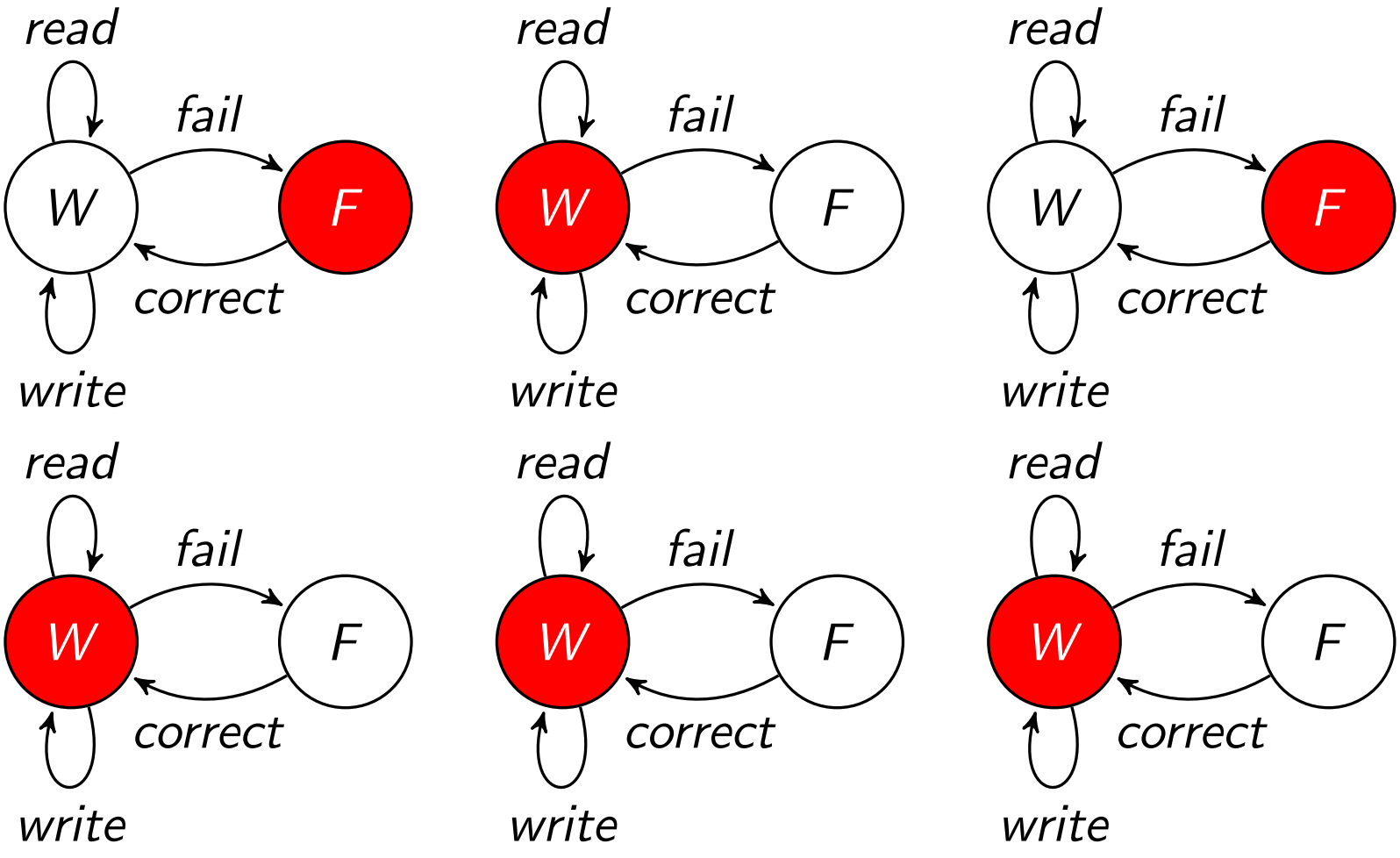
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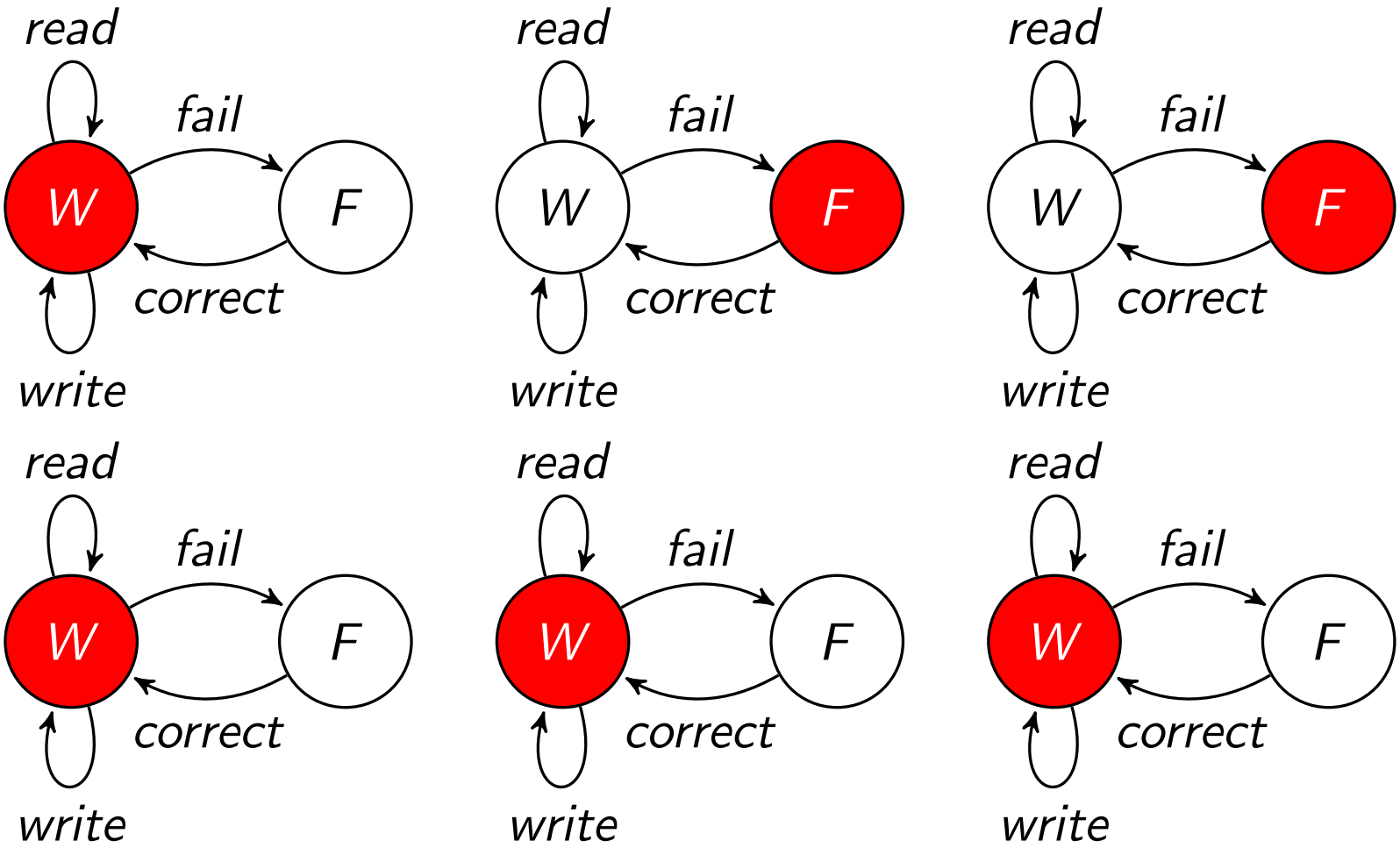
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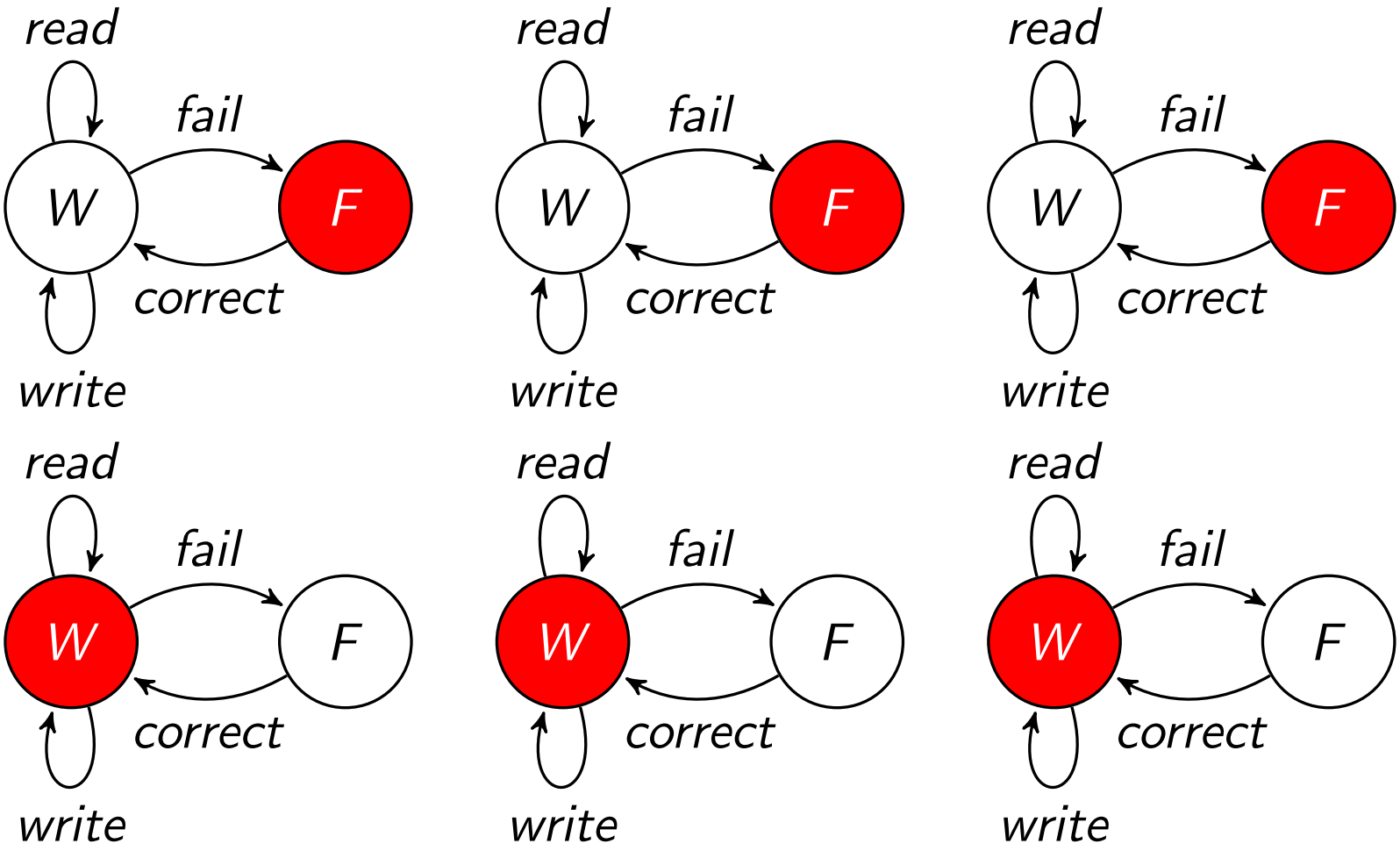
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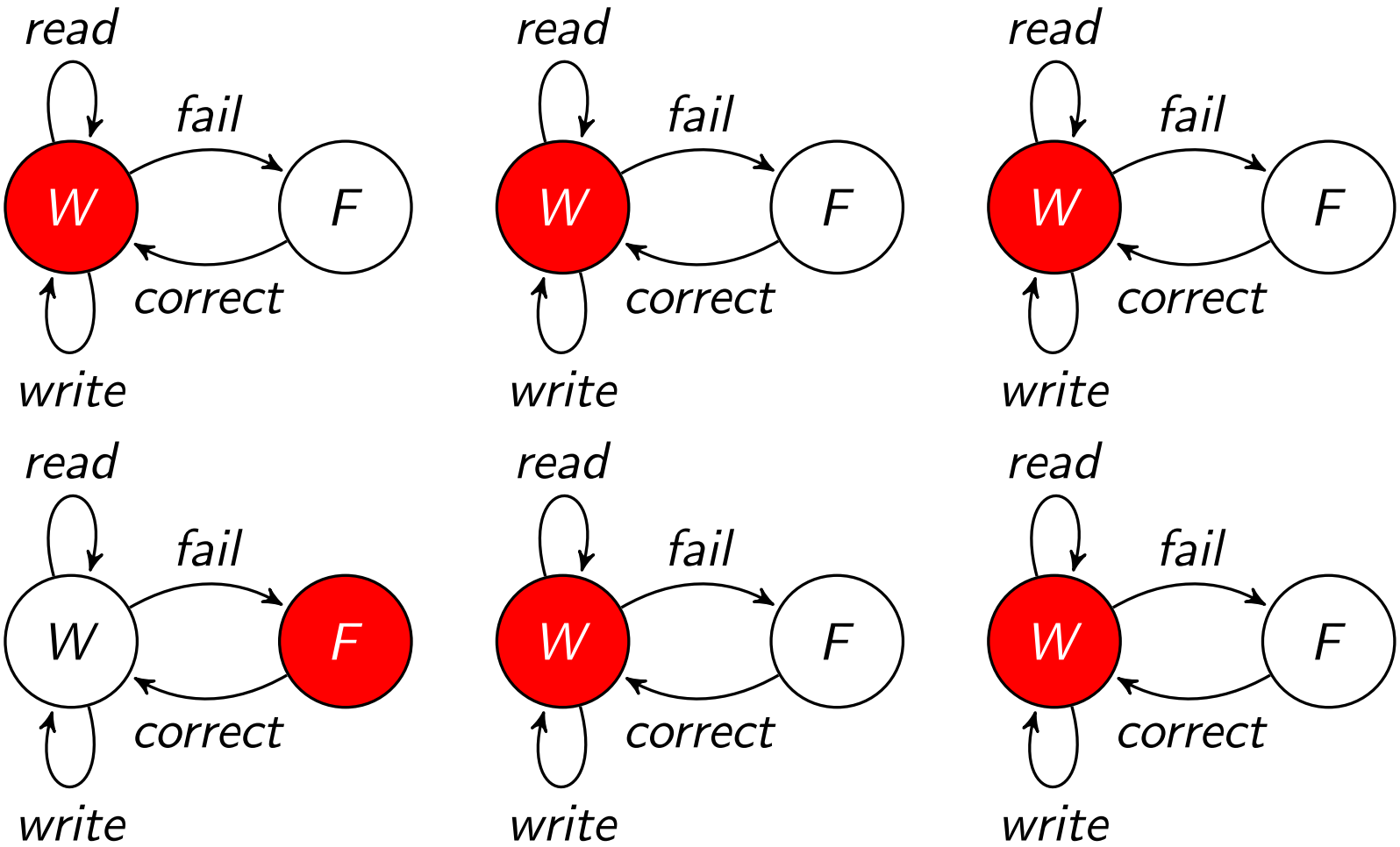
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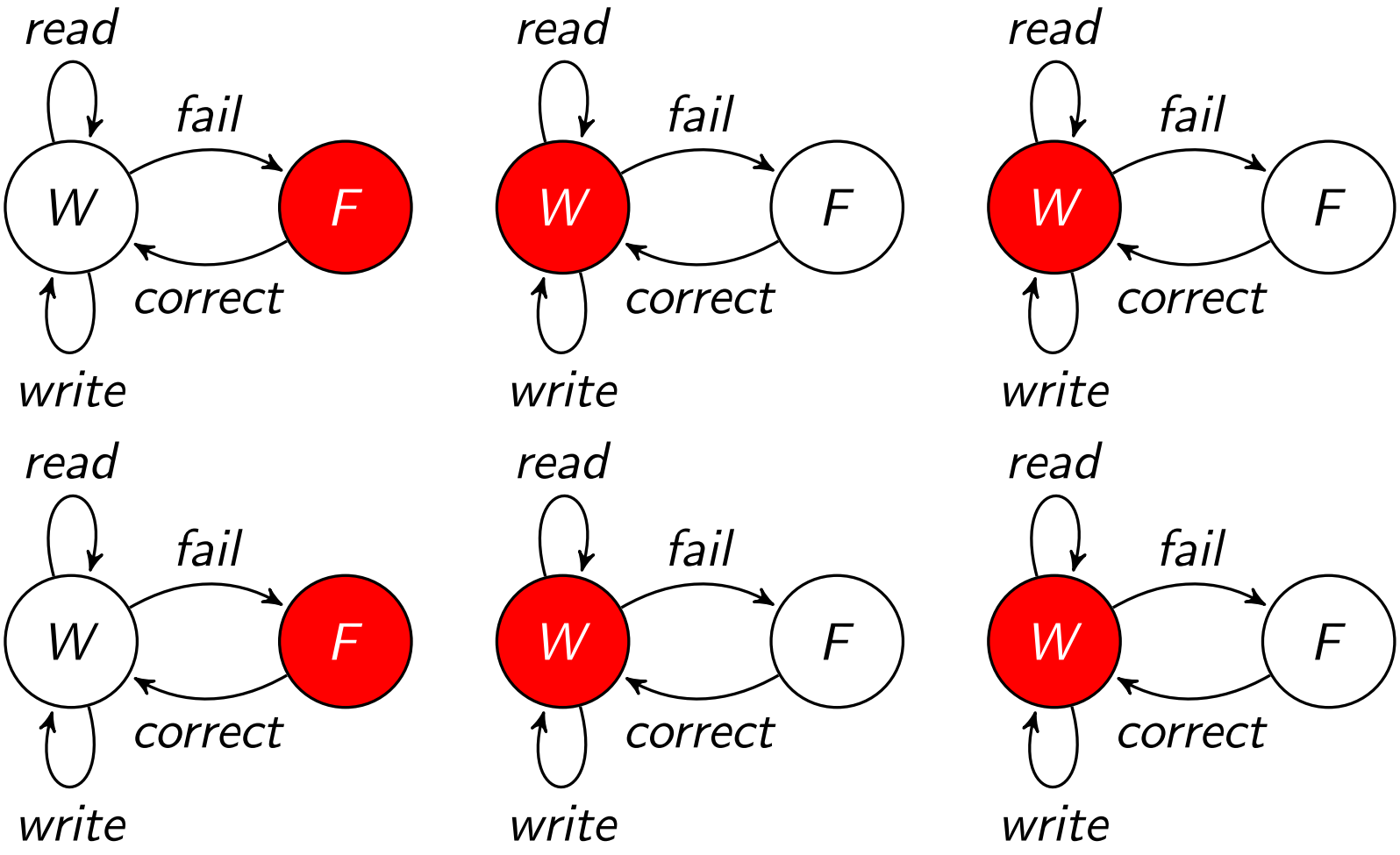
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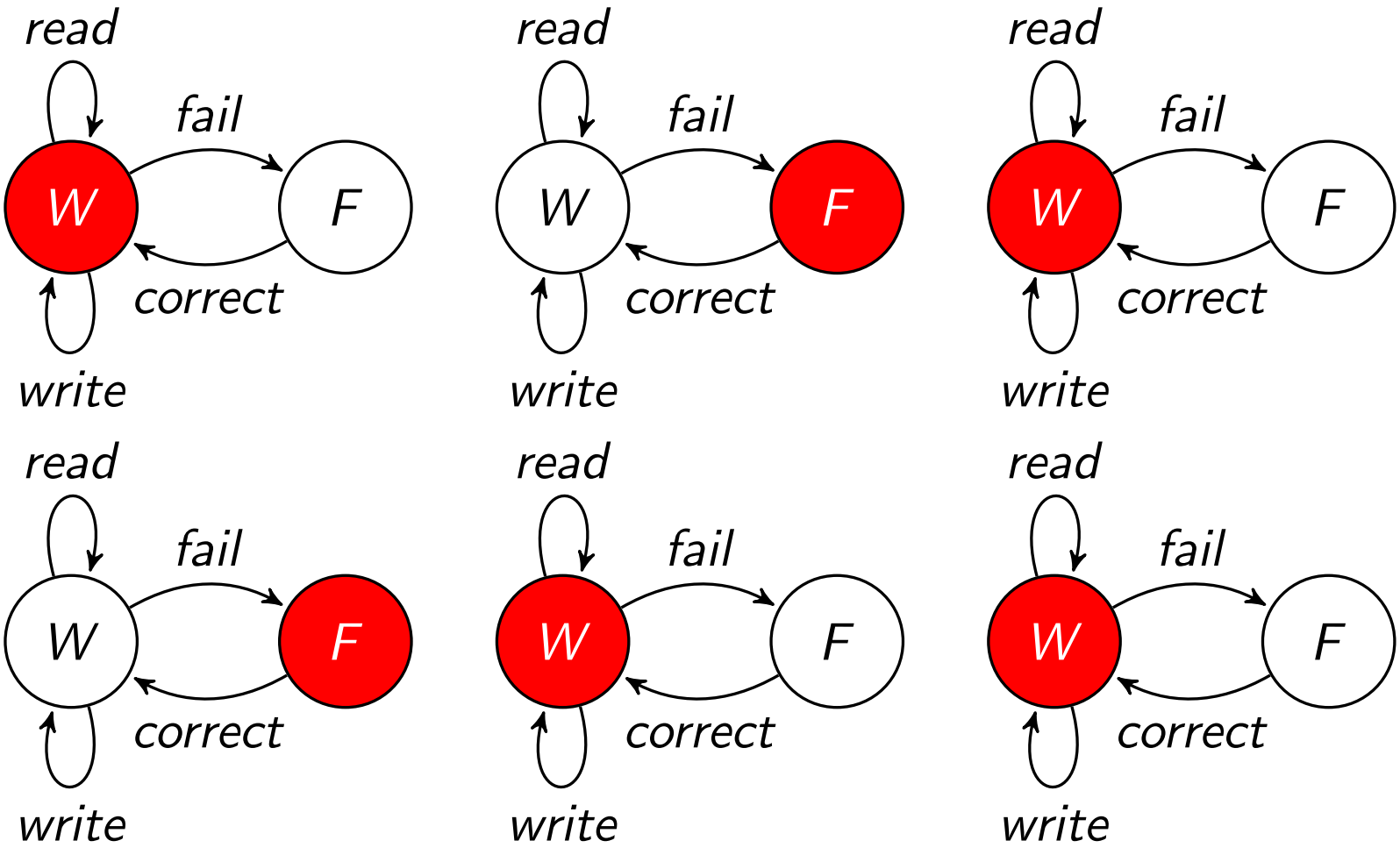
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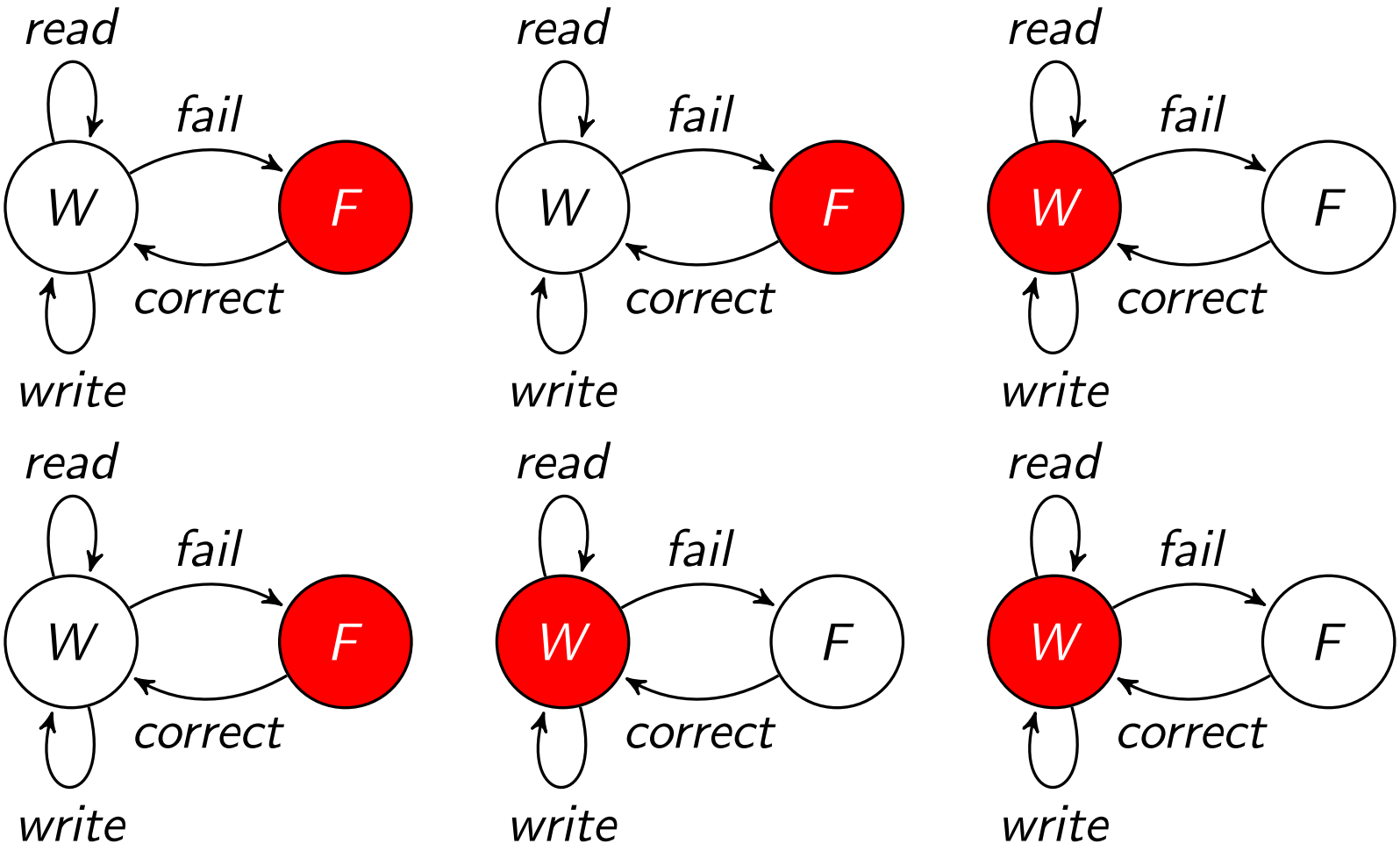
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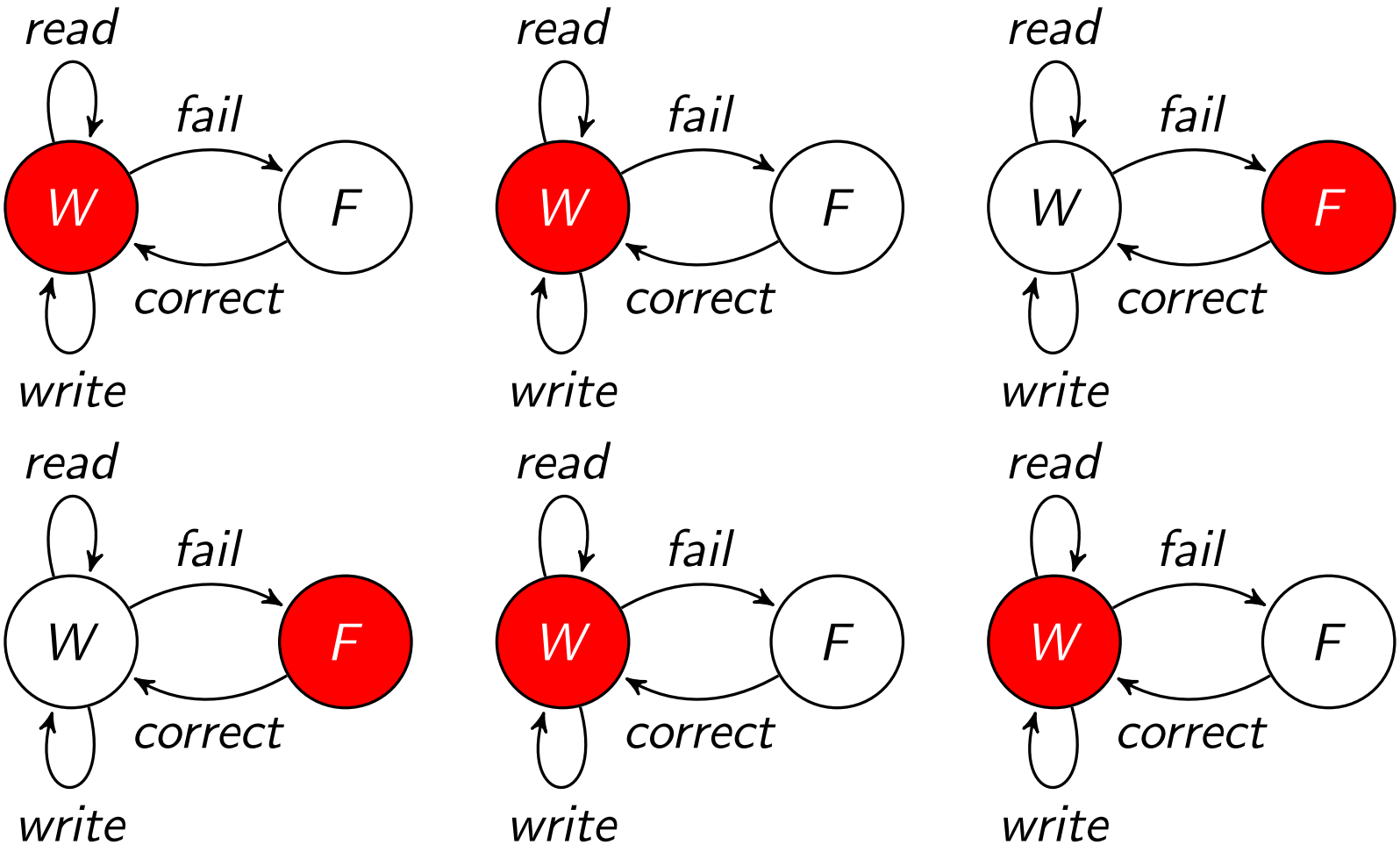
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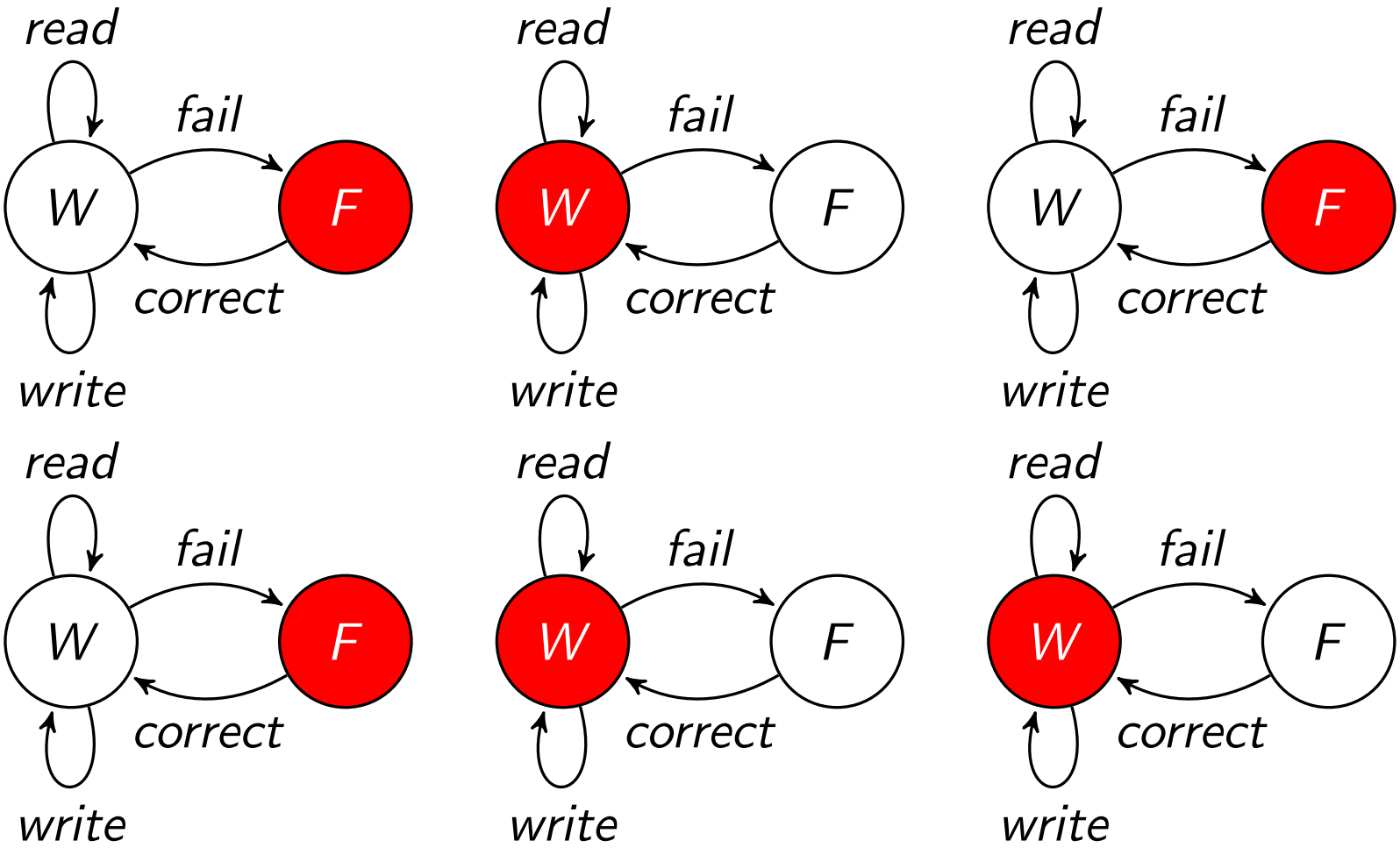
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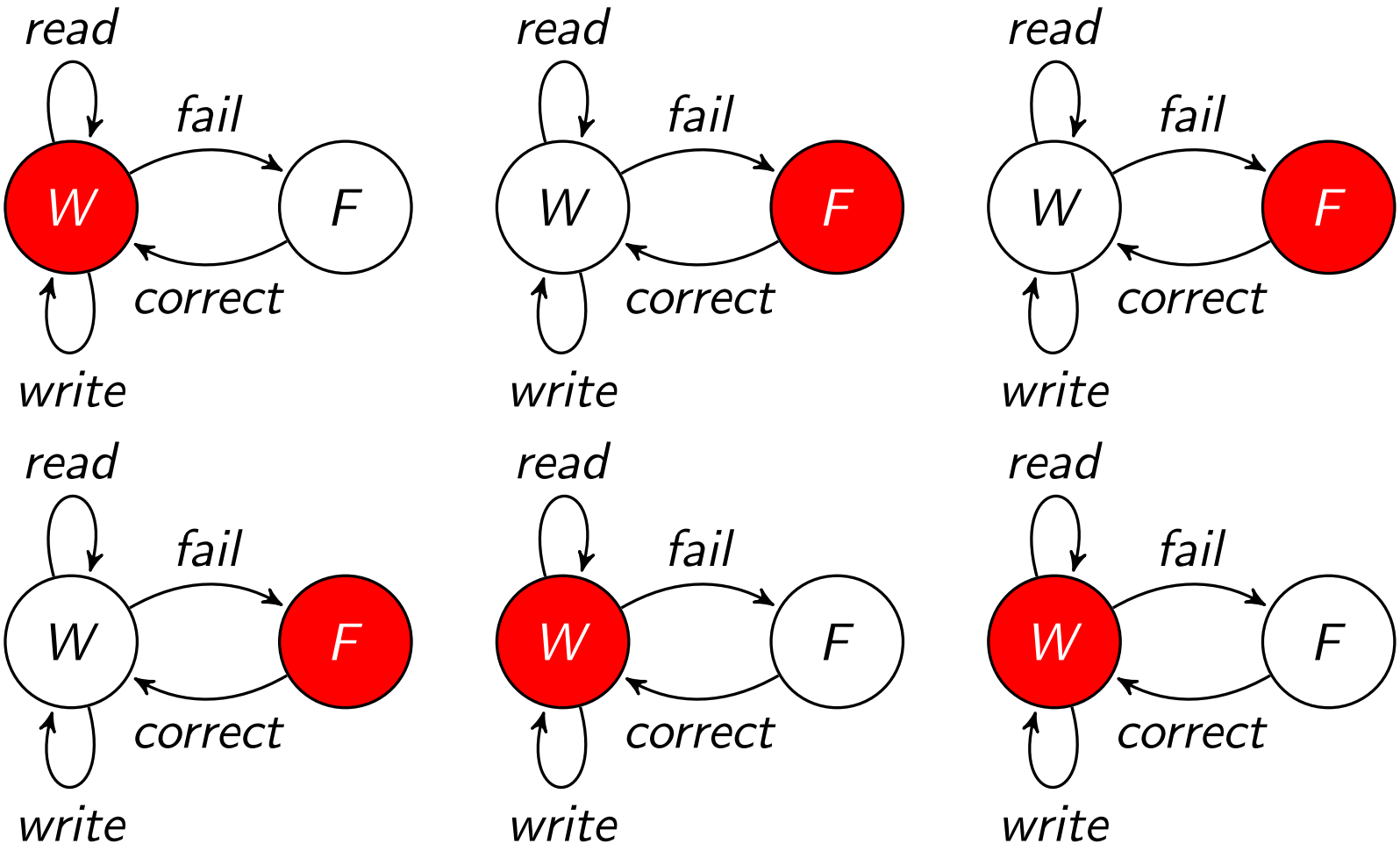
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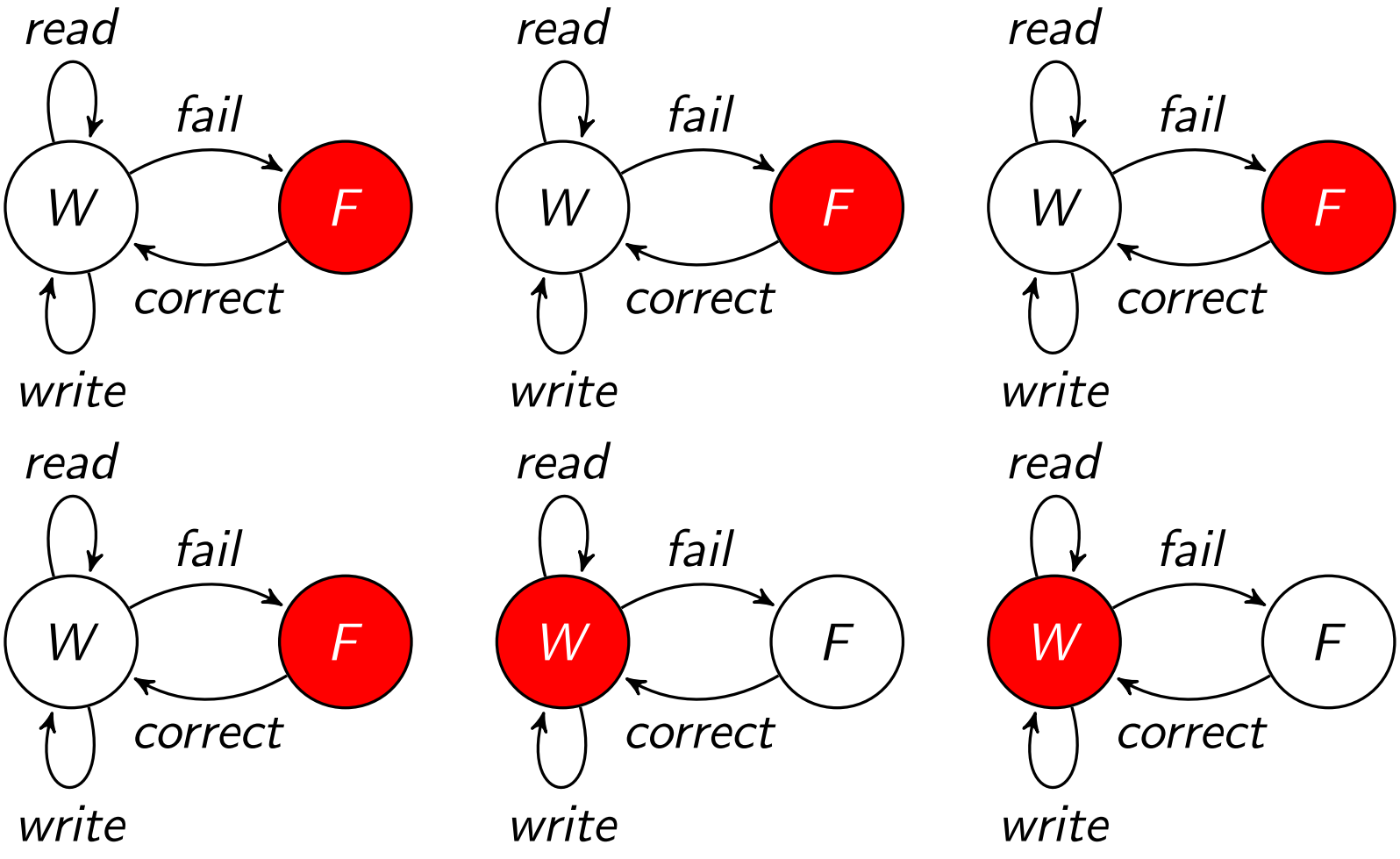
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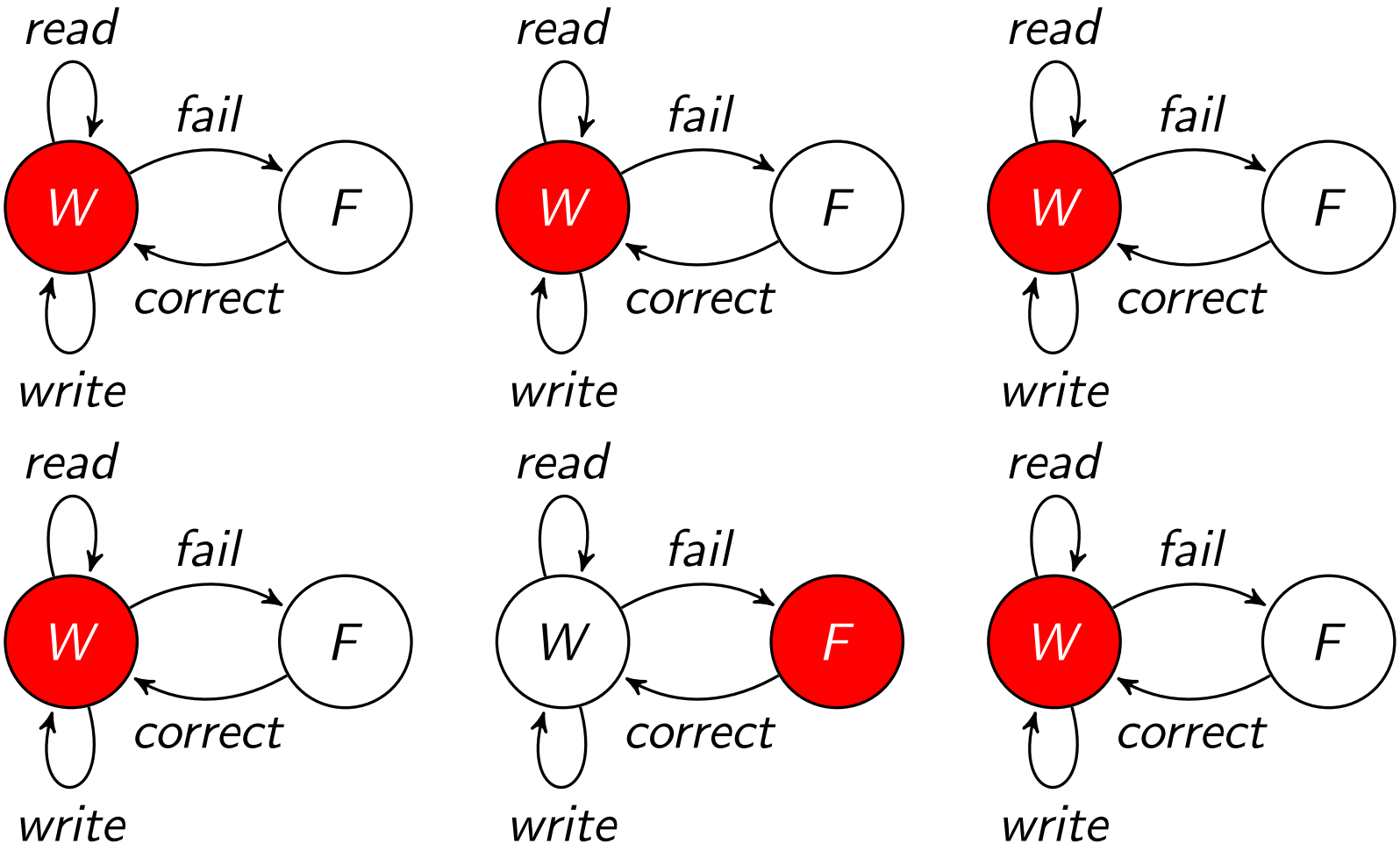
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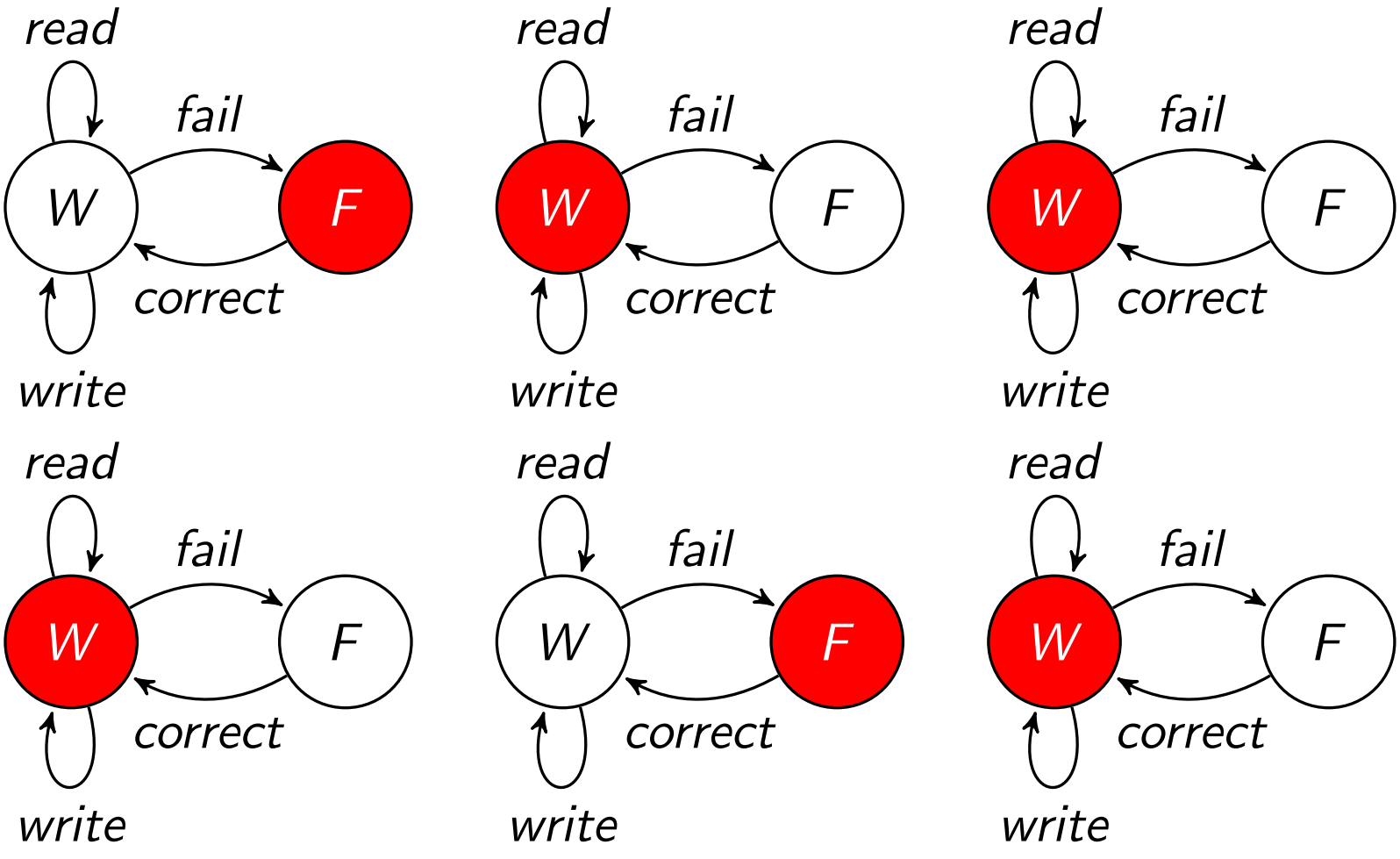
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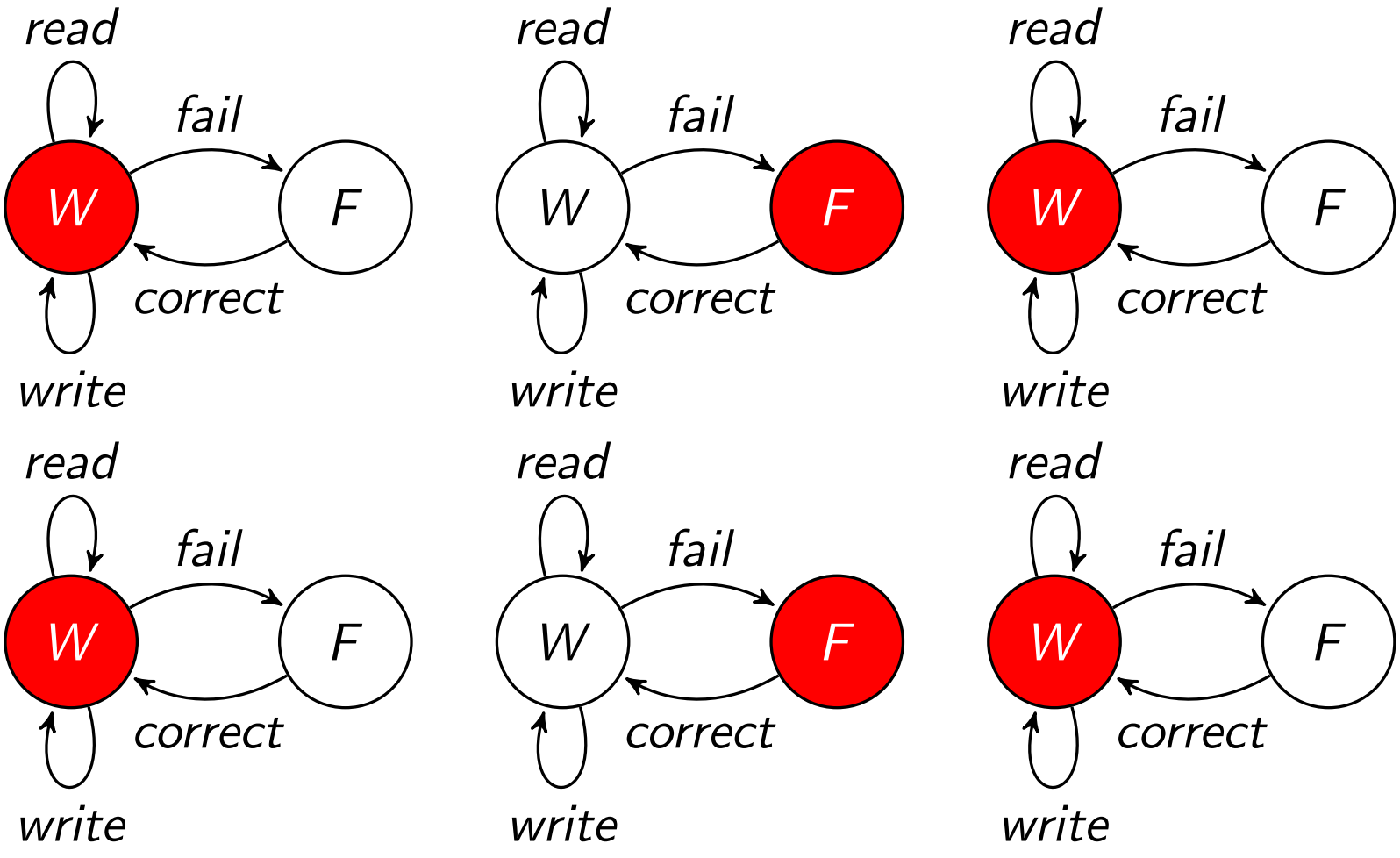
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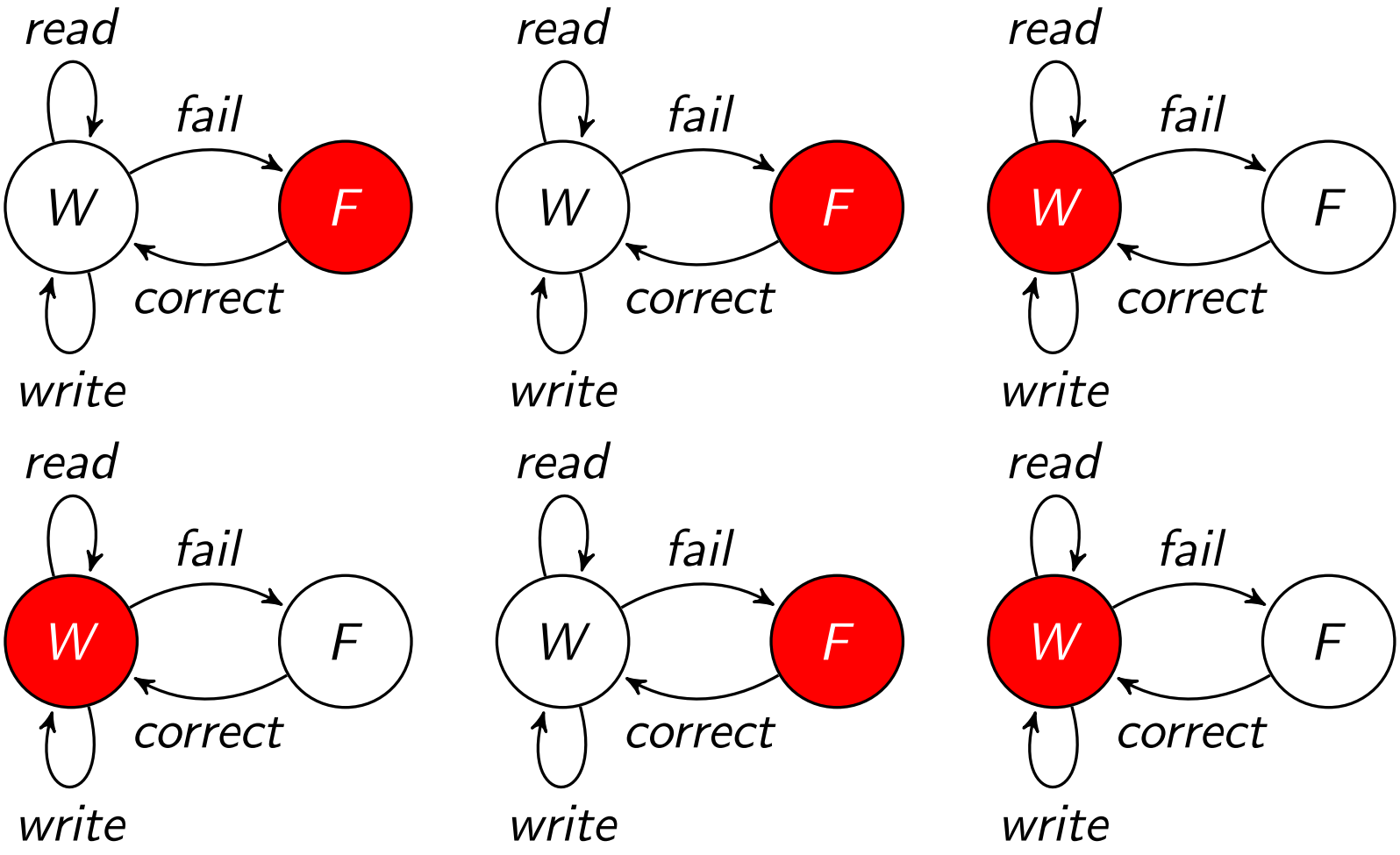
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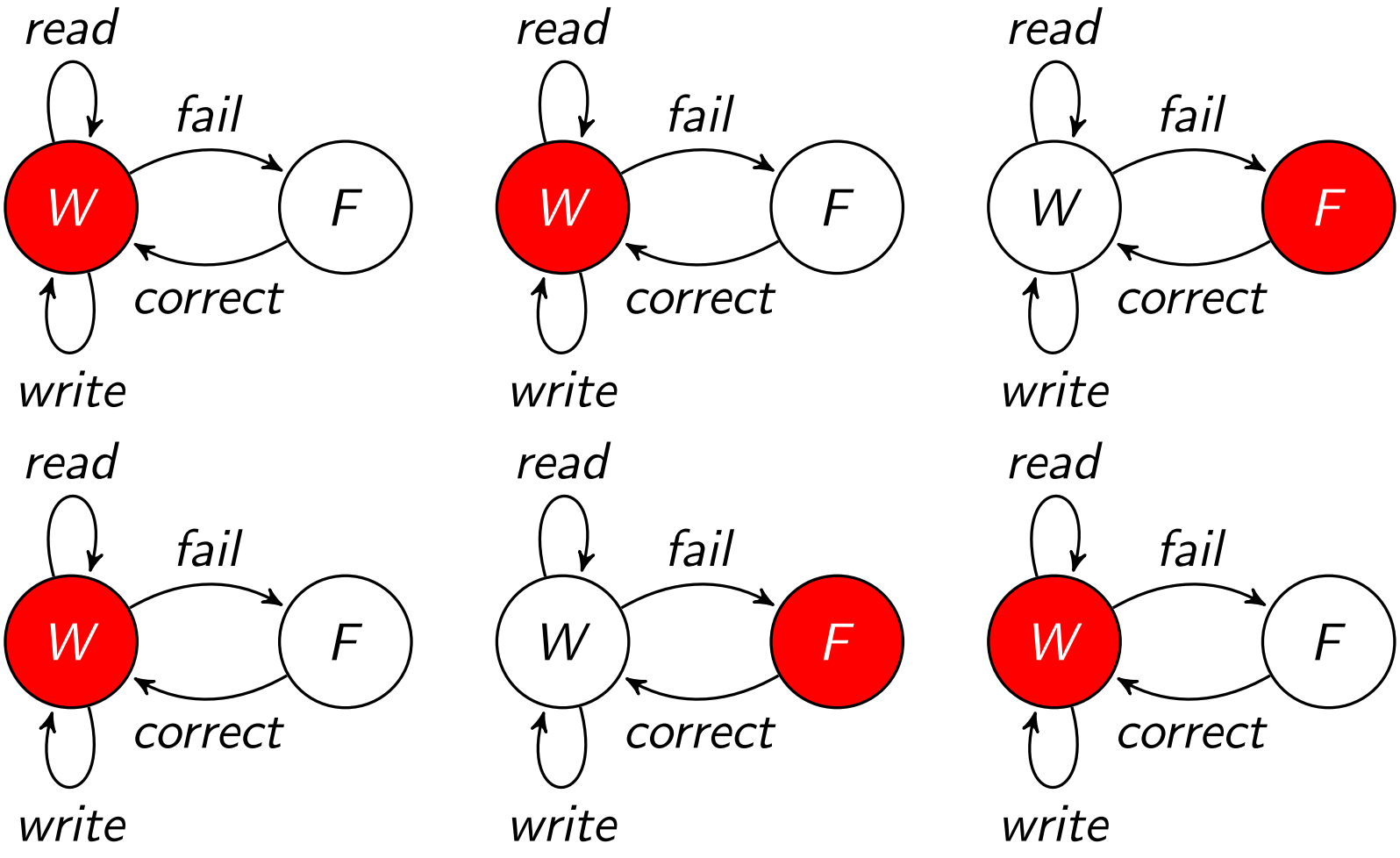
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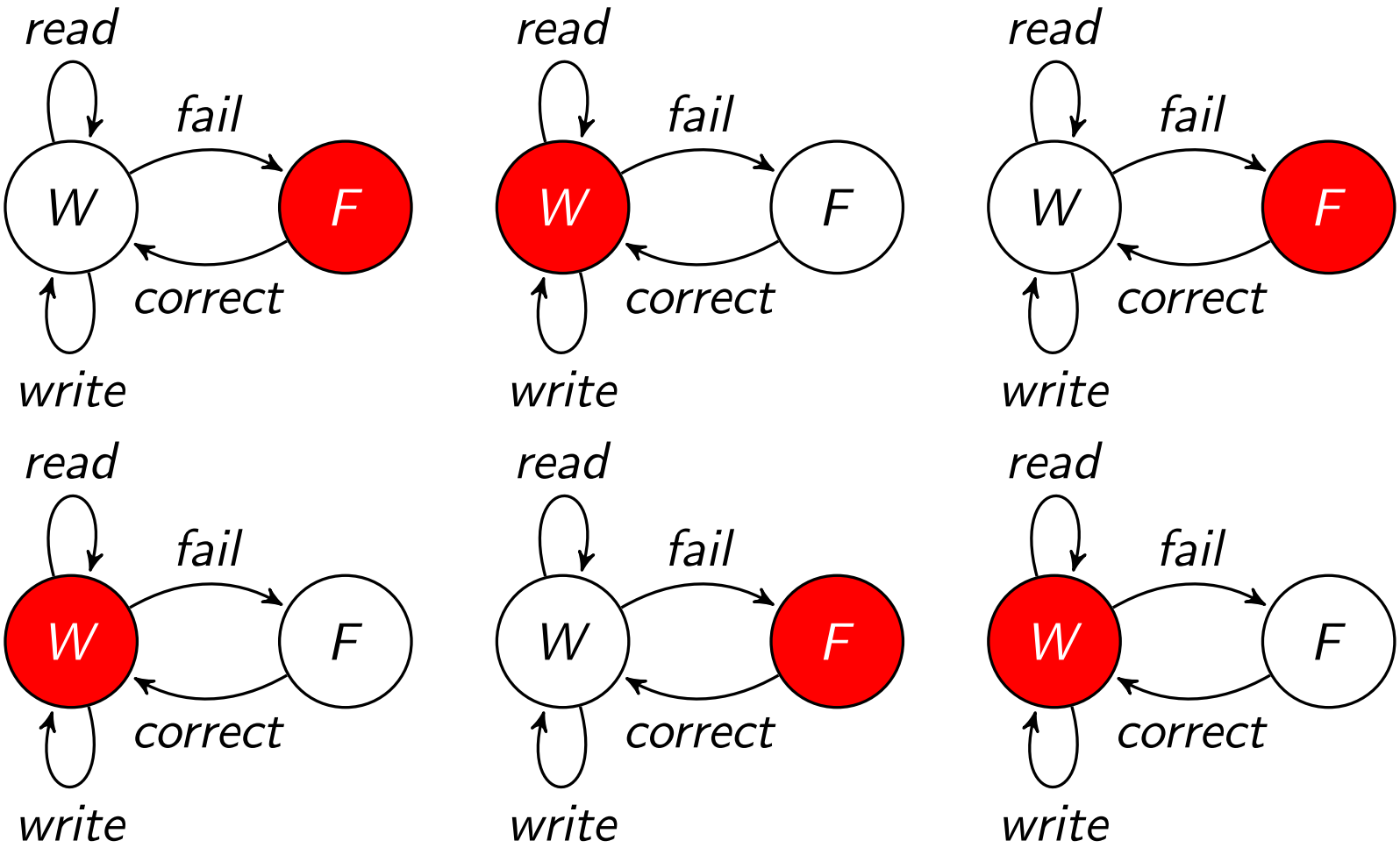
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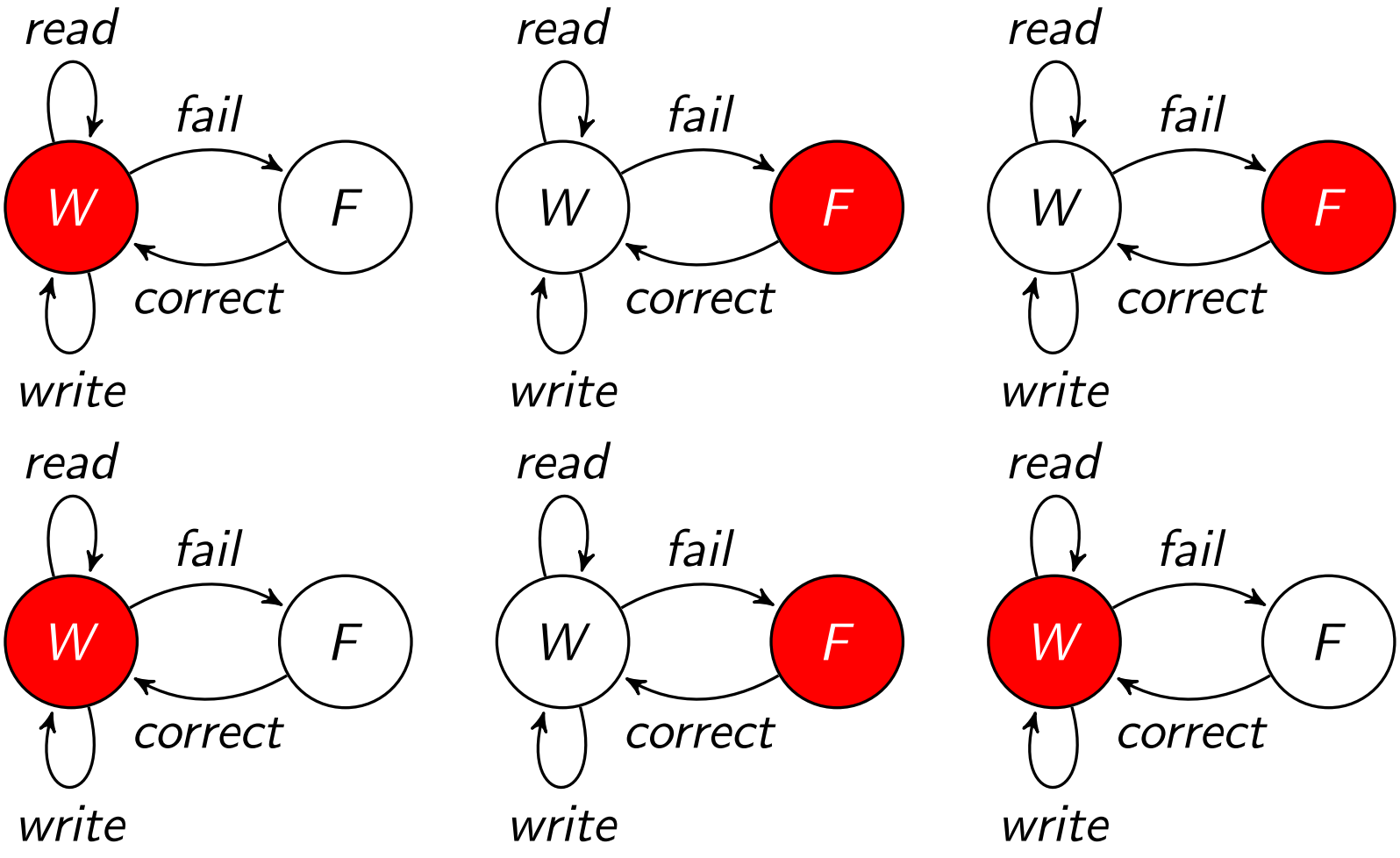
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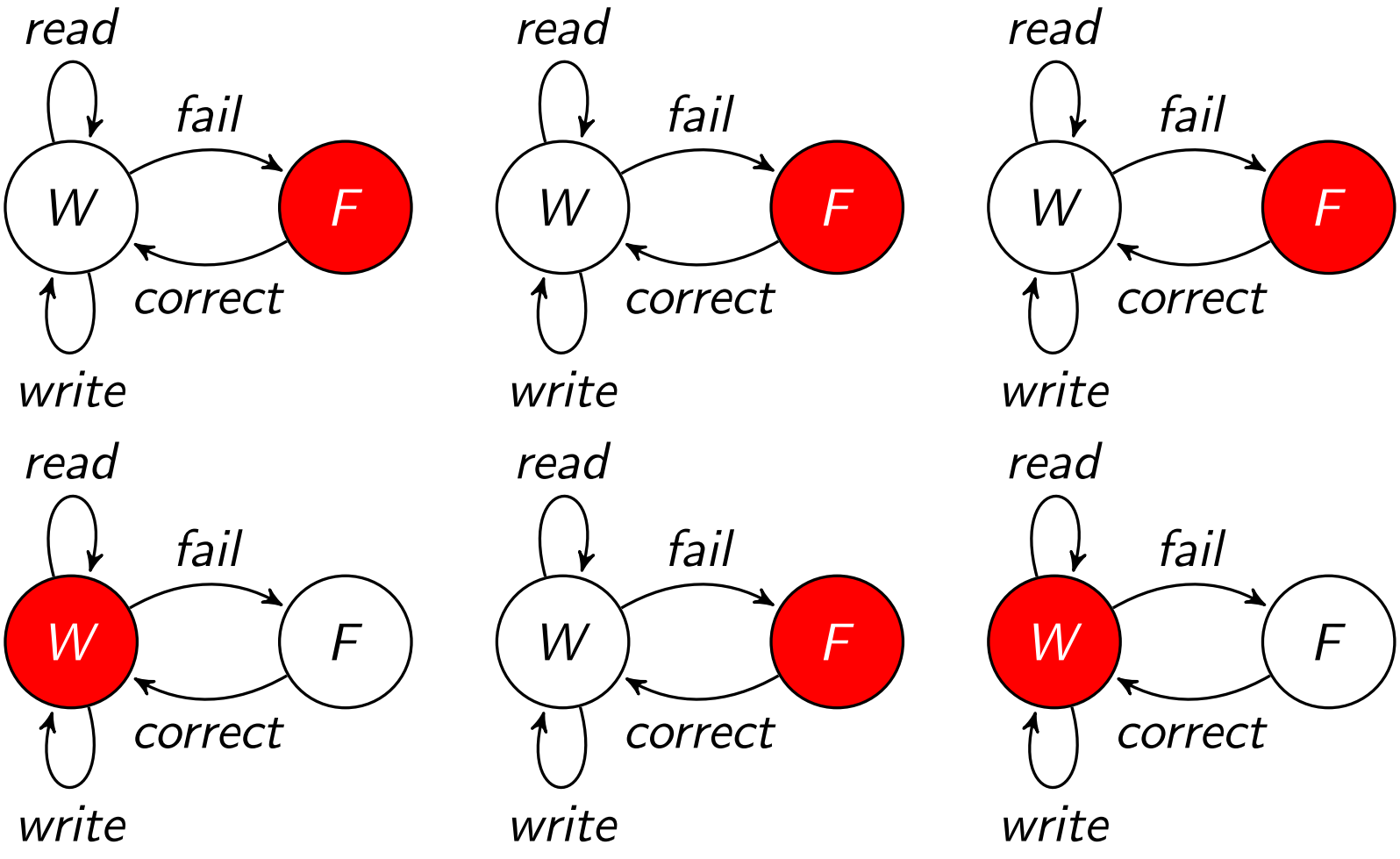
