Process algebras for quantitative analysis: Lecture 3 — Case Studies

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Outline

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- This can be viewed as a graph in which each node is a state of the model (comprised of the local states of each of the components) and the arcs represent the actions which can cause the move from one state to another.
- The language is also equipped with observational equivalence which can be used to compare models.

PEPA Eclipse Plug-In input

$$P_1 \stackrel{def}{=} (start, r_1).P_2 \qquad P_2 \stackrel{def}{=} (run, r_2).P_3 \qquad P_3 \stackrel{def}{=} (stop, r_3).P_1$$

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State space

1	$P_1 \parallel P_1$
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3	$P_2 \parallel P_1$
4	$P_1 \parallel P_3$
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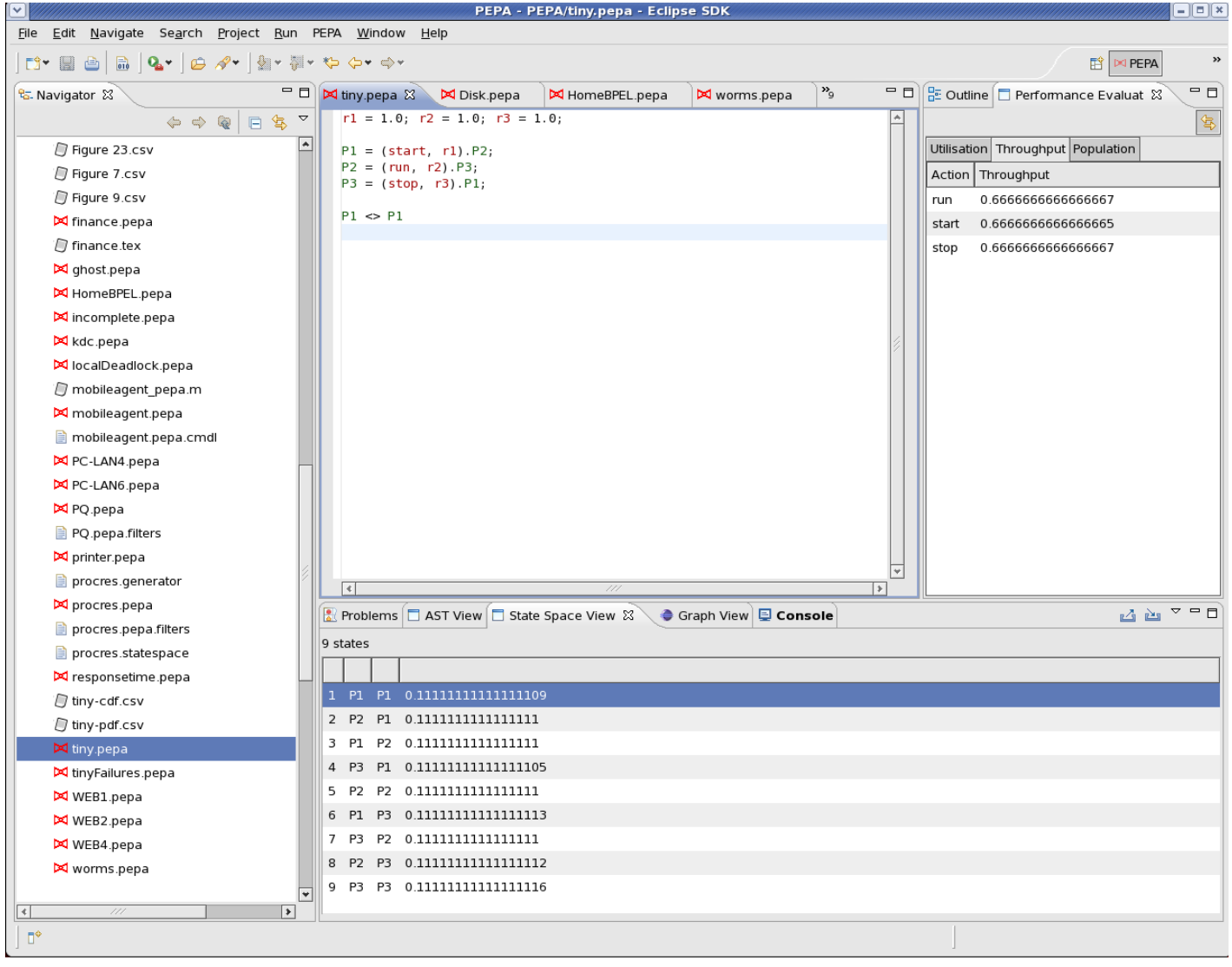
CTMC representation computed by the plug-in

$$\begin{pmatrix}
-2r_1 & r_1 & r_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -r_1 - r_2 & 0 & r_2 & r_1 & 0 & 0 & 0 & 0 \\
0 & 0 & -r_1 - r_2 & 0 & r_1 & r_2 & 0 & 0 & 0 \\
r_3 & 0 & 0 & -r_1 - r_3 & 0 & 0 & 0 & r_1 & 0 \\
0 & 0 & 0 & 0 & -2r_2 & 0 & r_2 & r_2 & 0 \\
r_3 & 0 & 0 & 0 & 0 & -r_1 - r_3 & r_1 & 0 & 0 \\
0 & r_3 & 0 & 0 & 0 & 0 & -r_2 - r_3 & 0 & r_2 \\
0 & 0 & r_3 & 0 & 0 & 0 & 0 & -r_2 - r_3 & r_2 \\
0 & 0 & 0 & r_3 & 0 & r_3 & 0 & 0 & -2r_3
\end{pmatrix}$$

The PEPA Eclipse Plug-in processing the model



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- Switch behaviour in active networks (Hillston, Kloul and Mokhtari, Edinburgh and Versailles)

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- Disease spread within animal populations (Benkrine, McCaig, Norman and Shankland, Stirling)

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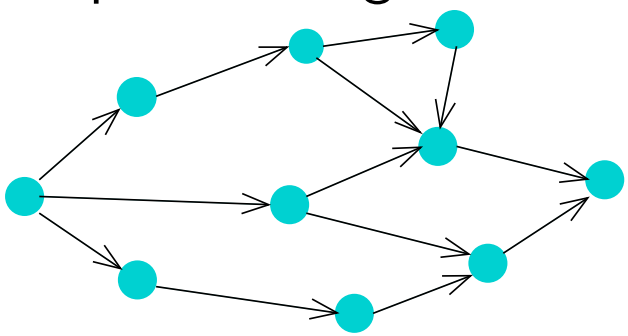

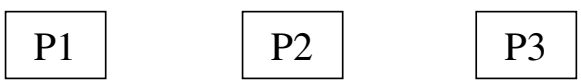

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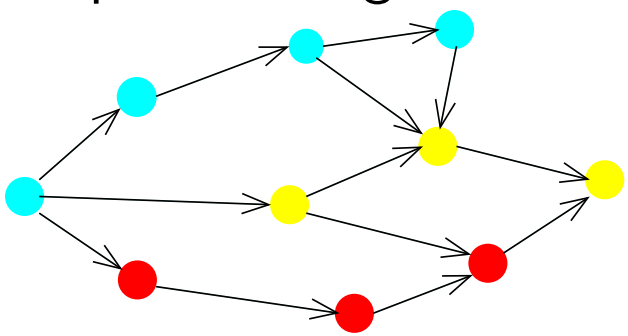

P1 P2 P3

A schedule maps tasks to processors

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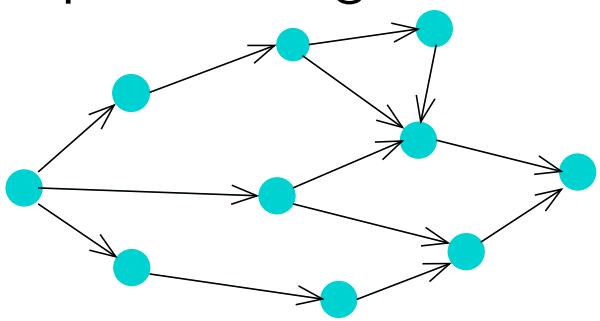

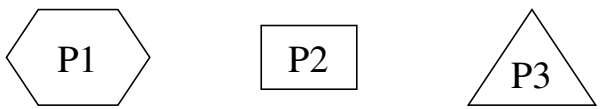

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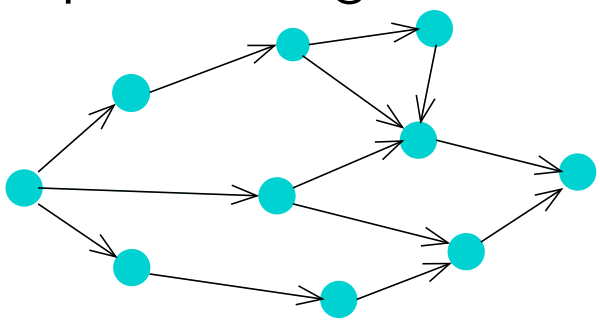

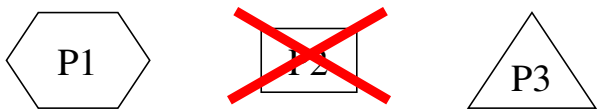

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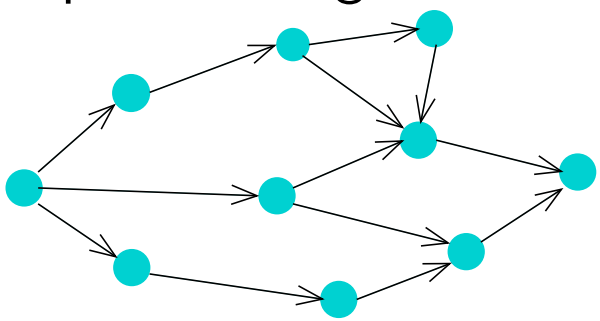

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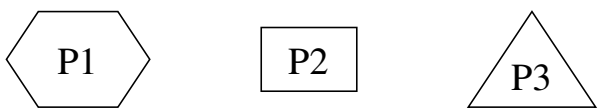

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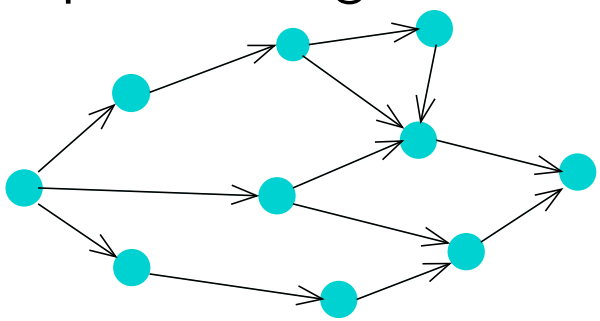