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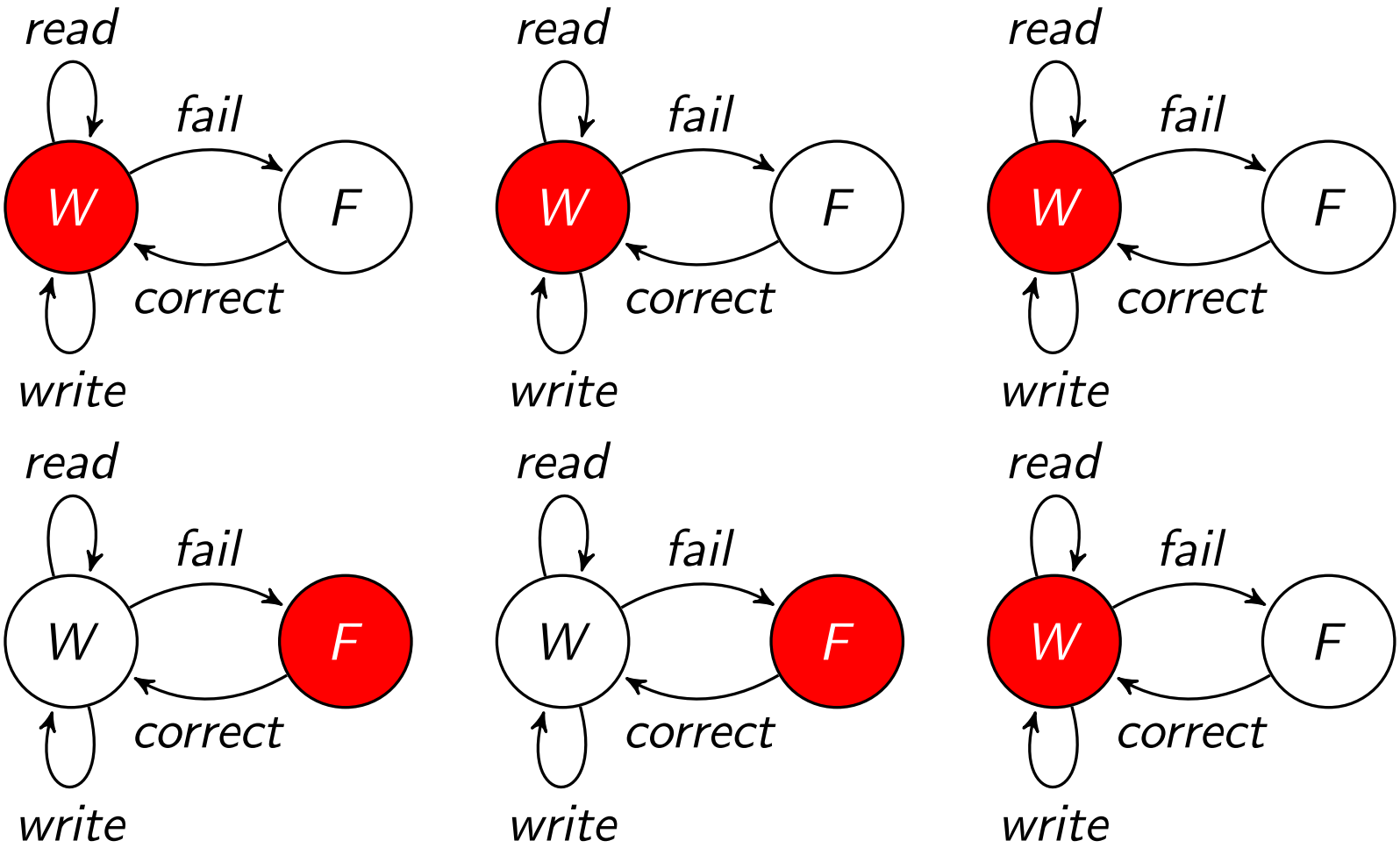
Six disks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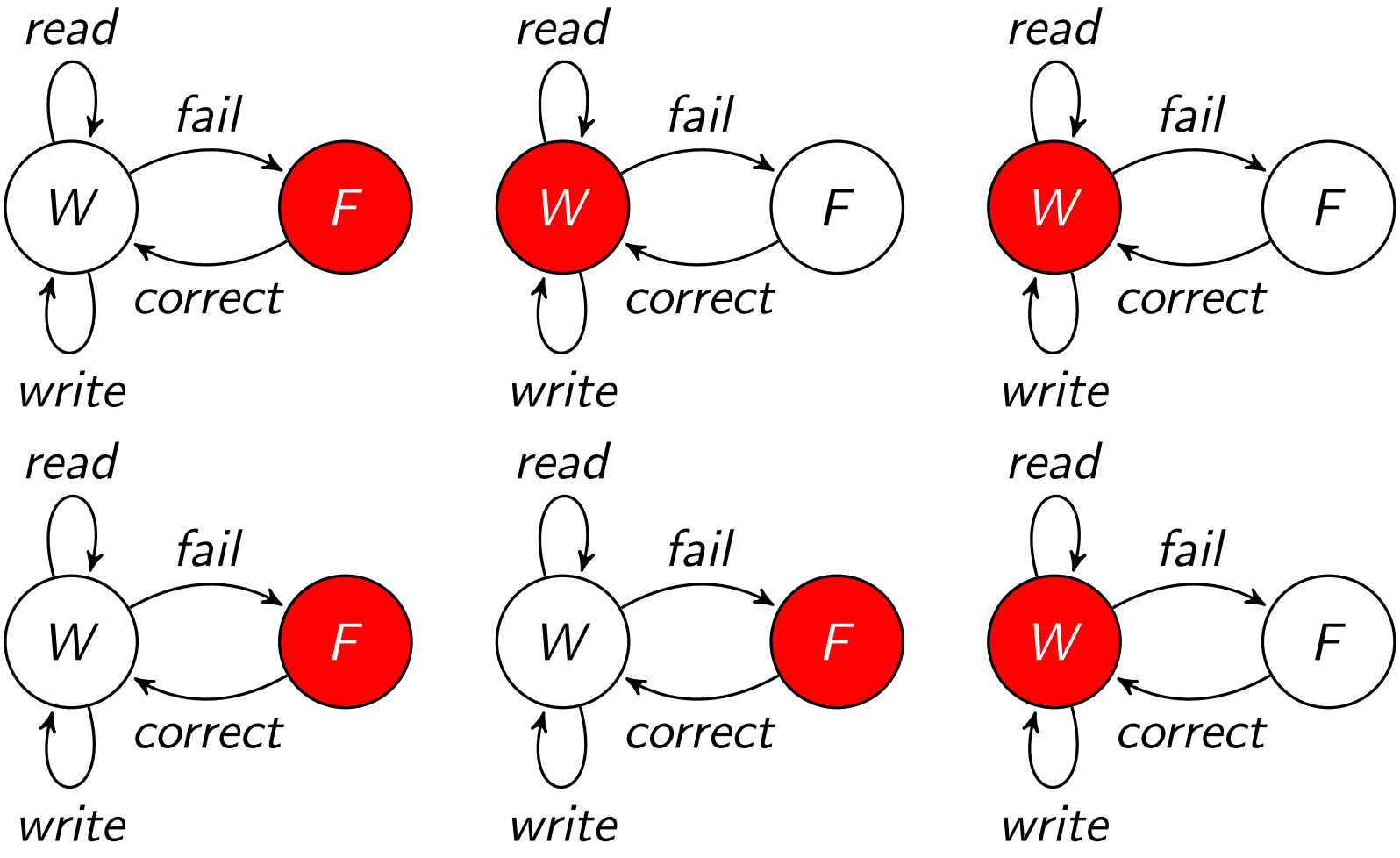
Six disks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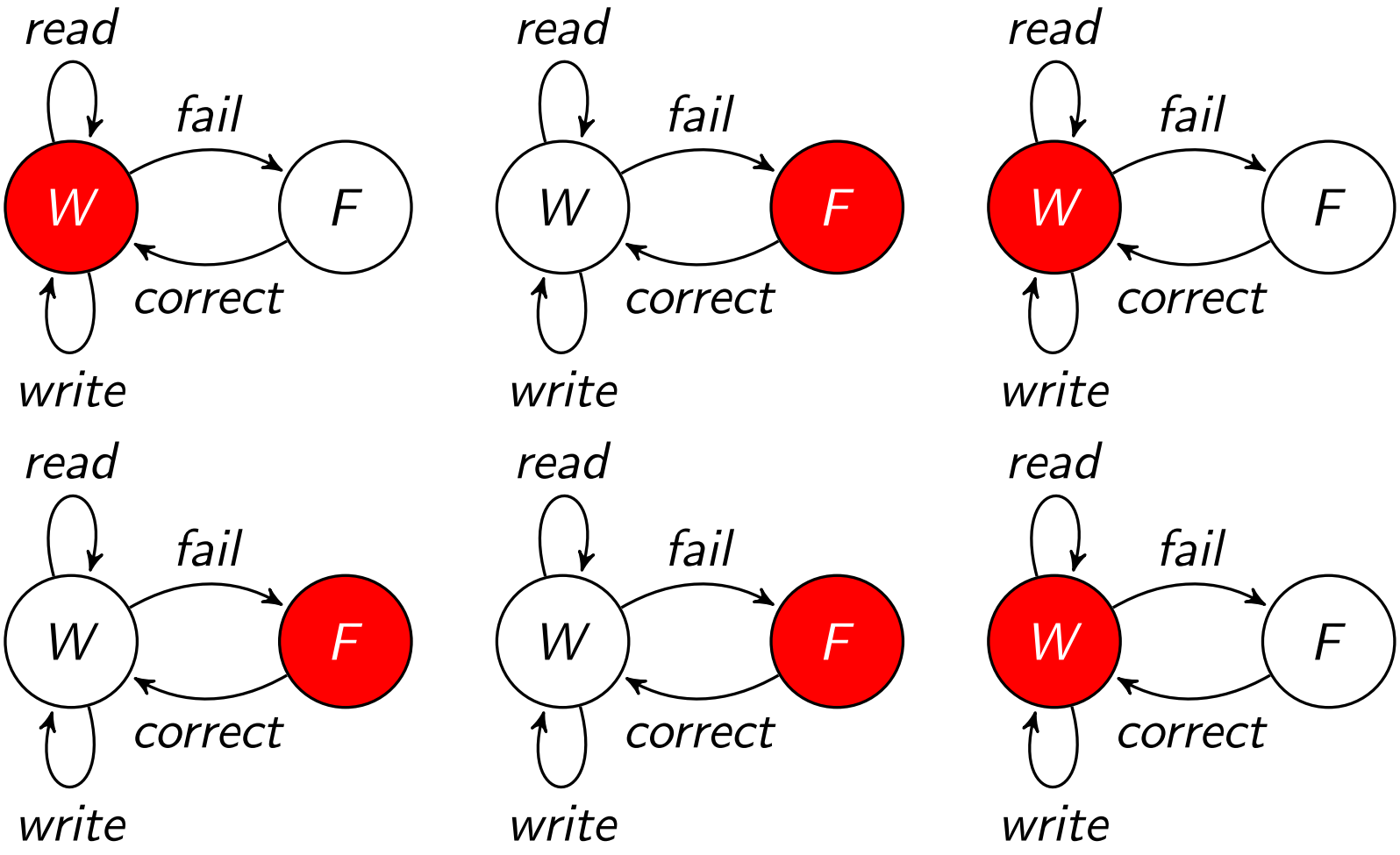
Six disks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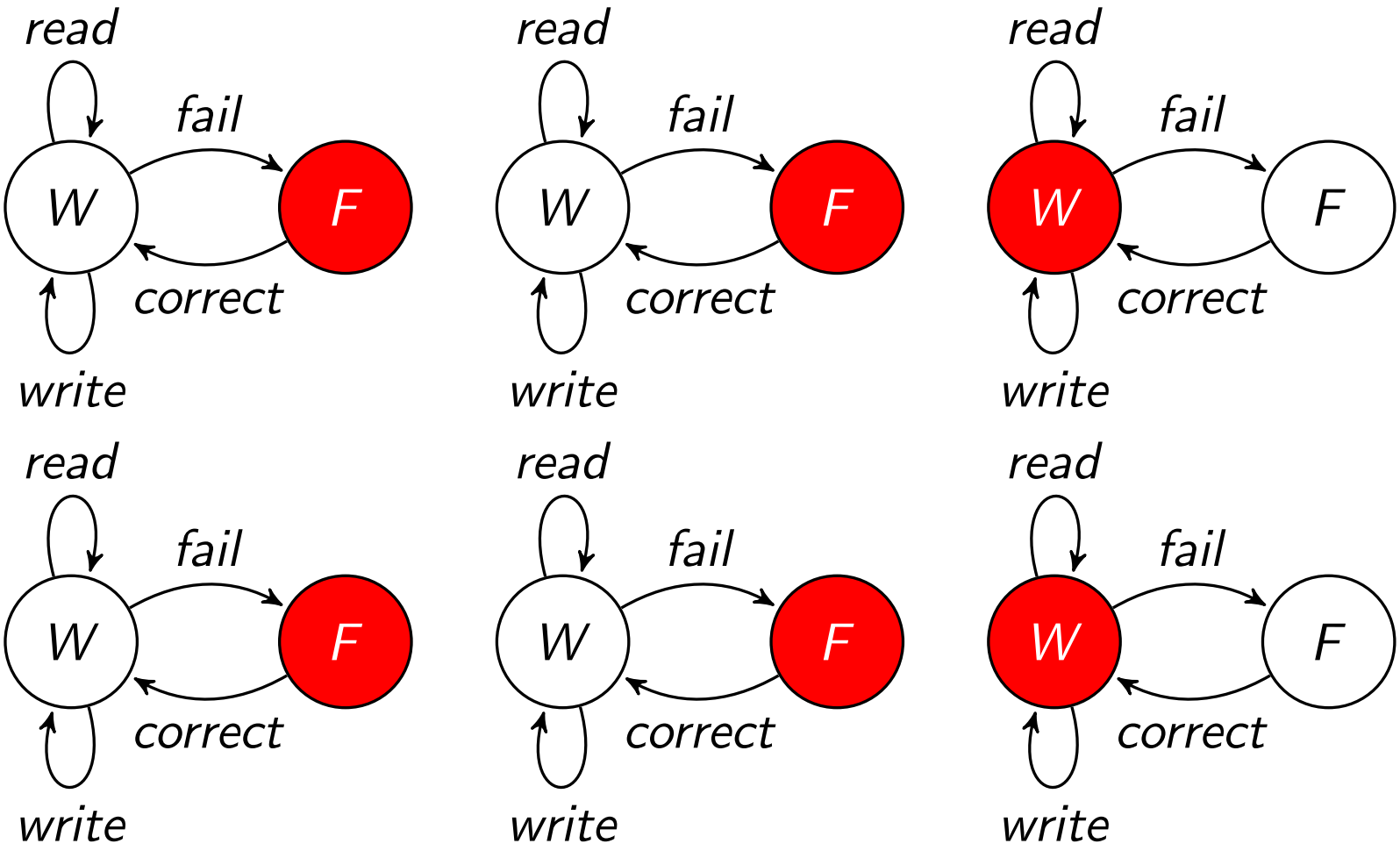
Six disks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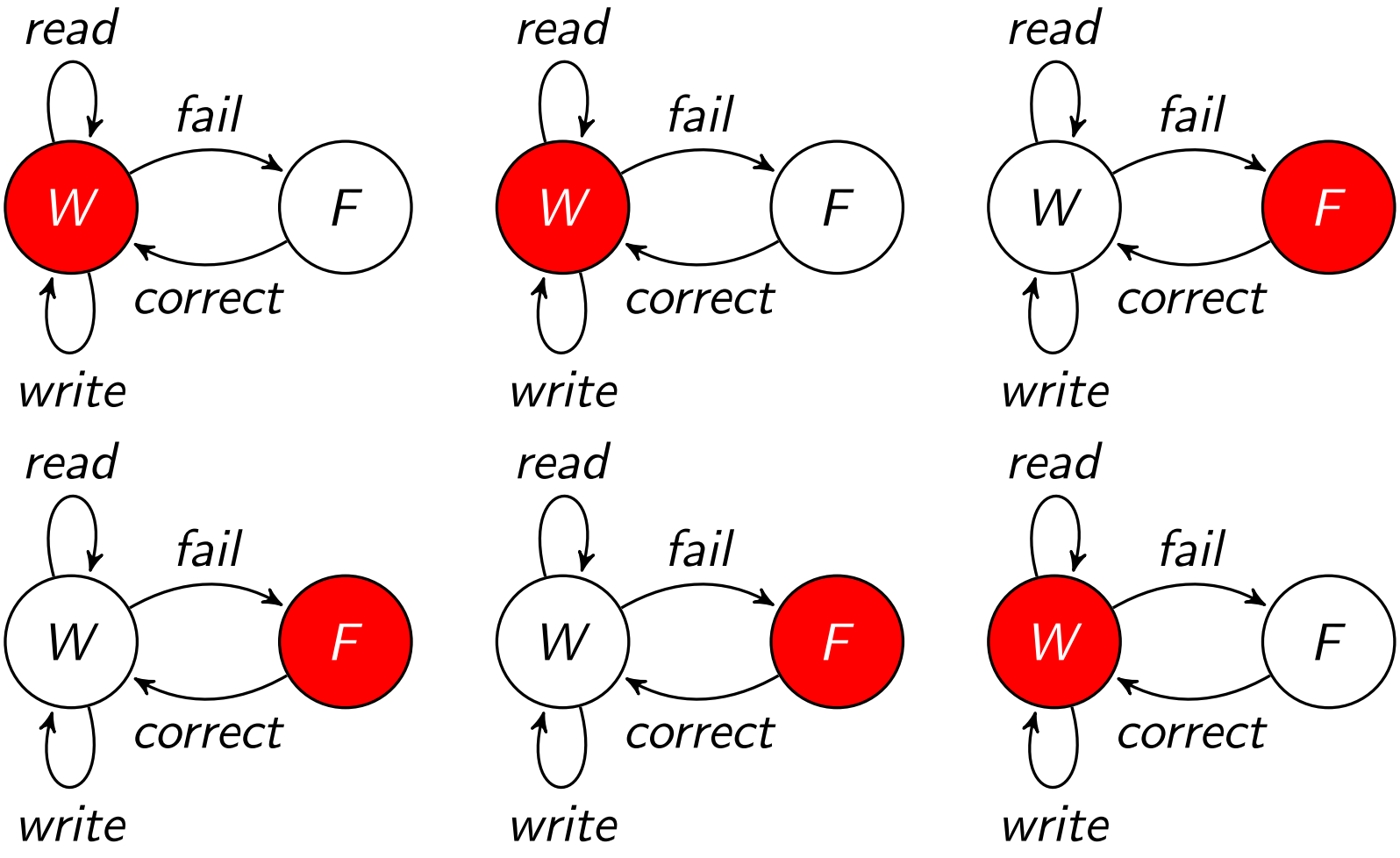
Six disks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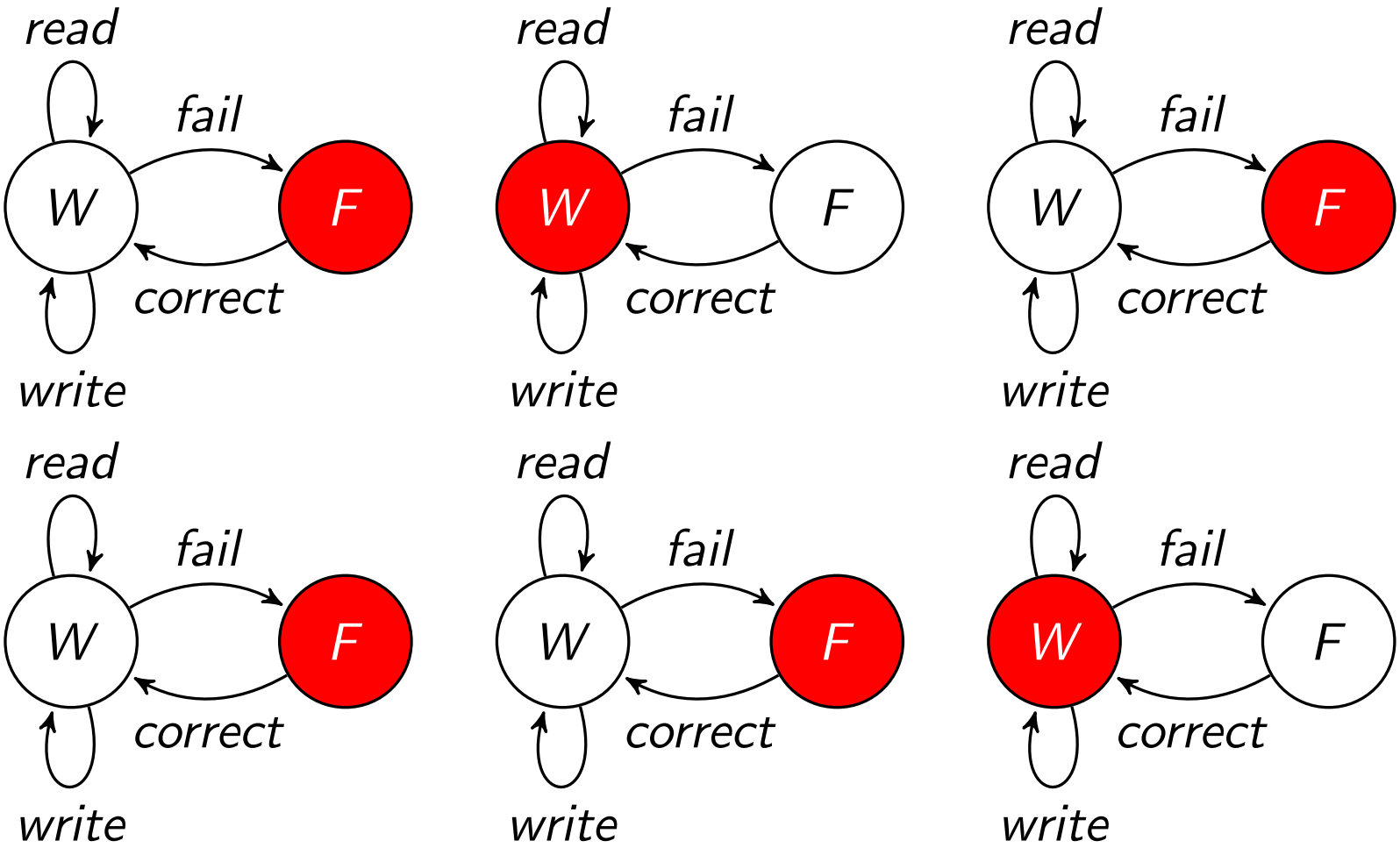
Six disks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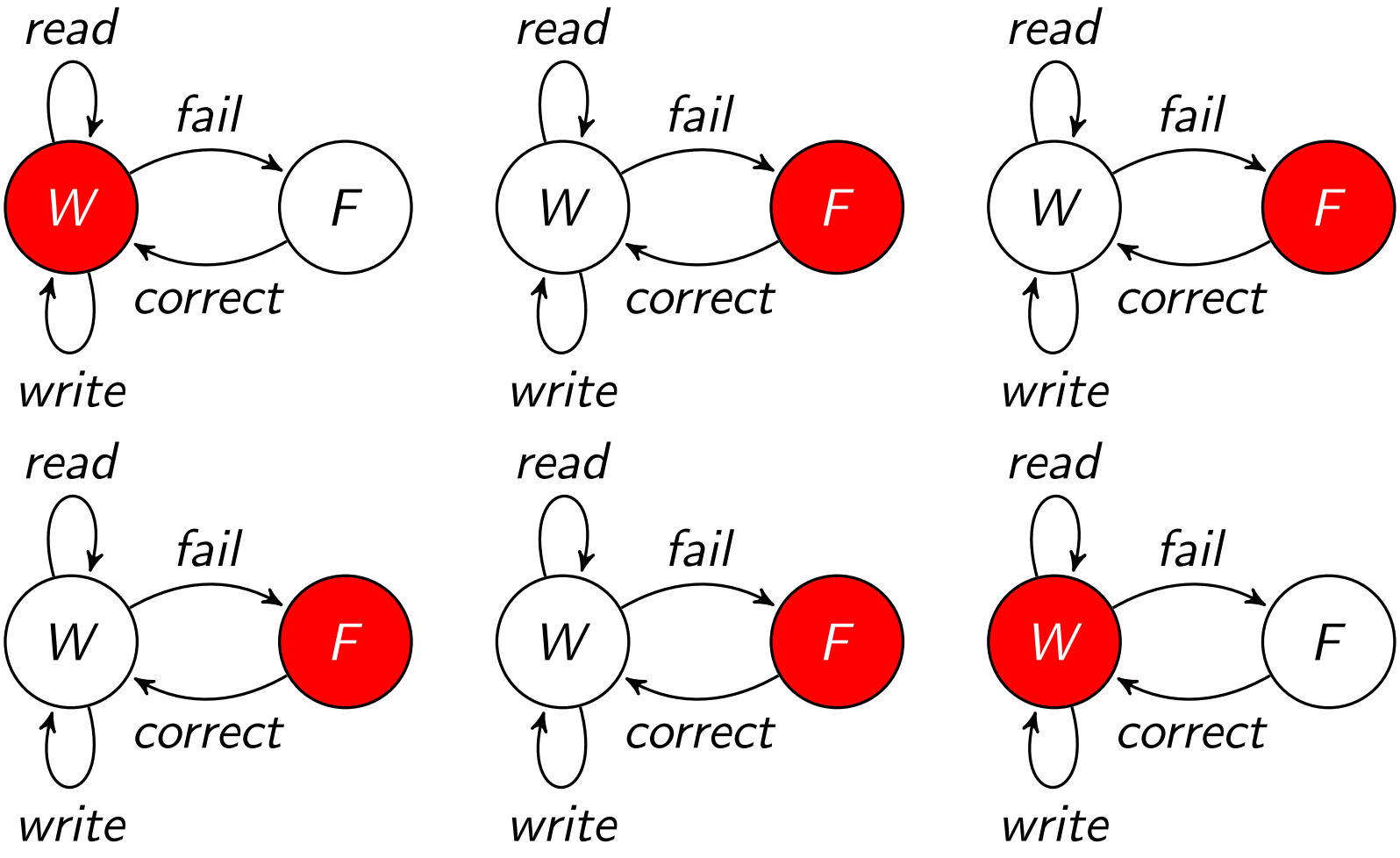
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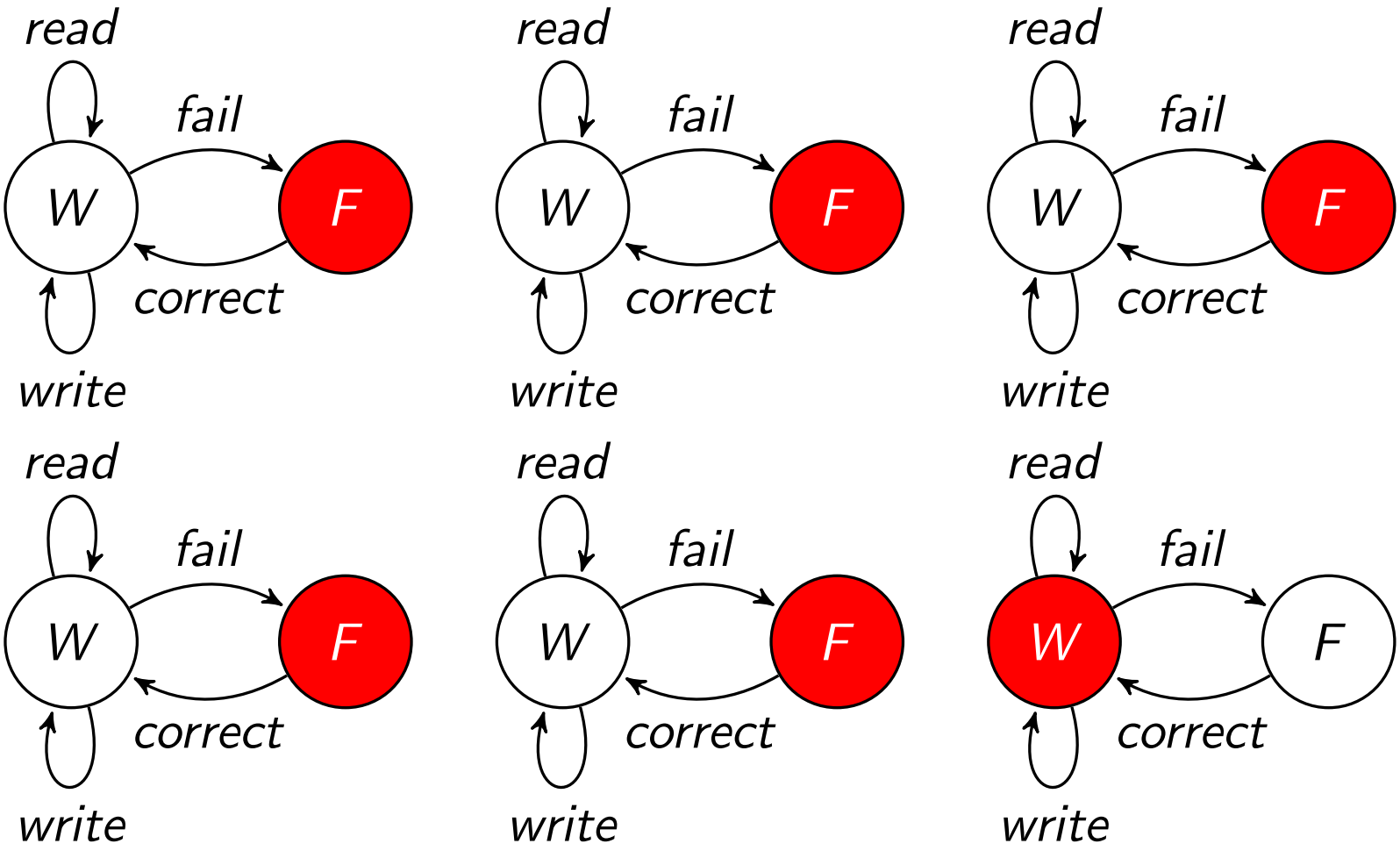
Six disks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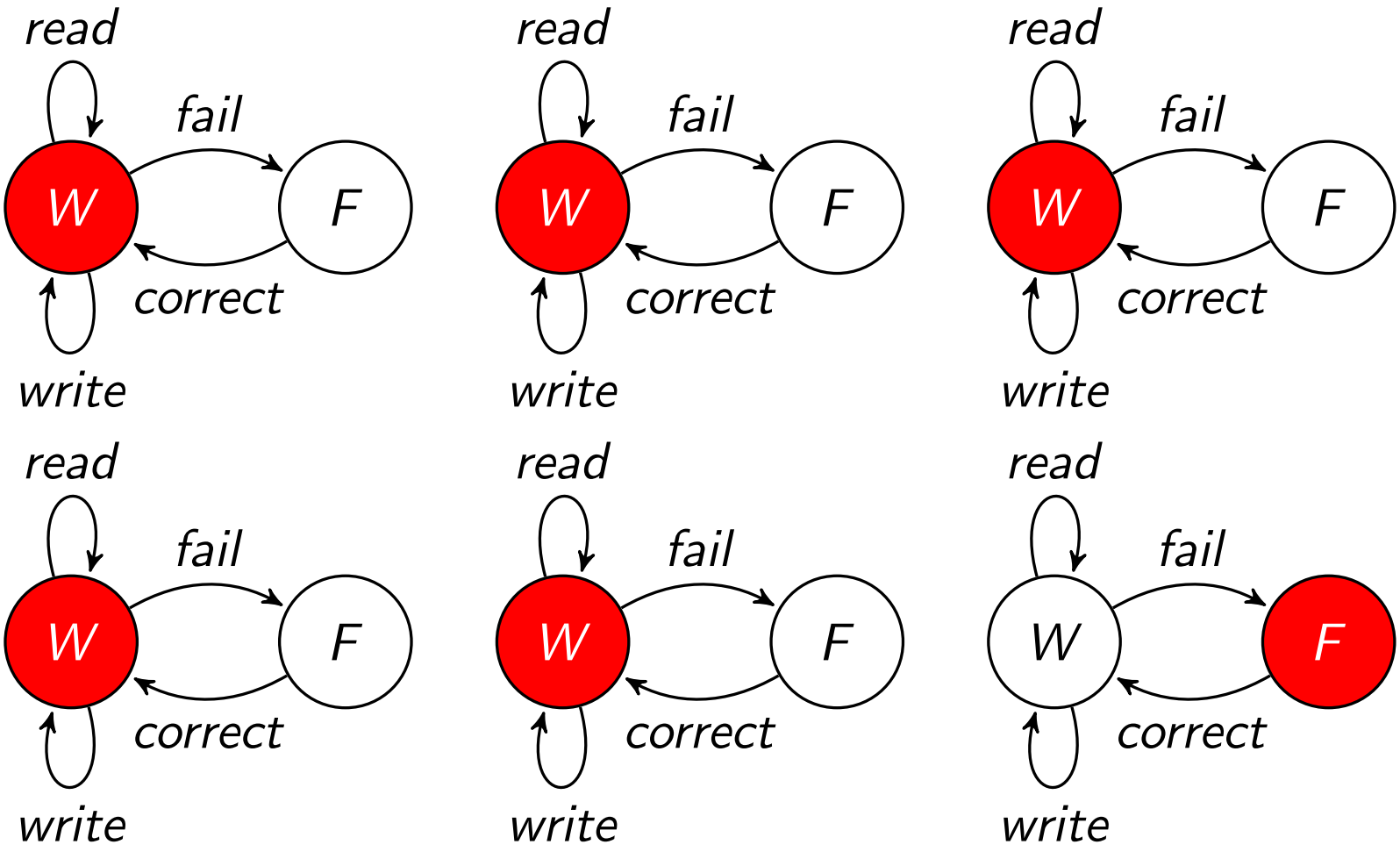
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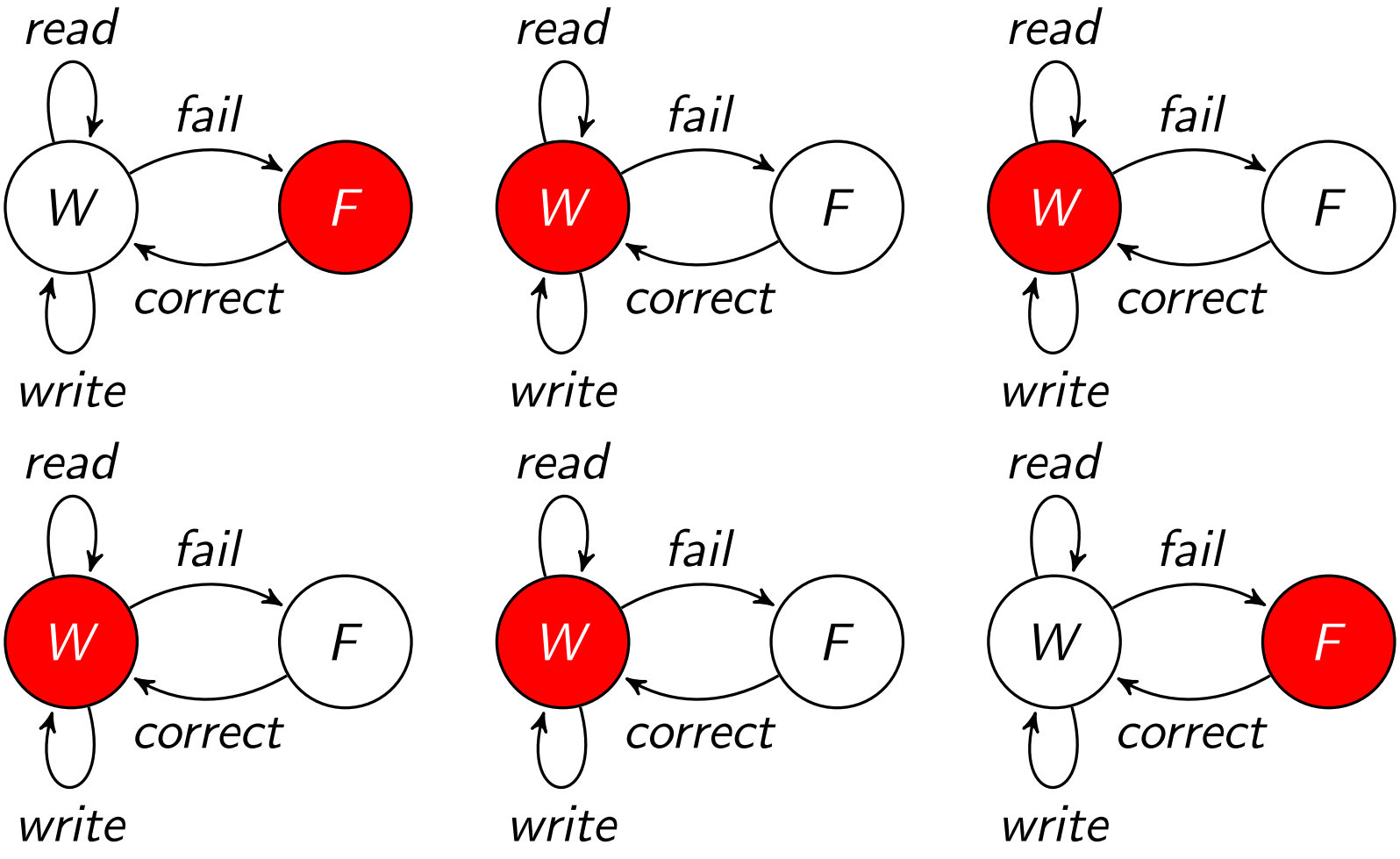
Six disks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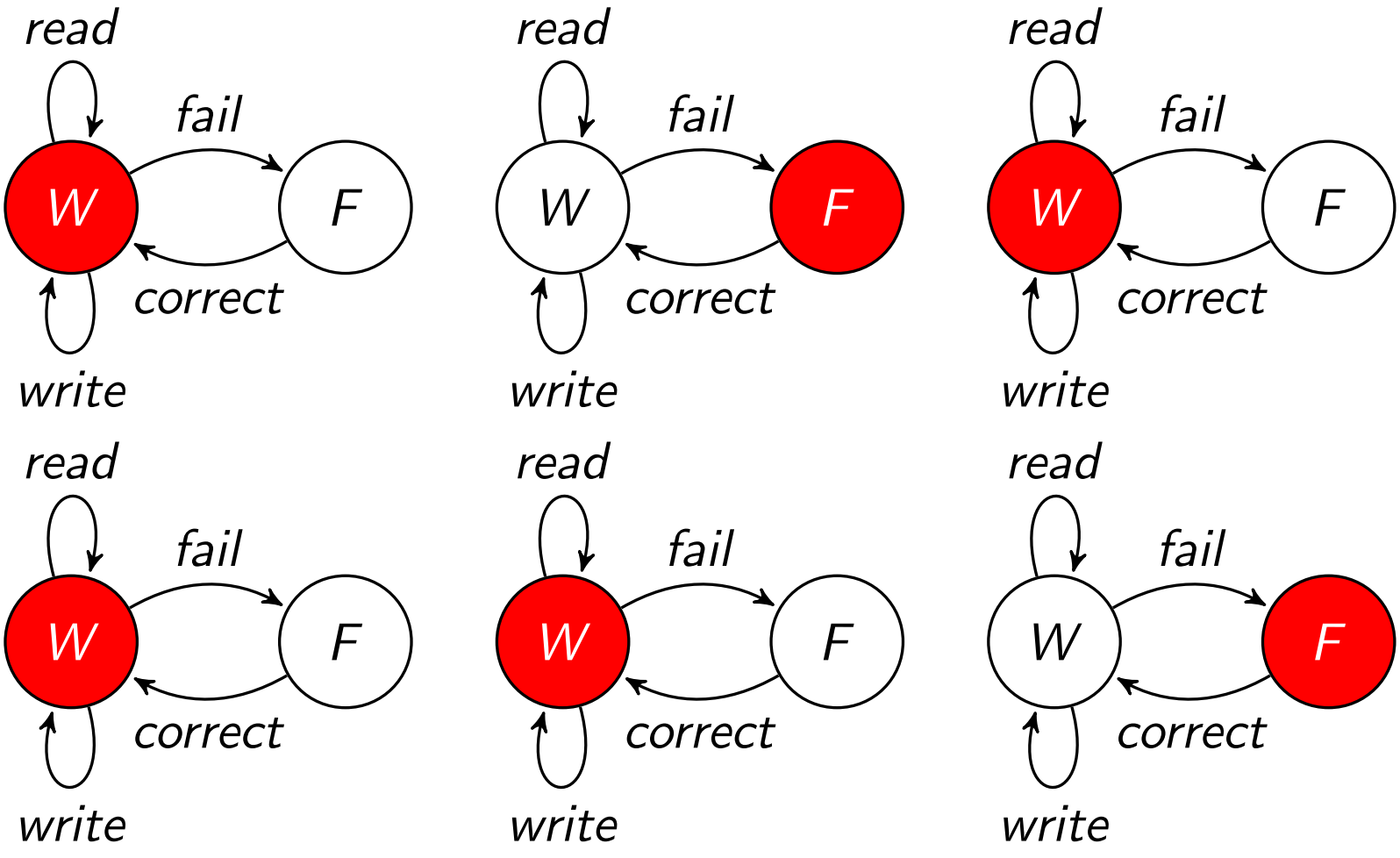
Six disks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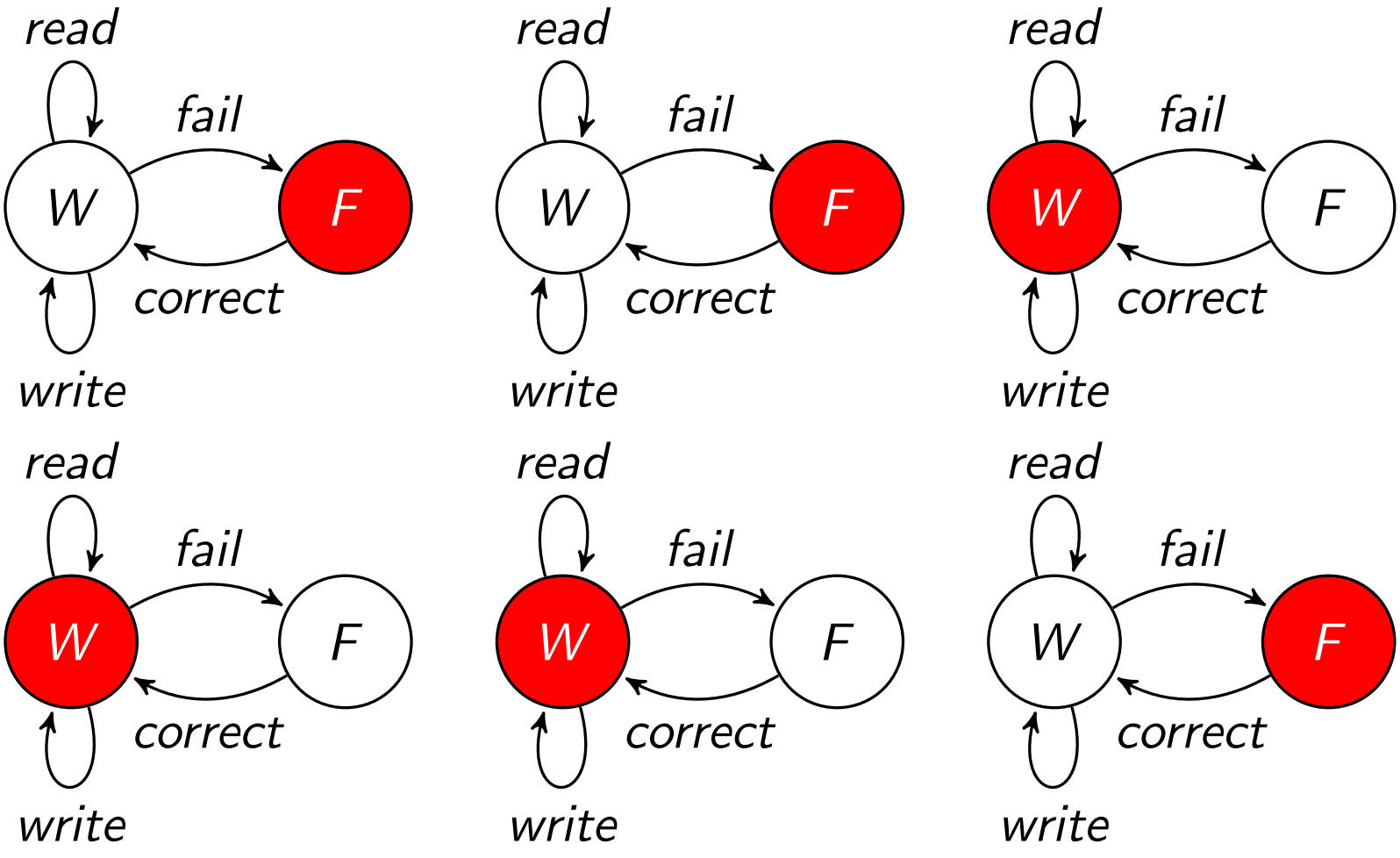
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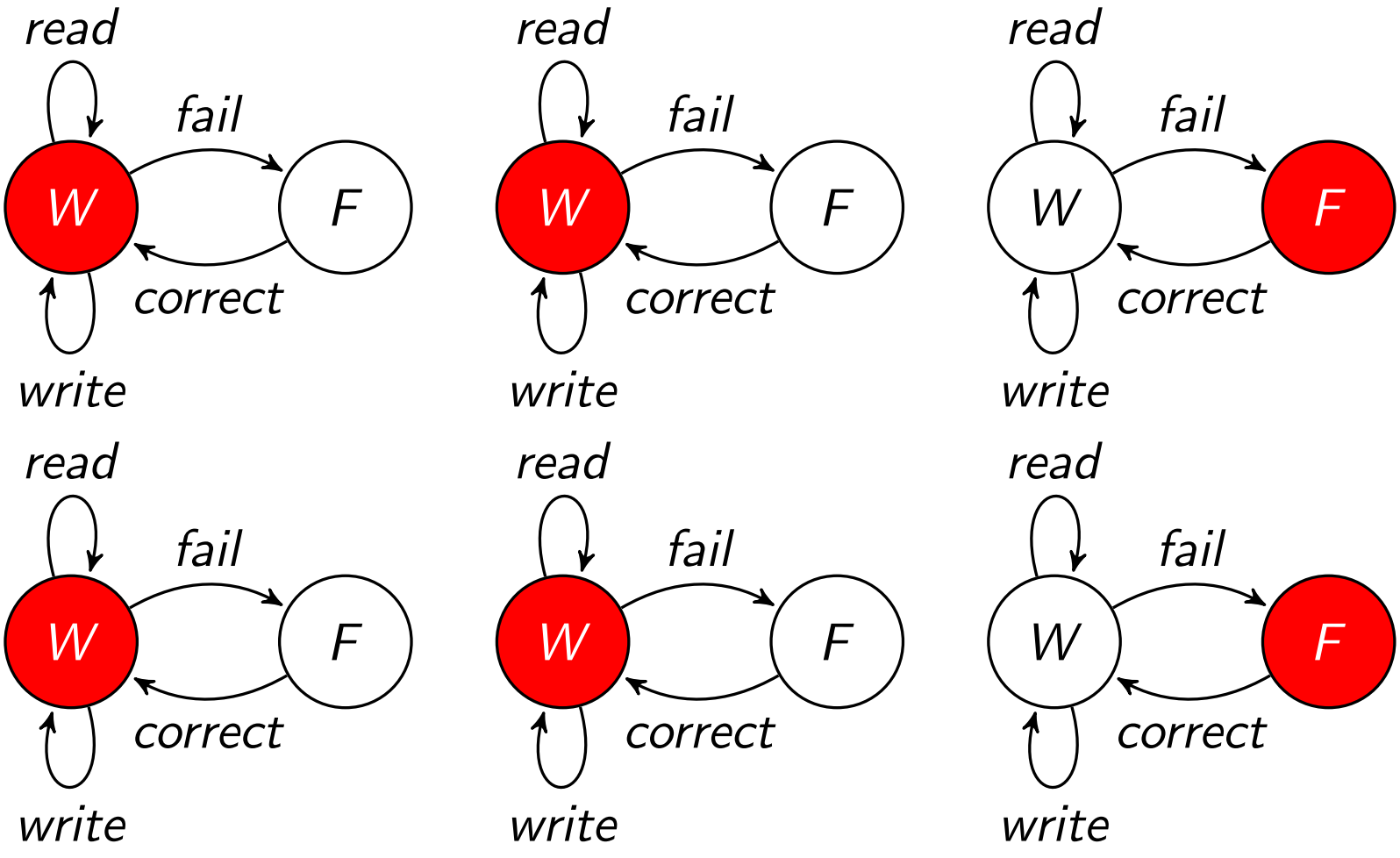
Six disks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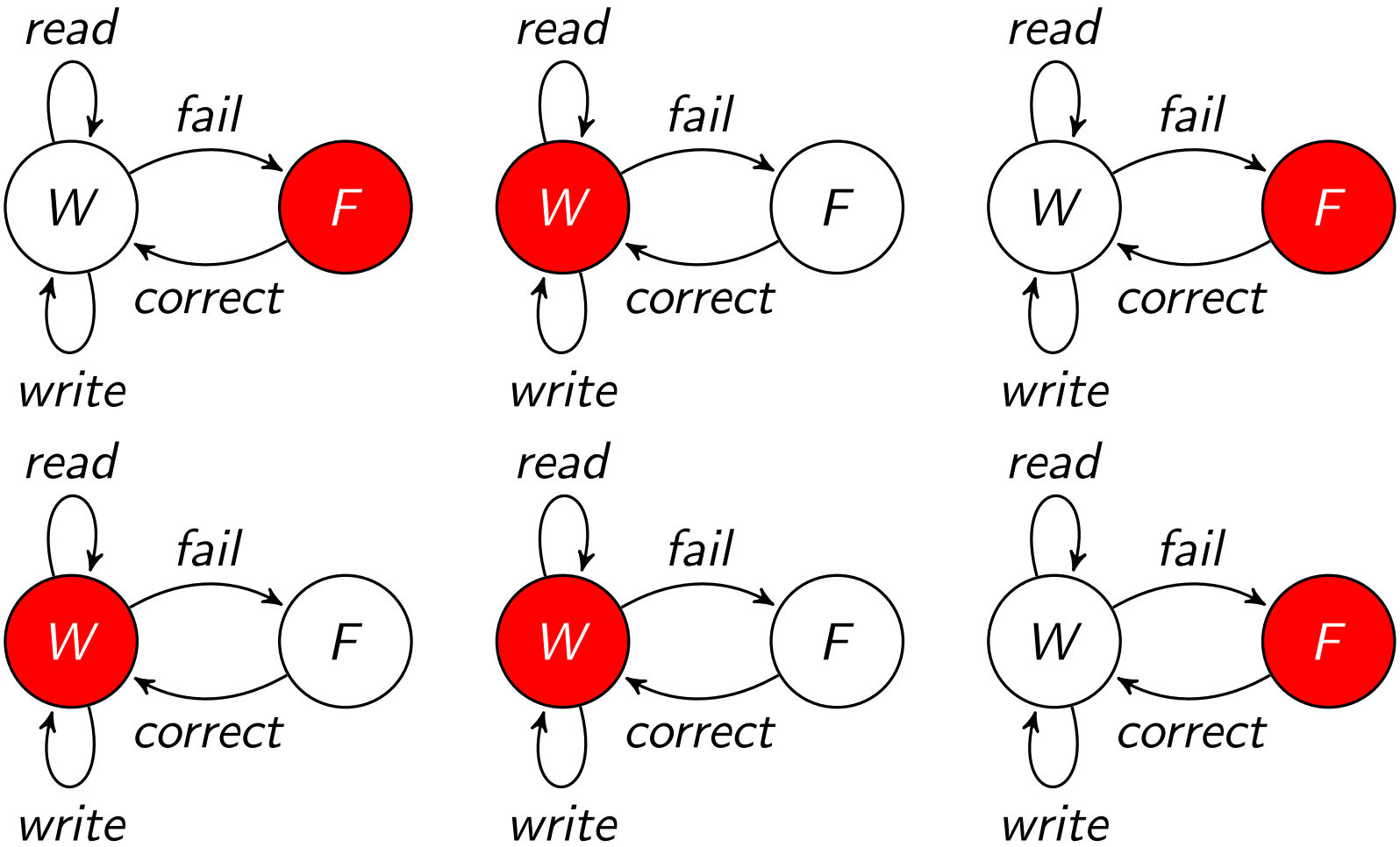
Six disks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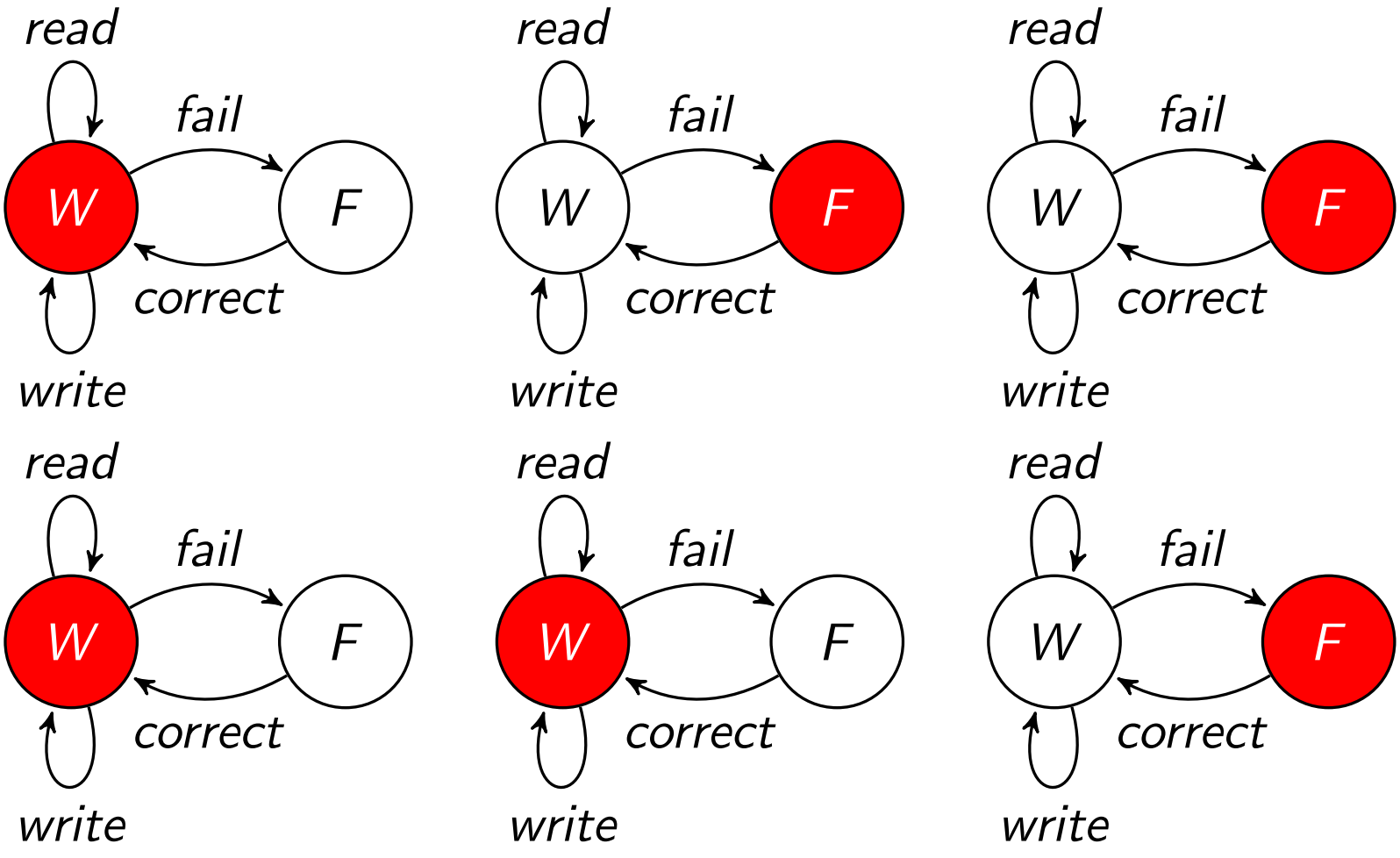
Six disks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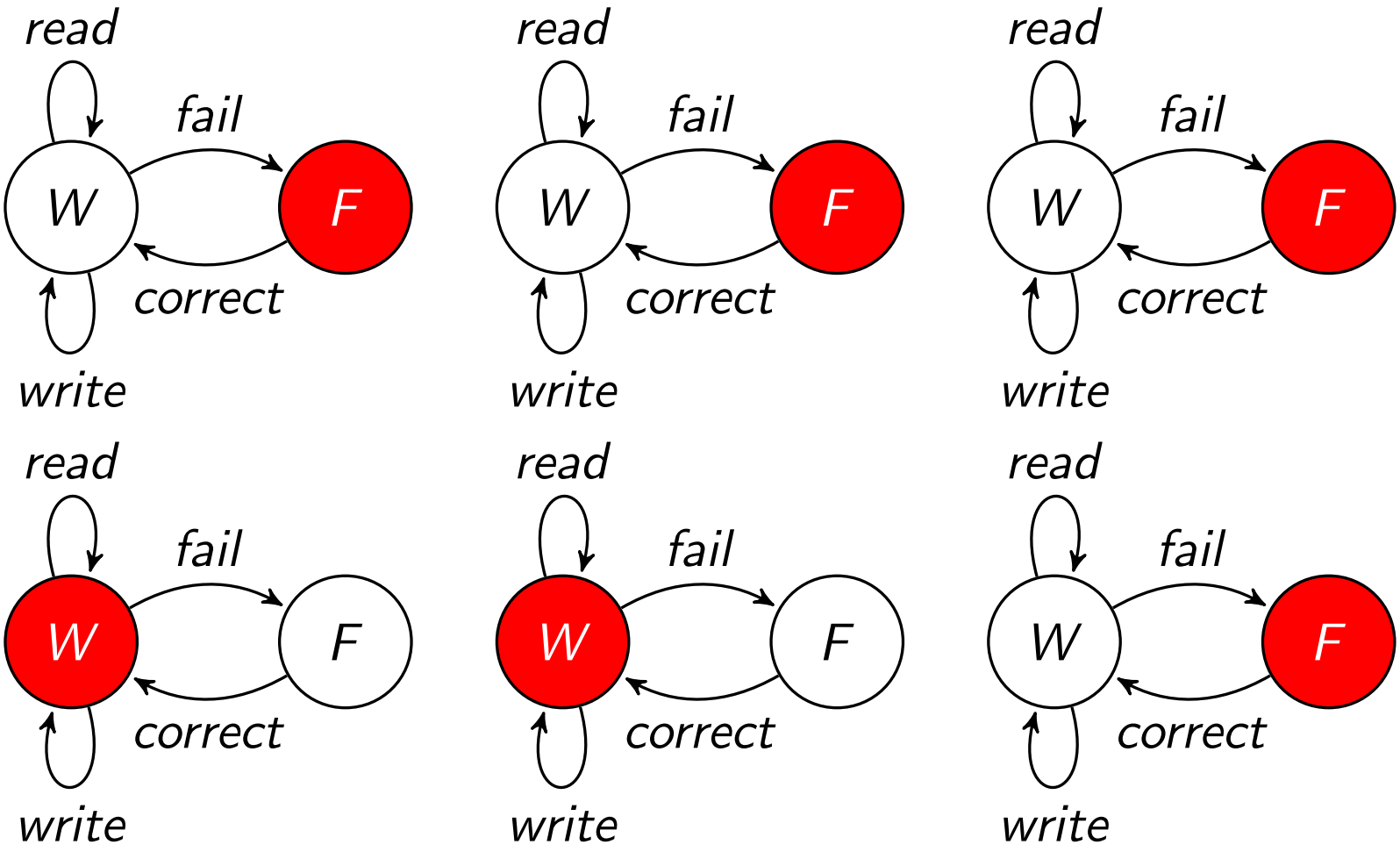
Six disks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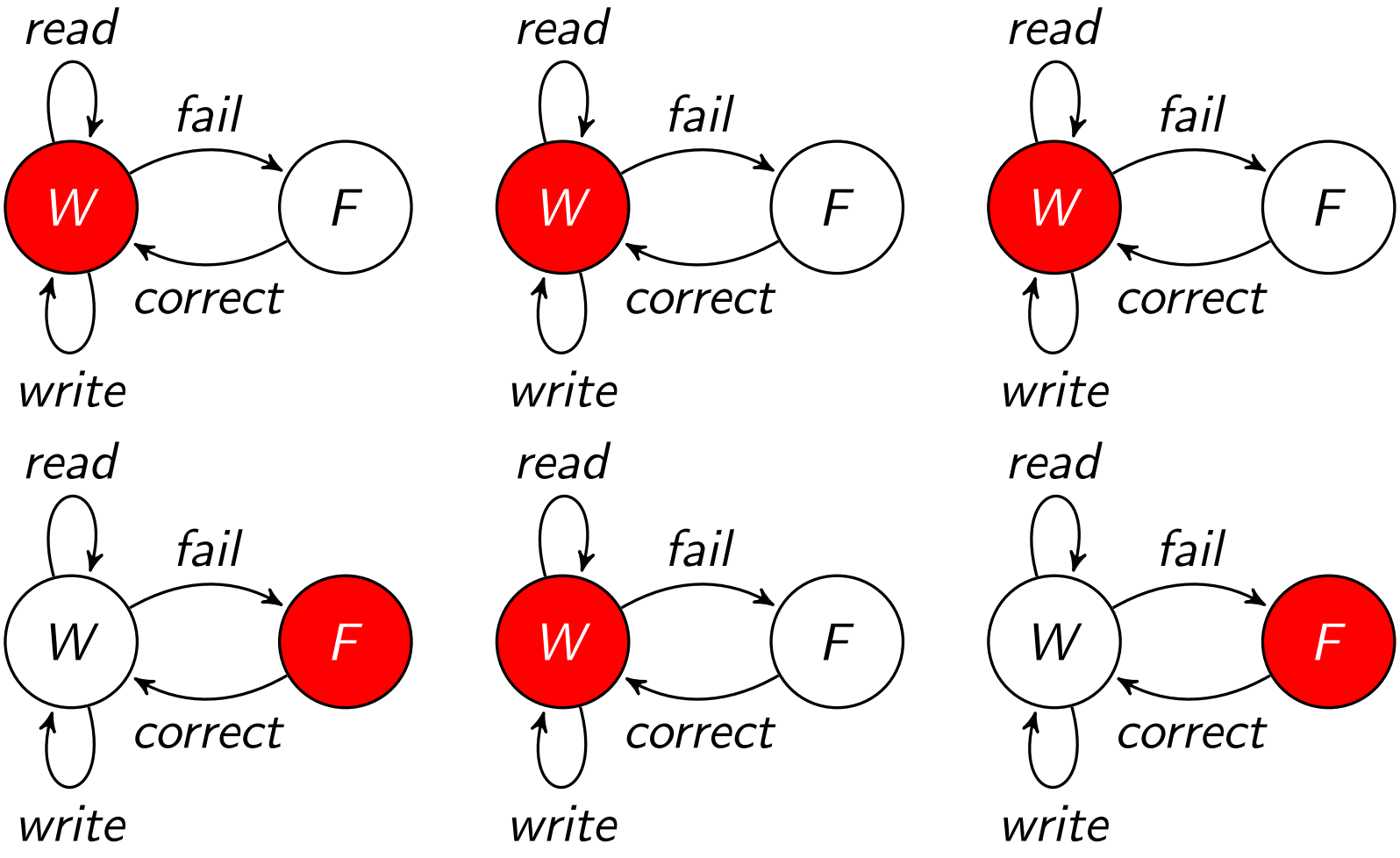