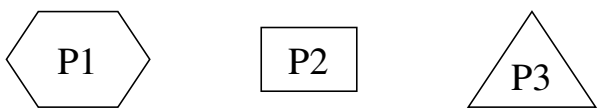

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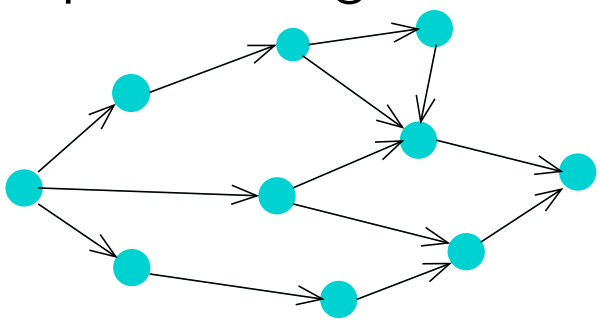

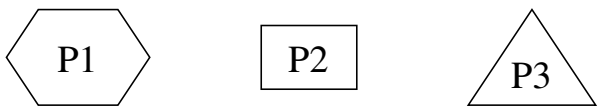

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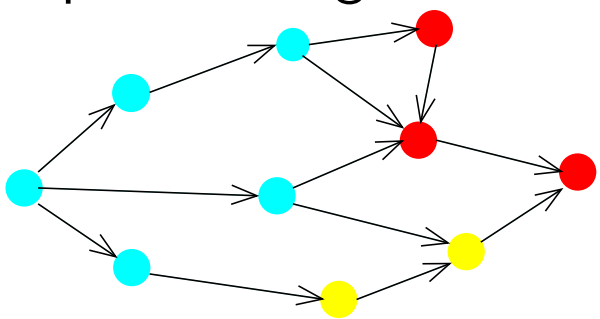

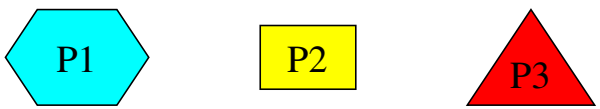

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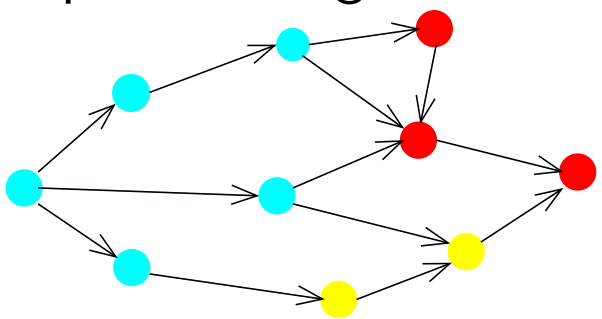

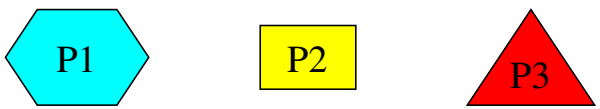

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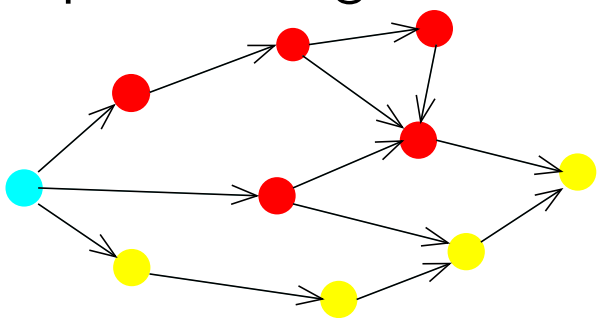

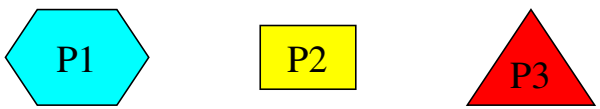

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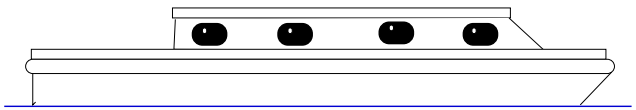

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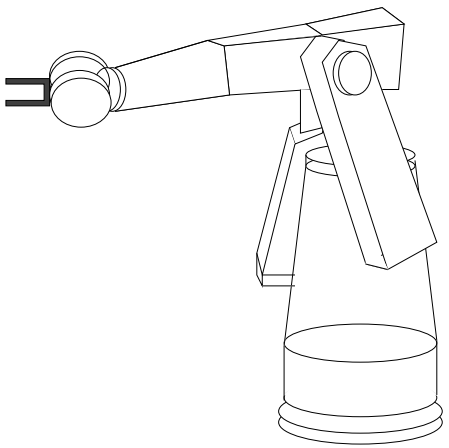

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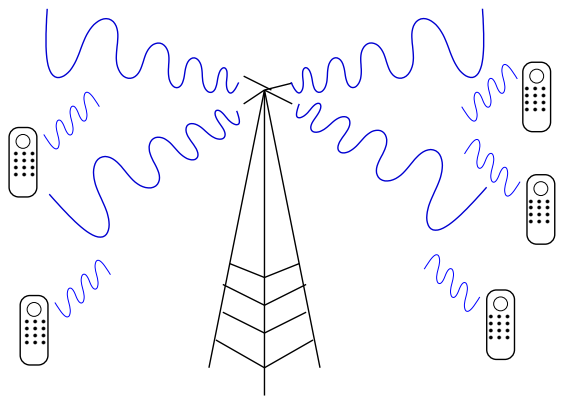

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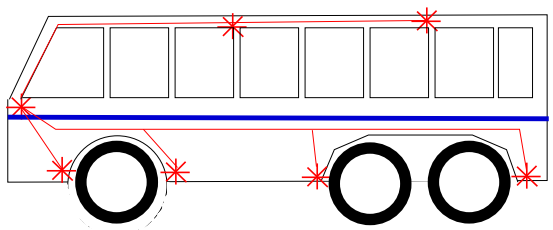

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Upgrading a PC LAN

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A node has control of the medium, i.e. it can transmit, only whilst it holds the token. In a PC LAN every PC corresponds to a node on the network. Other nodes on the network might be peripheral devices such as printers or faxes but for the purposes of this study we make no distinction and assume that all nodes are PCs.

Upgrading a PC LAN

There are currently four PCs (or similar devices) connected to the LAN in a small office, but the company has recently recruited two new employees, each of whom will have a PC. Our task is to find out how the delay experienced by data packets at each PC will be affected if another two PCs are added.

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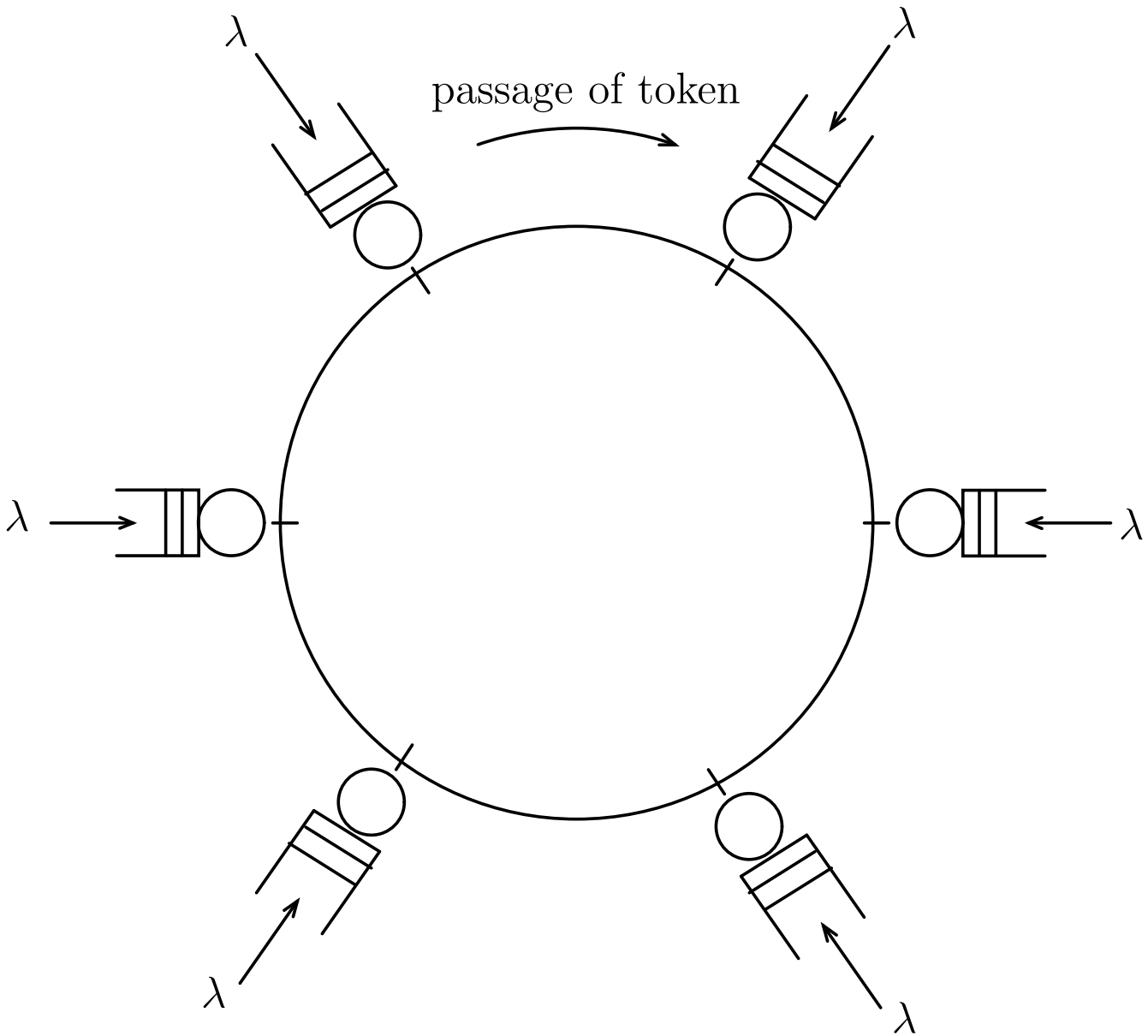
It is assumed that if another data packet is generated, whilst the PC is transmitting, this second data packet must wait for the next visit of the token before it can be transmitted. In other words, each PC can transmit at most one data packet per visit of the token.


λ
λ
passage of token
λ
λ
λ
λ

Modelling the system: choosing components

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We also need to represent the medium in some way. In fact for the token ring it is only necessary to represent the token as this is sufficient to determine whether a PC can engage in its desired actions or not.

Modelling the system: choosing actvities

The description of the PC is very simple in this case. It only has two activities which it can undertake:

- generate a data packet;
- transmit a data packet.

Moreover we are told that it can only hold one data packet at a time and so these activities must be undertaken sequentially.

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This suggests the following PEPA component for the ith PC:

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However we will see that this needs some refinement.