Six disks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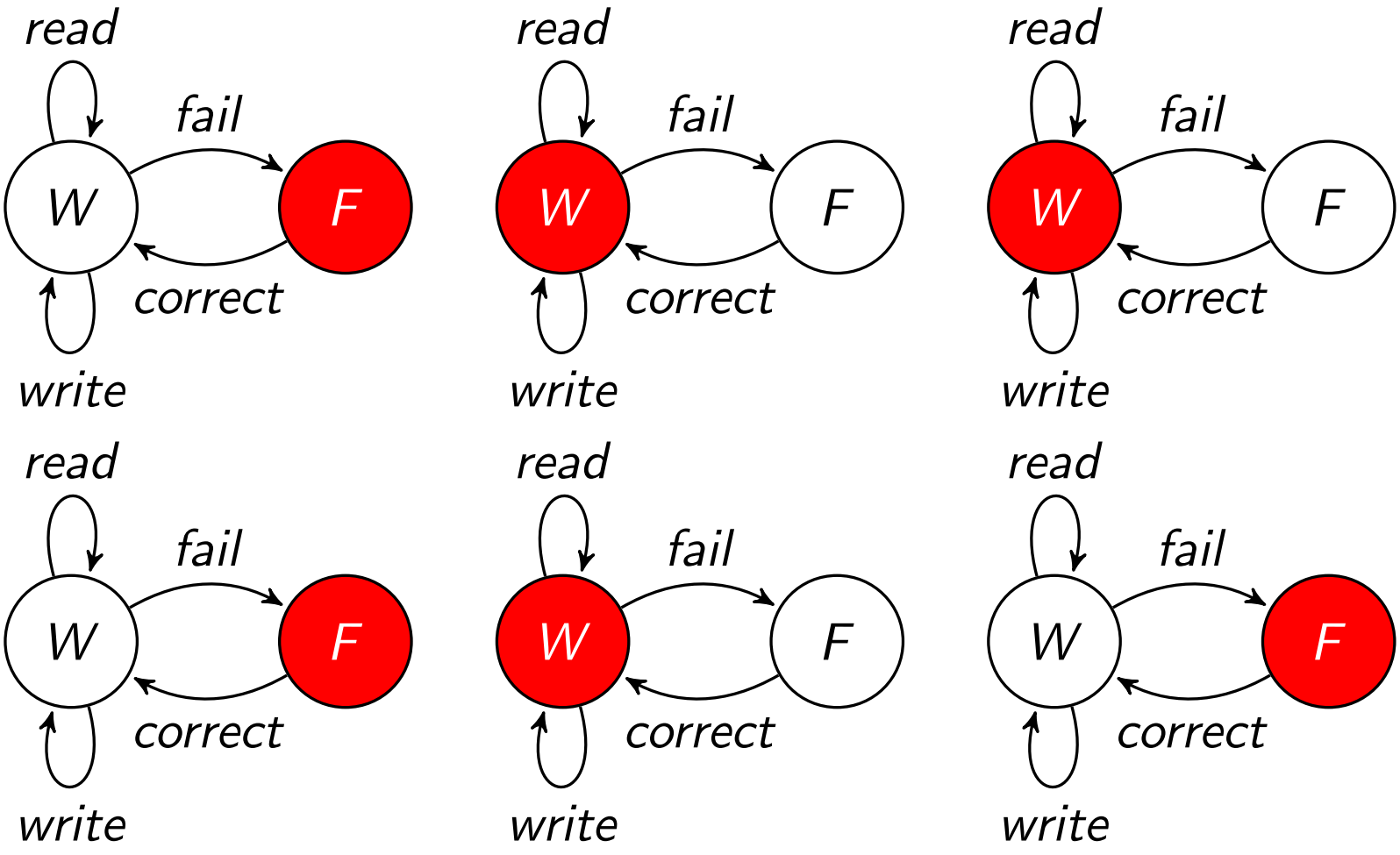
Six disks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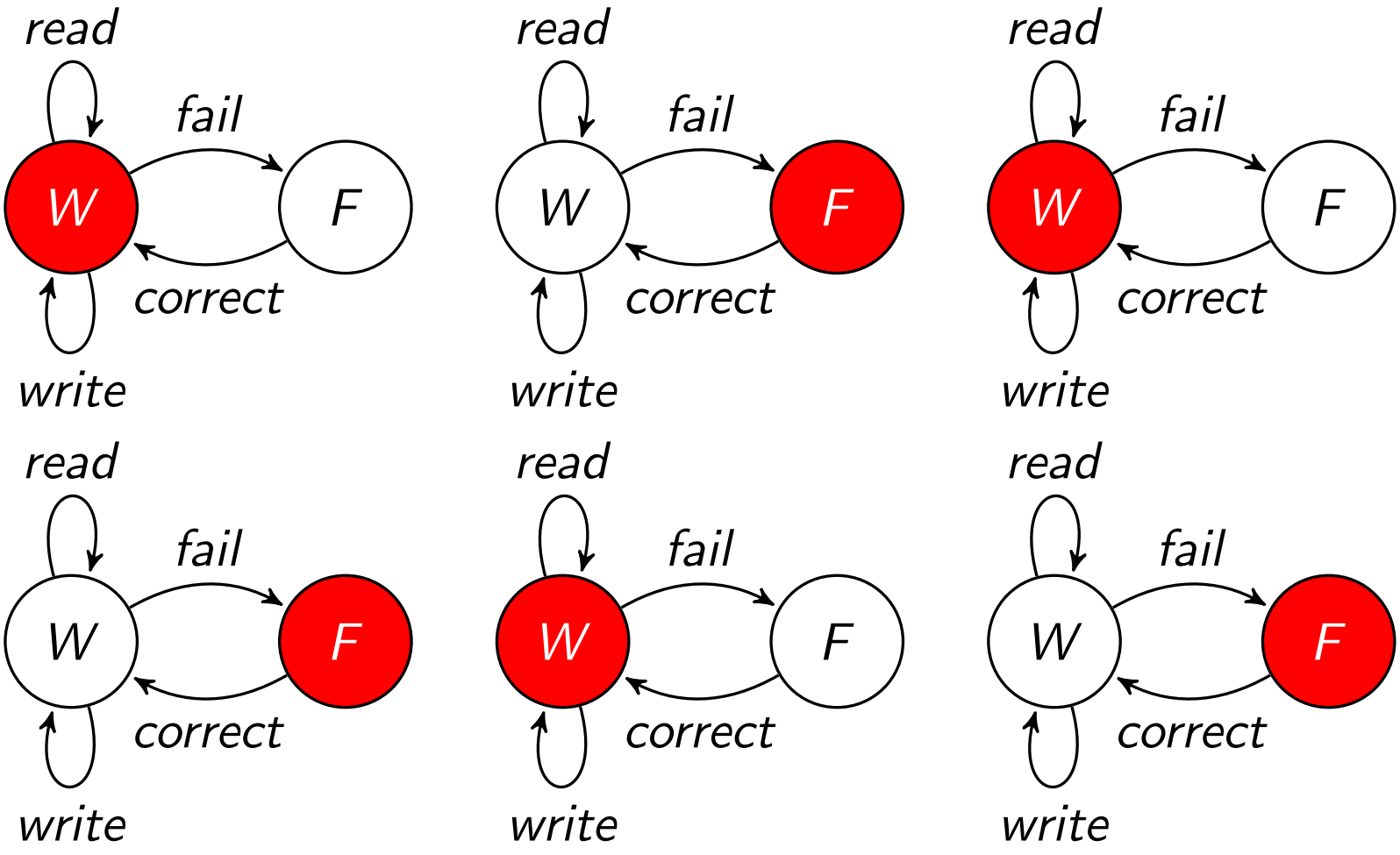
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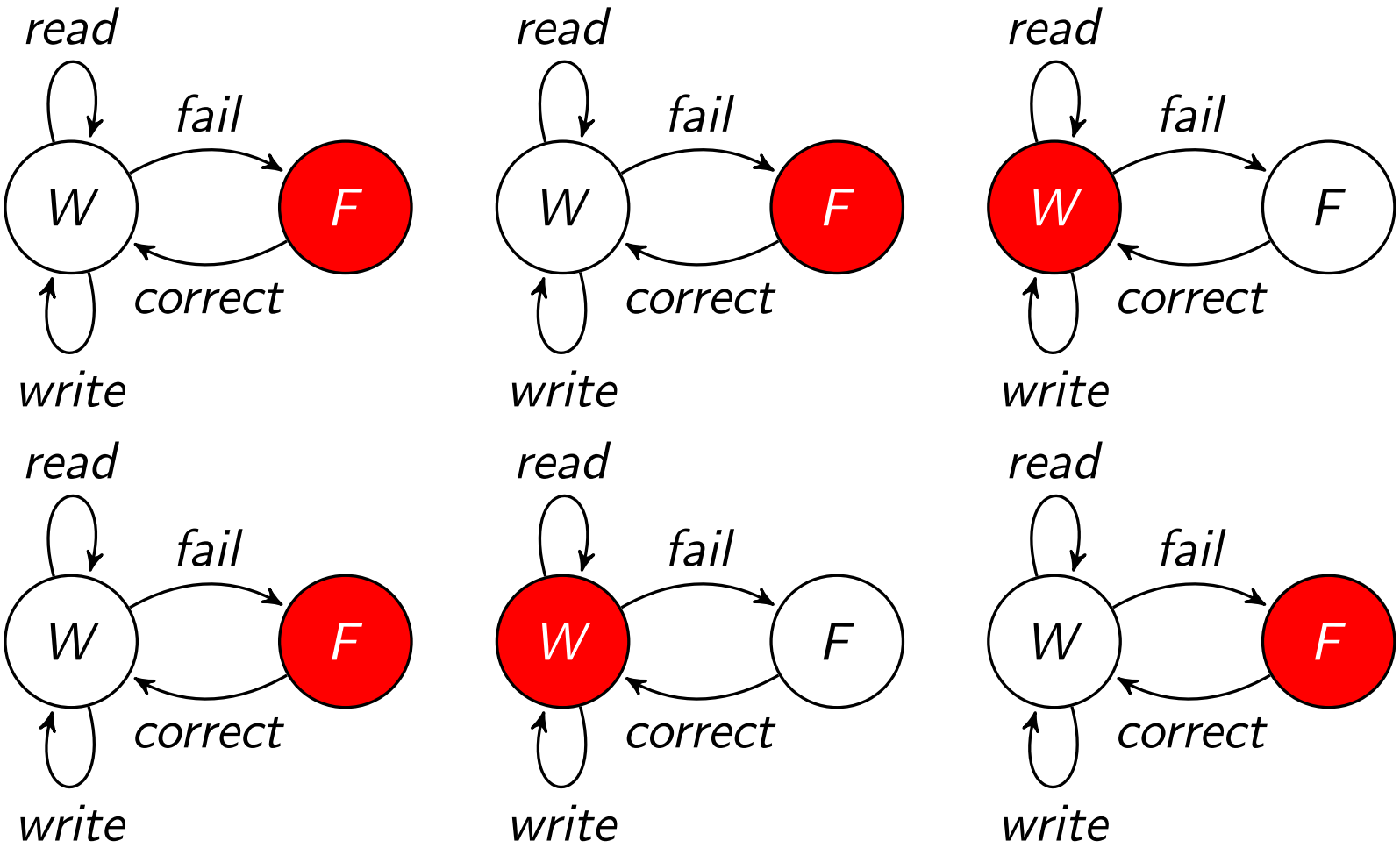
Six disks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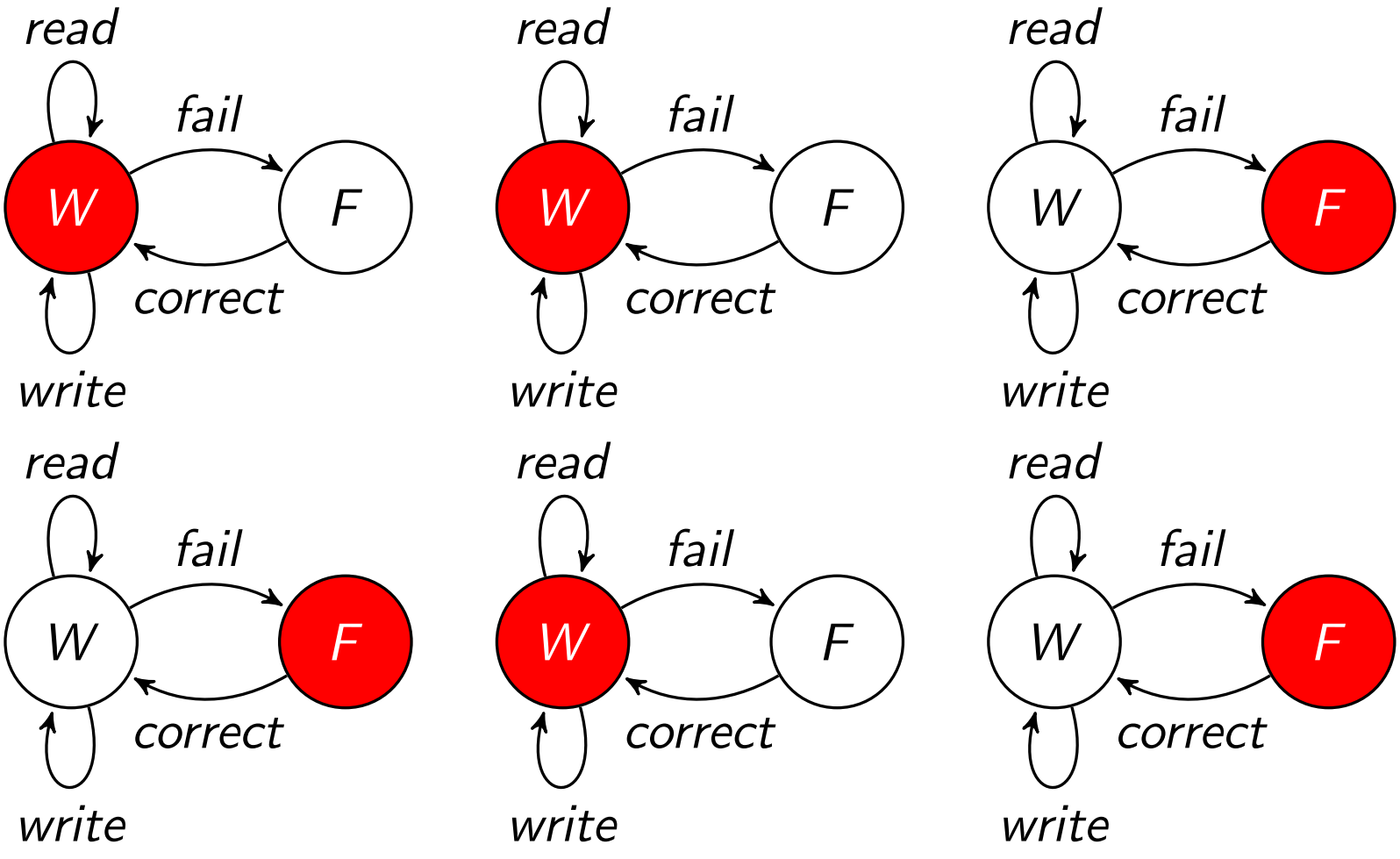
Six disks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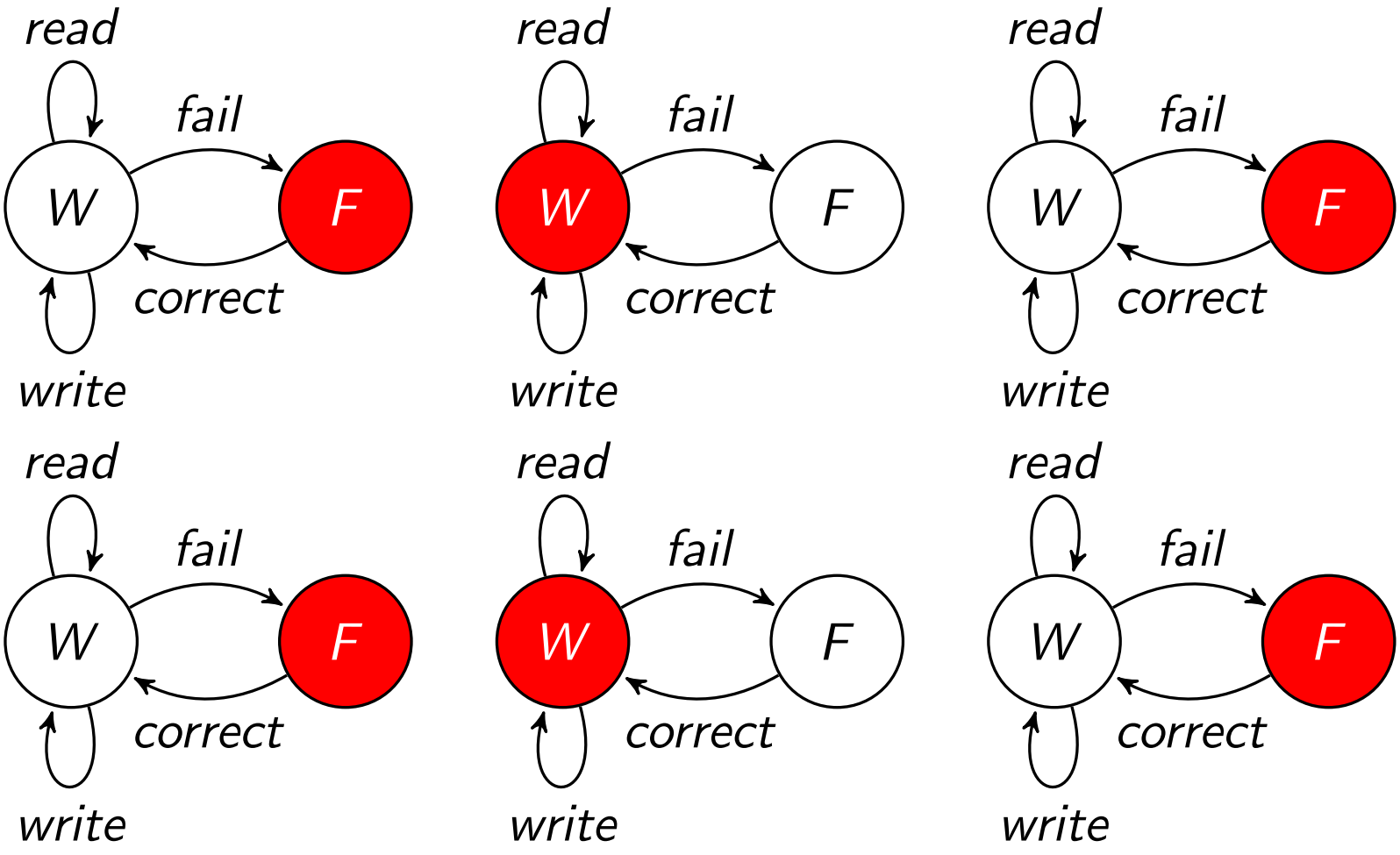
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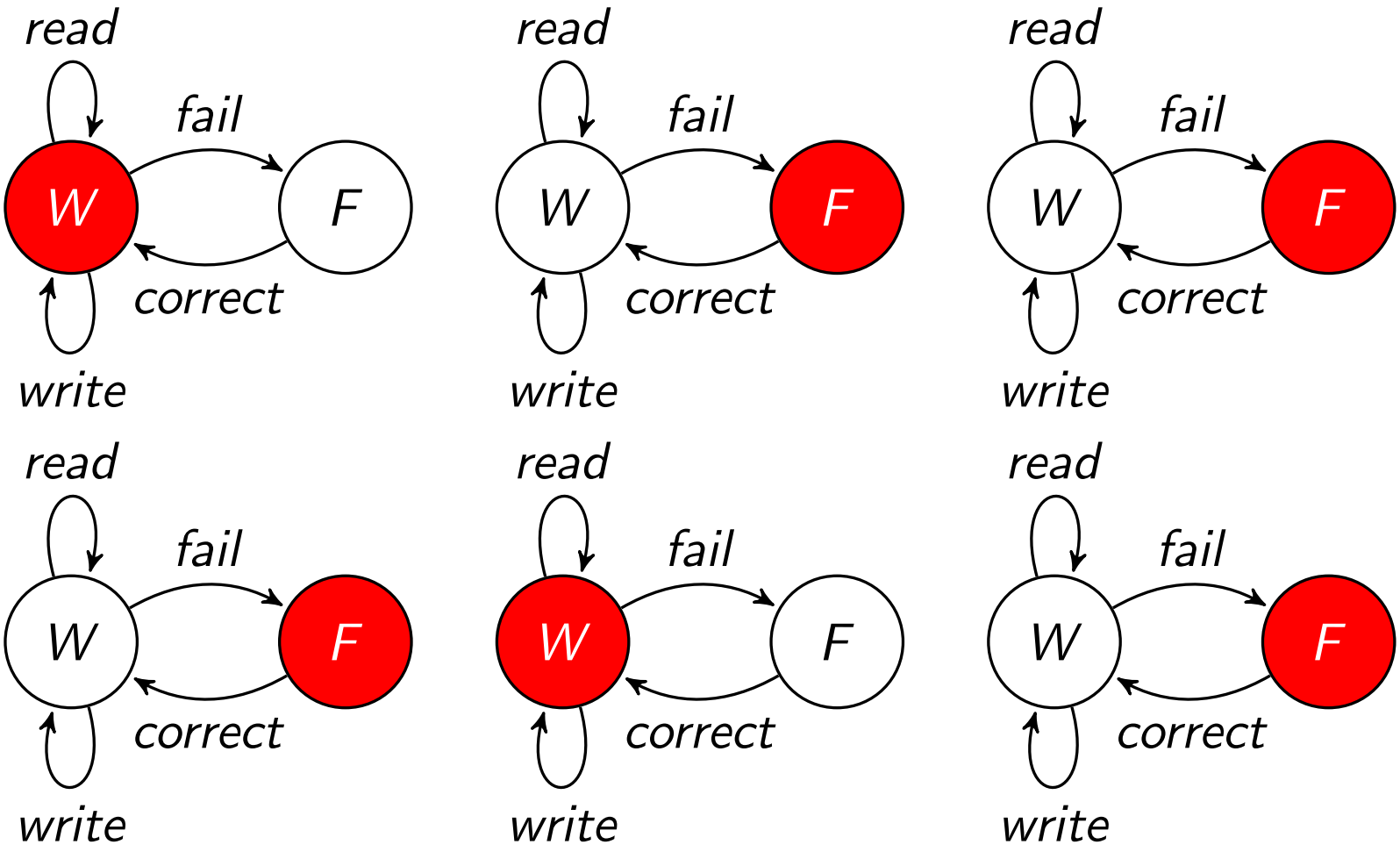
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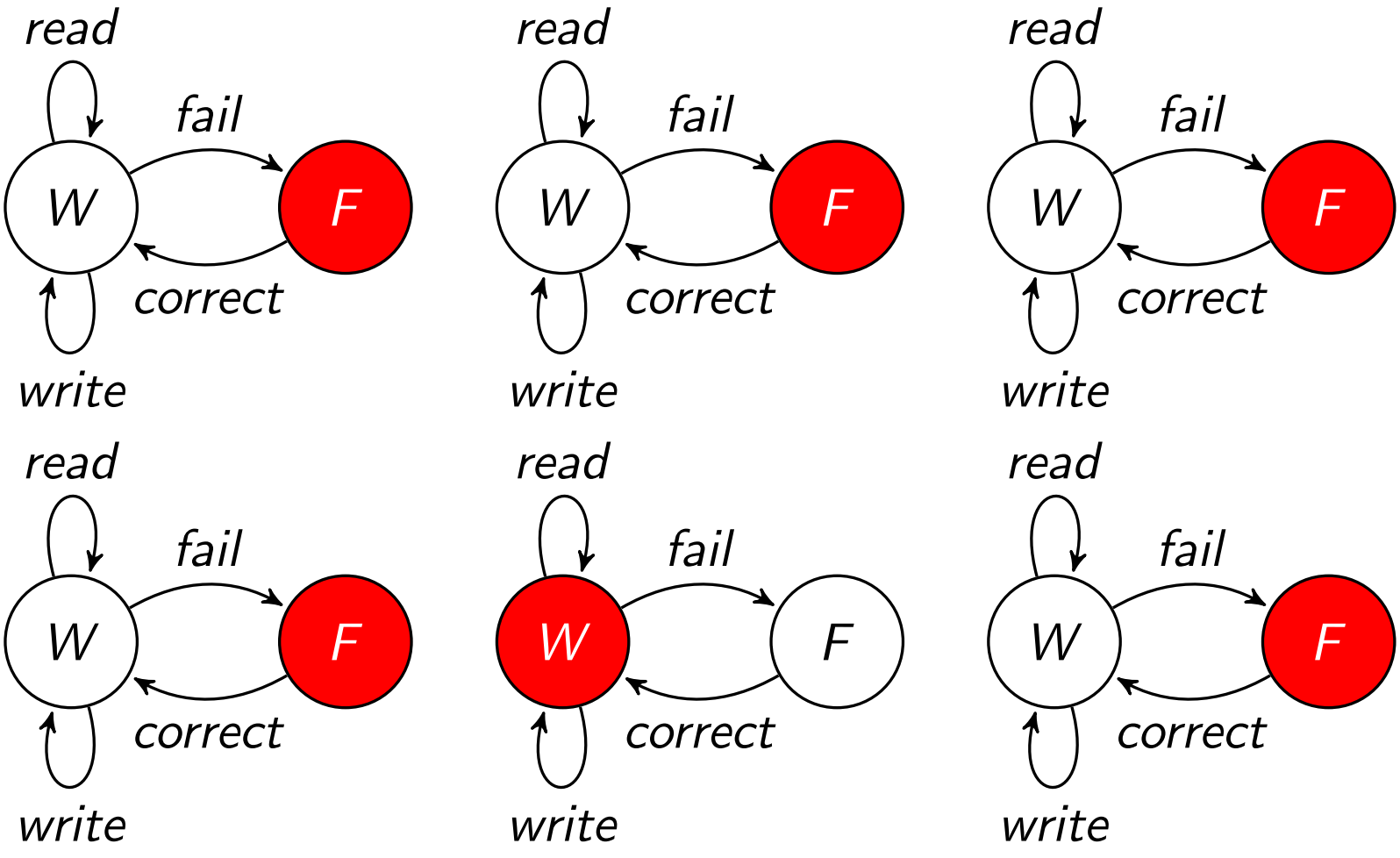
Six disks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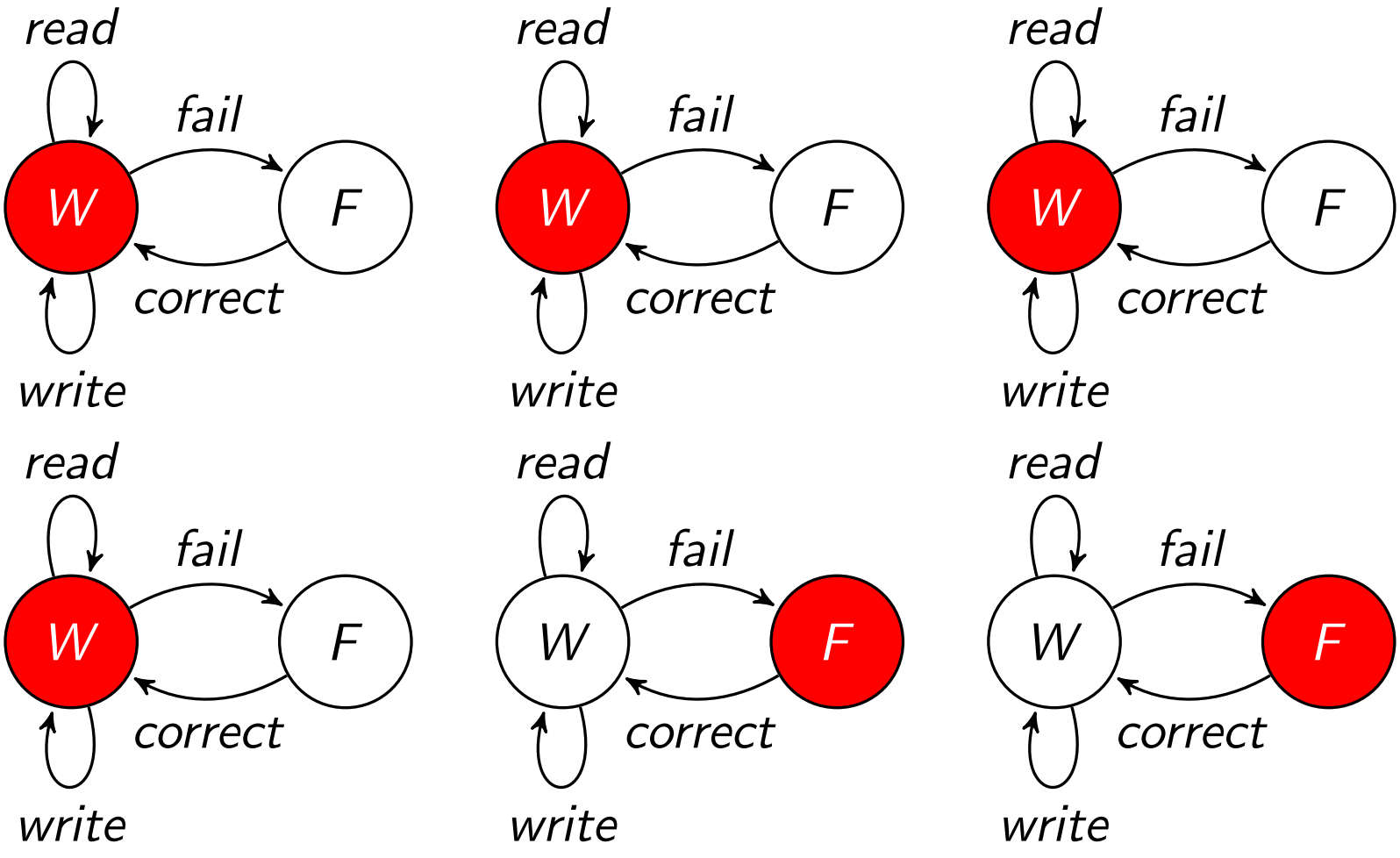
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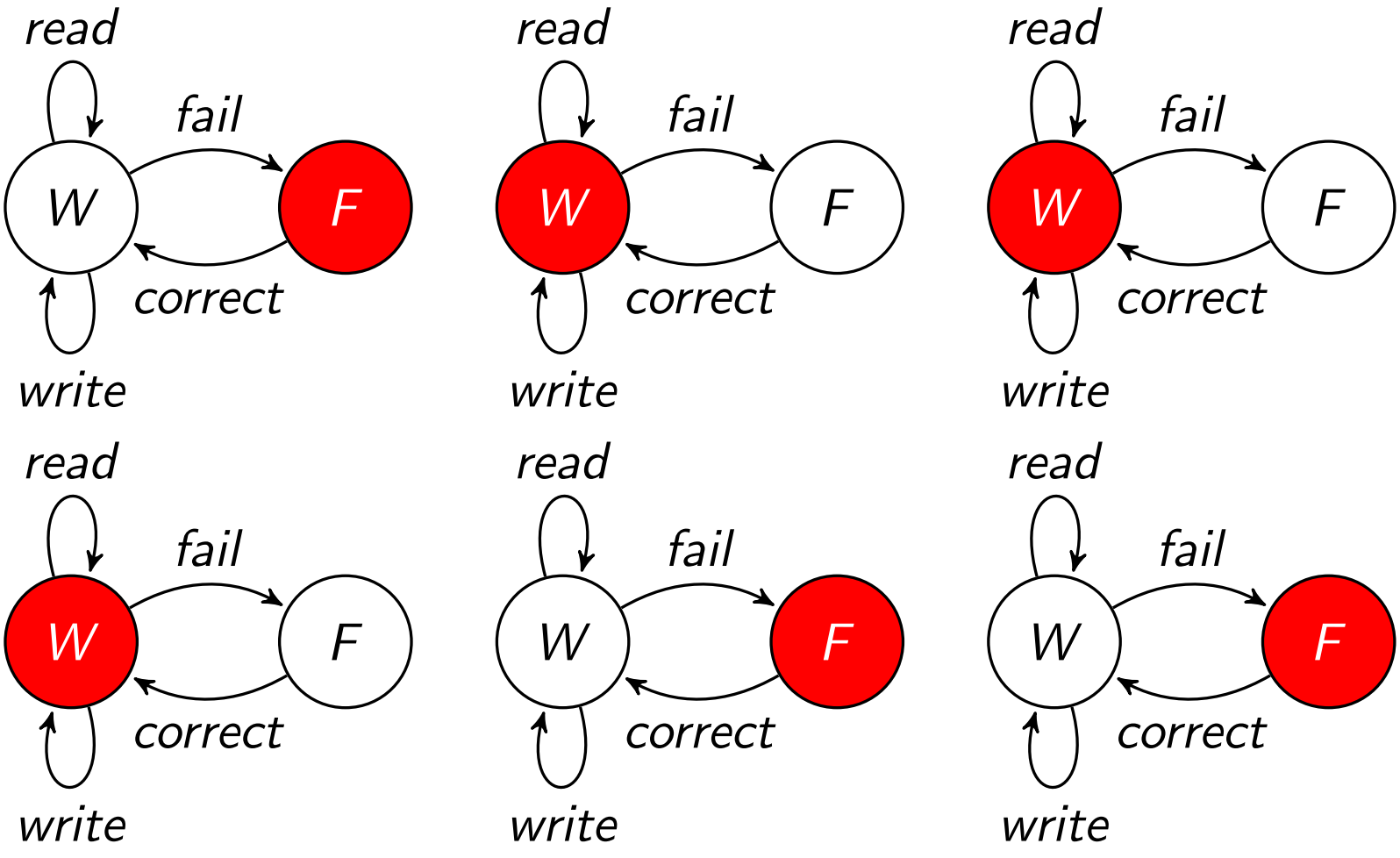
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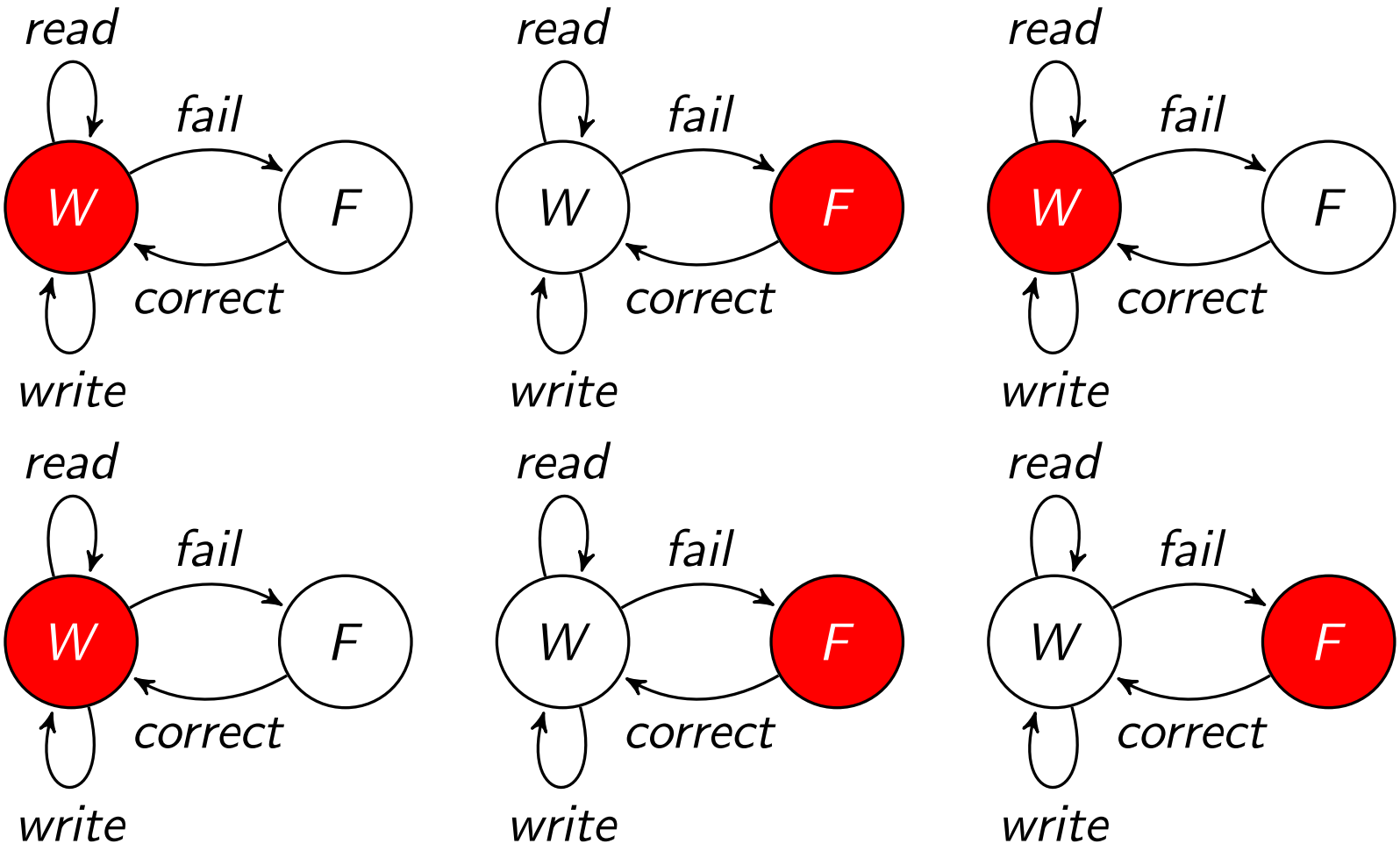
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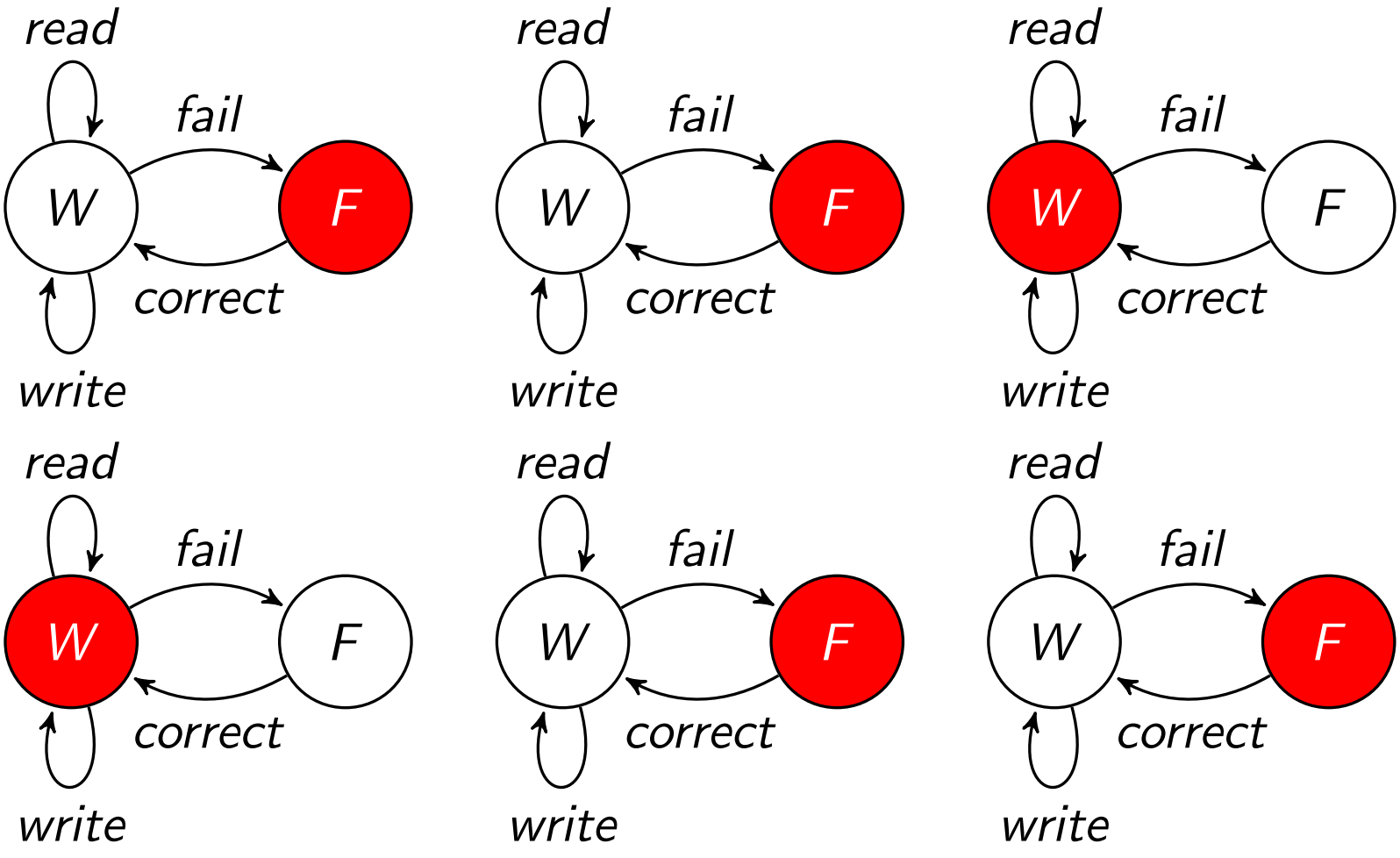
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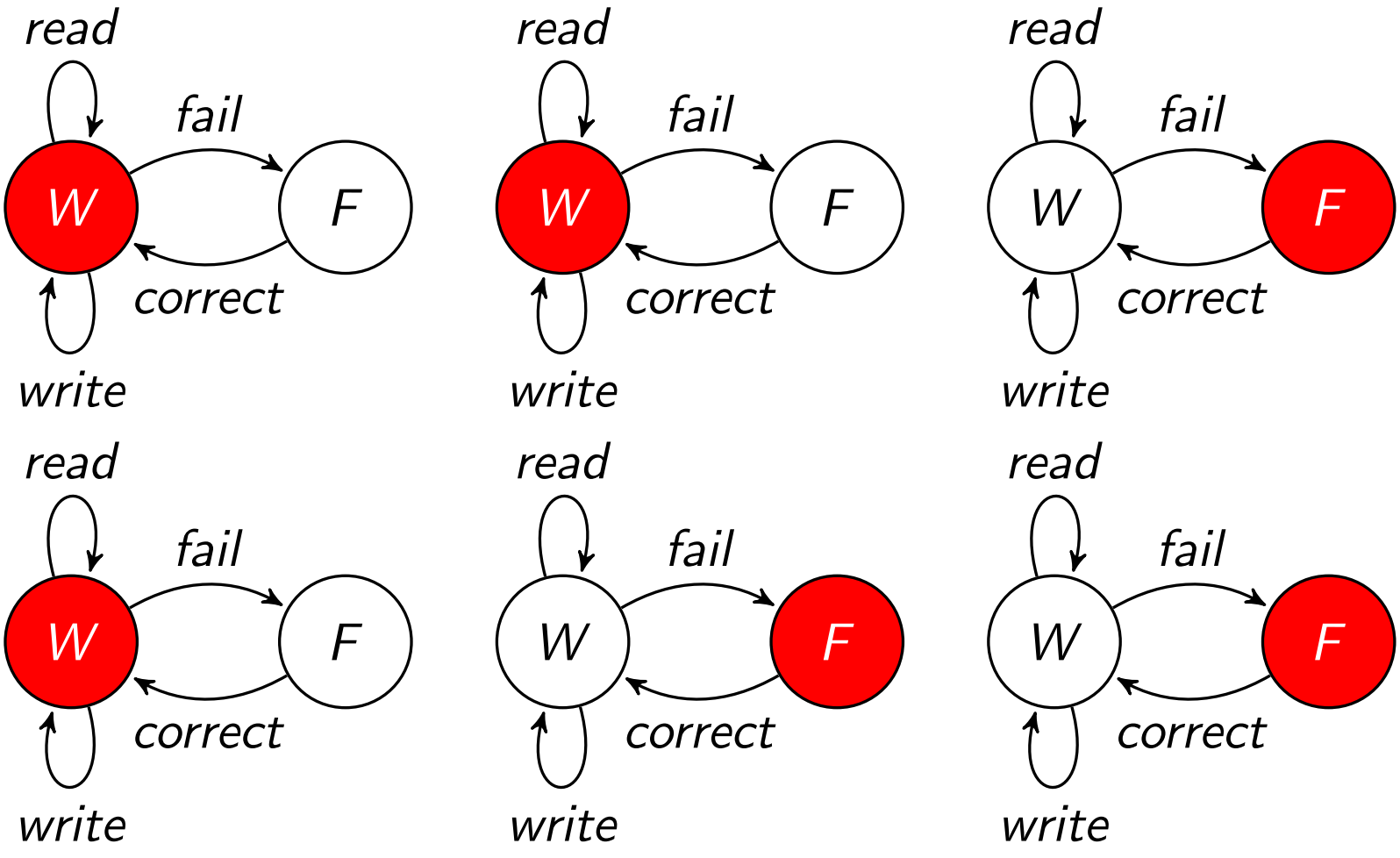
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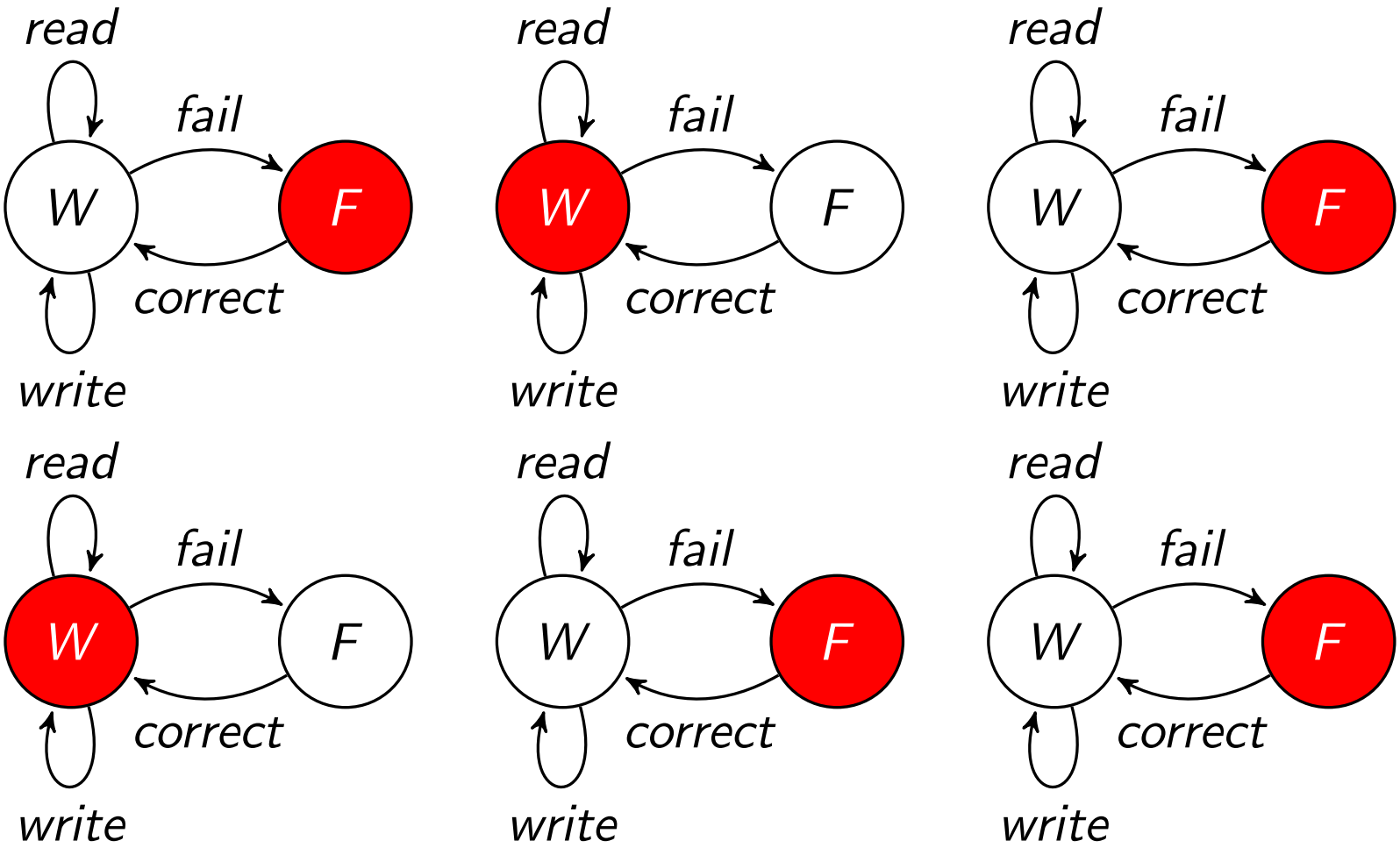
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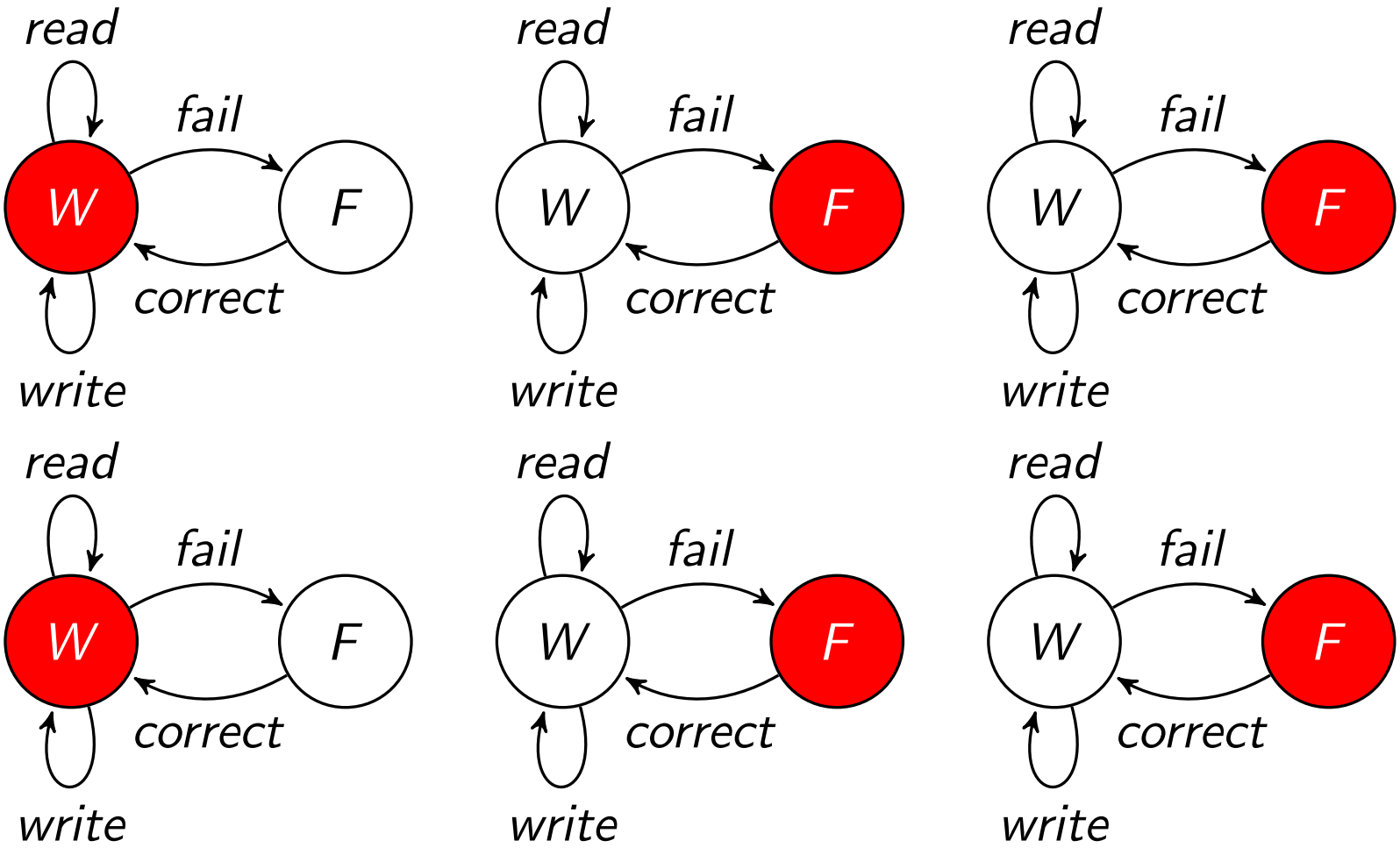
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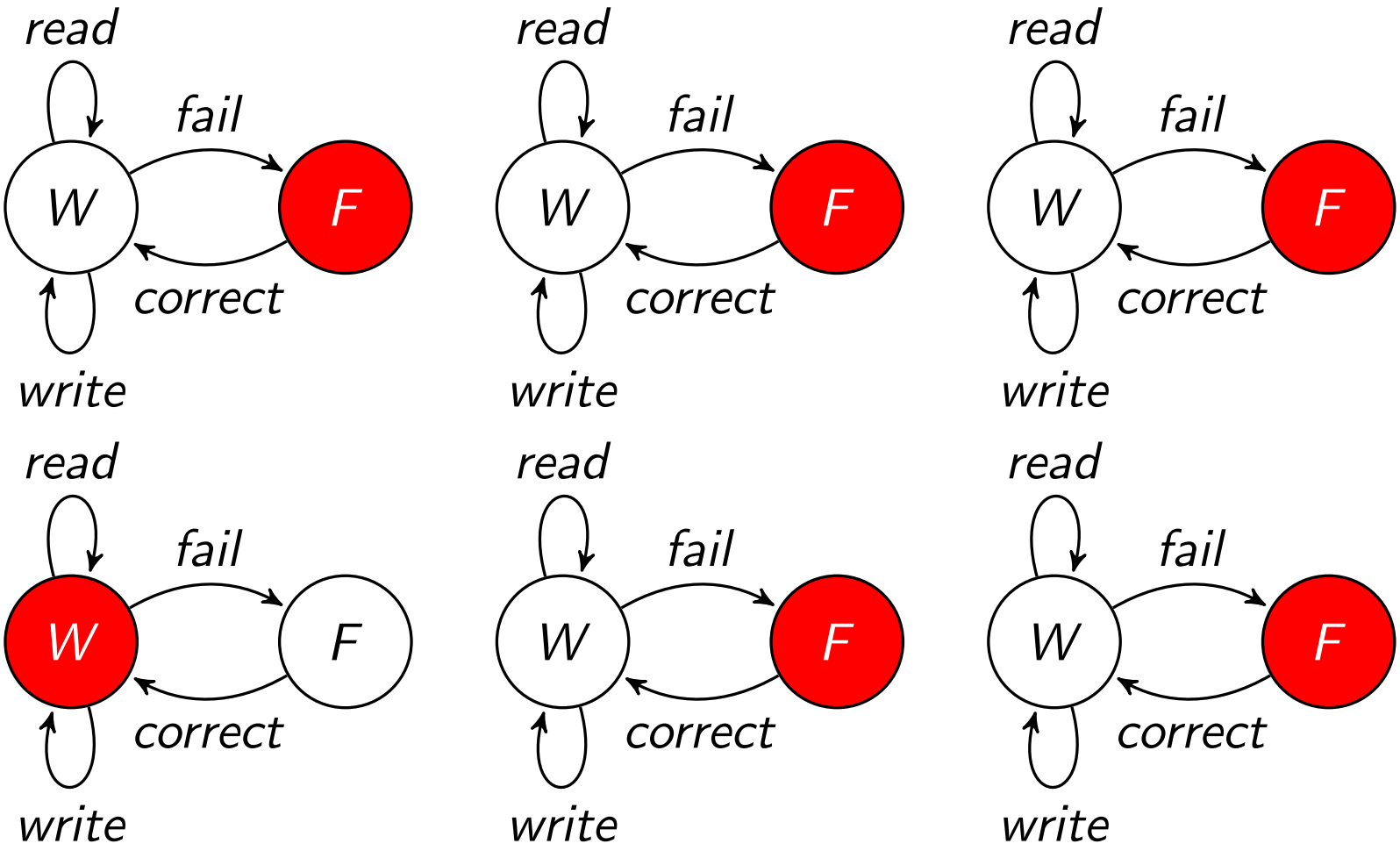
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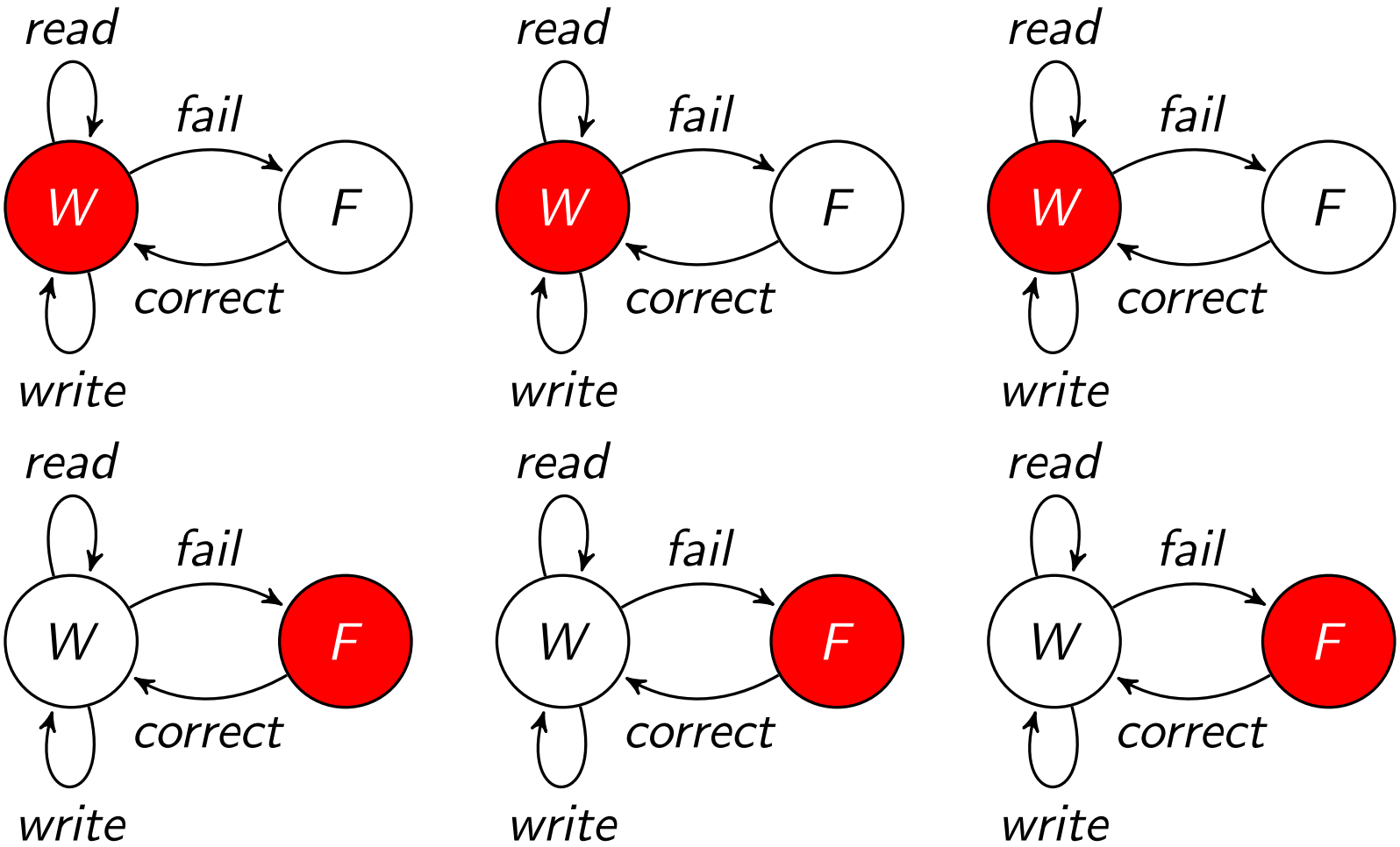
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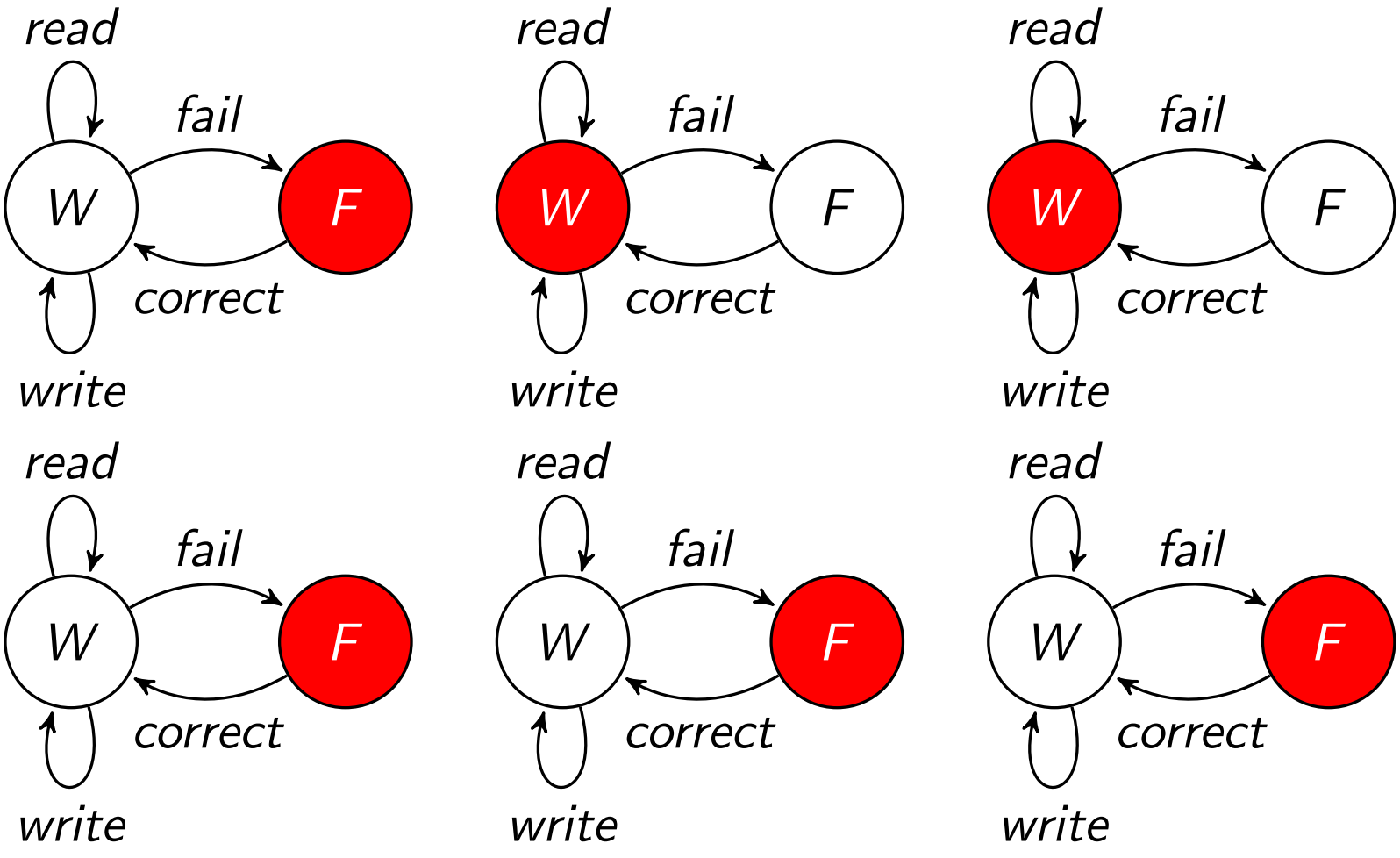
W
F
correct
write
read
fail
W
F
correct
write

Six disks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fail
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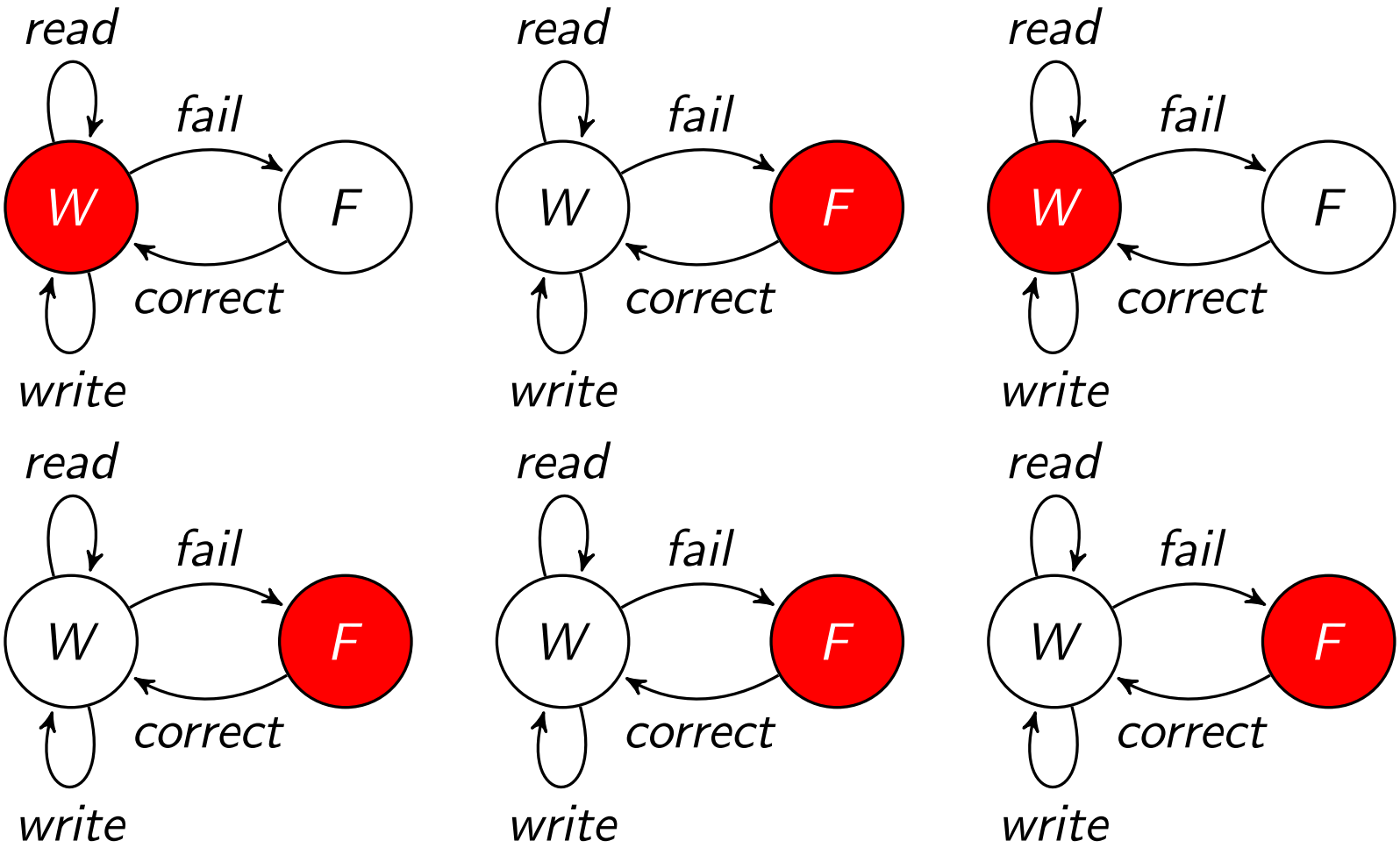
F
correct
write
read
fail
W
F
correct
write