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$$Token_i \stackrel{def}{=} (walkon_{i+1}, \omega).Token_{i+1} + (serve_i, \mu).(walk_{i+1}, \omega).Token_{i+1}$$

Refining the components

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$$PC_{i1} \stackrel{def}{=} (serve_i, \mu).PC_{i0}$$

Complete model: four PC case

$$
\begin{aligned}
PC_{10} &\stackrel{def}{=} (arrive, \lambda).PC_{11} + (walkon_2, \omega).PC_{10} \\
PC_{11} &\stackrel{def}{=} (serve_1, \mu).PC_{10}
\end{aligned}
$$

$$
\begin{aligned}
PC_{20} &\stackrel{def}{=} (arrive, \lambda).PC_{21} + (walkon_3, \omega).PC_{20} \\
PC_{21} &\stackrel{def}{=} (serve_2, \mu).PC_{20}
\end{aligned}
$$

$$
\begin{aligned}
PC_{30} &\stackrel{def}{=} (arrive, \lambda).PC_{31} + (walkon_4, \omega).PC_{30} \\
PC_{31} &\stackrel{def}{=} (serve_3, \mu).PC_{30}
\end{aligned}
$$

$$
\begin{aligned}
PC_{40} &\stackrel{def}{=} (arrive, \lambda).PC_{41} + (walkon_1, \omega).PC_{40} \\
PC_{41} &\stackrel{def}{=} (serve_4, \mu).PC_{40}
\end{aligned}
$$

$$
\begin{aligned}
Token_1 &\stackrel{def}{=} (walkon_2, \omega).Token_2 + (serve_1, \mu).(walk_2, \omega).Token_2 \\
Token_2 &\stackrel{def}{=} (walkon_3, \omega).Token_3 + (serve_2, \mu).(walk_3, \omega).Token_3 \\
Token_3 &\stackrel{def}{=} (walkon_4, \omega).Token_4 + (serve_3, \mu).(walk_4, \omega).Token_4 \\
Token_4 &\stackrel{def}{=} (walkon_1, \omega).Token_1 + (serve_4, \mu).(walk_1, \omega).Token_1
\end{aligned}
$$

$$
\begin{aligned}
LAN &\stackrel{def}{=} (PC_{10} \parallel PC_{20} \parallel PC_{30} \parallel PC_{40}) \underset{L}{\bowtie} Token_1 \\
&\quad \text{where } L = \{walkon_1, walkon_2, walkon_3, walkon_4, \\
&\qquad\qquad serve_1, serve_2, serve_3, serve_4\}.
\end{aligned}
$$

Here we have arbitrarily chosen a starting state in which all the PCs are empty and the Token is at PC1.

State space size

N	2	3	4	5	6	8
$\|S\|$	16	48	128	320	768	4096

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N	10	20	30
$\|S\|$	2048	4.194304×10^7	6.442450×10^{10}

Performance results

In order to calculate performance measures from the model we must first assign values to the parameters.

The following values were assigned to parameters of the model in both cases (the unit of time is assumed to be milliseconds),

$$\lambda = 0.01 \qquad d = 10 \qquad \mu = 0.1 \qquad m = 1.0 \qquad \omega = 1.0$$

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$$\lambda = 0.01 \qquad d = 10 \qquad \mu = 0.1 \qquad m = 1.0 \qquad \omega = 1.0$$

We wish to calculate the average waiting time of data packets at any PC in the network. Since all the PCs are statistically identical, we can arbitrary choose one as representative—we select PC_1.

Derivation of performance measures

Roughly speaking, there are three ways in which performance measures can be derived from the steady state distribution of a Markov process.

These different methods can be thought of as corresponding to different types of measure:

- state-based measures, e.g. utilisation;
- rate-based measures, e.g. throughput;
- other measures which fall outside the above categories, e.g. response time.

State-based measures

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Thus in order to calculare utilisation we sum the steady state probabilities of being in any of the states where the resource is in use.

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This will be the product of the rate of the event, and the probability that the event is enabled, i.e. the probability of being in one of the states from which the event can occur.

Thus to calculate the throughput of the transmission we consider the probability of being a state where tranmission can occur and multiply it by μ the transmission rate.

Calculating waiting time

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However, if we know the average number of data packets at the interface, and the average throughput, we can apply Little's law to calculate residence time

$$\mathcal{R} = \frac{\mathcal{N}}{X}$$

Calculating waiting time

Once we have the residence time the waiting time can be calculated as:

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Similarly, a data transmission can occur whenever there is a data packet waiting for transmission and the token is at PC1. So the average throughput X will be the rate at which transmission occurs, μ, multiplied by the total probability of being in one of these states.

Predicted waiting time

Based on these expressions and the calculated steady state distributions we find the following values:
for 4 PCs

average waiting time, $\mathcal{W} = \frac{0.1333}{0.008666} - \frac{1}{0.1} = 15.3862 - 10 = 5.3862$

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for 6 PCs

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$$\text{average waiting time, } \mathcal{W} = \frac{0.1719}{0.00828} - \frac{1}{0.1} = 20.7678 - 10 = 10.7678$$

Thus the average waiting time for data packets will almost double when two more PCs are added to the network.

Outline

Roland the Gunslinger

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  - illustrate the main features of the language,
  - give you some experience of how models are constructed, and
  - demonstrate a variety of solution techniques.

Roland alone

In the first scenario we consider Roland alone, with the single activity of firing his gun which is a six-shooter. When his gun is empty Roland will reload the gun and then continue shooting.

$$
\begin{aligned}
Roland_6 &\stackrel{def}{=} (fire, r_{fire}).Roland_5 \\
Roland_5 &\stackrel{def}{=} (fire, r_{fire}).Roland_4 \\
Roland_4 &\stackrel{def}{=} (fire, r_{fire}).Roland_3 \\
Roland_3 &\stackrel{def}{=} (fire, r_{fire}).Roland_2 \\
Roland_2 &\stackrel{def}{=} (fire, r_{fire}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (fire, r_{fire}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_6
\end{aligned}
$$

Roland with two guns

All self-respecting gun-slingers have one gun in each hand. If we suppose that Roland has two guns then he should be allowed to fire either gun independently. A simplistic way to model this is to have two instances of *Roland* in parallel:

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However, this model does not capture the fact that Roland needs both hands in order to reload either gun. The simplest way to fix this is to assume that Roland only reloads both guns when both are empty.

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$$Roland_6 \underset{\{reload\}}{\bowtie} Roland_6$$

From now on we restrict Roland to his shotgun, which has two bullets and requires both hands for firing.

Roland meets an Enemy

- Upon his travels Roland encounters some enemies and when he does so he must fight them.
- Roland is the wildest gunslinger in the west so we assume that no enemy has the skill to seriously harm Roland.
- Each time Roland fires he might miss or hit his target.
- But with nothing to stop him he will keep firing until he successfully hits (and kills) the enemy.
- We assume that some sense of cowboy honour prevents any enemy attacking Roland if he is already involved in a gun fight.

The model

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, r_{attack}).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\quad +(miss, r_{miss}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\quad +(miss, r_{miss}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2
\end{aligned}
$$

Parameter settings for the $Roland_2$ model

parameter	*value*	*explanation*
r_{fire}	1.0	Roland can fire the gun once per-second
$p_{hit\text{-}success}$	0.8	Roland has an 80% success rate
r_{hit}	0.8	$r_{fire} \times p_{hit\text{-}success}$
r_{miss}	0.2	$r_{fire} \times (1 - p_{hit\text{-}success})$
r_{reload}	0.3	It takes Roland about 3 seconds to reload
r_{attack}	0.01	Roland is attacked once every 100 seconds

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```
 State Measure 'roland peaceful'
mean                9.5490716180e-01
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```
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```

$>$ 95% chance that Roland is not currently involved in a gun battle.

Passage-Time Analysis

Passage-time analysis allows us to calculate measures such as the probability that Roland has killed his enemy at a given time after he is attacked.

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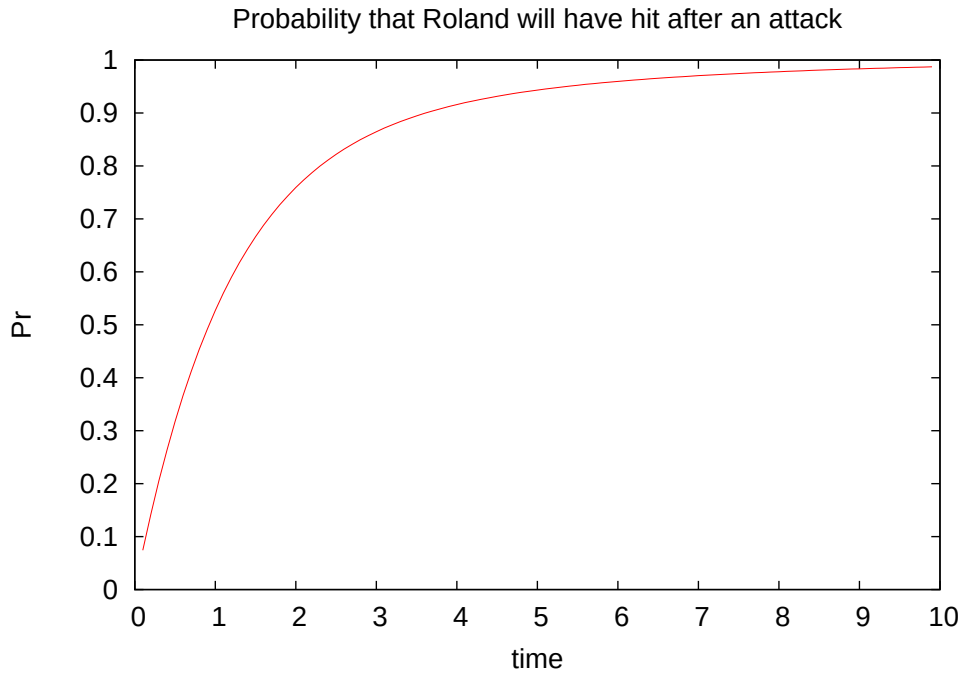
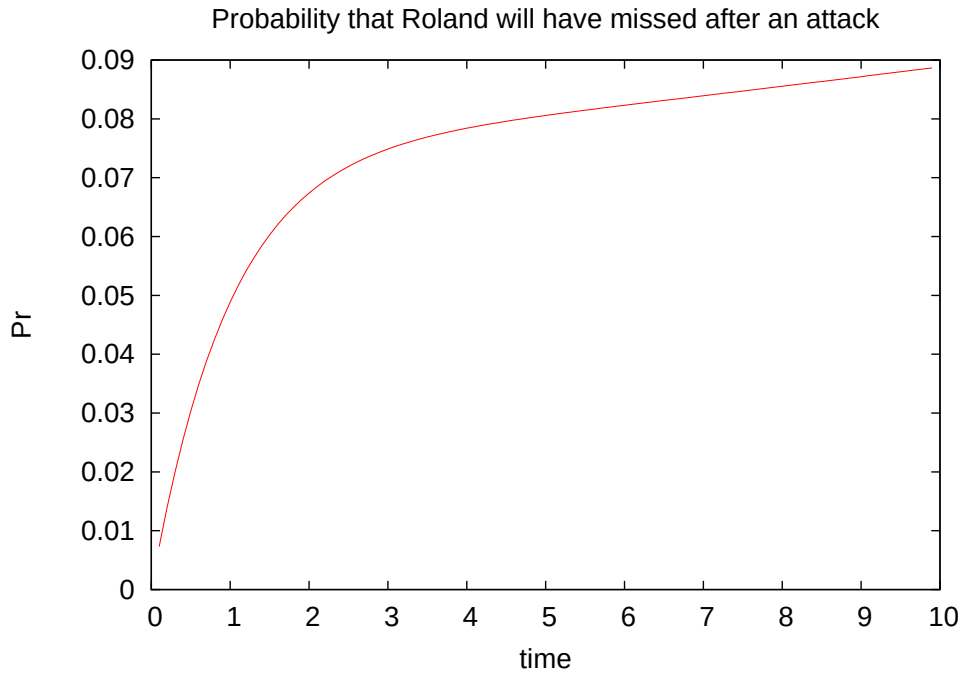
This would involve calculating the probability that the model performs a *hit* action within the given time after performing an *attack* action.

Passage-Time Analysis results



The probability that Roland will successfully perform a *hit* action a given time after an *attack*. Gun battles typically last about five seconds and one occurs about once every 100 seconds. The probability that Roland has performed a *hit* action five seconds after an *attack* action is $\approx 90\%$

Passage-Time Analysis results



The probability that Roland has performed a *miss* action a given time after an *attack* action. These probabilities are much lower because Roland's probability of success are high and if he successfully kills an enemy we must wait for the next attack in order to have a chance of seeing a *miss*.

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- We now consider a model in which the enemies appear explicitly and allow them to fight back.
- However for now we still assume that they are rather ineffectual and so they never seriously injure Roland.
- This model can be used to calculate properties such as the likelihood that an enemy will manage to fire one shot before they are killed by Roland.

Revised Model

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\qquad + (miss, r_{miss}).Roland_1 \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&\qquad + (miss, r_{miss}).Roland_{empty} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2 \\
\\
Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\
Enemies_{attack} &\stackrel{def}{=} (fire, r_{e\text{-}miss}).Enemies_{attack} \\
&\qquad + (hit, \top).Enemies_{idle} \\
\\
Roland_2 &\underset{\{hitattack\}}{\bowtie} Enemies_{idle}
\end{aligned}
$$

Additional parameters

parameter	*value*	*explanation*
r_{attack}	0.01	Roland is attacked once every 100 seconds
$r_{e\text{-}miss}$	0.3	Enemies can fire only once every 3 seconds

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- This model can be thought of as an approximation to a more complicated component similar to the one which models Roland.
- Here the rate at which the enemy fires encompasses all of the actions, including the reloading of an empty gun.
- We may choose to model a component in such an abstract way when the focus of our modelling is really elsewhere in the model.

Model Validation

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For example in this model we could make such a sanity check by calculating the probability that the model is in a state in which Roland is idle but the enemies are not, or vice versa.

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For example in this model we could make such a sanity check by calculating the probability that the model is in a state in which Roland is idle but the enemies are not, or vice versa.

This should never occur and hence the probability should be zero.

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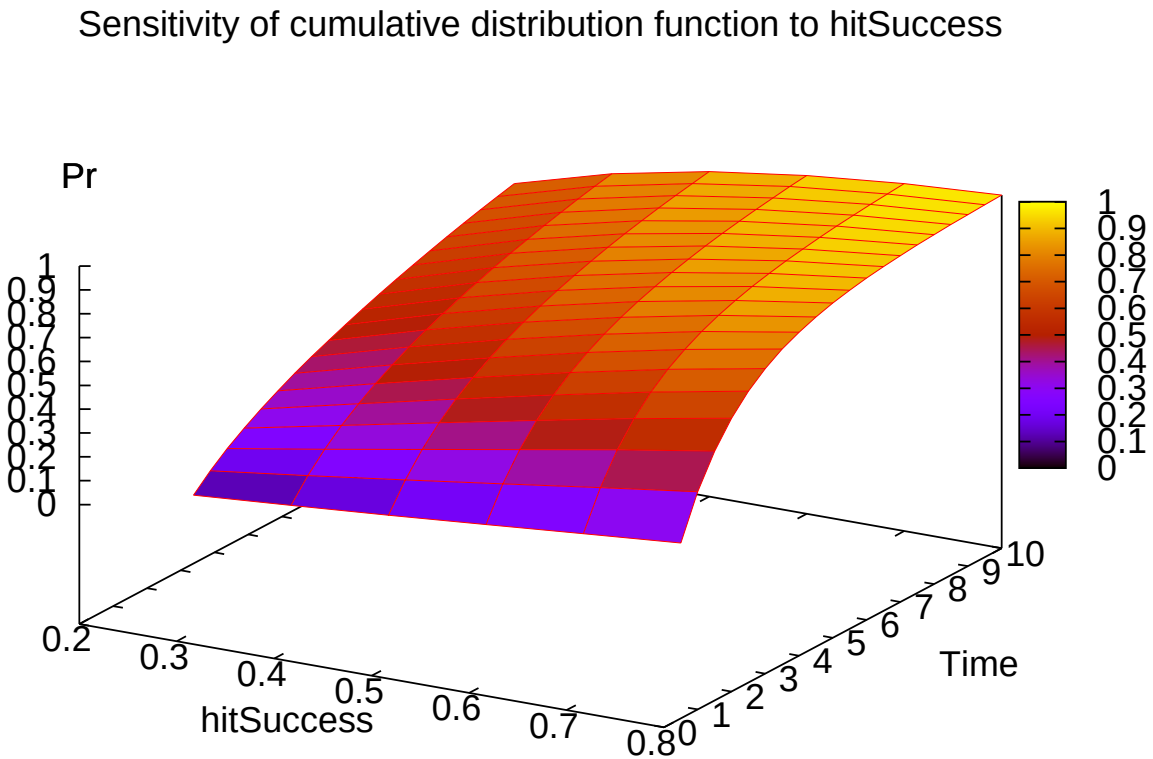
- Sensitivity analysis is performed by solving the model many times while varying the rates slightly.
- For this model we chose to vary three of the rates involved and measured the passage time between an *attack* and a *hit* activity, for each combintation of rates.

Sensitivity Analysis: Results



The effect of varying the $p_{hit\text{-}success}$ parameter.

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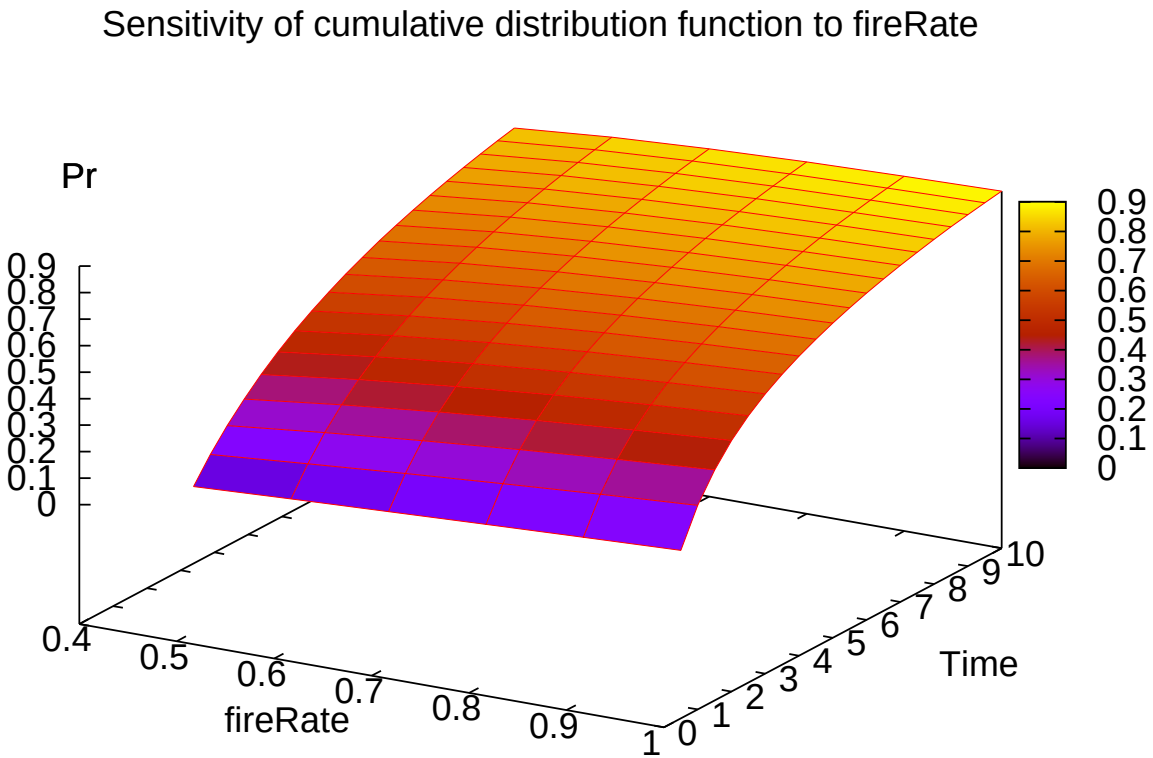

The effect of varying the r_{fire} parameter.

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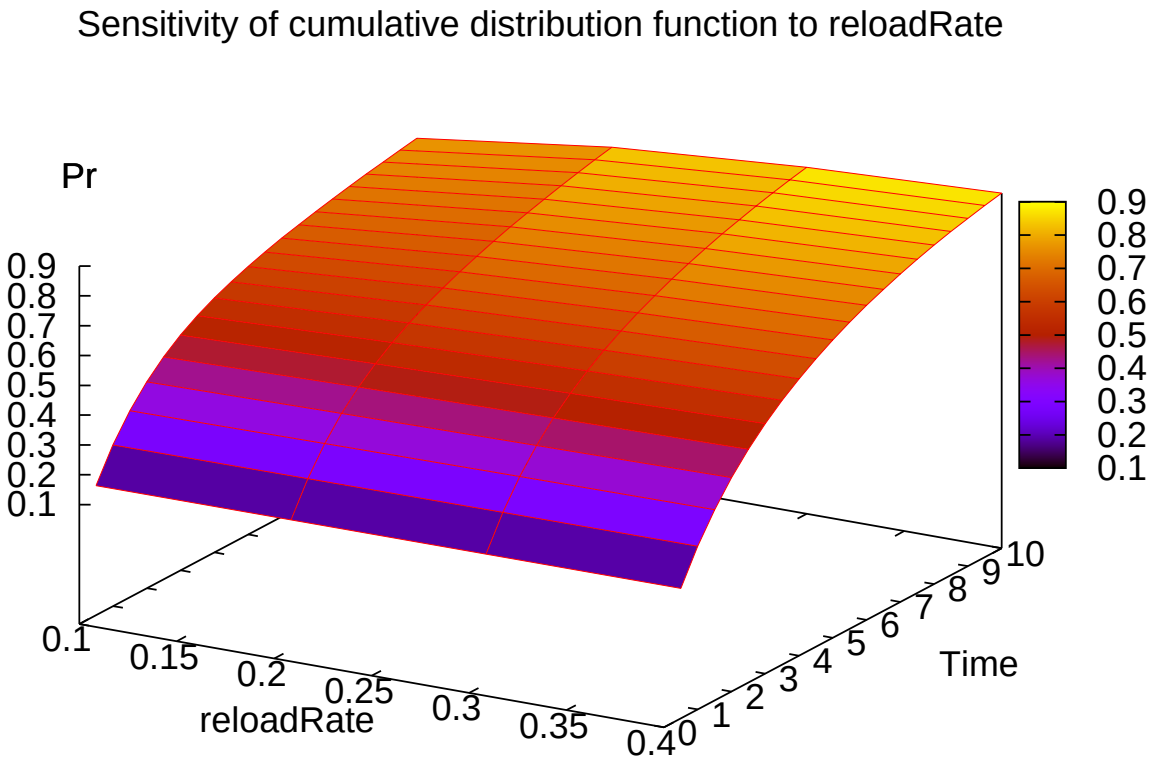

The effect of varying the r_{reload} parameter.