Six disks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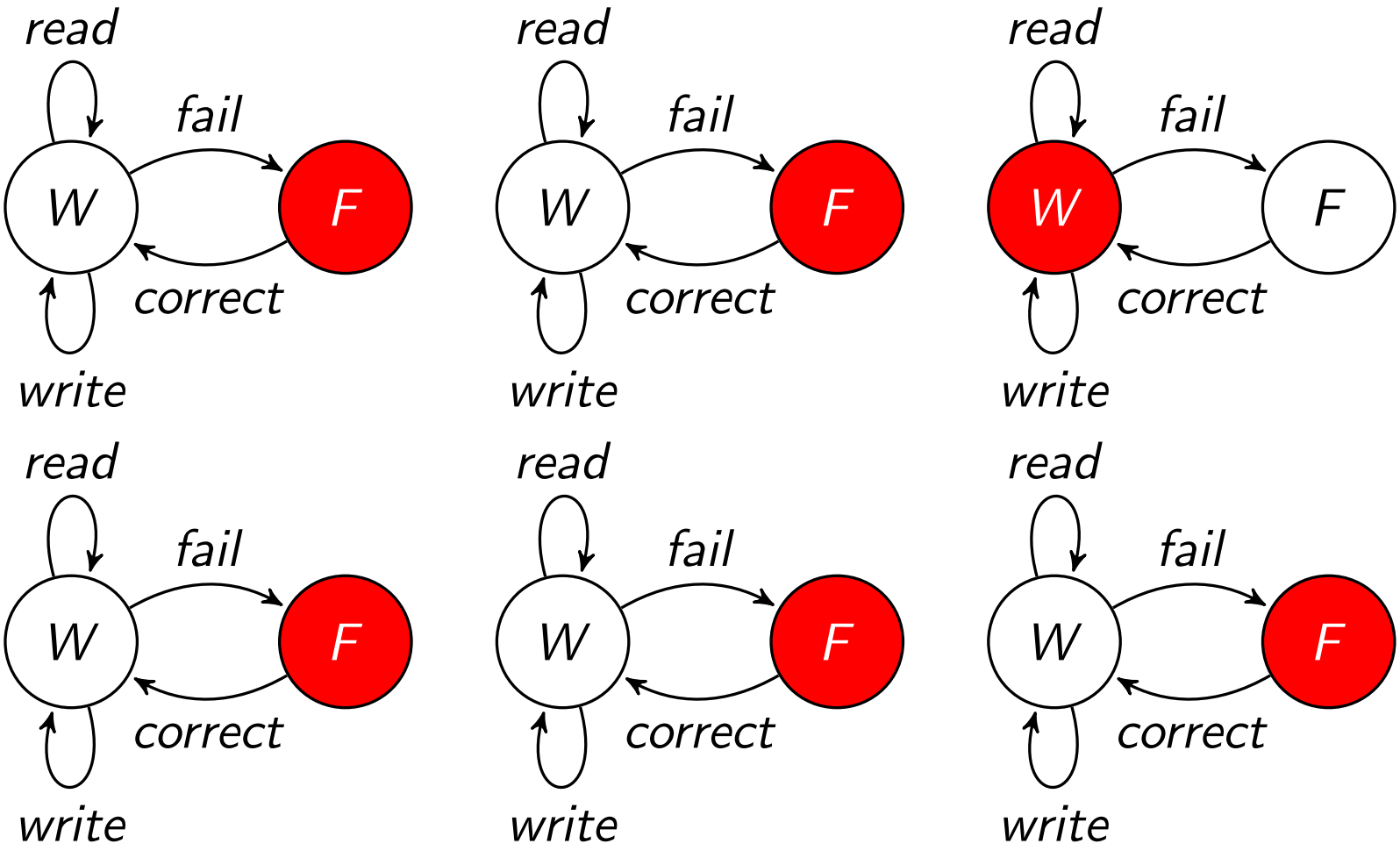
Six disks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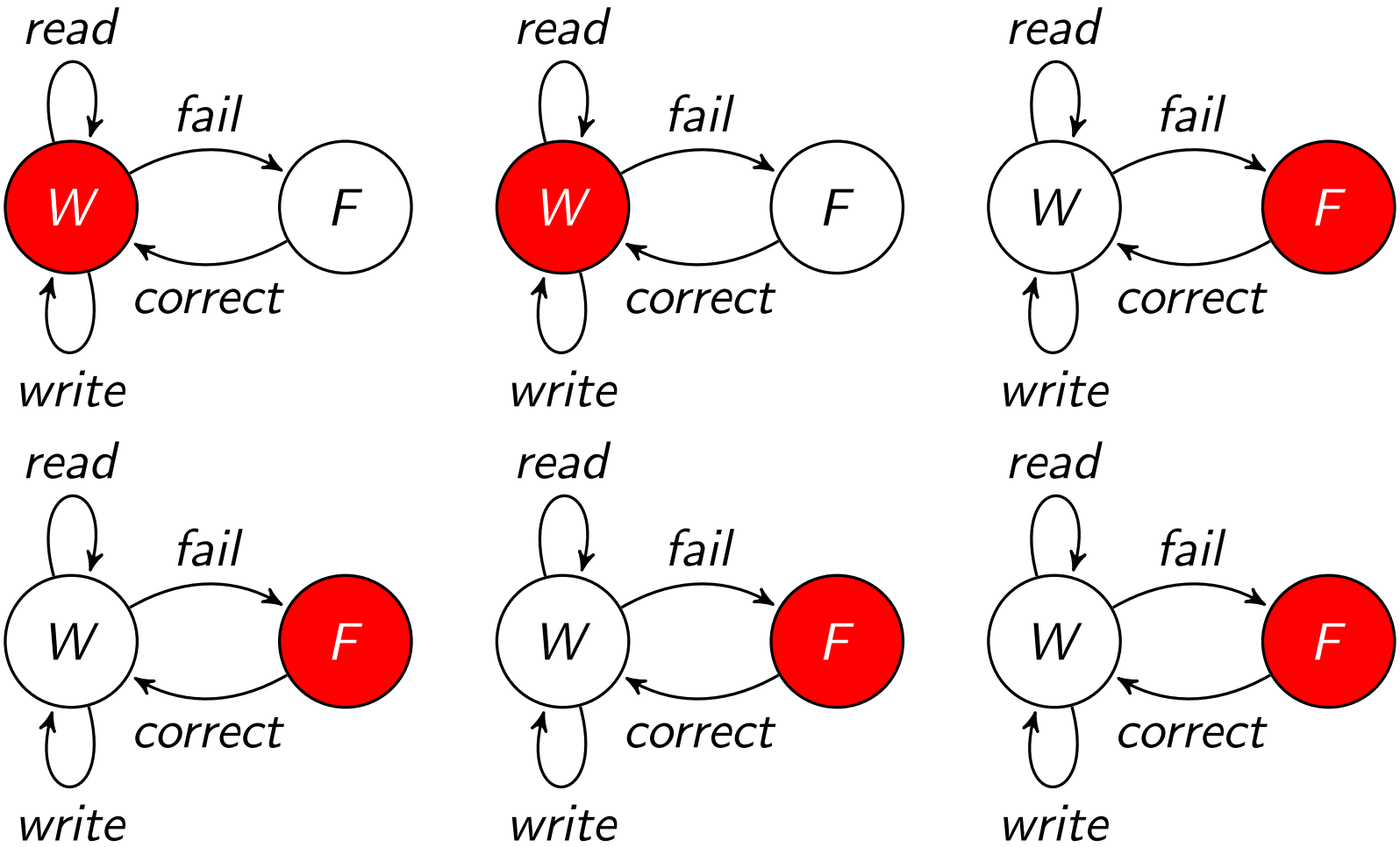
Six disks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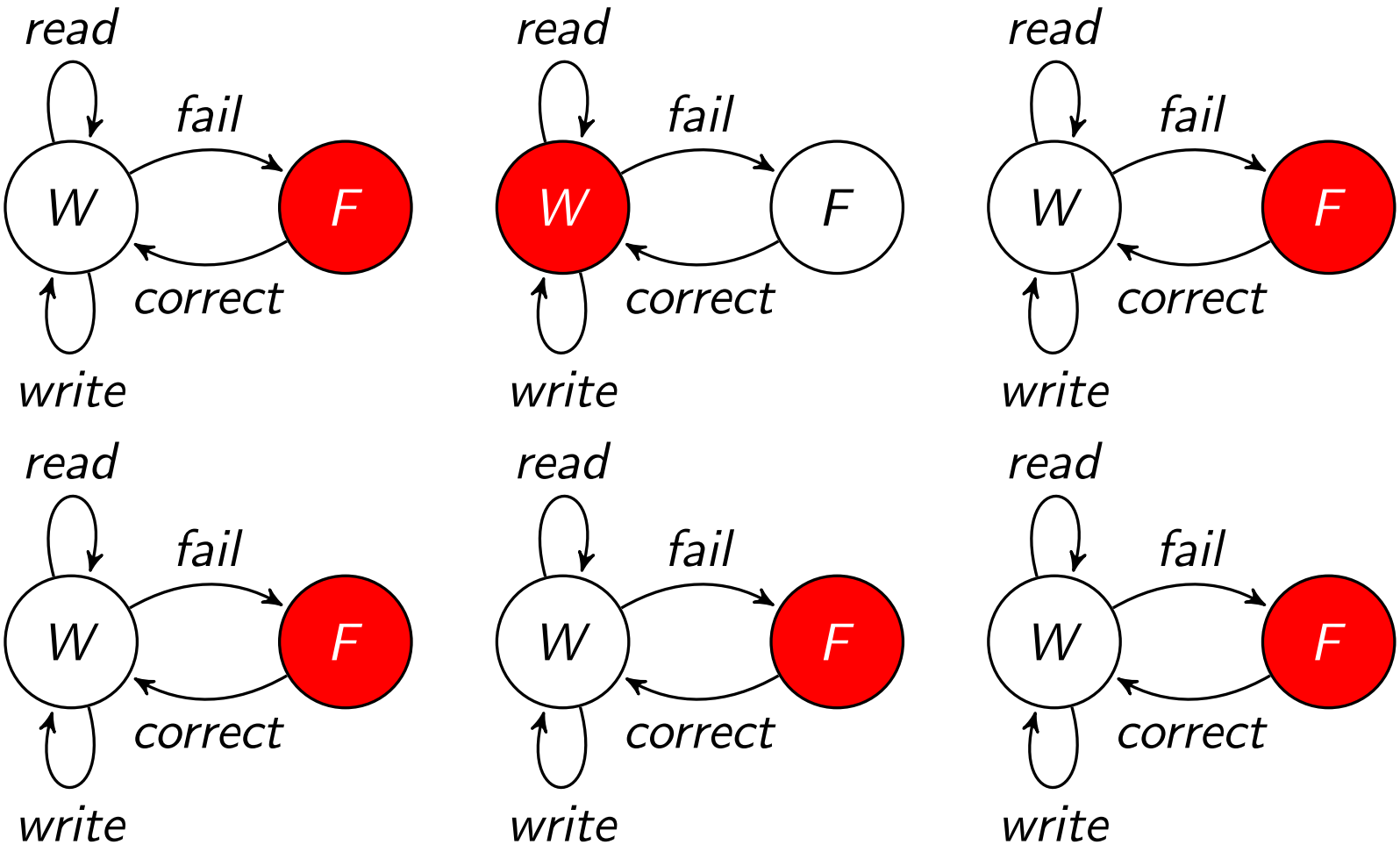
Six disks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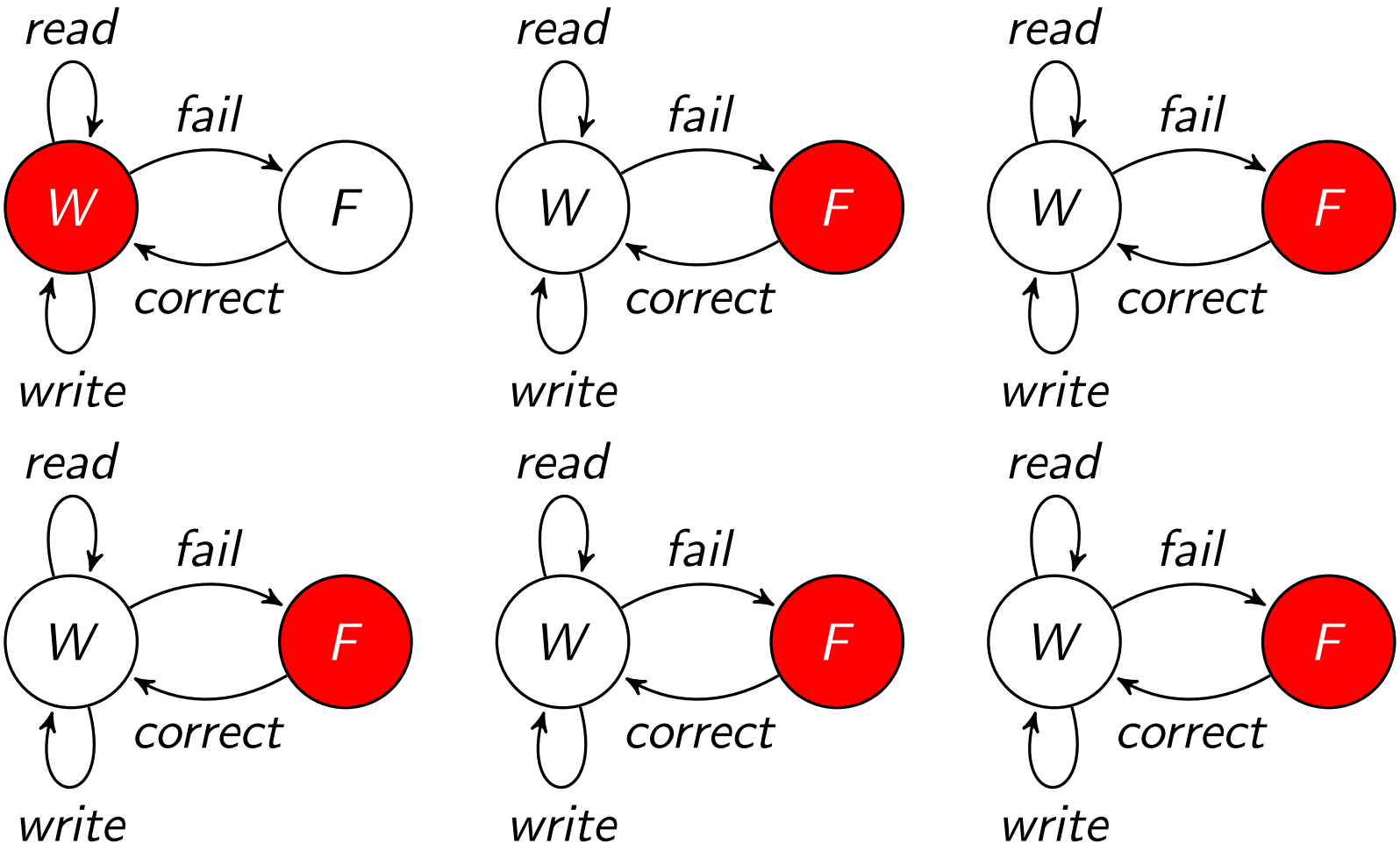
Six disks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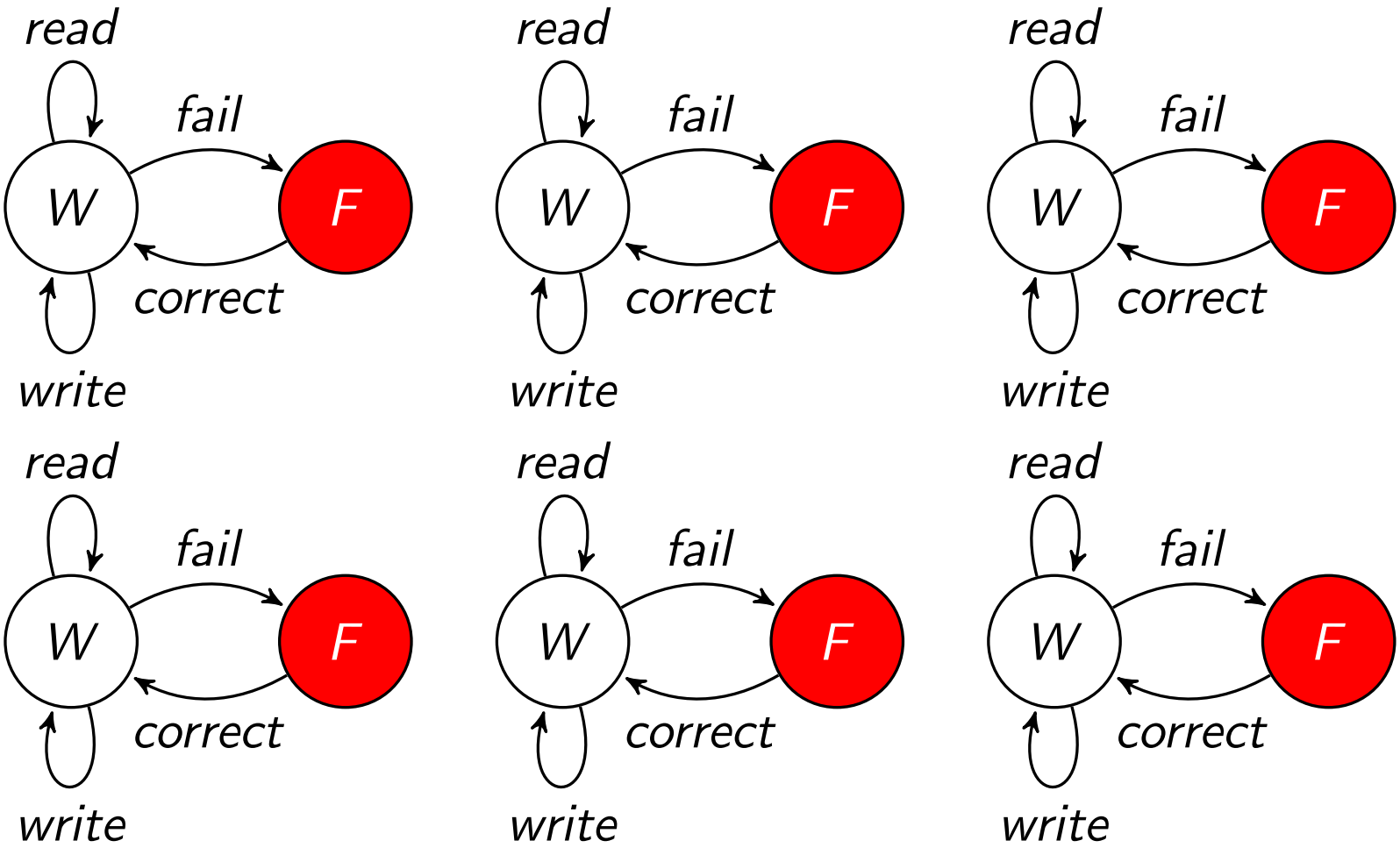
Six disks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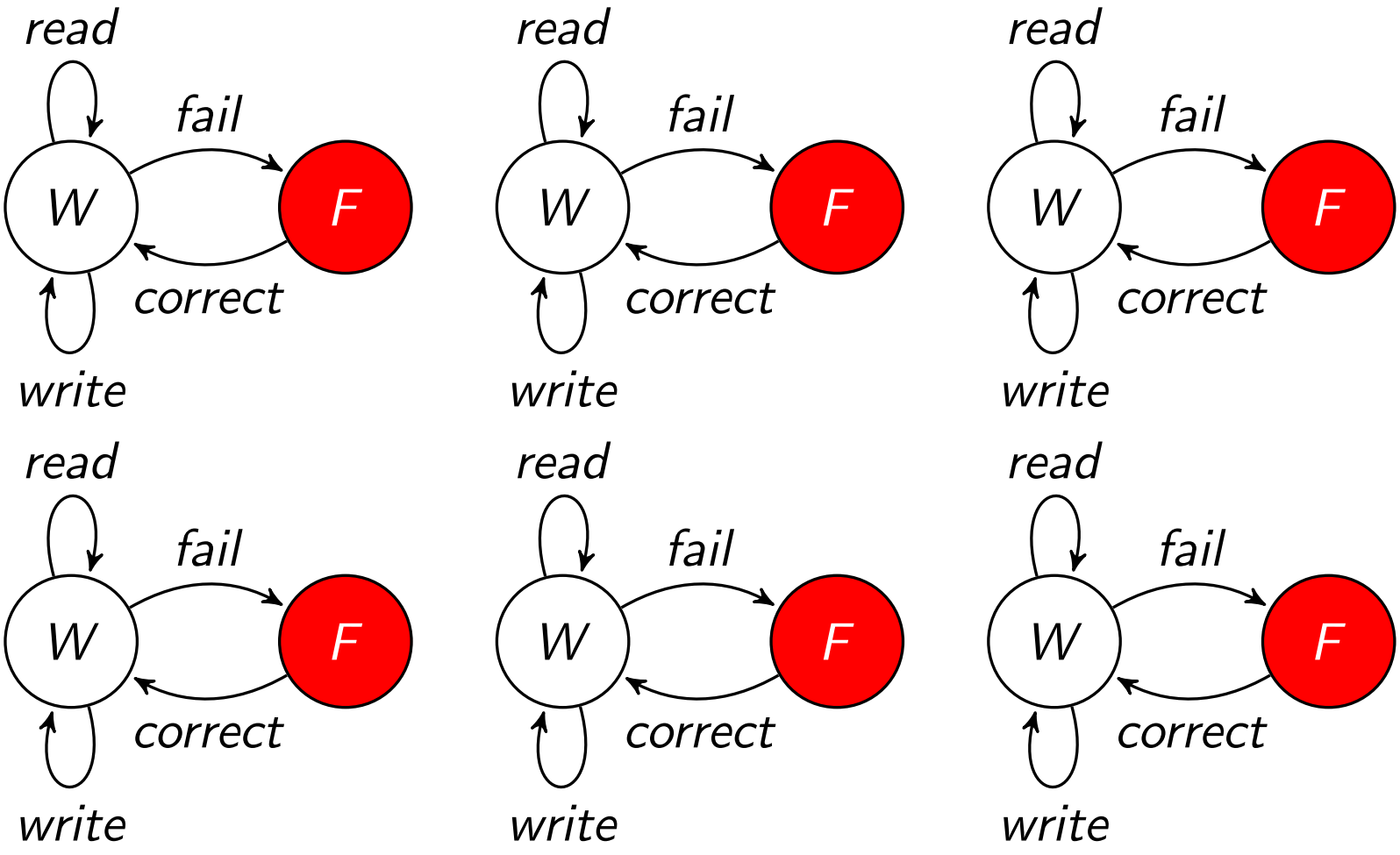
Six disks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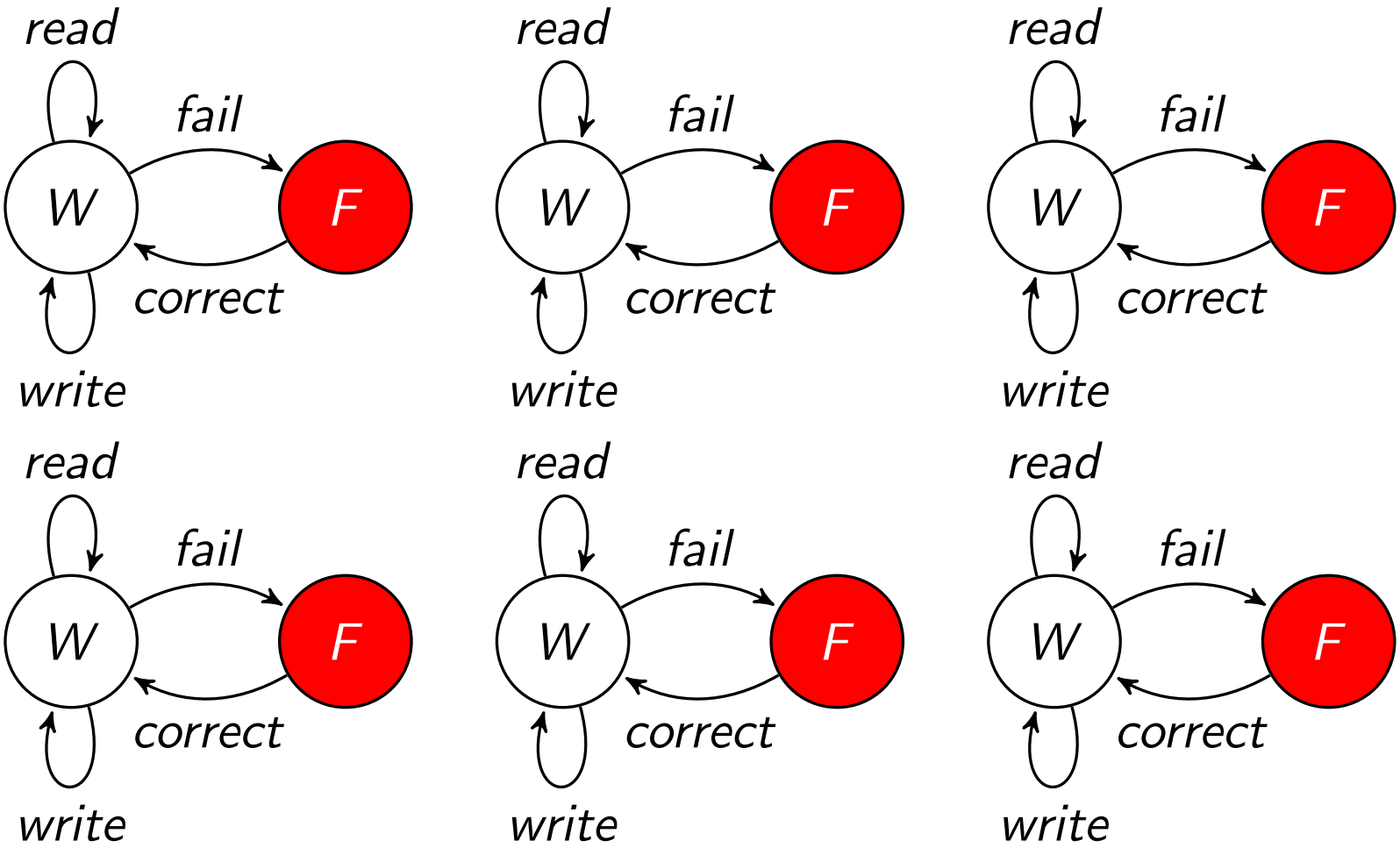
Six disks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write
read
fail
W
F
correct
write
read
fail
W
F
correct
write