Accurate Enemies

- We now allow the enemies of Roland to actually hit him. This means that Roland may die. It is important to note that this has the consequence that the model will always deadlock. The underlying Markov process is no longer ergodic.

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- The only new parameter is $r_{e\text{-}hit}$ which is assigned a value 0.02 to reflect this assumption.

New Roland

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&+ (miss, r_{miss}).Roland_1 \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).(reload, r_{reload}).Roland_{idle} \\
&+ (miss, r_{miss}).Roland_{empty} \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).(reload, r_{reload}).Roland_2 \\
&+ (e\text{-}hit, \top).Roland_{dead} \\
Roland_{dead} &\stackrel{def}{=} Stop
\end{aligned}
$$

New Enemy

$$
\begin{aligned}
Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\
Enemies_{attack} &\stackrel{def}{=} (e\text{-}hit, r_{e\text{-}hit}).Enemies_{idle} \\
&\quad + \quad (hit, \top).Enemies_{idle}
\end{aligned}
$$

$$
Roland_2 \underset{\{hit, attack, e\text{-}hit\}}{\bowtie} Enemies_{idle}
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Transient Analysis Transient analysis on this model can be used to calculate the probability that Roland is dead after a given amount of time. As time increases this should tend towards probability 1.

Passage-Time Analysis Passage-time analysis could be used to calculate the probability of a given event happening at a given time after another given event, e.g. from an attack on Roland until he dies or wins the gun fight.

Roland makes a friend

In the next revision of the model we introduce an accomplice who is befriended by Roland and who, when Roland is attacked, fights alongside him.

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In this scenario cooperation is used to synchronise between components of the model such that they observe events which they neither directly cause nor are directly affected by.

Whenever either Roland or the accomplice kills the enemy the other must witness this action, so as to stop firing at a dead opponent (it would be a waste of ammunition!).

A new component for Roland

$$
\begin{aligned}
Roland_{idle} &\stackrel{def}{=} (attack, \top).Roland_2 \\
&+ (befriend, r_{befriend}).Roland_{idle} \\
Roland_2 &\stackrel{def}{=} (hit, r_{hit}).Roland_{hit} + (miss, r_{miss}).Roland_1 \\
&+ (a\text{-}hit, \top).Roland_{idle} \\
Roland_1 &\stackrel{def}{=} (hit, r_{hit}).Roland_{hit} \\
&+ (miss, r_{miss}).Roland_{empty} \\
&+ (a\text{-}hit, \top).(reload, r_{reload}).Roland_{idle} \\
Roland_{hit} &\stackrel{def}{=} (enemy\text{-}die, \top).(reload, r_{reload}).Roland_{idle} \\
Roland_{empty} &\stackrel{def}{=} (reload, r_{reload}).Roland_2 \\
&+ (a\text{-}hit, \top).(reload, r_{reload}).Roland_{idle}
\end{aligned}
$$

Synchronising Roland and the Accomplice

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$$\begin{aligned} Acmpl_{abs} \stackrel{def}{=} & (befriend, r_{befriend}).Acmpl_{idle} \\ & + (hit, \top).Acmpl_{abs} \\ & + (attack, \top).Acmpl_{abs} \end{aligned}$$

Component for the Accomplice

$$
\begin{aligned}
Acmpl_{idle} &\stackrel{def}{=} (attack, \top).Acmpl_2 \\
Acmpl_2 &\stackrel{def}{=} (a\text{-}hit, r_{a\text{-}hit}).Acmpl_{hit} + (hit, \top).Acmpl_{idle} \\
&+ (miss, r_{miss}).Acmpl_1 \\
&+ (enemy\text{-}hit, \top).Acmpl_{abs} \\
Acmpl_1 &\stackrel{def}{=} (a\text{-}hit, r_{a\text{-}hit}).Acmpl_{hit} \\
&+ (hit, \top).(reload, r_{a\text{-}reload}).Acmpl_{idle} \\
&+ (miss, r_{miss}).Acmpl_{empty} \\
&+ (enemy\text{-}hit, \top).Acmpl_{abs} \\
Acmpl_{hit} &\stackrel{def}{=} (enemy\text{-}die, \top).(reload, r_{a\text{-}reload}).Acmpl_{idle} \\
Acmpl_{empty} &\stackrel{def}{=} (reload, r_{a\text{-}reload}).Acmpl_2 \\
&+ (enemy\text{-}hit, \top).Acmpl_{abs} \\
&+ (hit, \top).(reload, r_{a\text{-}reload}).Acmpl_{idle}
\end{aligned}
$$

Parameter Settings for the Accomplice

parameter	*value*	*explanation*
$r_{befriend}$	0.001	Roland befriends a stranger once every 1000 seconds
$r_{a\text{-}fire}$	1.0	the accomplice can also fire once per second
$p_{a\text{-}hit\text{-}success}$	0.6	the accomplice has a 60 percent accuracy
$r_{a\text{-}hit}$	0.6	$r_{fire} \times p_{hit\text{-}success}$
$r_{a\text{-}miss}$	0.4	$r_{fire} \times (1.0 - p_{hit\text{-}success})$
$r_{a\text{-}reload}$	0.25	it takes the accomplice 4 seconds to reload

Component for the Enemy

The component representing the enemy is similar to before.

$$\begin{aligned} Enemies_{idle} &\stackrel{def}{=} (attack, r_{attack}).Enemies_{attack} \\ Enemies_{attack} &\stackrel{def}{=} (enemy\text{-}hit, r_{e\text{-}hit}).Enemies_{attack} \\ &+ (enemy\text{-}die, \top).Enemies_{idle} \end{aligned}$$

The system equation is as follows:

$$(Roland_2 \underset{\{attack, hit, a\text{-}hit, befriend\}}{\bowtie} Acmpl_{abs}) \underset{\{attack, enemy\text{-}die, enemy\text{-}hit\}}{\bowtie} Enemies_{idle}$$

Model Analysis

Steady-State Analysis

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- As before we can determine the probability that Roland is involved in a gun battle at an arbitrary time.
- We could also determine the likelihood that Roland has an accomplice at an arbitrary time.
- Since Roland cannot perform a befriending action while currently involved in a battle, the probabilty that Roland is in such a battle clearly affects the probability that he is alone in his quest.
- So, for example, if Roland's success rate is reduced then gun battles will take longer to resolve, hence Roland will be involved in a gun battle more often, and therefore he will befriend fewer accomplices.

Model Analysis

Transient Analysis

An example transient analysis would be to determine the expected time after Roland has set off before he meets his first accomplice.

Model Analysis

Passage-Time Analysis

- An example analysis would be to calculate the passage-time from an *attack* action until the death of the enemy or of the accomplice.

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Passage-Time Analysis

- An example analysis would be to calculate the passage-time from an *attack* action until the death of the enemy or of the accomplice.
- Since all gun battles now end in the enemy being killed stopping the analysis there would give us the expected duration of any one gun battle.
- There is also the possibility to start the analysis from the *befriend* action and stop it with the death of the accomplice.

Hiding

- Currently there is nothing in the model to stop an enemy from disrupting the interaction between Roland and his accomplice, e.g. by performing a *befriend* action.

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Hiding

- Currently there is nothing in the model to stop an enemy from disrupting the interaction between Roland and his accomplice, e.g. by performing a *befriend* action.
- One way to avoid this is to 'hide' those actions only Roland and the accomplice should cooperate on.
- To do this for our model we can simply change the system equation:

$$((Roland_2 \underset{L_1}{\bowtie} Acmpl)/L_1) \underset{L_2}{\bowtie} Enemies_{idle}$$

where $L_1 = \{hit, a\text{-}hit, befriend\}$ and
$L_2 = \{attack, enemy\text{-}die, enemy\text{-}hit\}$.