State-space explosion

Disks	States
1	2

State-space explosion

Disks	States
1	2
2	4

State-space explosion

Disks	States
1	2
2	4
6	64

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states $= 2^{152}$ bytes

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states $= 2^{152}$ bytes $= 2^{82} \times 2^{70}$ bytes

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

$$2^{150} \text{ states} = 2^{152} \text{ bytes} = 2^{82} \times 2^{70} \text{ bytes} = 2^{82} \text{ zettabytes}$$

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states $= 2^{152}$ bytes $= 2^{82} \times 2^{70}$ bytes $= 2^{82}$ zettabytes

Been there . . .

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states $= 2^{152}$ bytes $= 2^{82} \times 2^{70}$ bytes $= 2^{82}$ zettabytes

Been there . . . done that . . .

State-space explosion

Disks	States
1	2
2	4
6	64
10	1024
20	1048576
50	1125899906842624
100	1267650600228229401496703205376
150	1427247692705959881058285969449495136382746624

2^{150} states $= 2^{152}$ bytes $= 2^{82} \times 2^{70}$ bytes $= 2^{82}$ zettabytes

Been there . . . done that . . . got the t-shirt!

The t-shirt

The t-shirt

Nice t-shirt . . .

The t-shirt

Nice t-shirt . . . wish it came in a larger size.

Disk model in SRMC

```
DiskA::{
  Working = (read, r).Working
          + (write, w).Working
          + (fail, f).Failed;
  Failed  = (correct, c).Working;
};
```

- We have W working disks and F failed ($W + F = N$).

Disk model in SRMC

```
DiskA::{
  Working = (read, r).Working
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  Failed  = (correct, c).Working;
};
```

- We have W working disks and F failed ($W + F = N$).
- Working disks fail at rate $f \times W$.

Disk model in SRMC

```
DiskA::{
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- Failures are corrected at rate $c \times F$.

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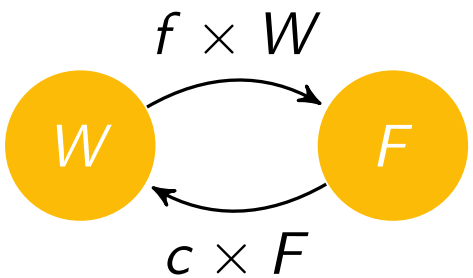

Disk model in SRMC

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- We have W working disks and F failed ($W + F = N$).
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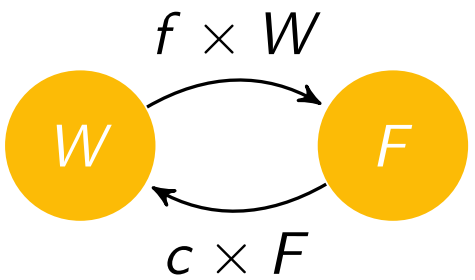
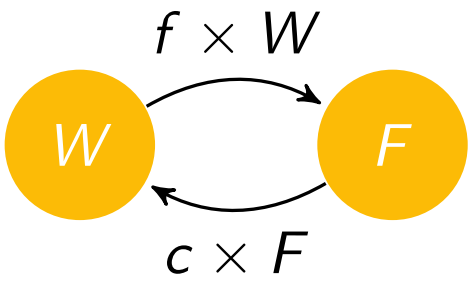


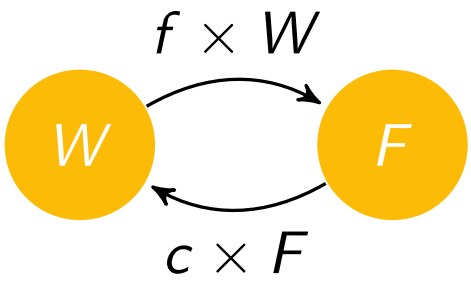
$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

Fluid-flow disk model



$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

Fluid-flow disk model



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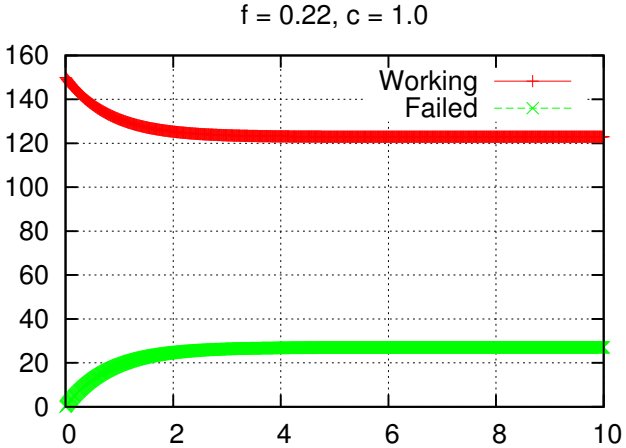
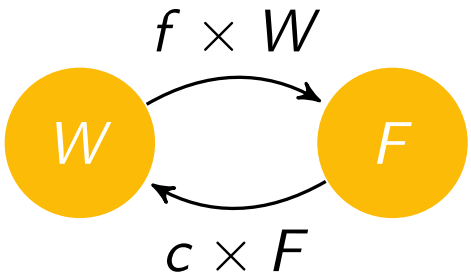
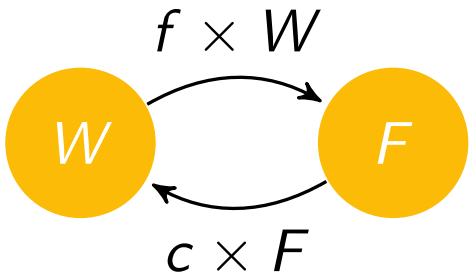


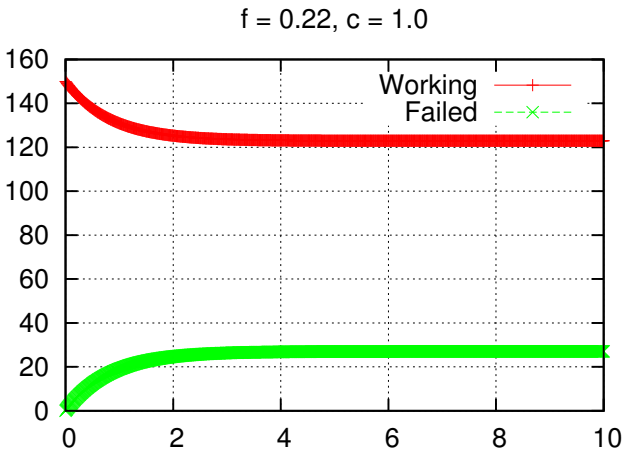
Fluid-flow disk model



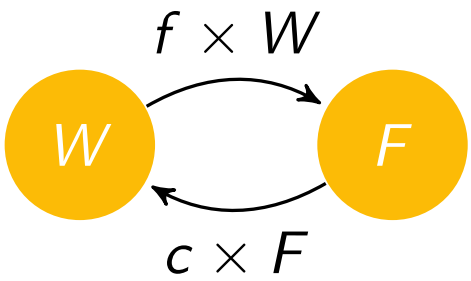


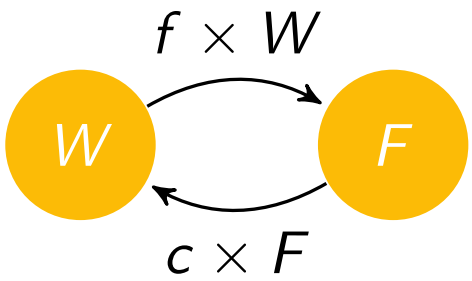
$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

$$0 = -f \times W + c \times F$$
$$0 = f \times W - c \times F$$



Fluid-flow disk model





$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
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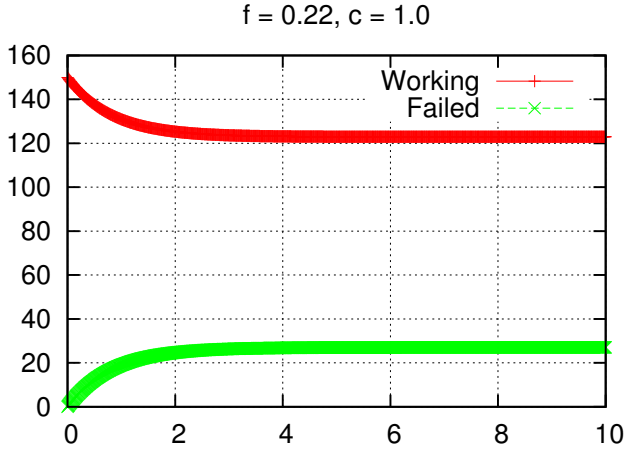
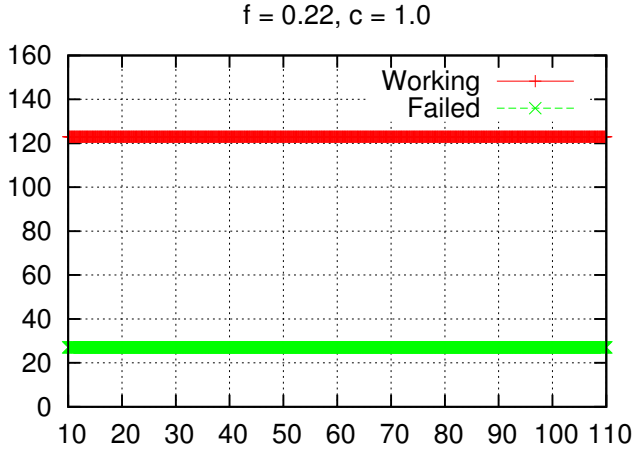
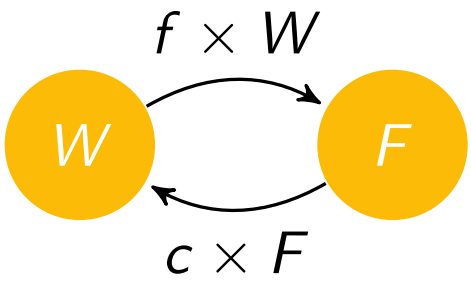
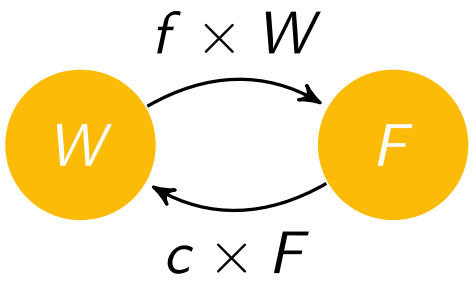




Fluid-flow disk model





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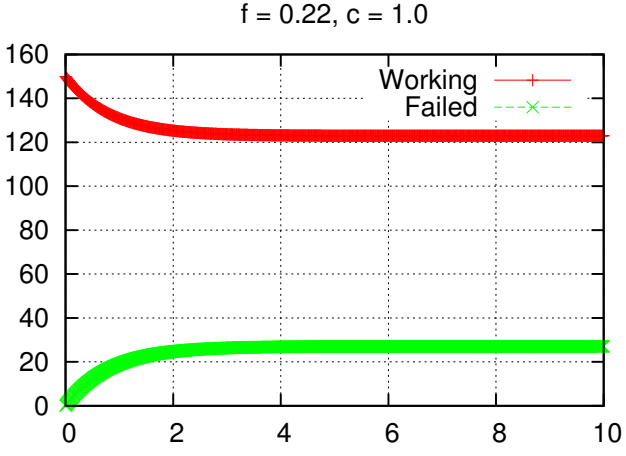
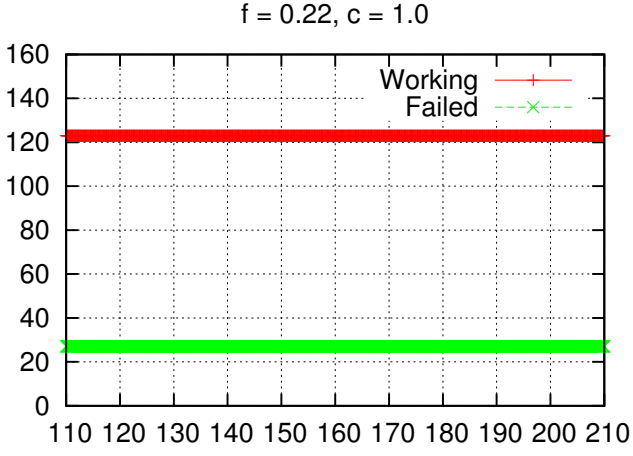
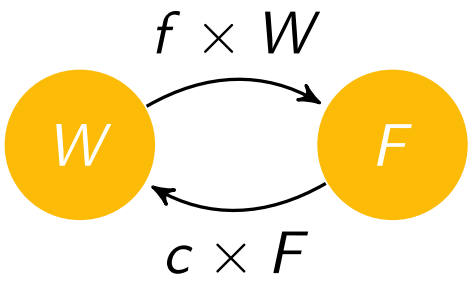
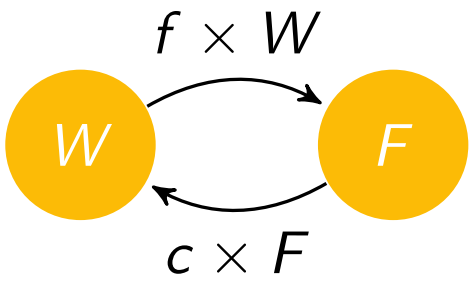




Fluid-flow disk model





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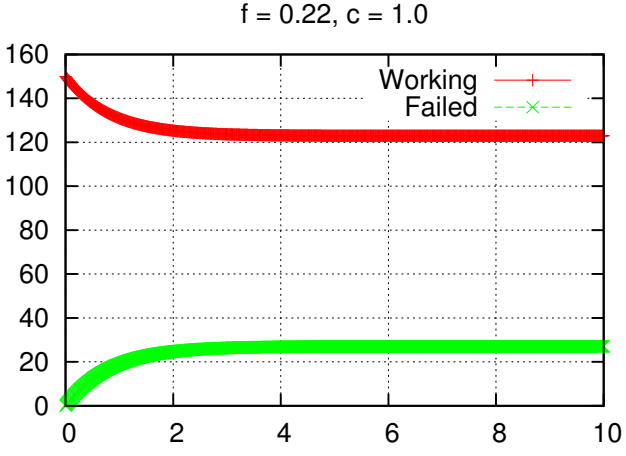
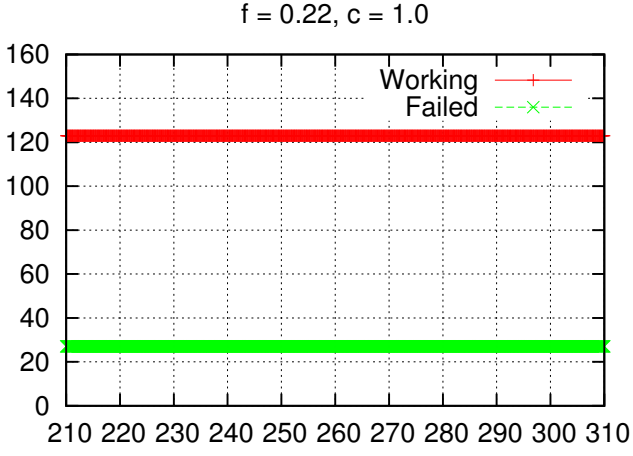
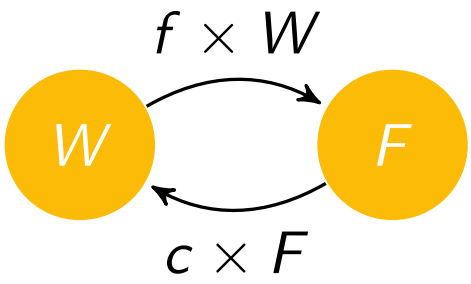
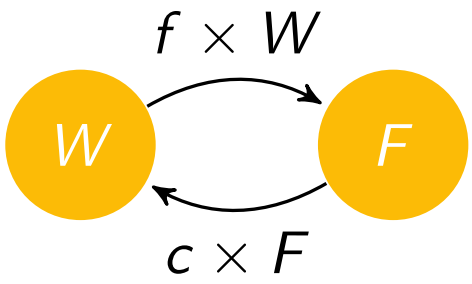




Fluid-flow disk model





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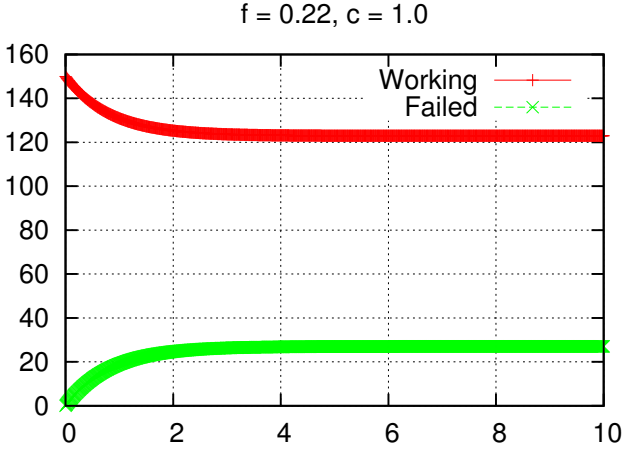
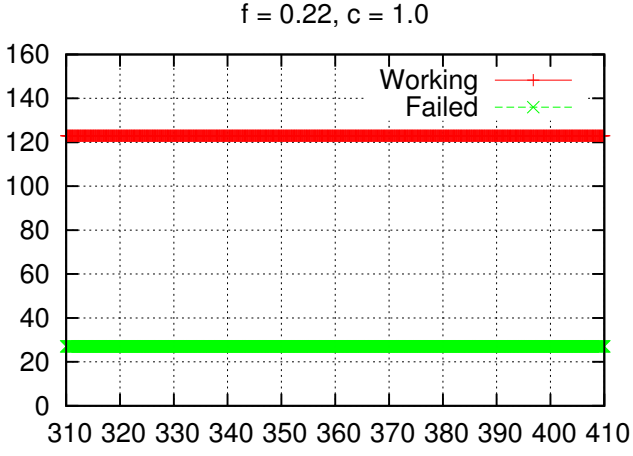
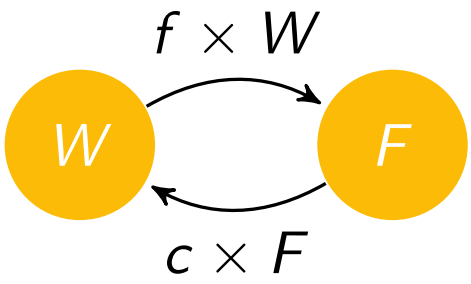
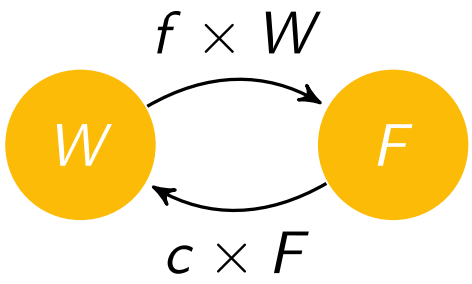




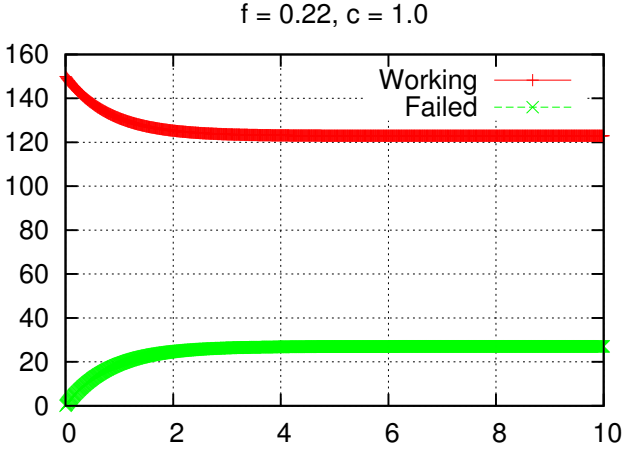
Fluid-flow disk model

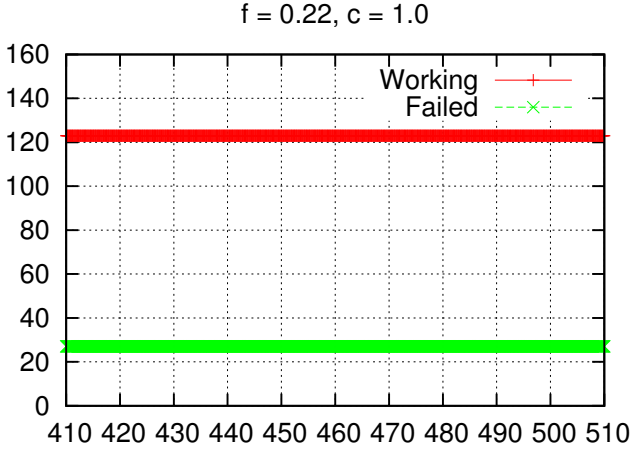




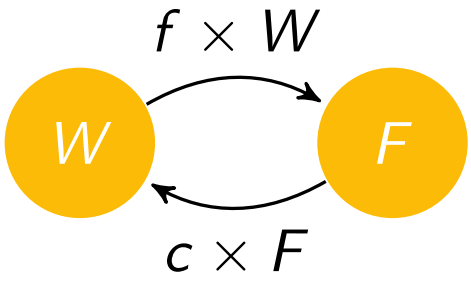
$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

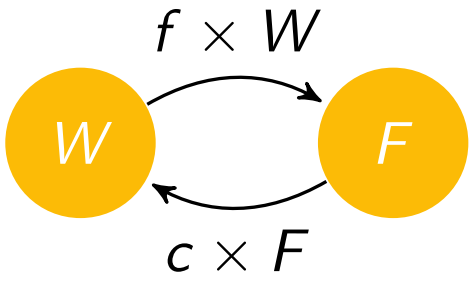
$$0 = -f \times W + c \times F$$
$$0 = f \times W - c \times F$$





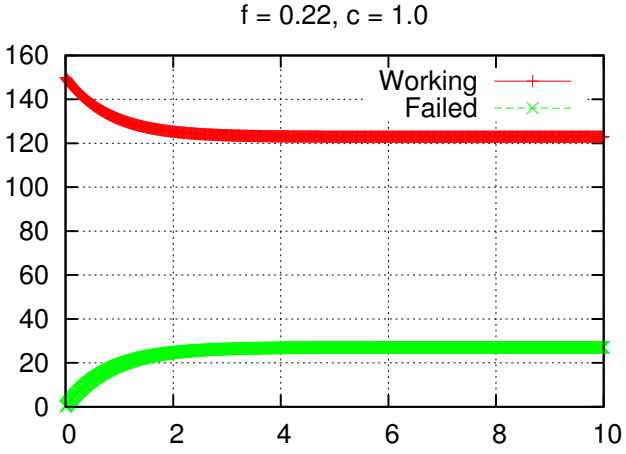
Fluid-flow disk model

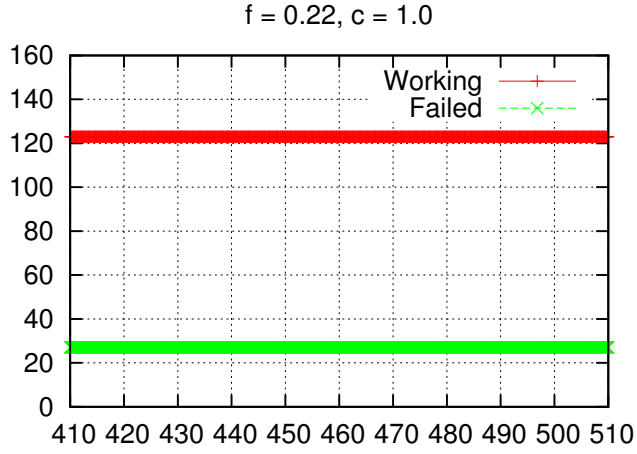




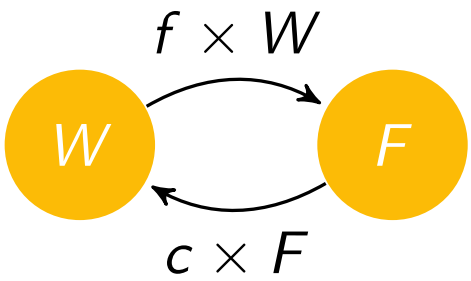
$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

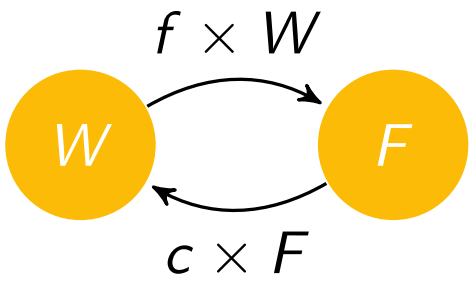
$$\left.\begin{aligned} W &= c/f \times F \\ F &= f/c \times W \end{aligned}\right\} W + F = N$$





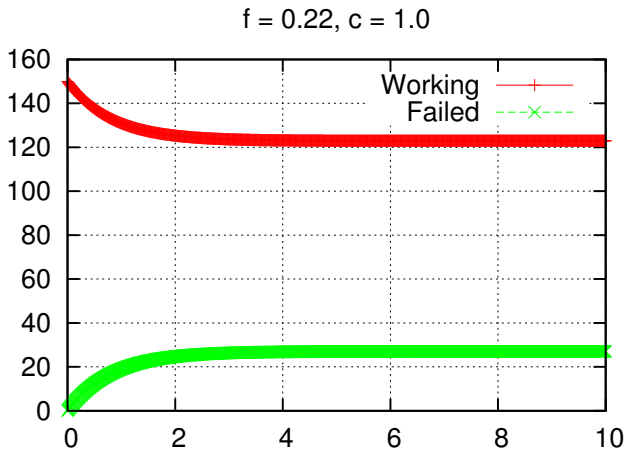
Fluid-flow disk model

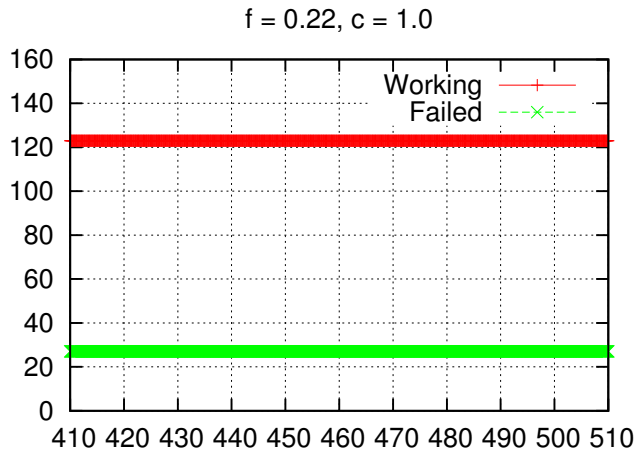




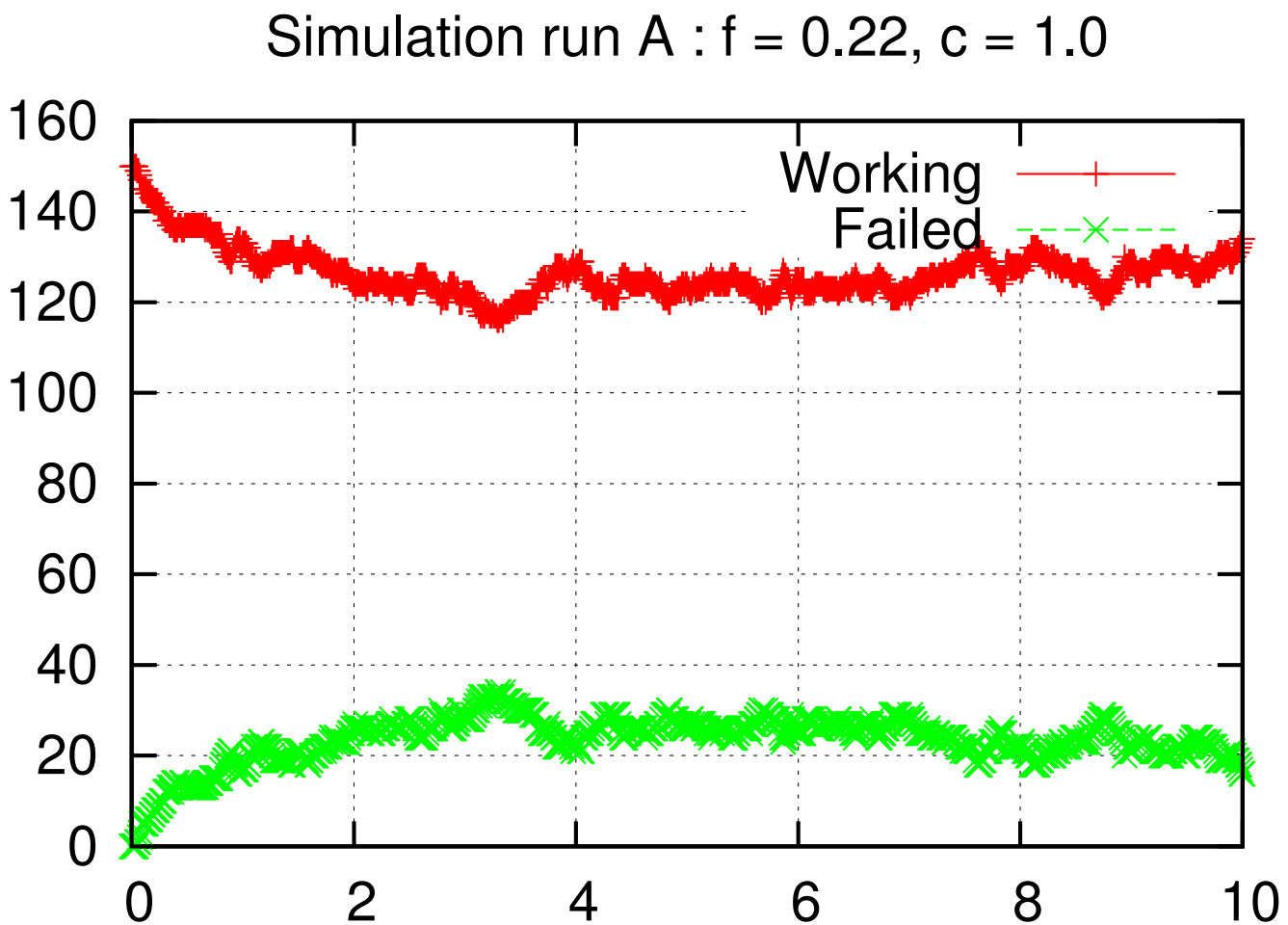
$$\mathrm{d}W/\mathrm{d}t = -f \times W + c \times F$$
$$\mathrm{d}F/\mathrm{d}t = f \times W - c \times F$$

$$W = N/(f/c + 1)$$
$$F = N/(c/f + 1)$$

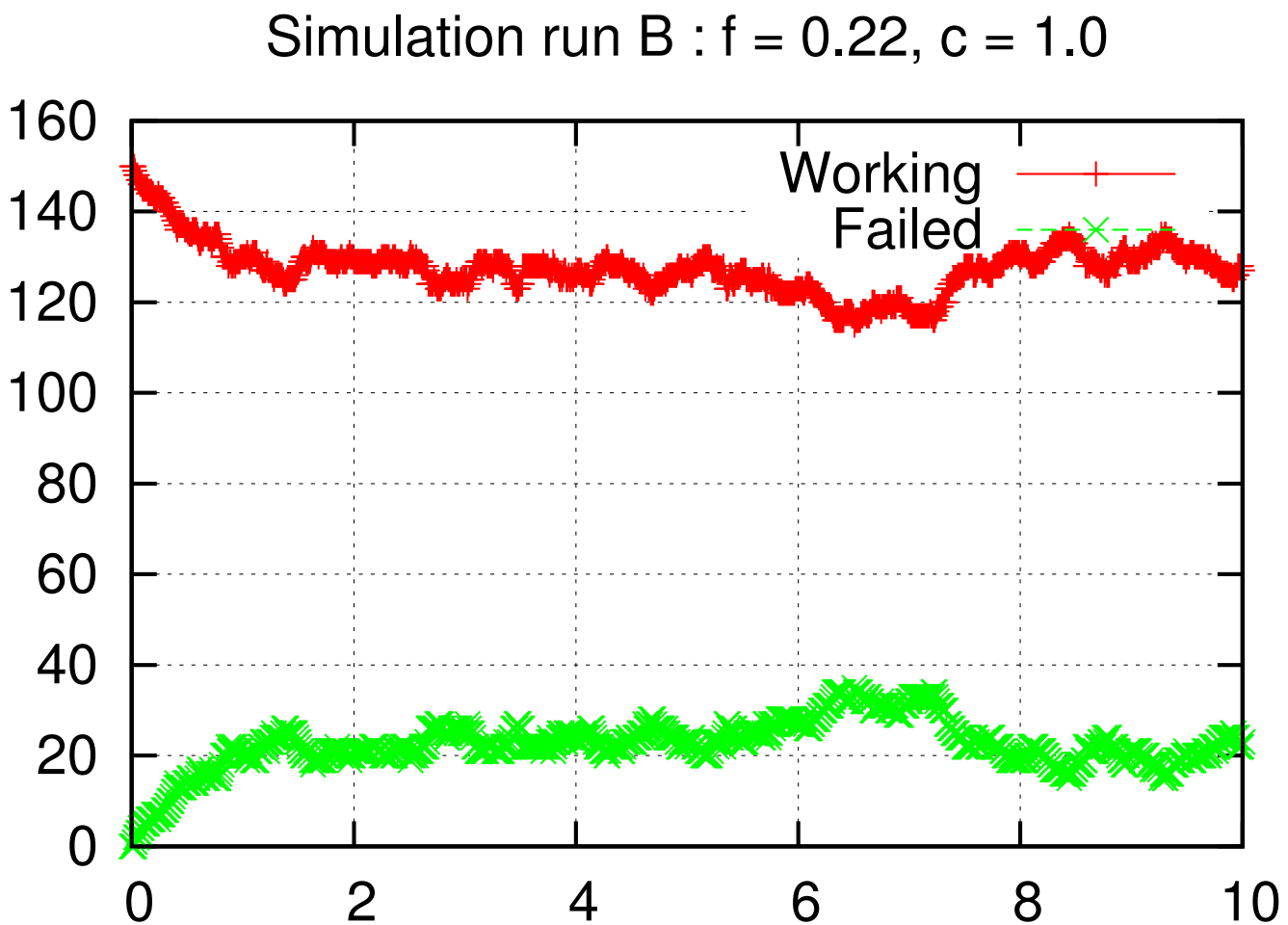




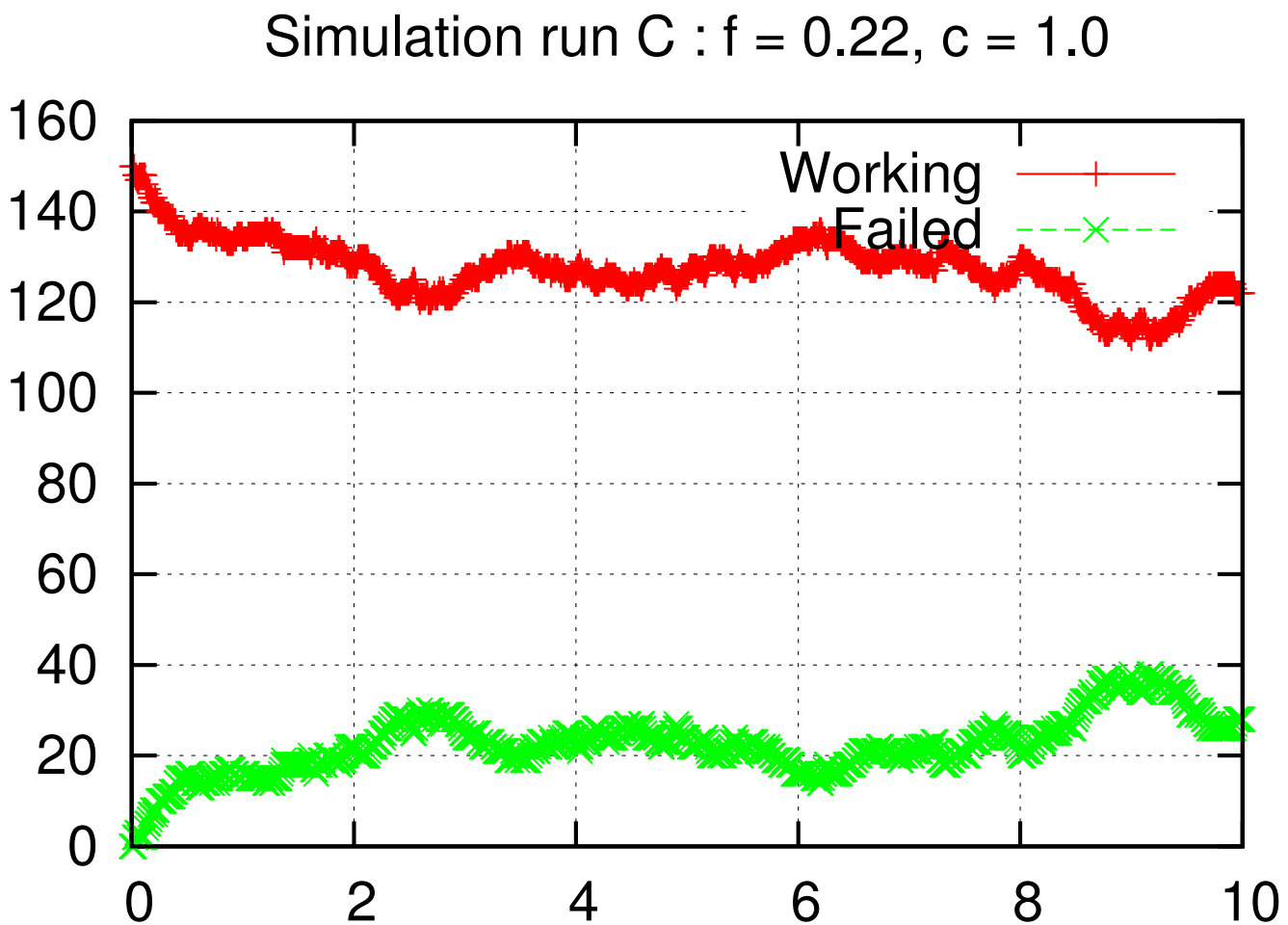
Markov chain simulation of the disk model



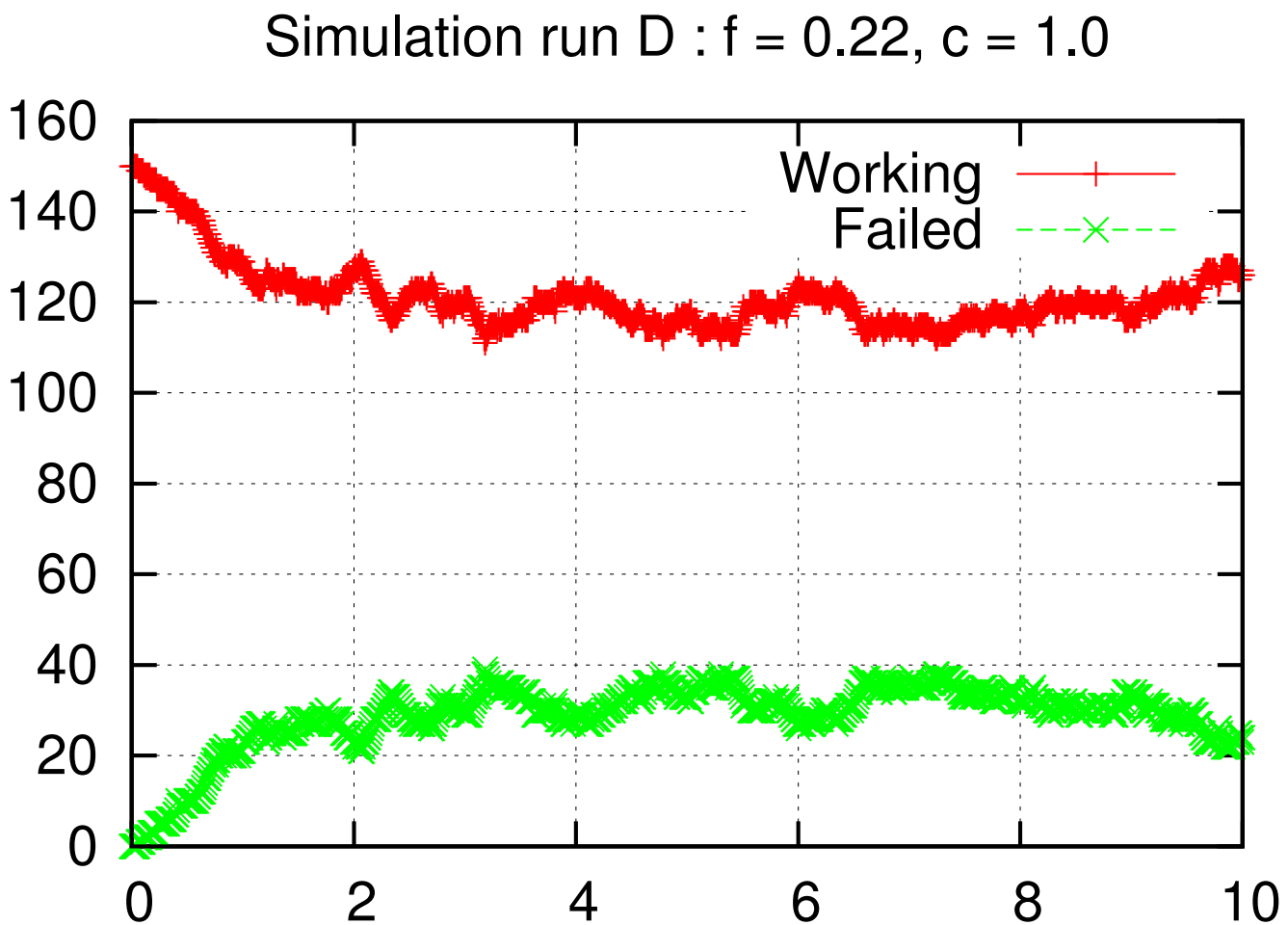
Markov chain simulation of the disk model



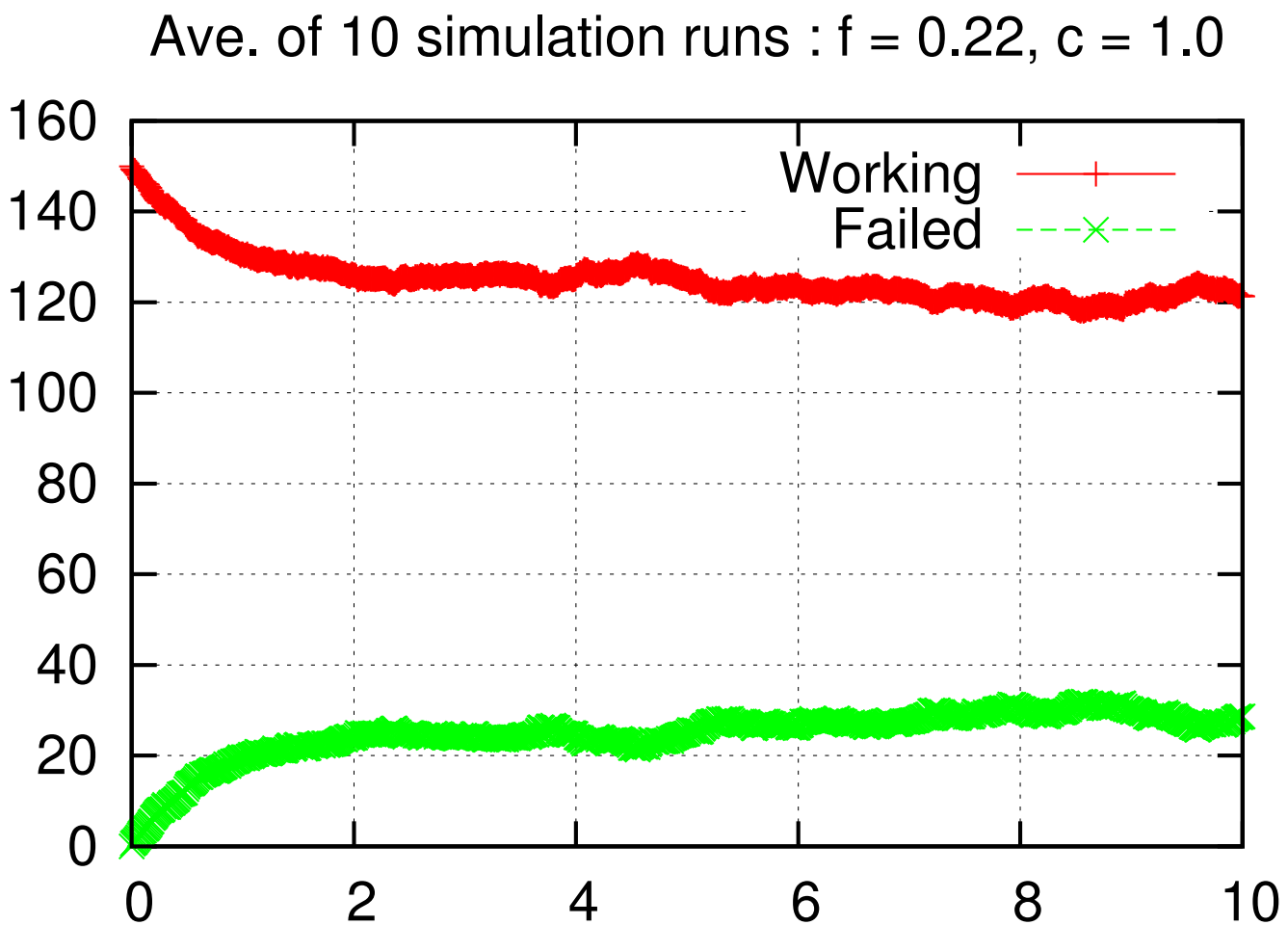
Markov chain simulation of the disk model



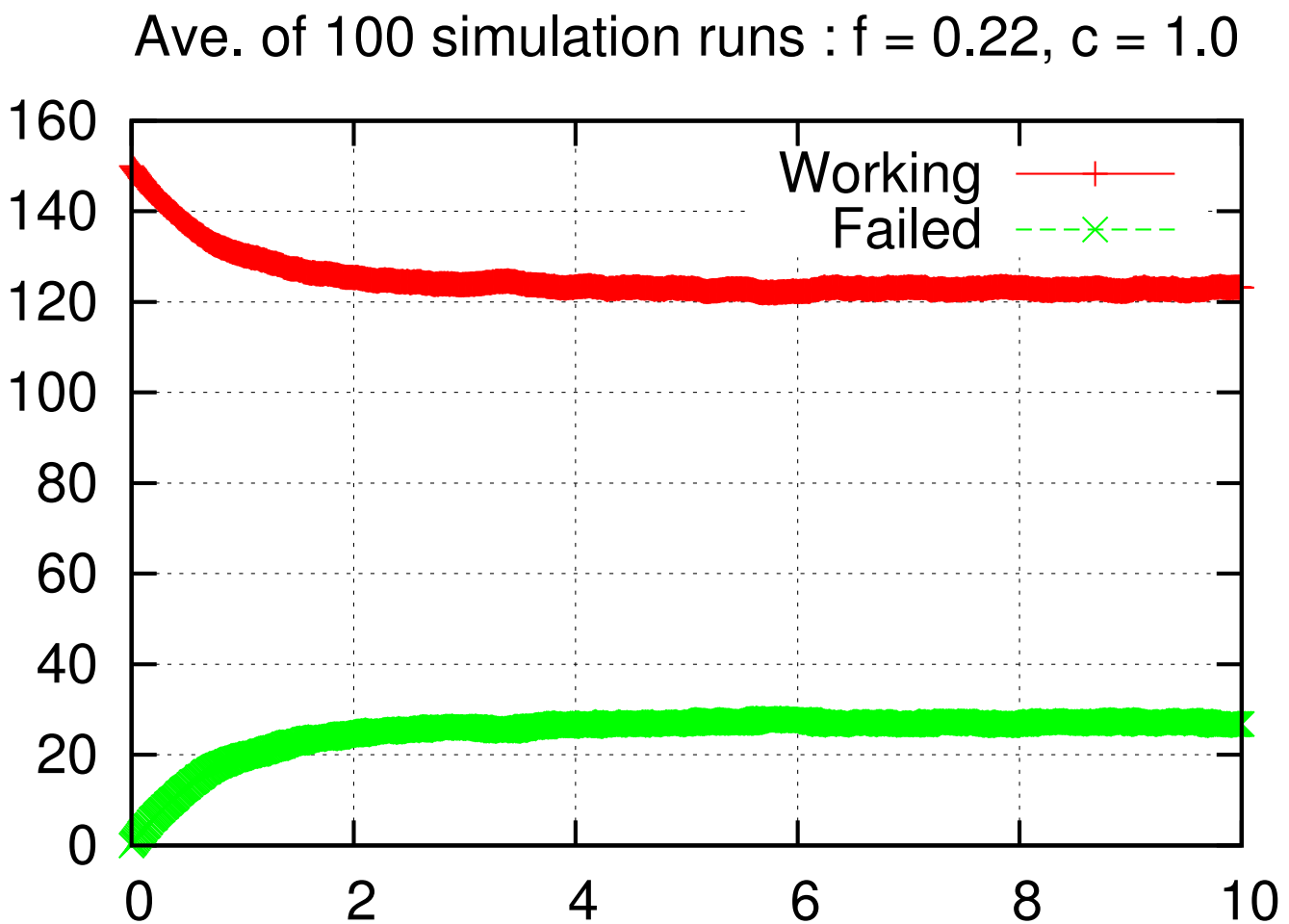
Markov chain simulation of the disk model



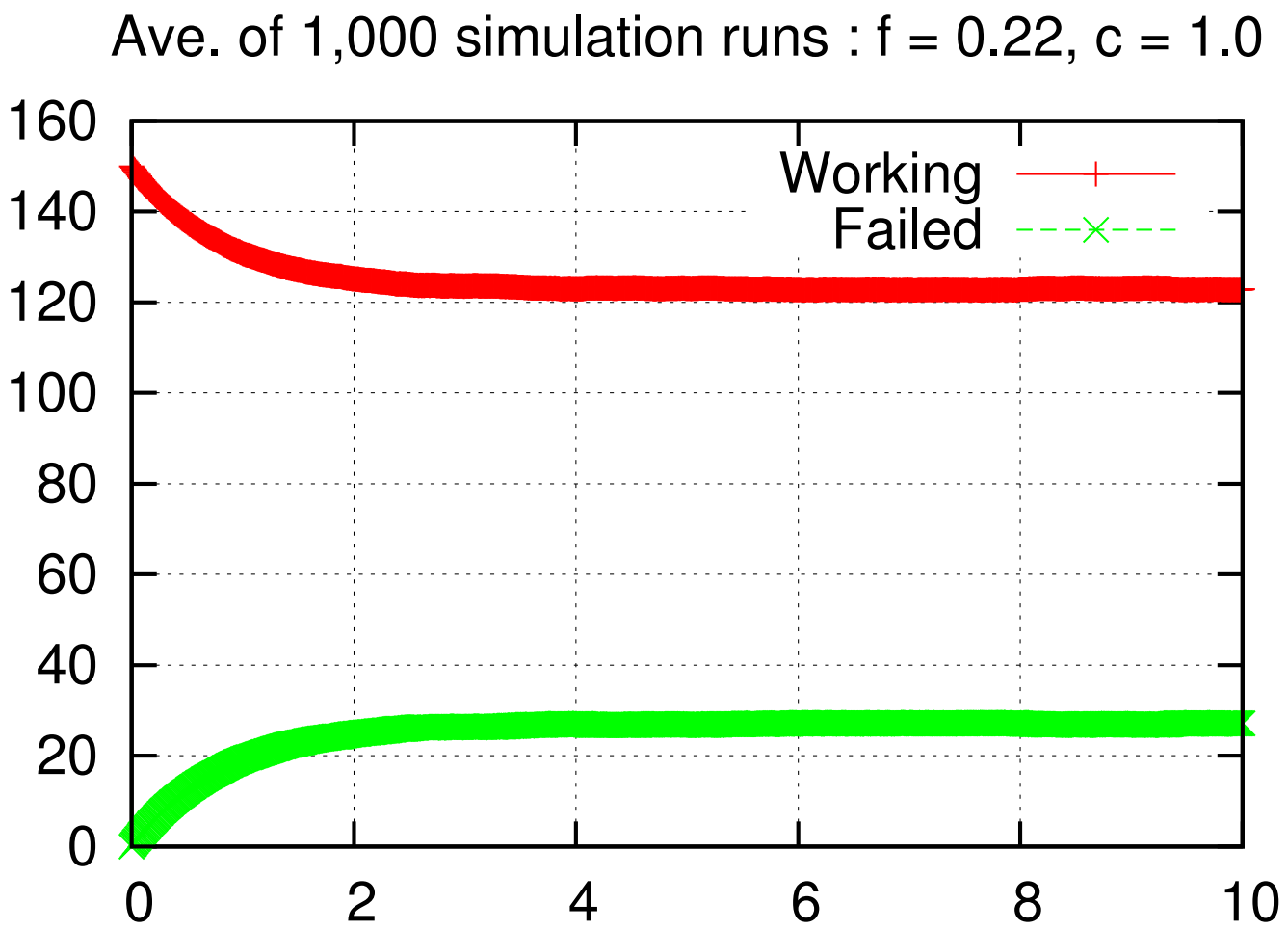
Markov chain simulation of the disk model



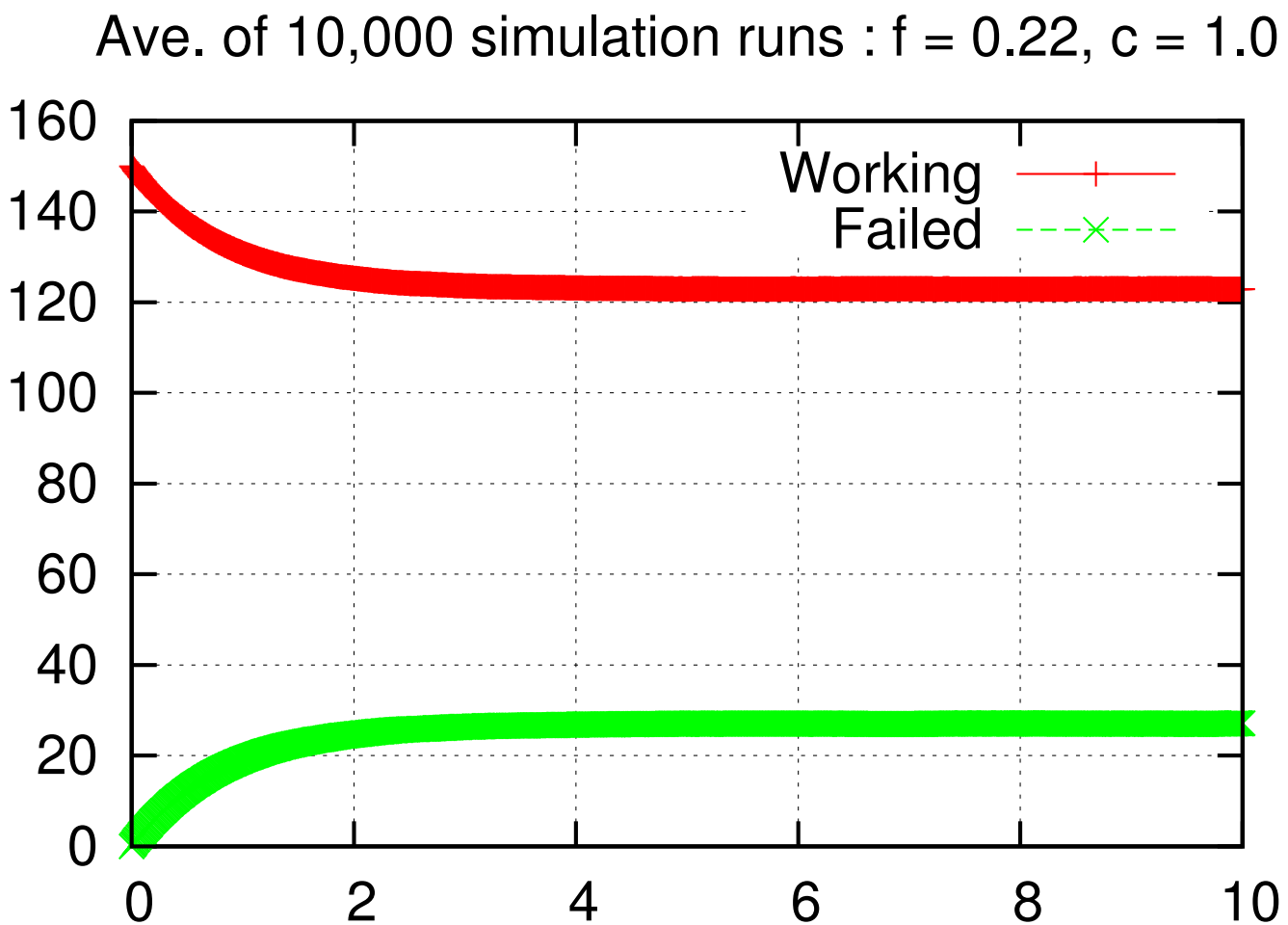
Markov chain simulation of the disk model



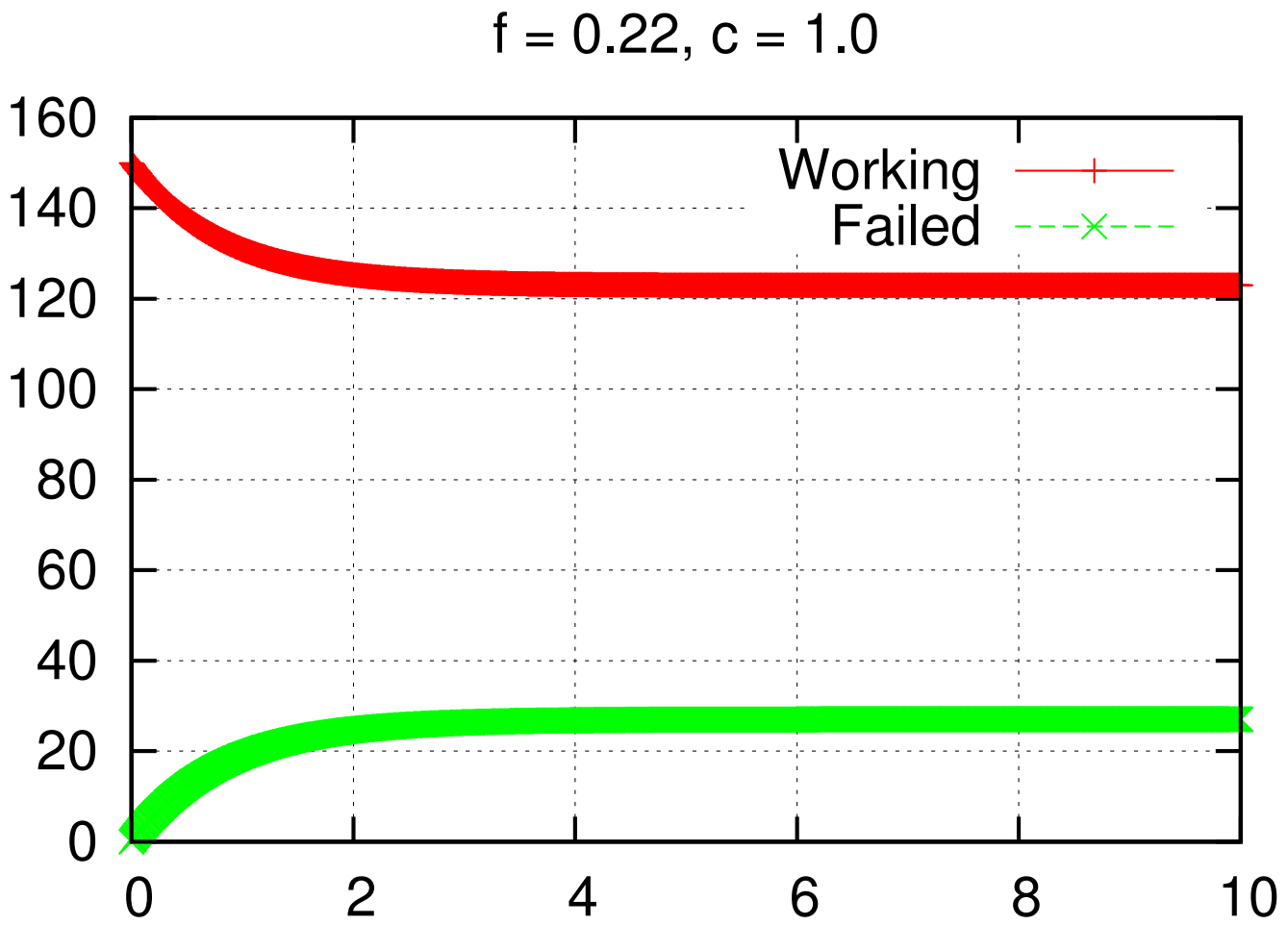
Markov chain simulation of the disk model



Markov chain simulation of the disk model



ODE analysis of the disk model



The new t-shirt

The new t-shirt

Nice t-shirt . . .

The new t-shirt

Nice t-shirt . . . available in XXXL!

Breakthrough paper

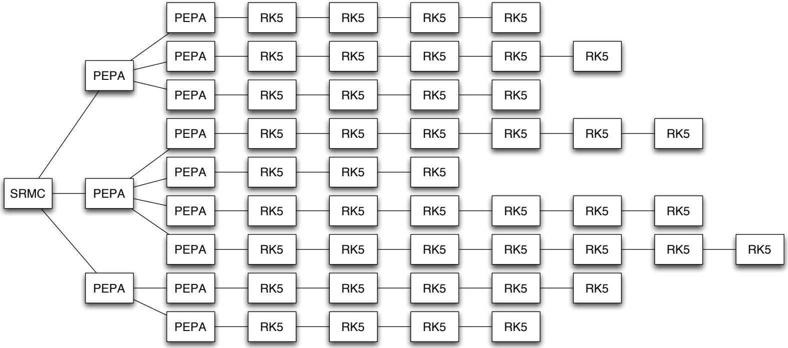
Jane Hillston.
Fluid flow approximation of PEPA models.
Proceedings of the Second International Conference on the Quantitative Evaluation of Systems, Torino, Italy, 2005.

Outline

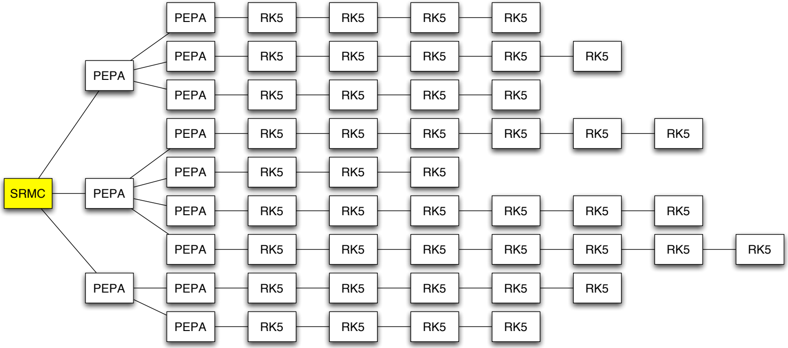
Large-scale systems

- Size is not the only difficulty in modelling large-scale systems.
- We need to accept that large-scale systems are naturally heterogeneous.
- We need to represent uncertainty about components, performance and configuration.
- We would like our analysis to be robust in the face of likely changes to the system.

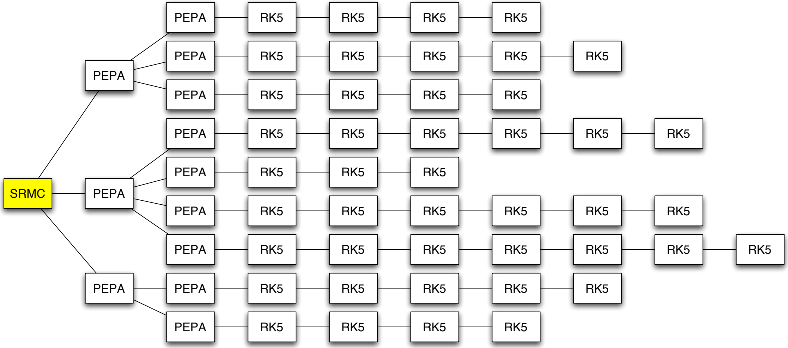
The evaluation model



The evaluation model

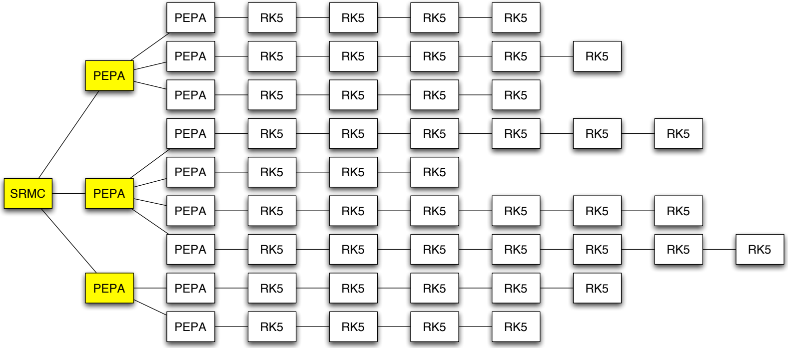


The evaluation model

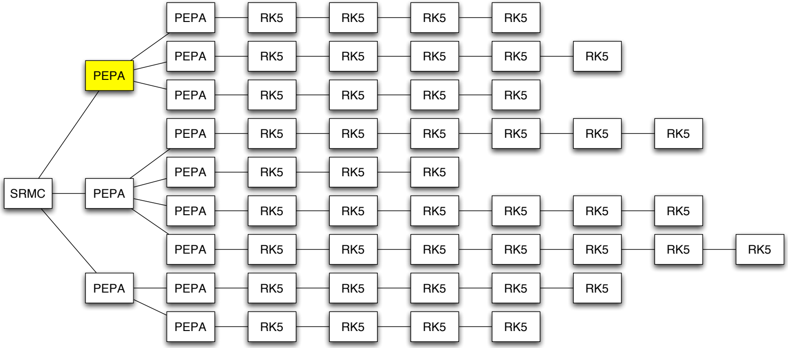