Outline

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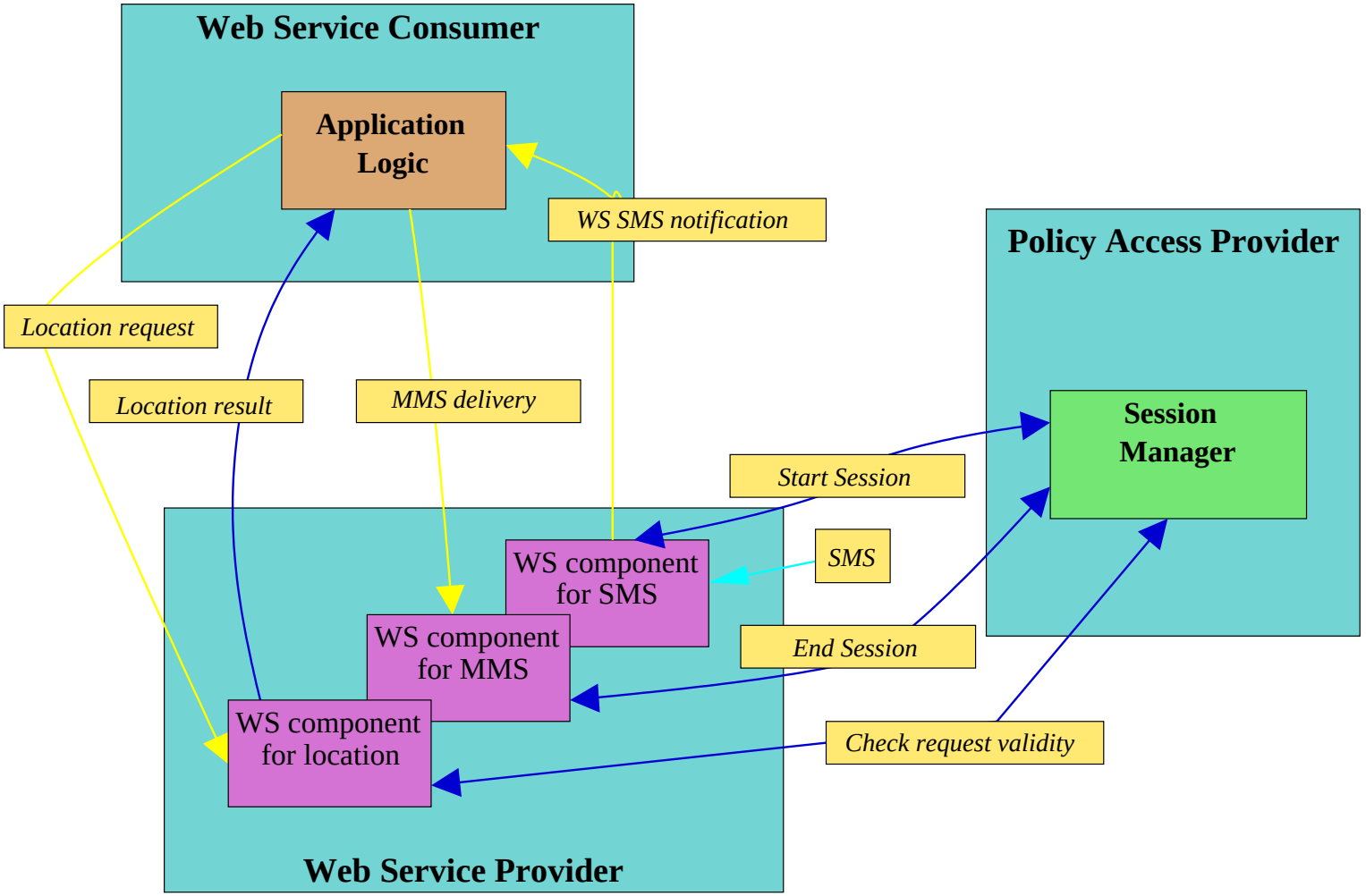
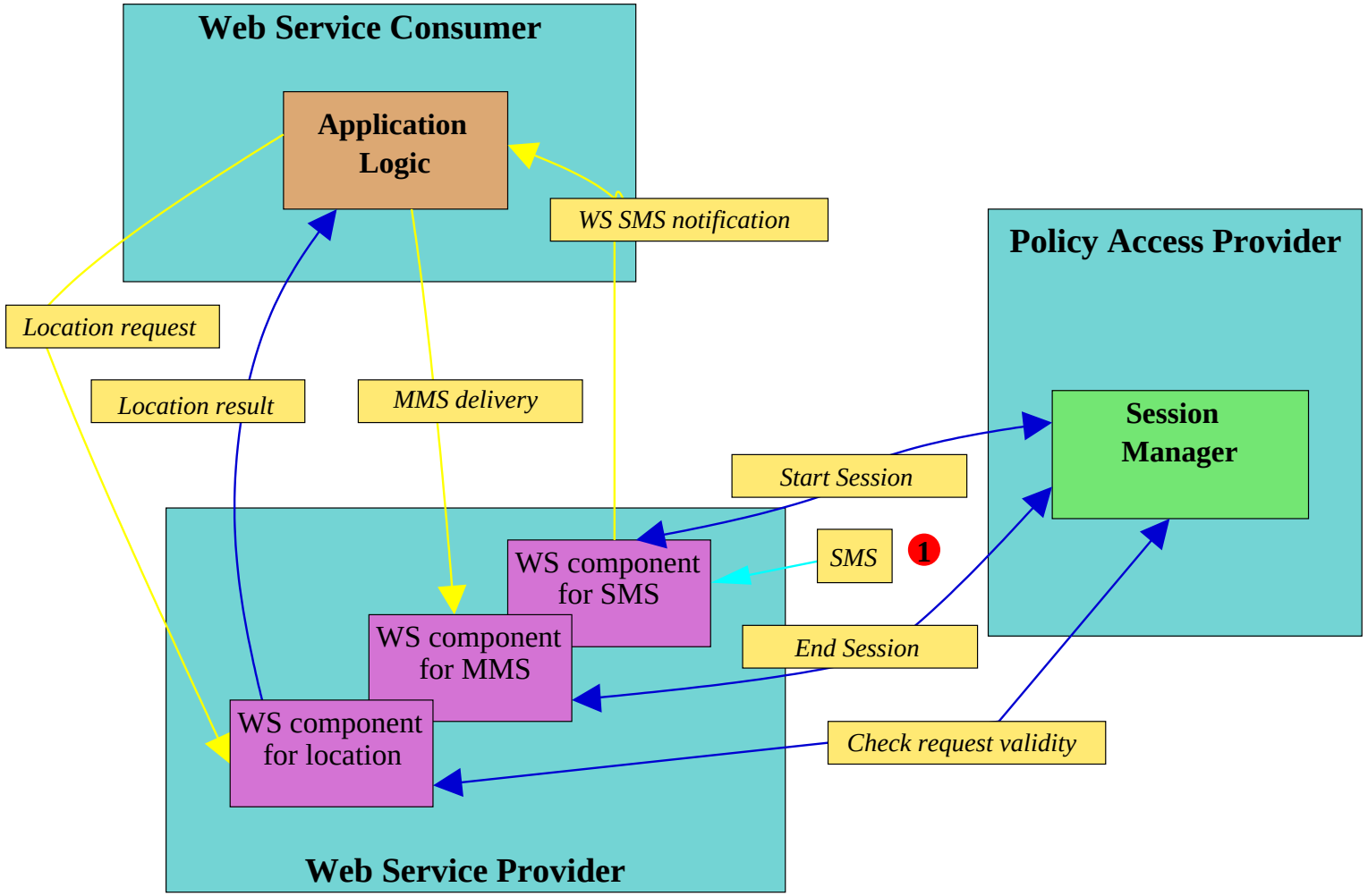
Moreover the service provider imposes a restriction that only one request may be handled for each SMS message received.

Schematic view



Schematic view



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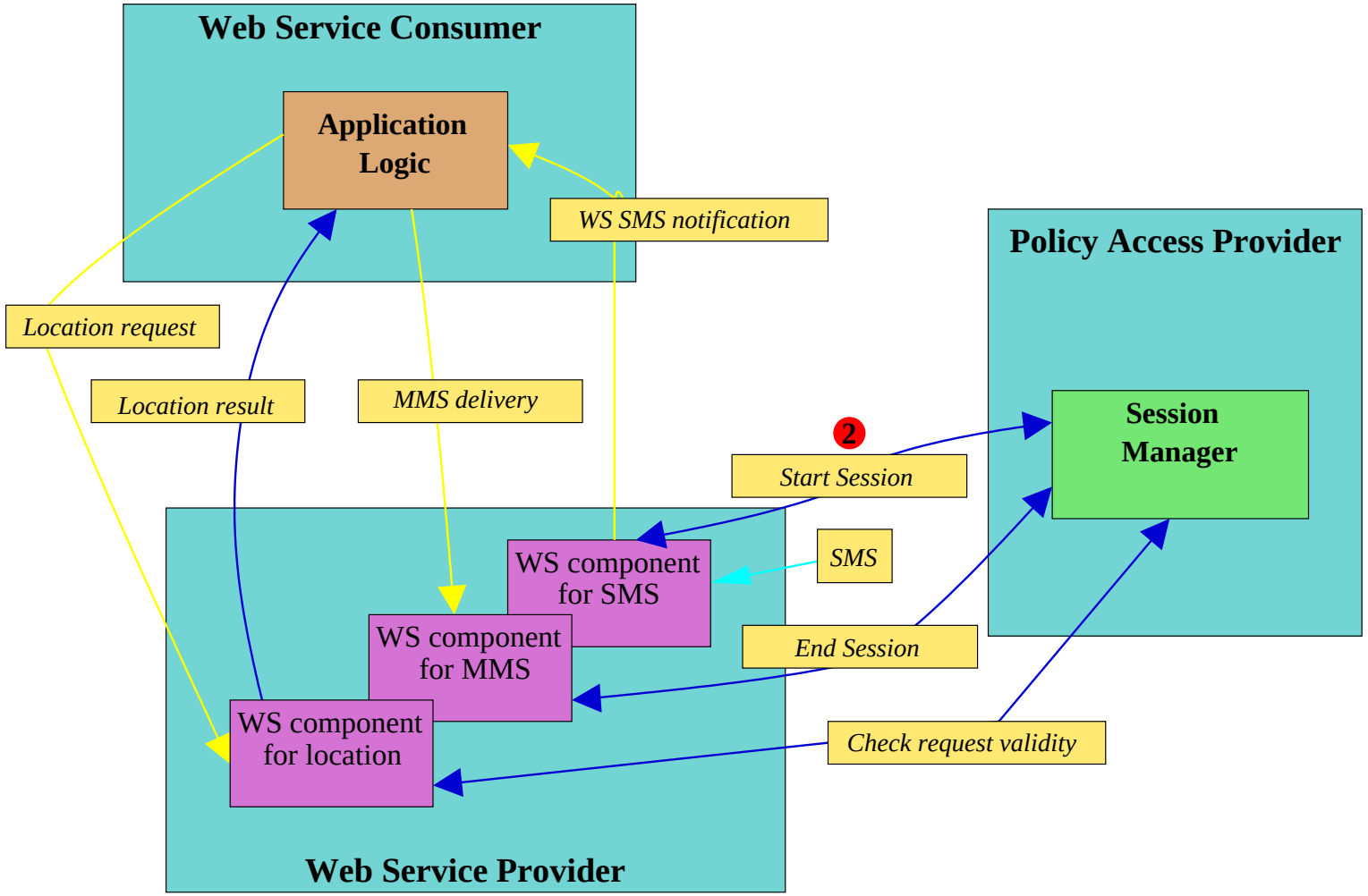

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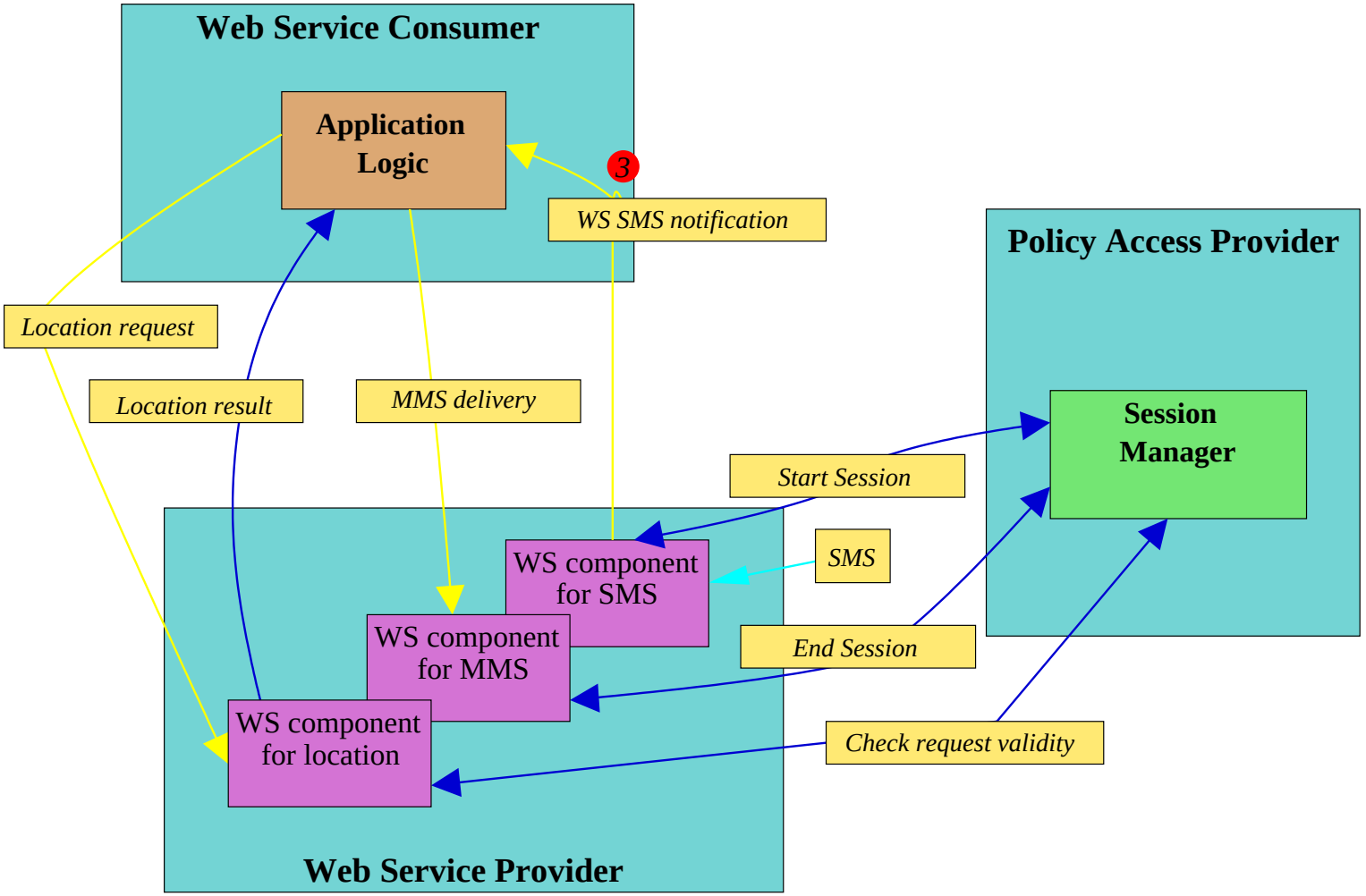

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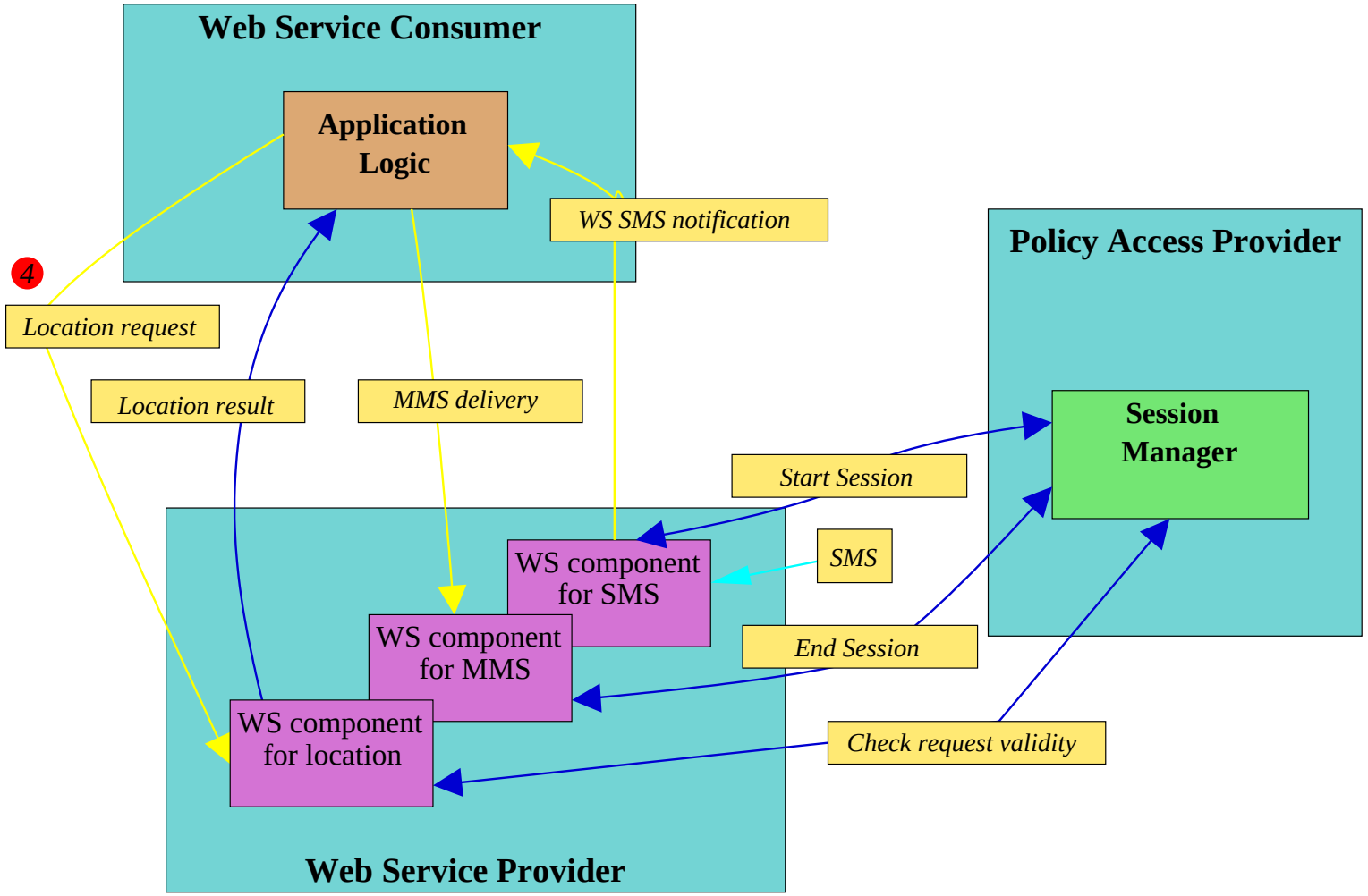

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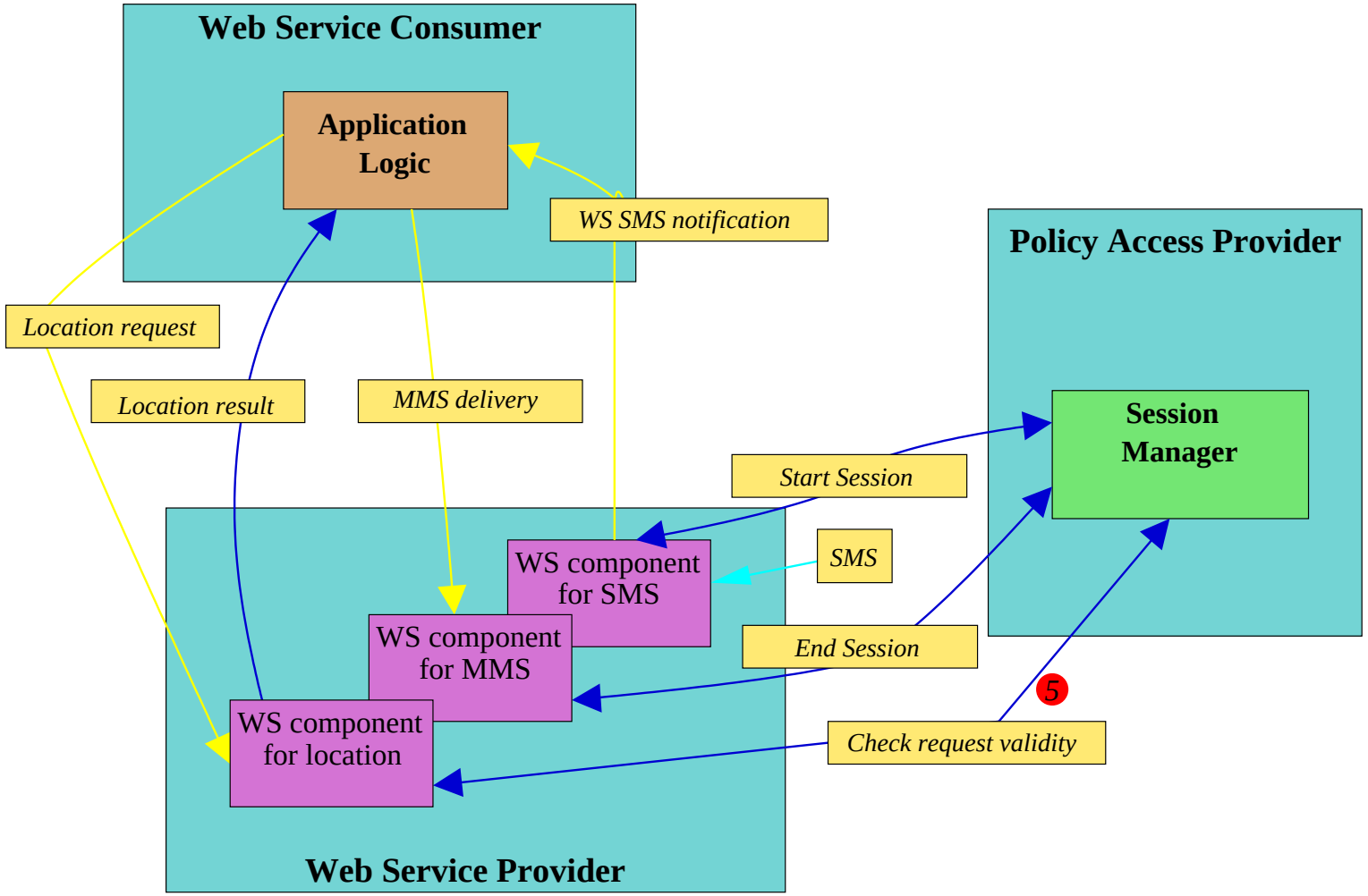