```
Disk ::= { DiskA, DiskB, DiskC }
```

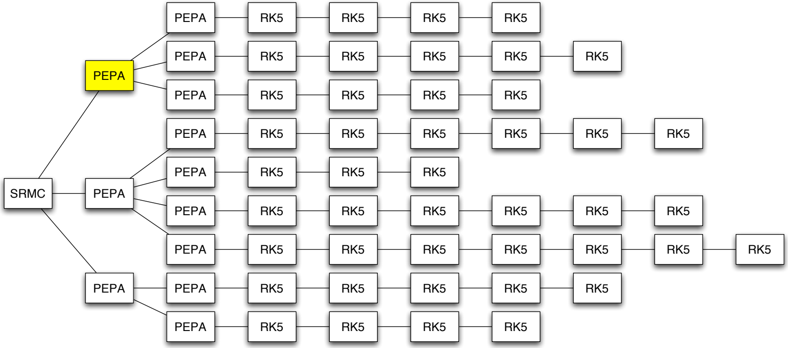
The evaluation model



The evaluation model

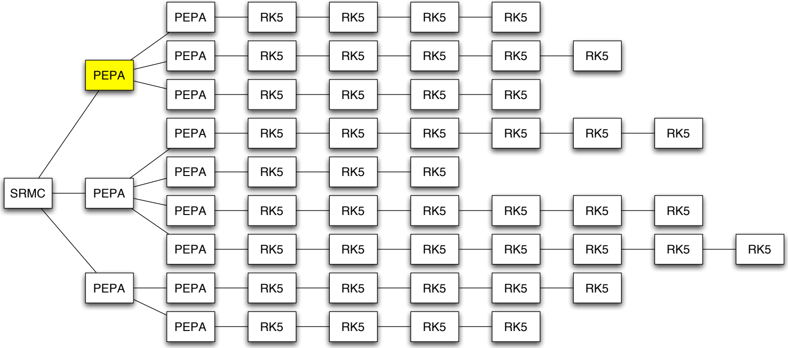


The evaluation model



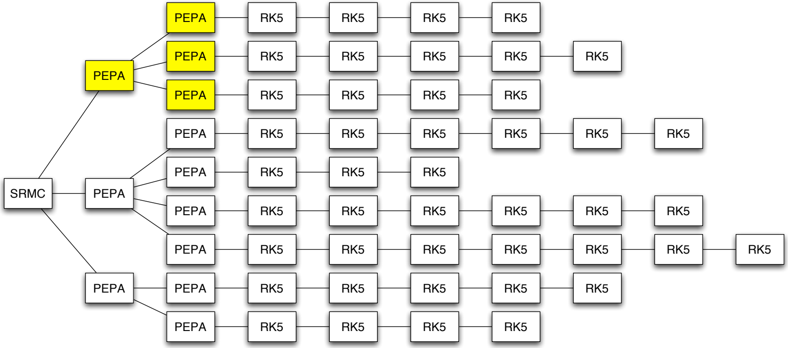

```
HTTPserver[?n+1] ==> HTTPserver[?n]
```

The evaluation model

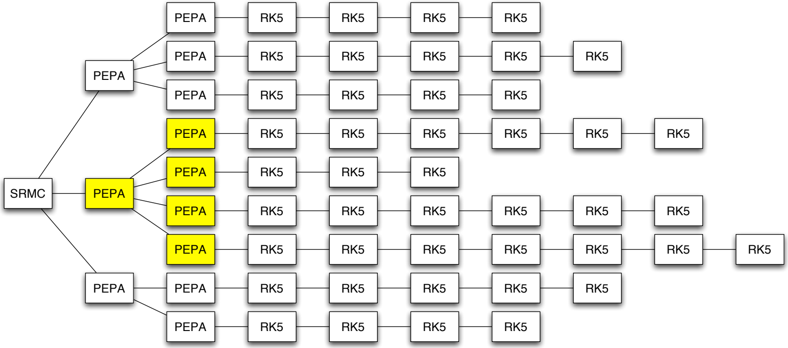



```
FTPserver[?n] ==> FTPserver[?n+1]
```

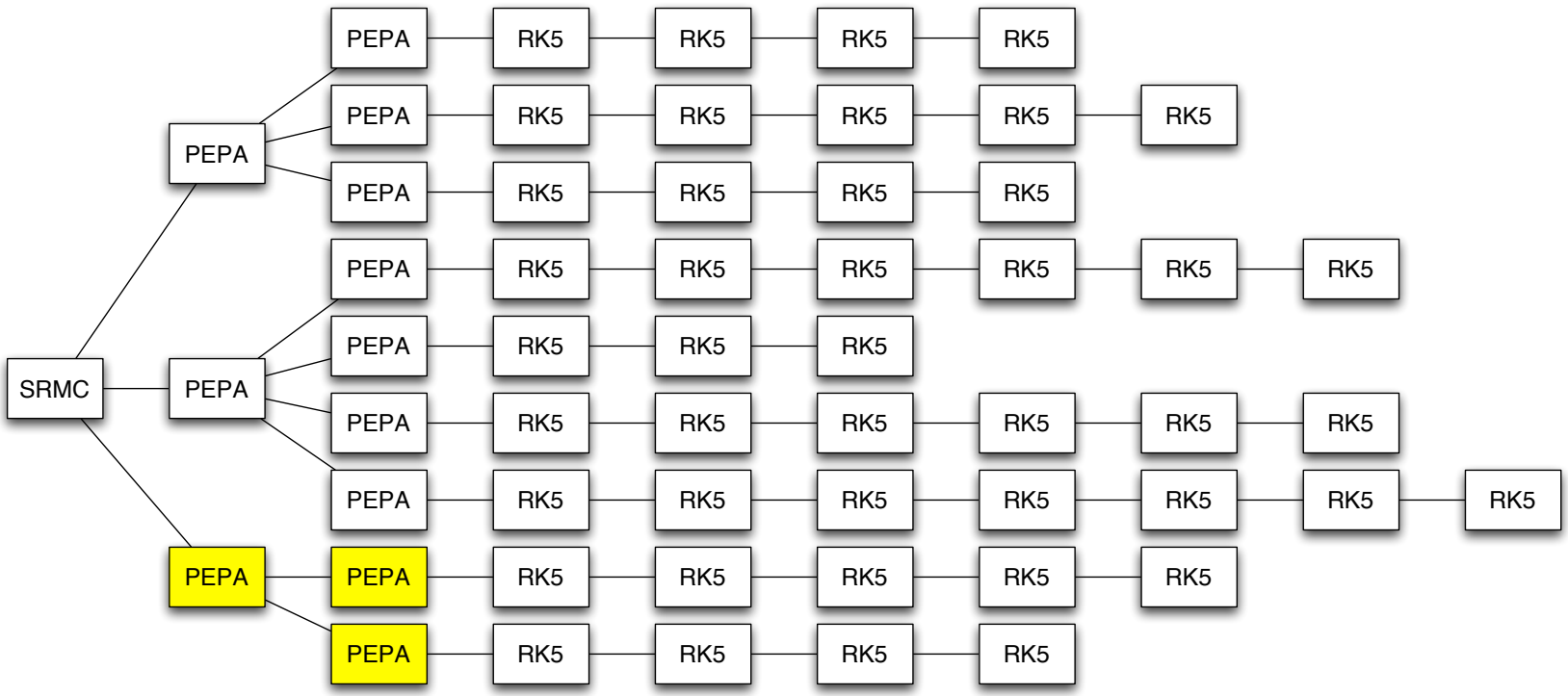
The evaluation model



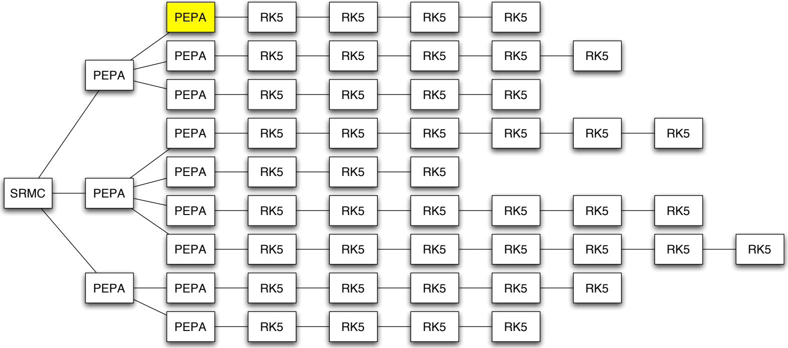
The evaluation model



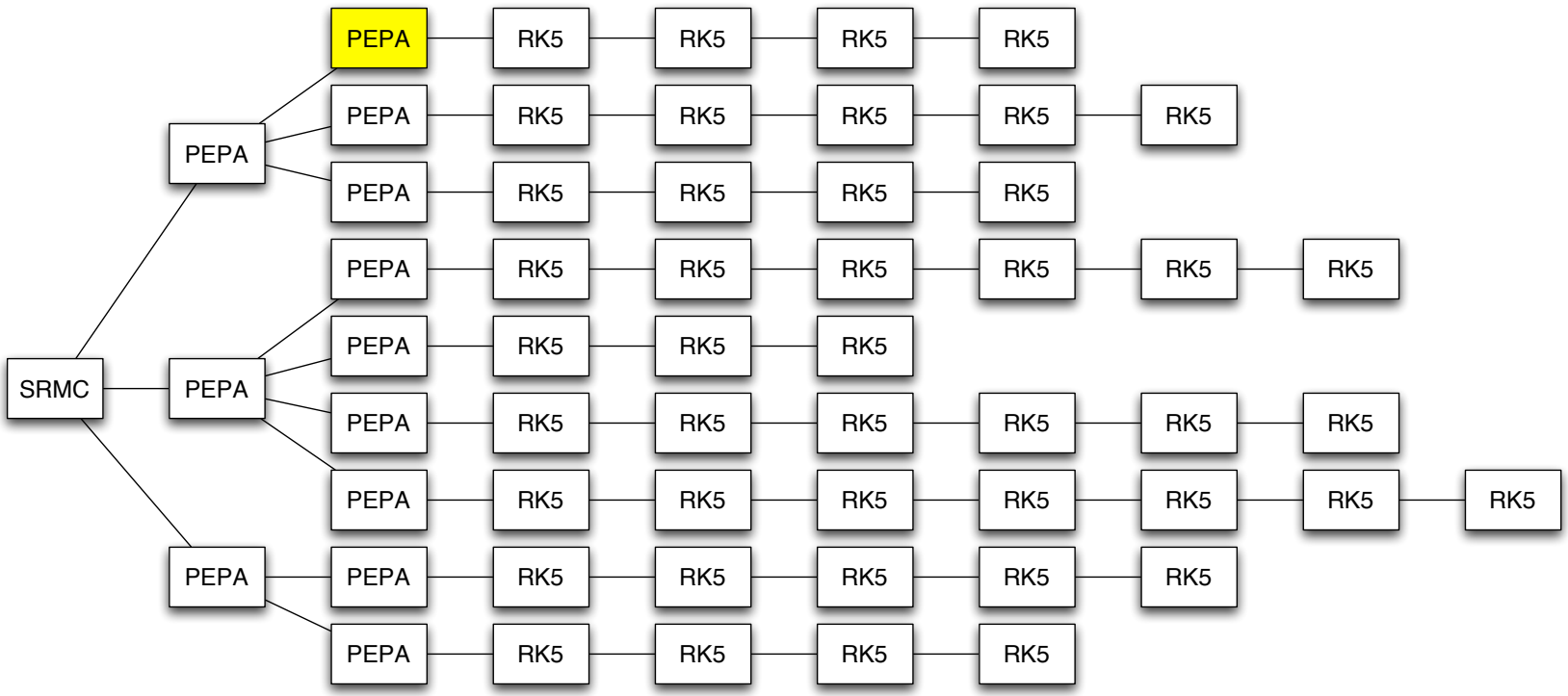
The evaluation model



The evaluation model

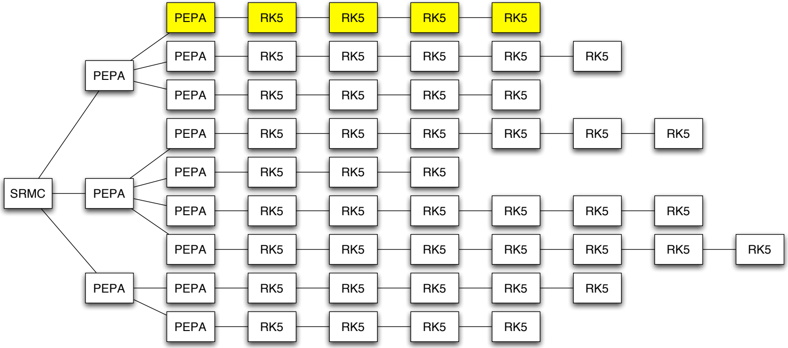


The evaluation model



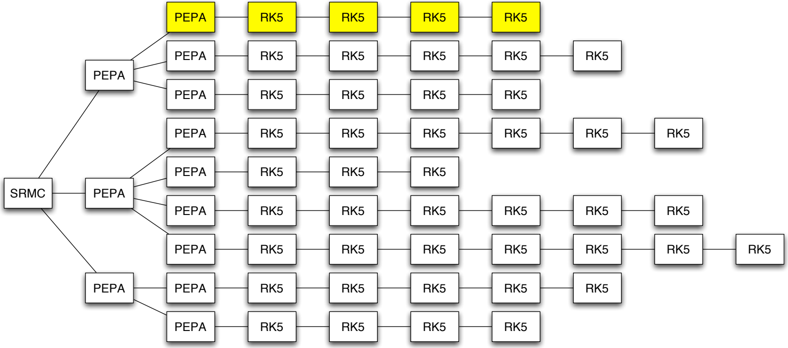
$$f = \{\ 0.075,\ 0.1,\ 0.22,\ 0.325\}$$

The evaluation model



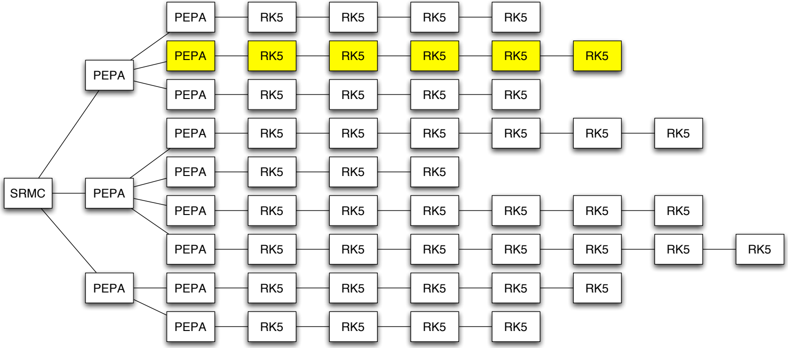

```
f = { 0.075, 0.1, 0.22, 0.325}
```

The evaluation model



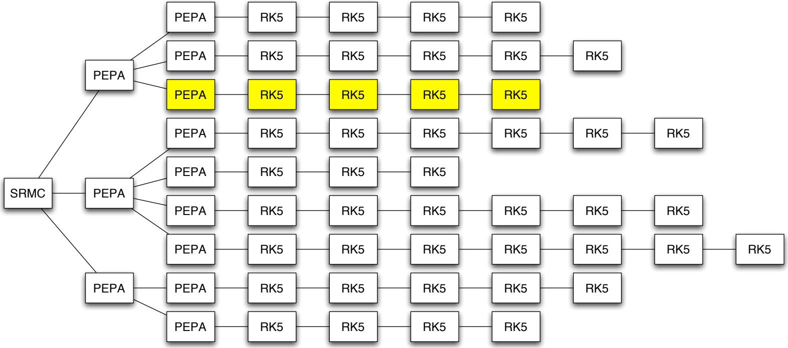
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



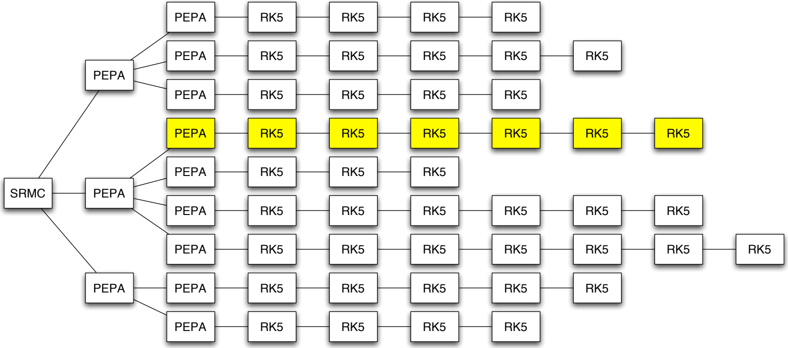
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



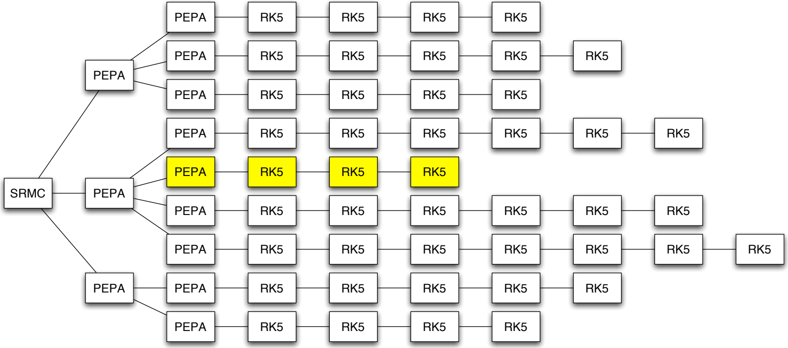
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



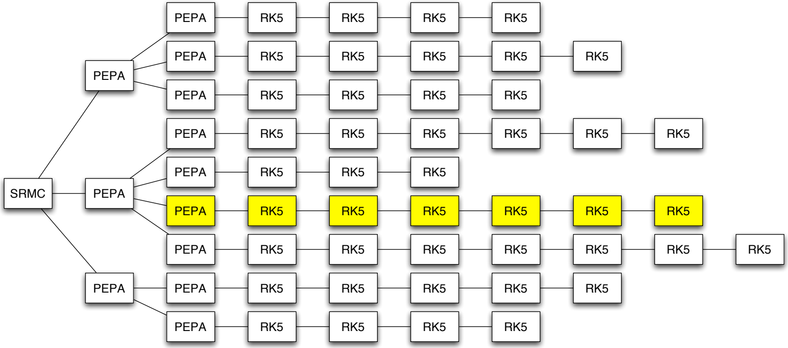
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



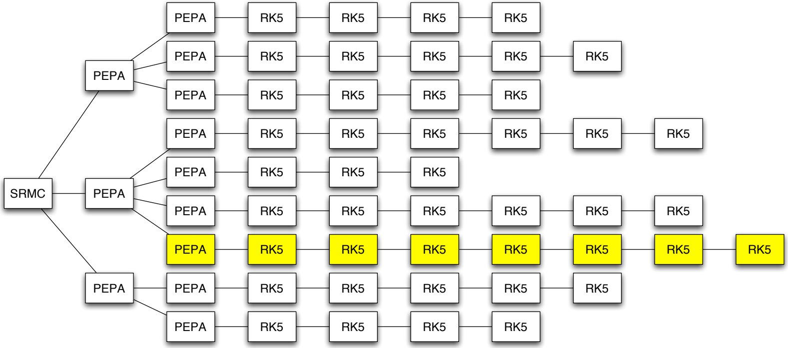
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



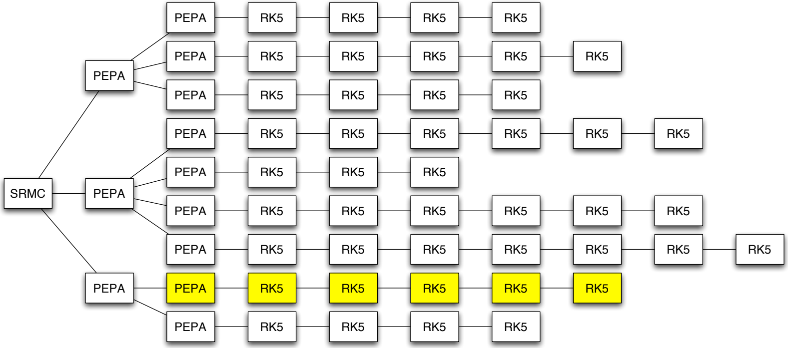
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



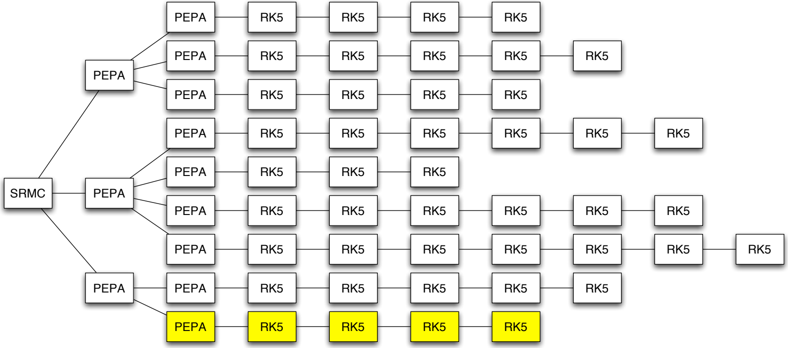
Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



Execute fifth-order Runge-Kutta integrator (RK5)

The evaluation model



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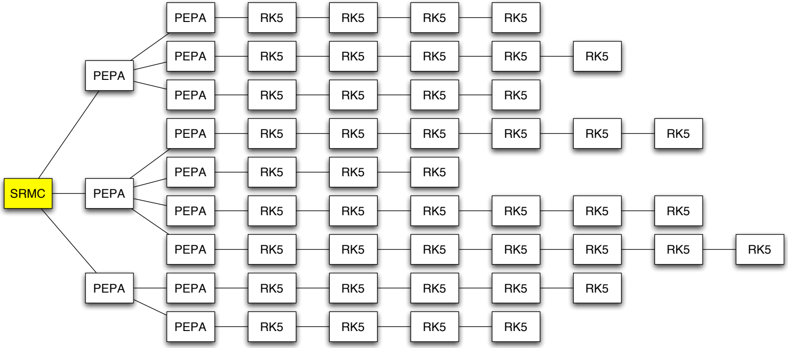
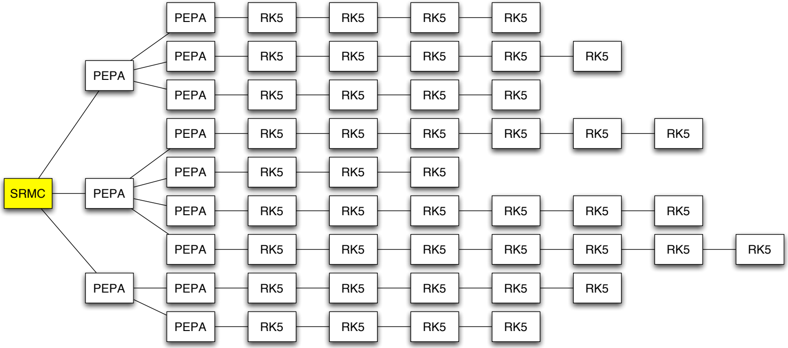
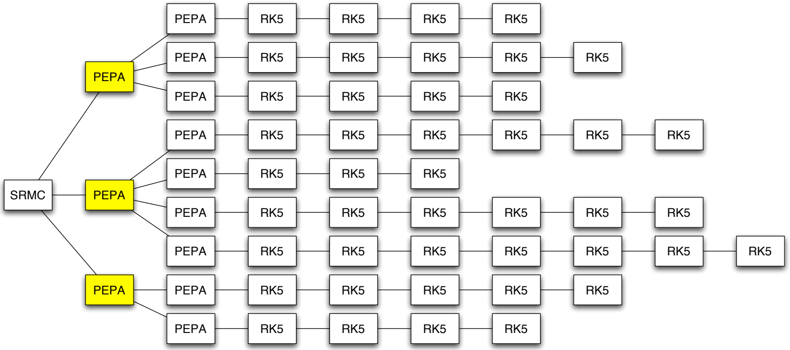


The evaluation model



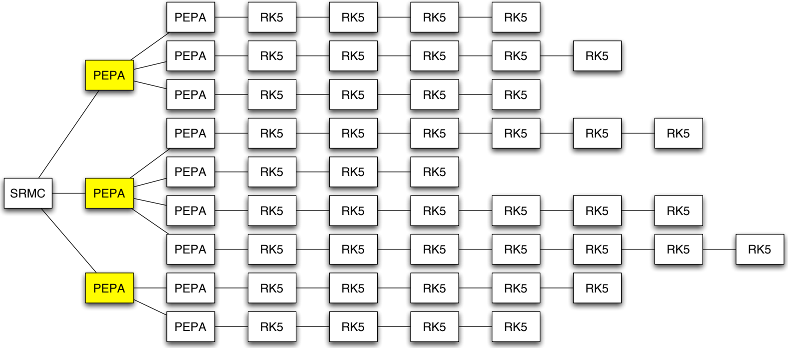
Resolve bindings

The evaluation model



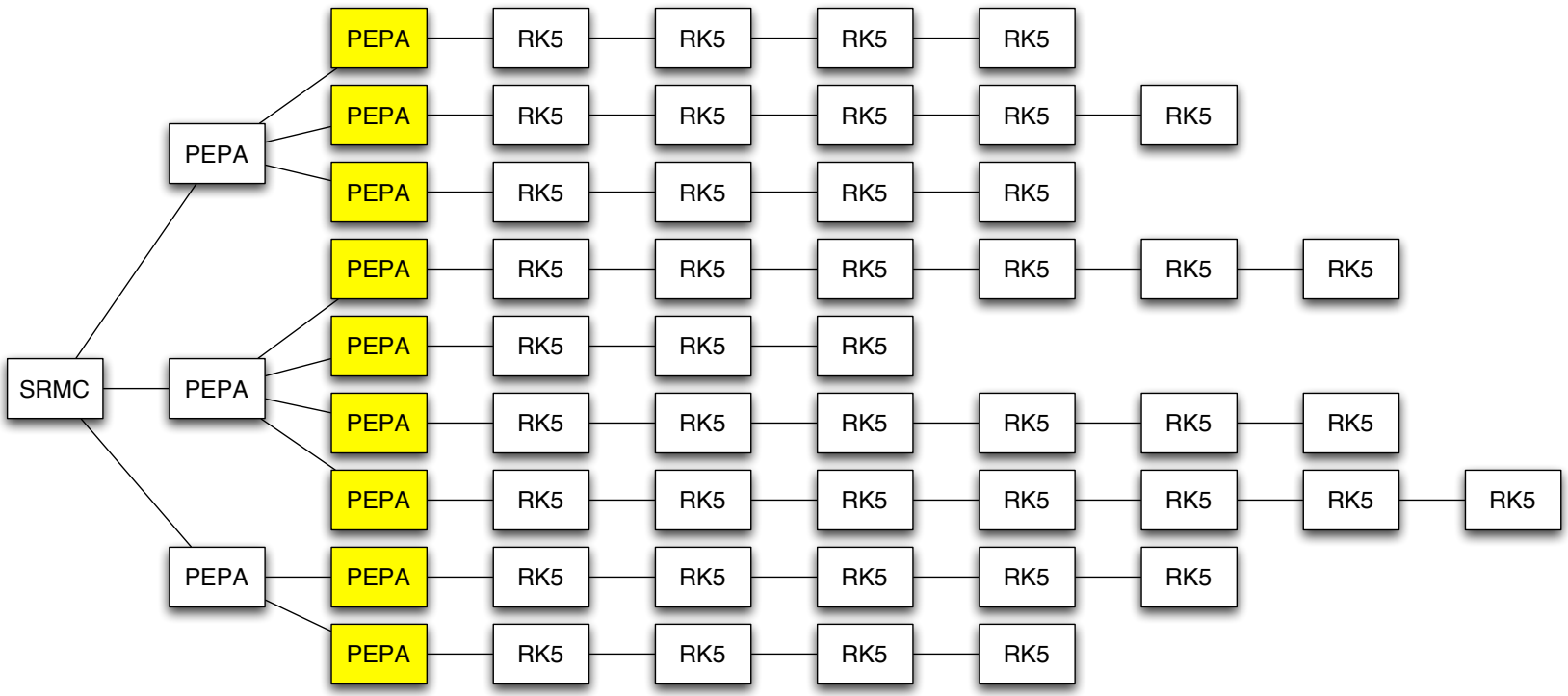
Resolve bindings

The evaluation model



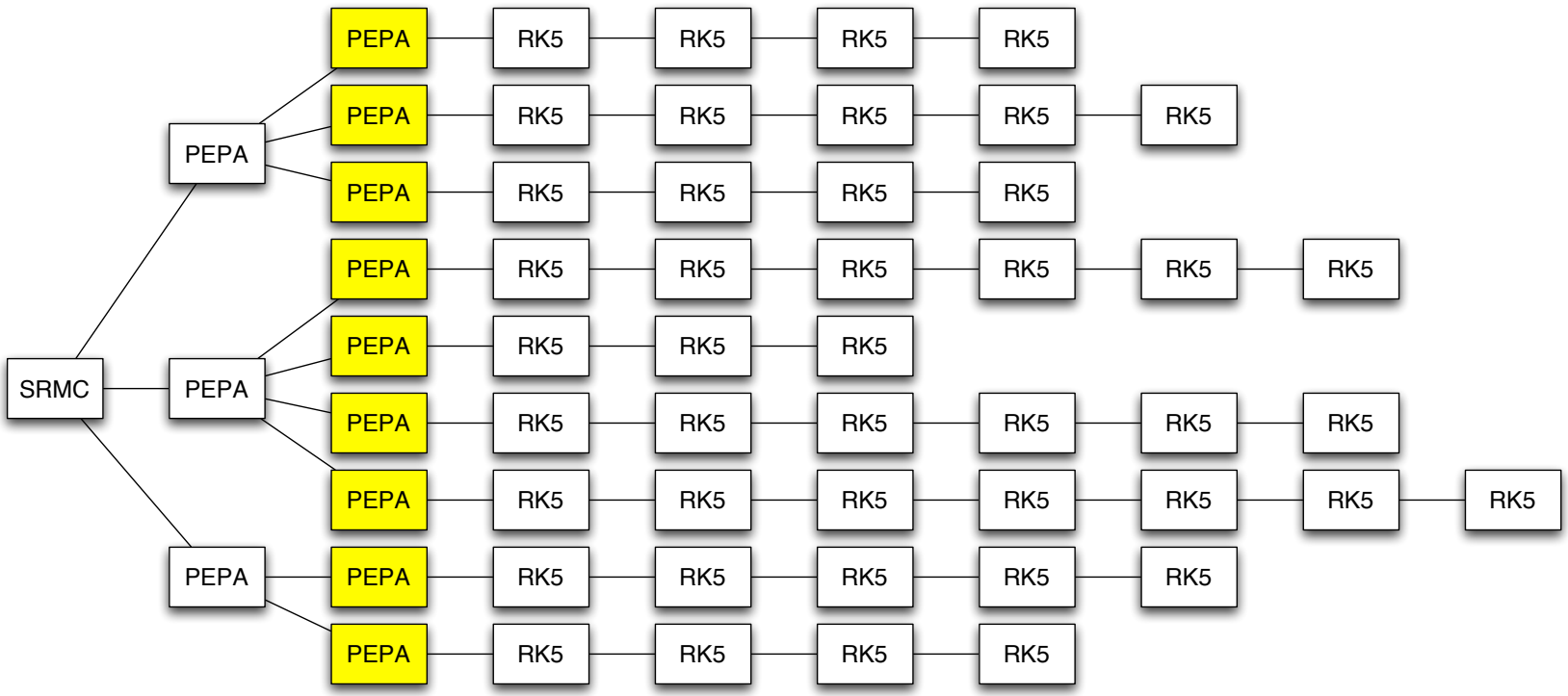
Apply model transformations

The evaluation model



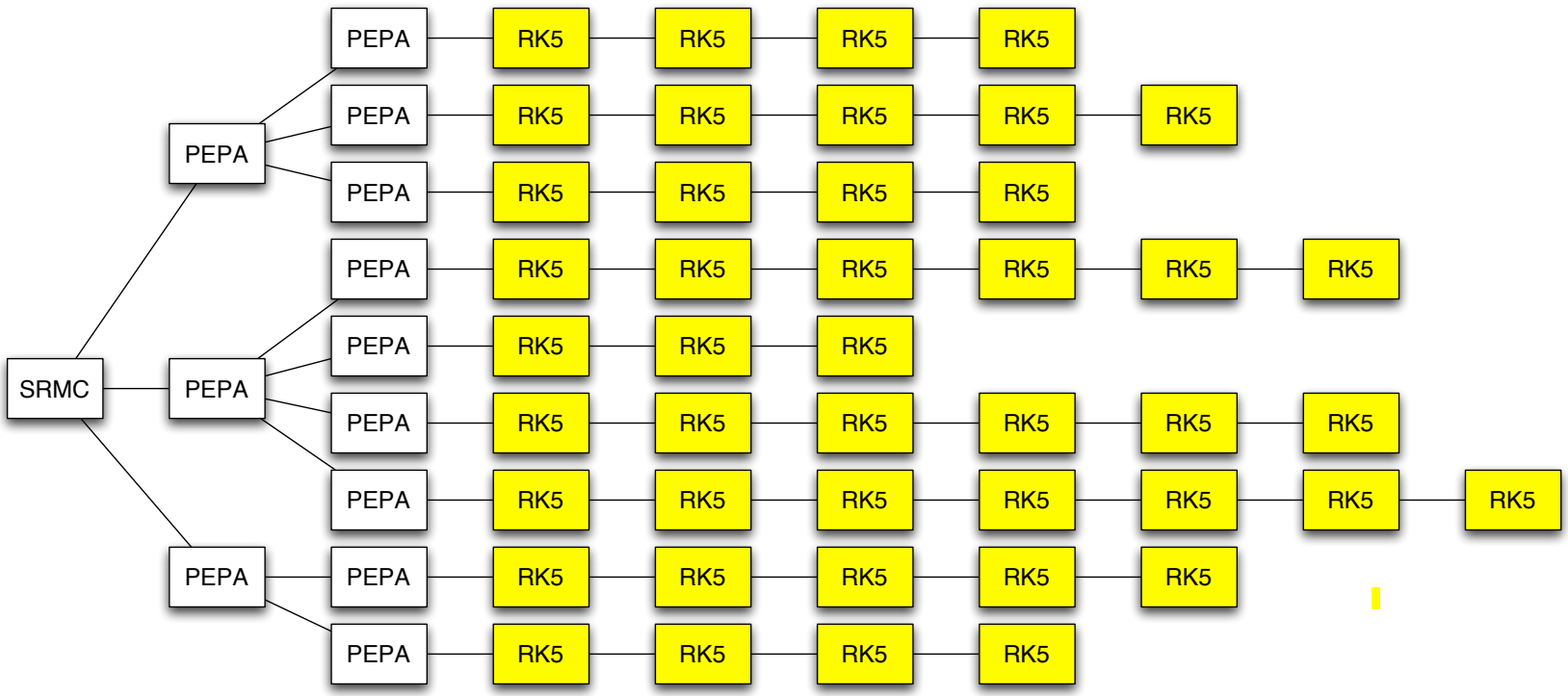
Apply model transformations

The evaluation model



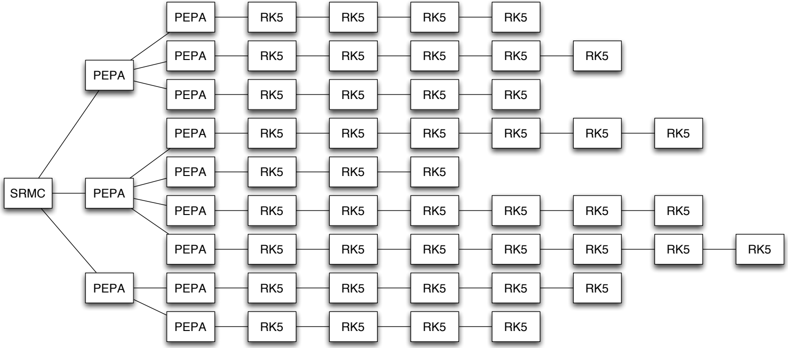
Parameter sweep

The evaluation model



Parameter sweep

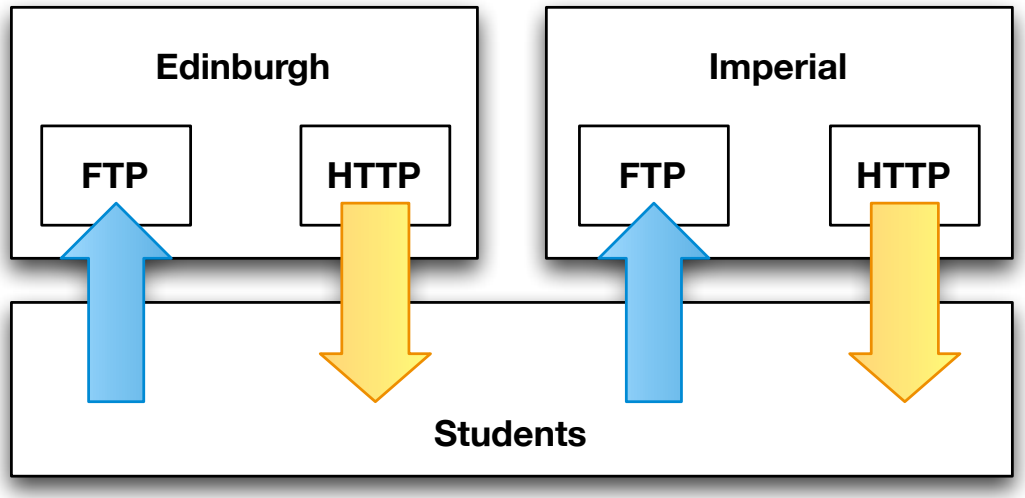
The evaluation model



Outline

Virtual university

Virtual university



- Two sites: Edinburgh and Imperial
- Students upload coursework to FTP servers
- Students download course content from HTTP servers
- We consider the system under different workloads

Edinburgh site

```
Edinburgh::{
  mu = 0.0001; gamma = 0.125; // rates of fail and repair
  avail = {0.6,0.7,0.8,0.9,1.0}; // server availability
  phi = 10.0; psi = 7.0; // rates for download and upload

  // The HTTP server
  Http::{
    Idle = (download, avail * phi).Idle + (fail, mu).Broken;
    Broken = (repair, gamma).Idle;
  };
  // The FTP server
  Ftp::{
    Idle = (upload, avail * psi).Idle + (fail, mu).Broken;
    Broken = (repair, gamma).Idle;
  };
};
```

Imperial site

```
Imperial::{
  mu = 0.006; gamma = 0.125; // failures are more likely
  avail = {0.6,0.7,0.8,0.9,1.0}; // availability is the same
  phi = 20.0; psi = 15.0; // download and upload are faster

  // The HTTP server
  Http::{
    Idle = (download, avail * phi).Idle + (fail, mu).Broken;
    Broken = (repair, gamma).Idle;
  };
  // The FTP server
  Ftp::{
    Idle = (upload, avail * psi).Idle + (fail, mu).Broken;
    Broken = (repair, gamma).Idle;
  };
};
```

Harry, a client

```
Harry::{
  c  = { 0.01, 0.02, 0.03 };  d = 1.0;
  δ  = { 0.01, 0.02, 0.03 }; μ = 1.0;

  // Model components
  Idle = (connect, c/2).Upload + (connect, c/2).Download;
  Upload = (upload, μ).Disconnect;
  Download = (download, δ).Disconnect;
  Disconnect = (disconnect, d).Idle;

  // Virtual components
  Uploading   = [ Upload ];
  Downloading = [ Download ];
  Inservice   = [ Uploading + Downloading ];
};
```

Sally, another client

```
Sally::{
  c  = { 0.009, 0.0095, 0.01 };   d = 0.5;
  δ = { 0.01, 0.02, 0.03 }; μ = 0.2;

  Idle = (connect, c/3).Upload
       + (connect, c/3).Download1
       + (connect, c/3).Download2 ;
  Upload    = (upload, μ).Disconnect;
  Download1  = (download, δ).Download2;
  Download2  = (download, δ).Disconnect;
  Disconnect = (disconnect, d).Idle;

  Uploading  = [ Upload ];
  Downloading = [ Download1 + Download2 ];
  Inservice  = [ Uploading + Downloading ];
};
```

Model composition

Configuration

```
Client ::= { Harry, Sally };
Http   ::= { Edinburgh::Http, Imperial::Http };
Ftp    ::= { Edinburgh::Ftp, Imperial::Ftp };
```

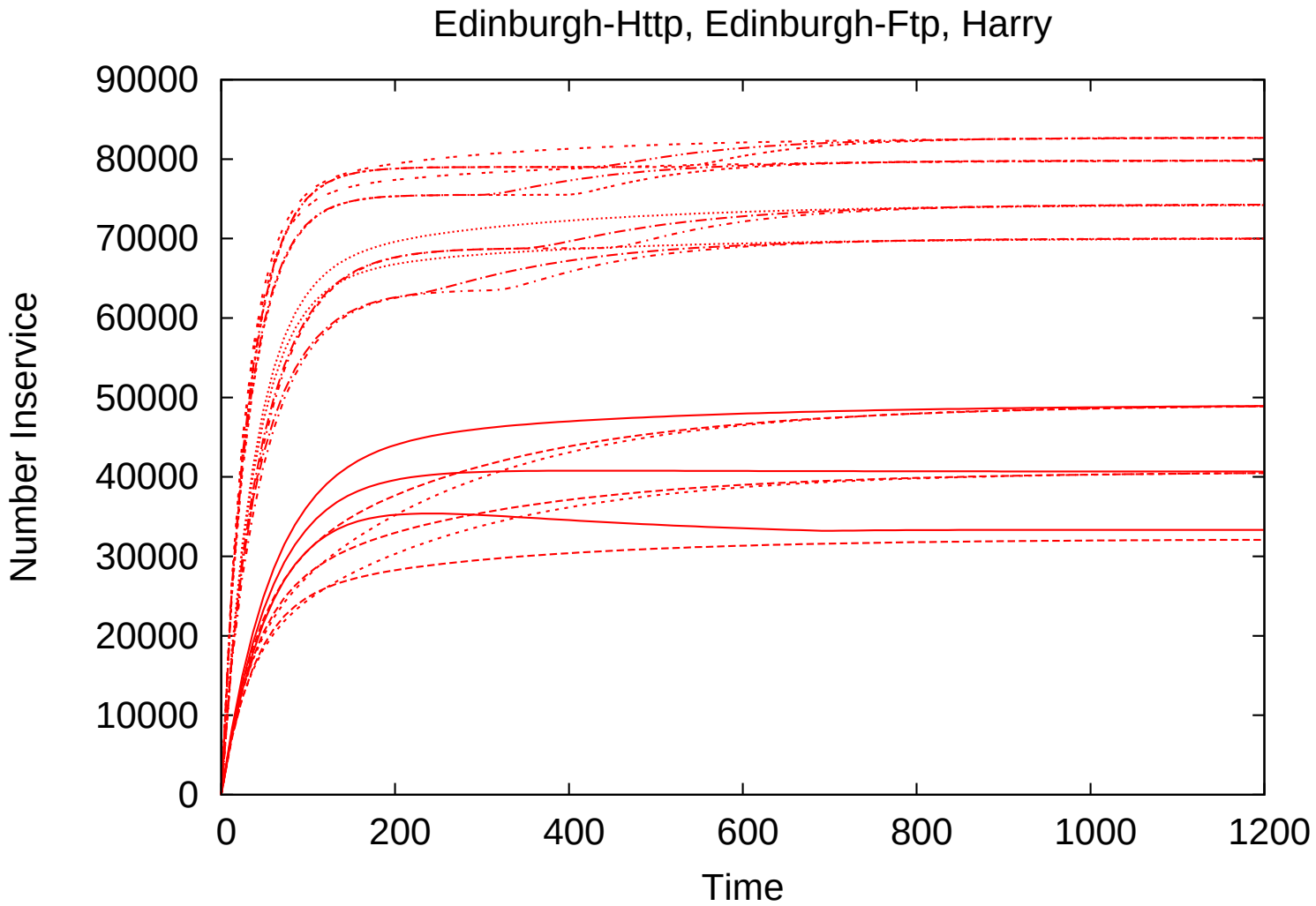
Parameters

```
servers = { 3, 4, 5, 6 }; threads = 20; clients = 100000;
```

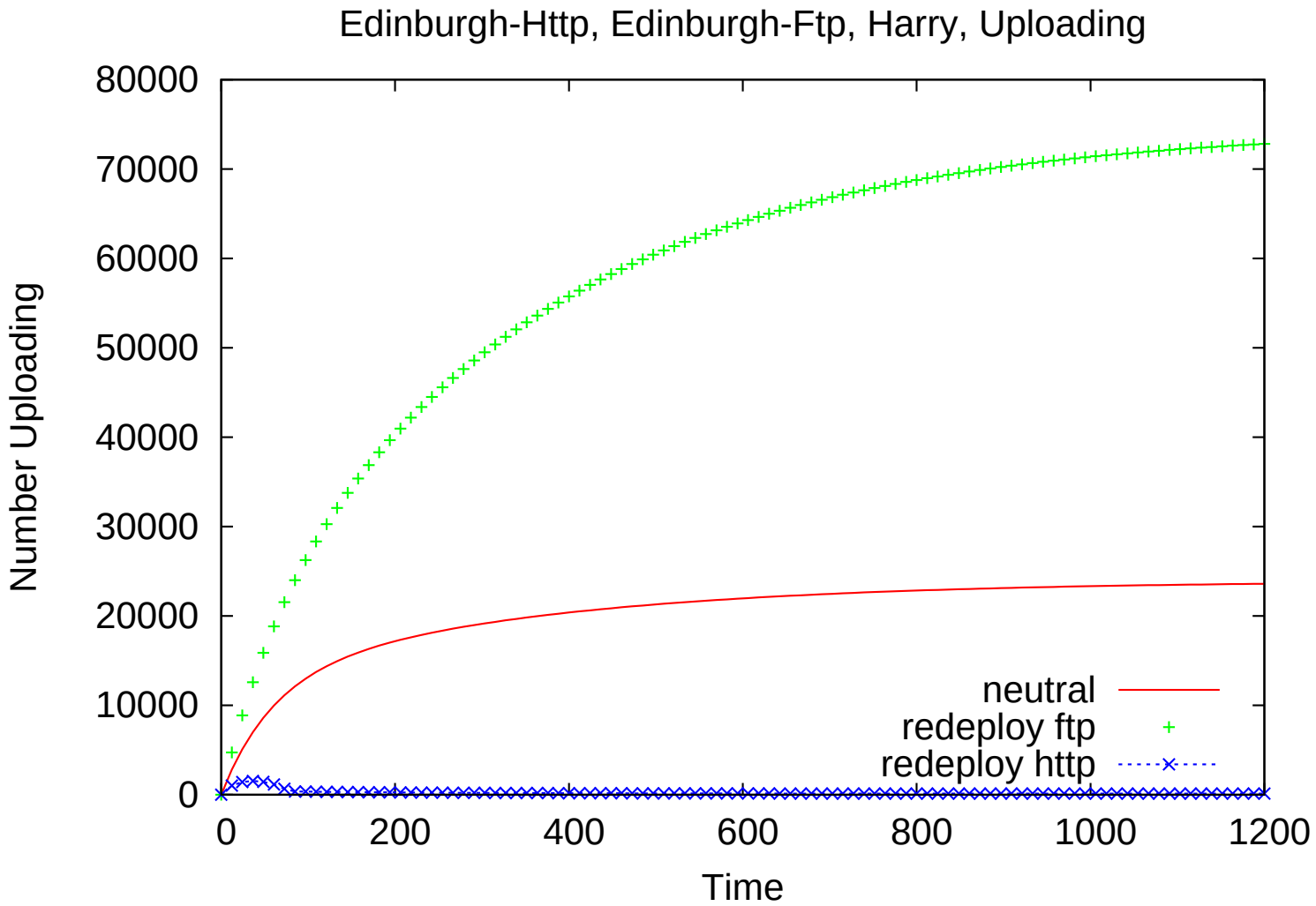
Model

```
Client::Idle[clients]
          <download, upload> (Http::Idle[servers * threads]
                             ||Ftp::Idle[servers * threads])
```

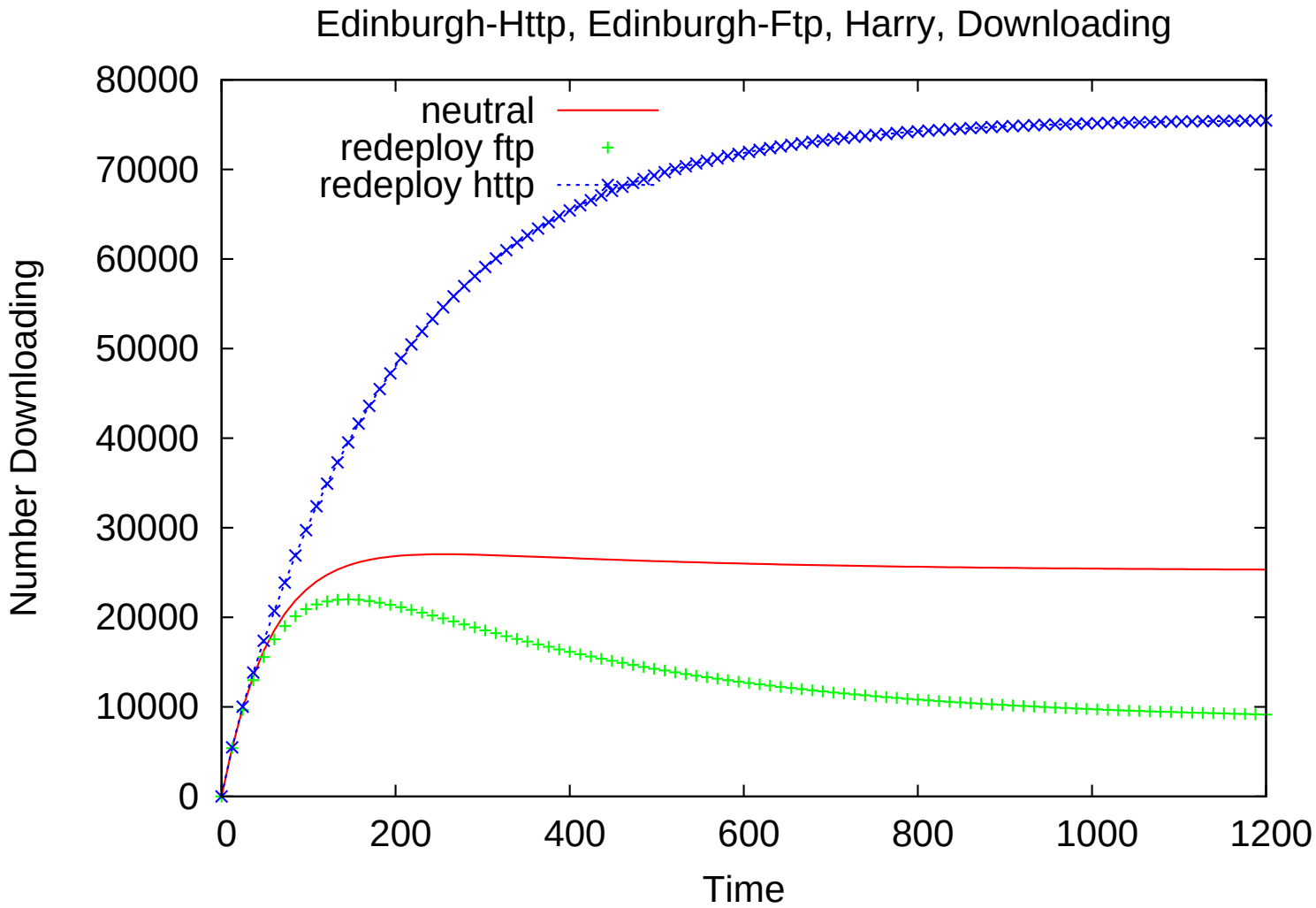
First configuration



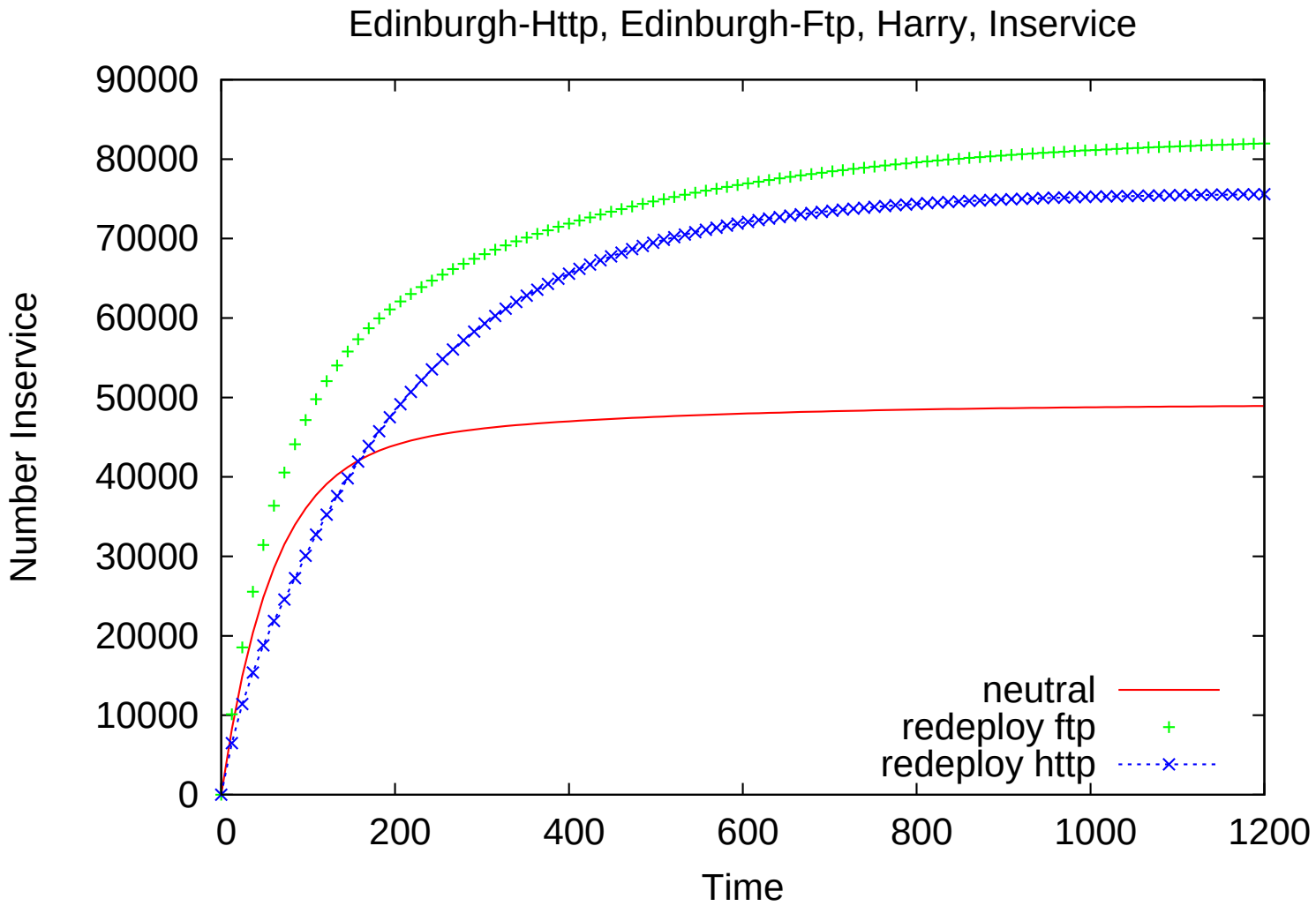
First configuration, uploading



First configuration, downloading



First configuration, number in service



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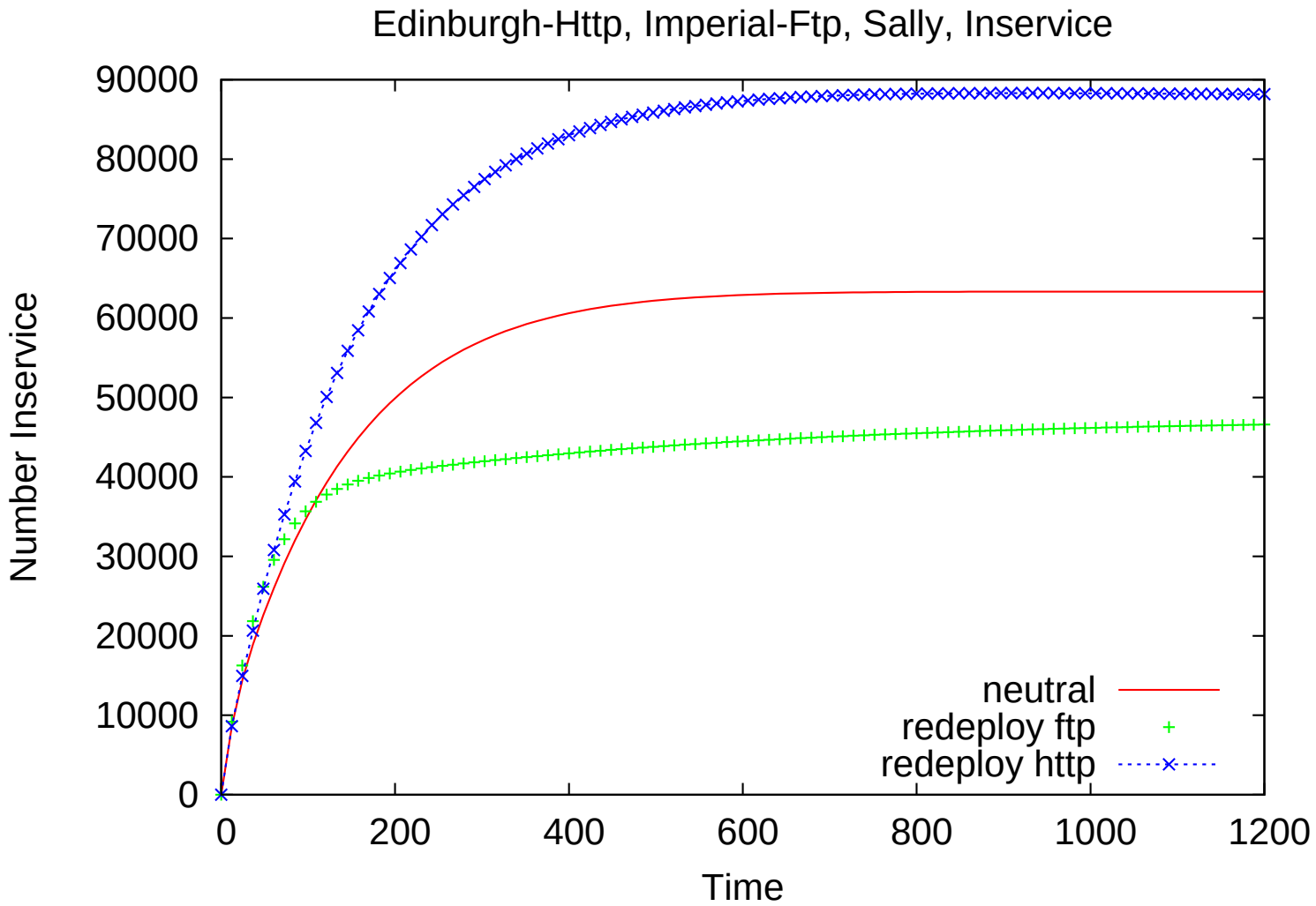
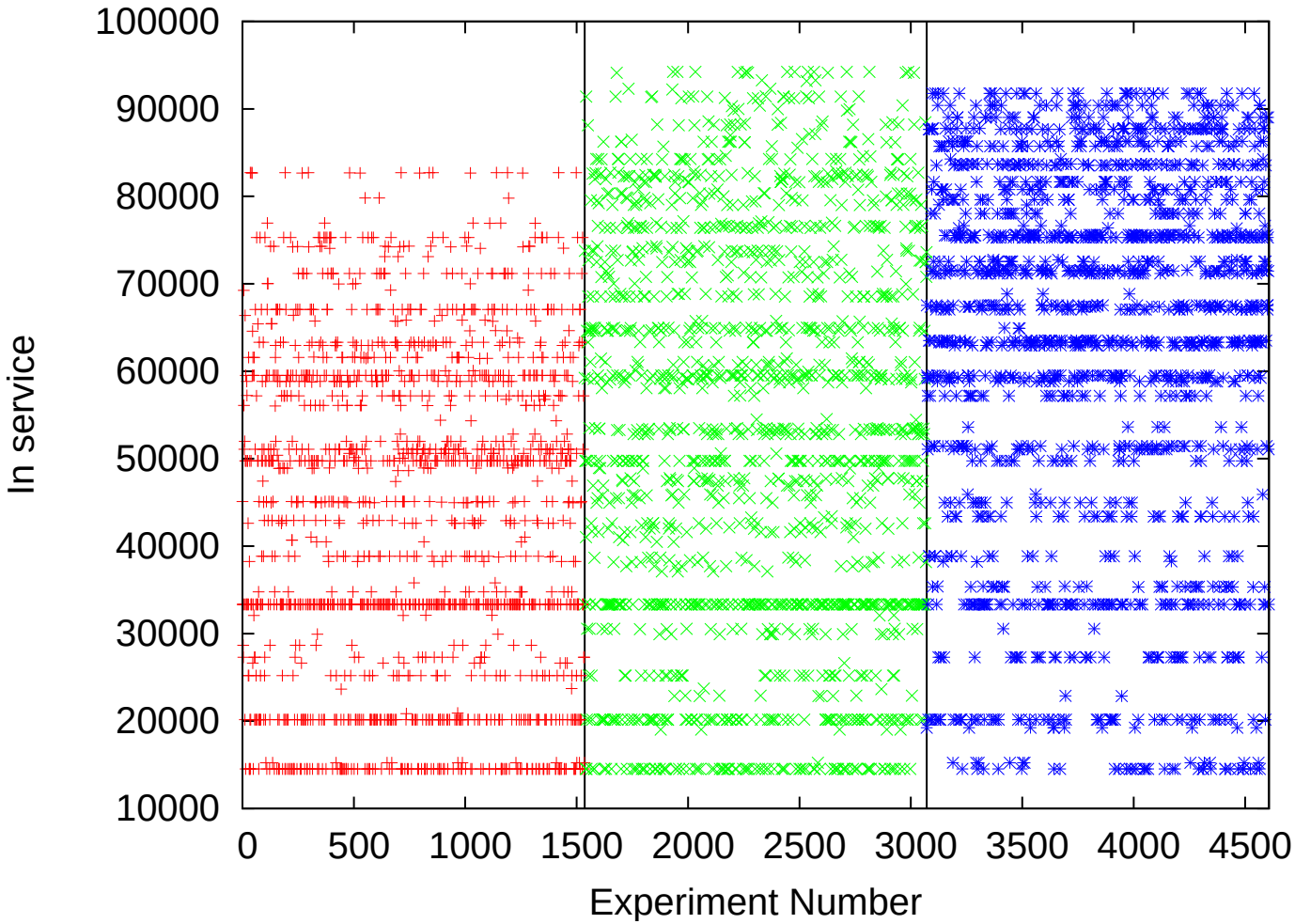
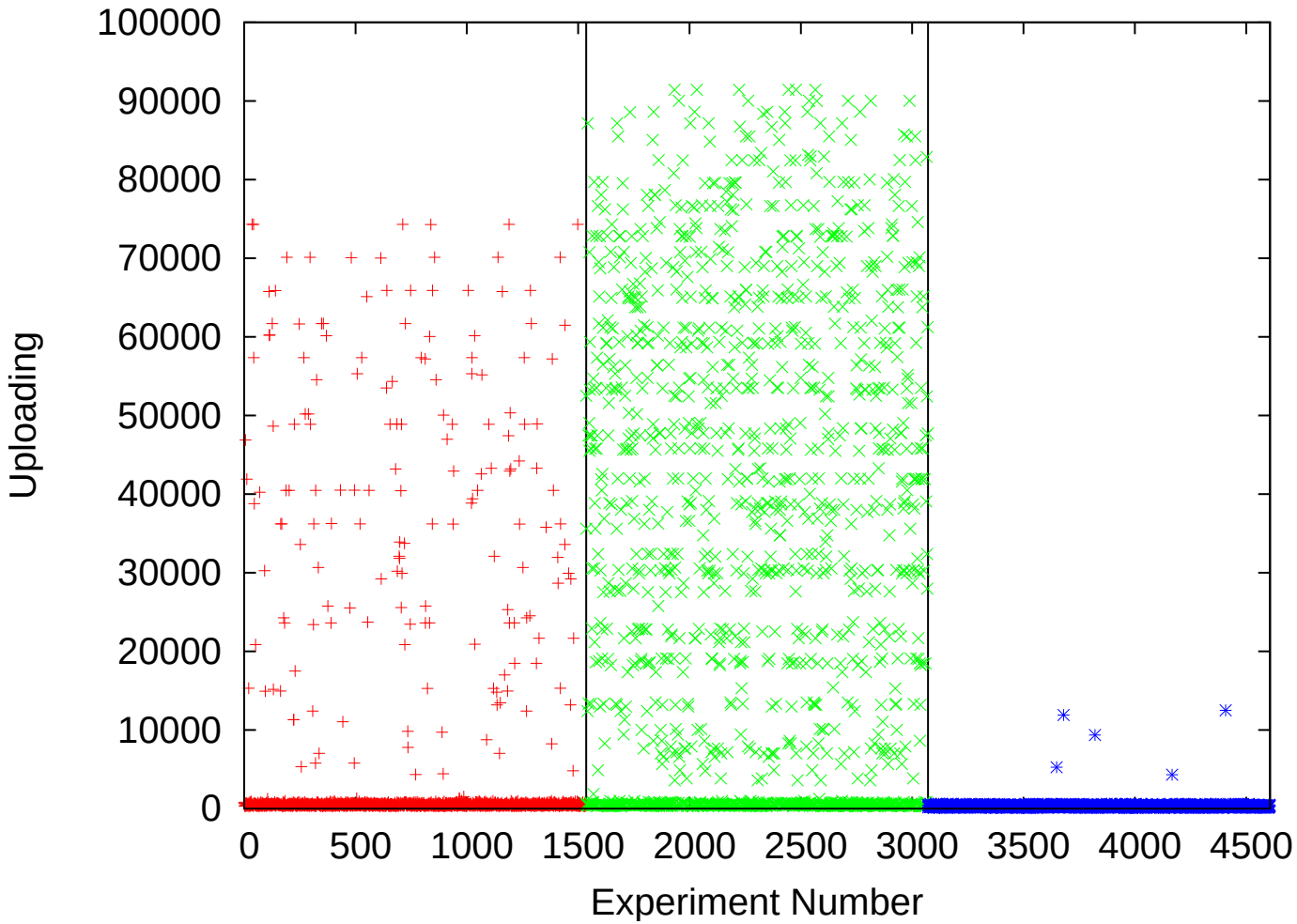


Effect of redeployments on number in service



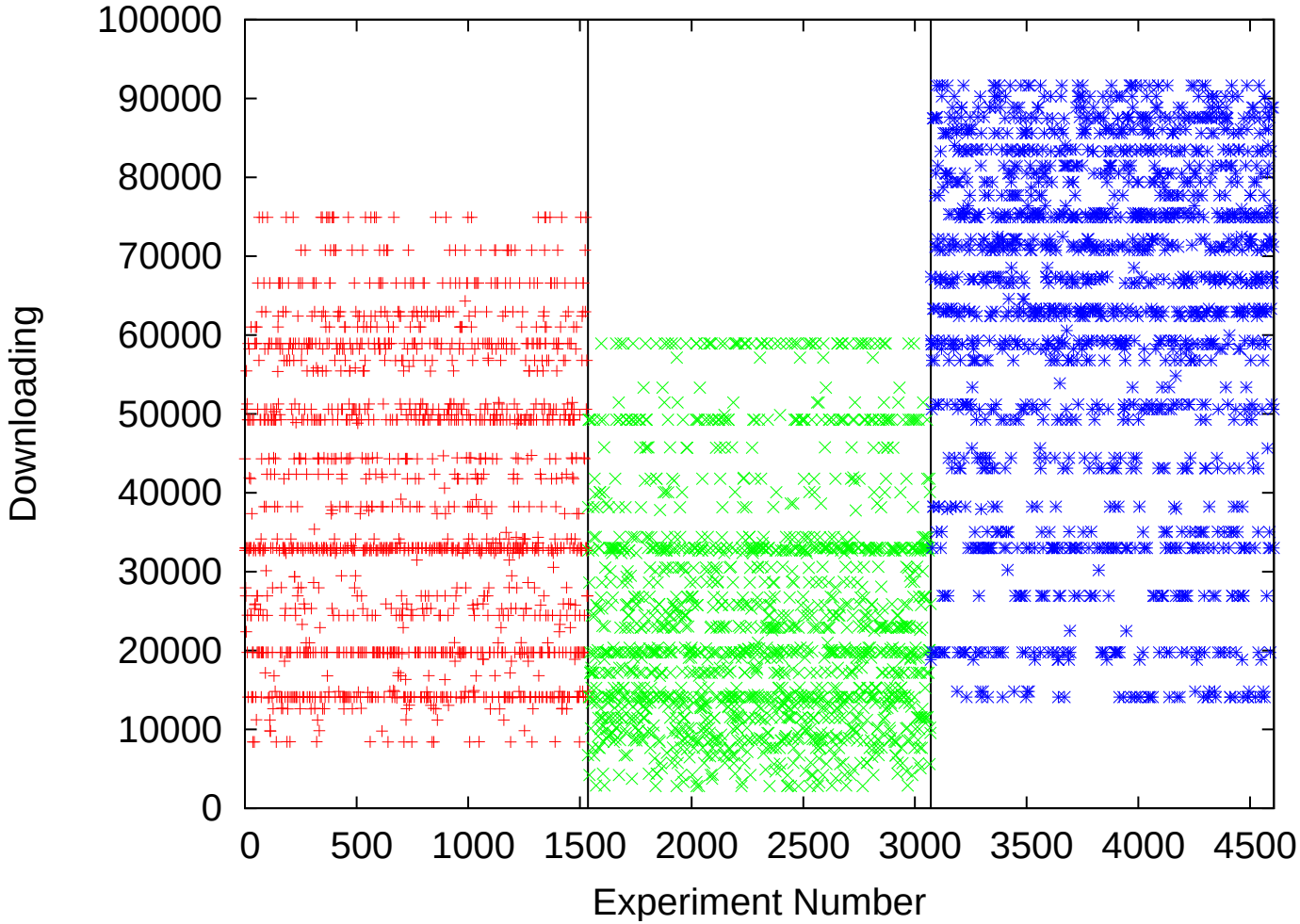
Red: no change, Green: redeploy FTP, Blue: redeploy HTTP

Effect of redeployments on uploading



Red: no change, Green: redeploy FTP, Blue: redeploy HTTP

Effect of redeployments on downloading



Red: no change, Green: redeploy FTP, Blue: redeploy HTTP

Conclusions

- Scalable systems are difficult to develop and difficult to analyse.
- Component-based modelling supported by divide-and-conquer analysis seems a promising approach.
- We analyse configurations by cases and perform parameter sweep across uncertain parameters.
- Hopefully a “real world” approach to a “real world” problem.

Website

http://groups.inf.ed.ac.uk/srmc/

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