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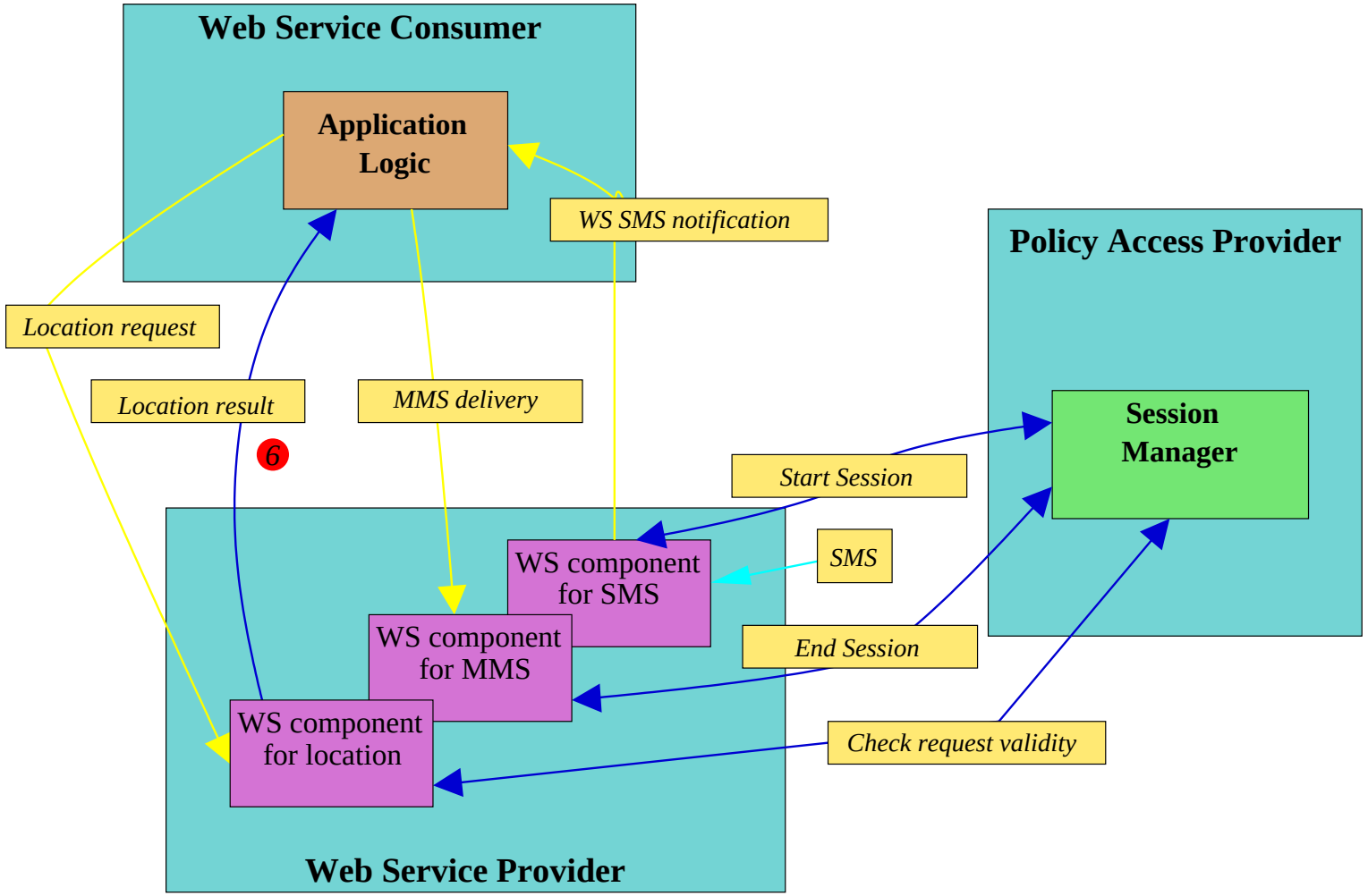

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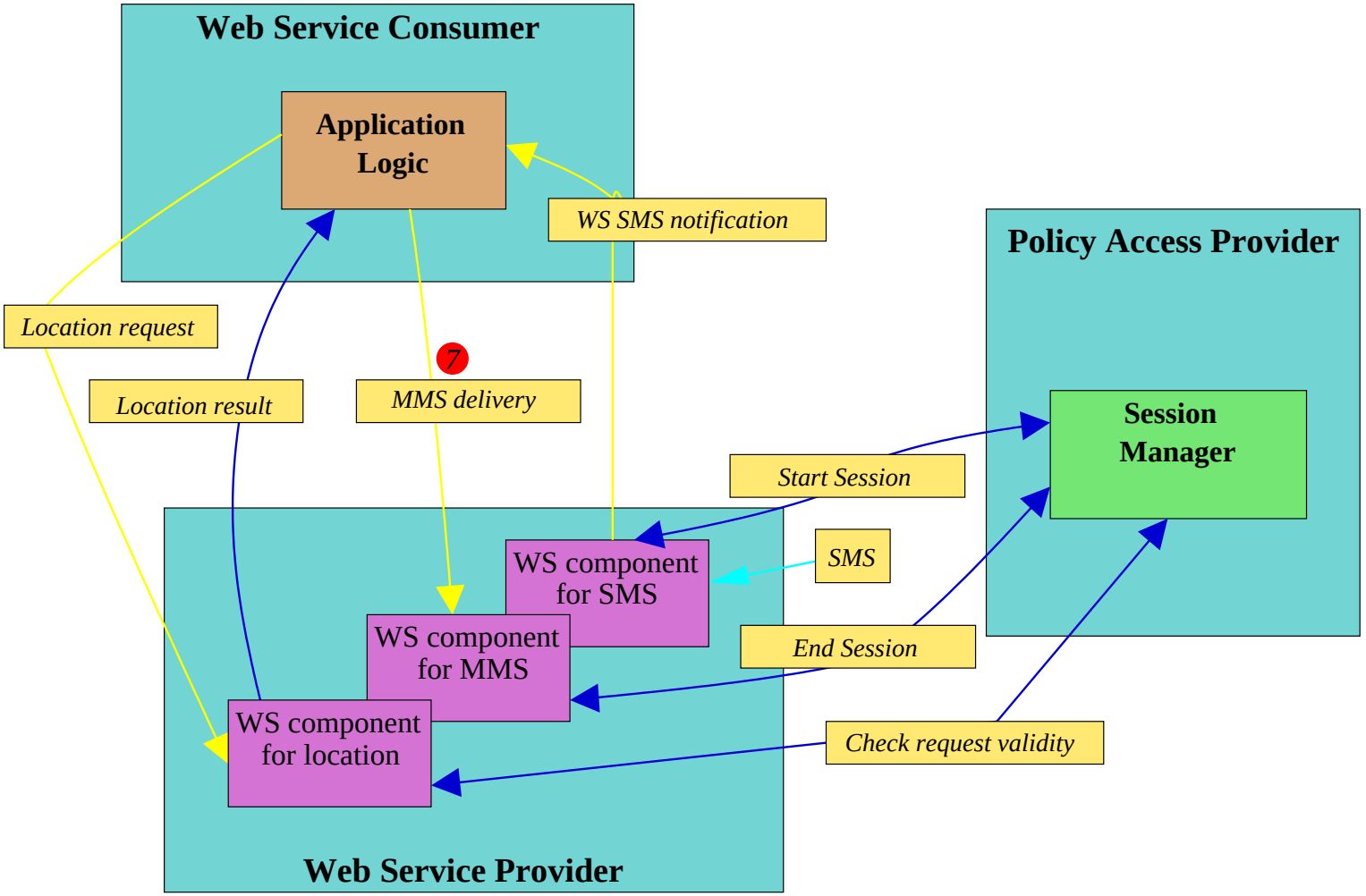

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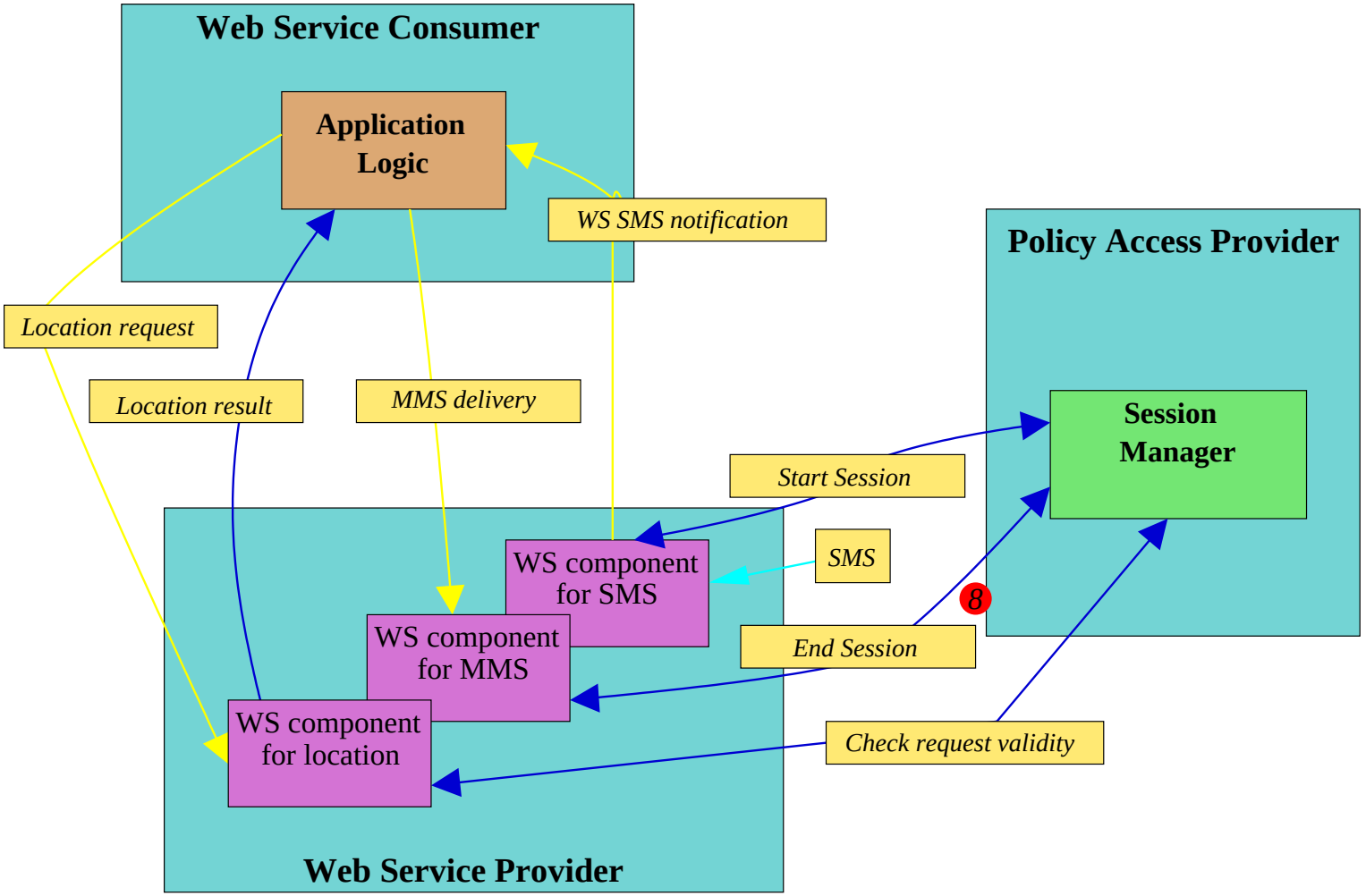

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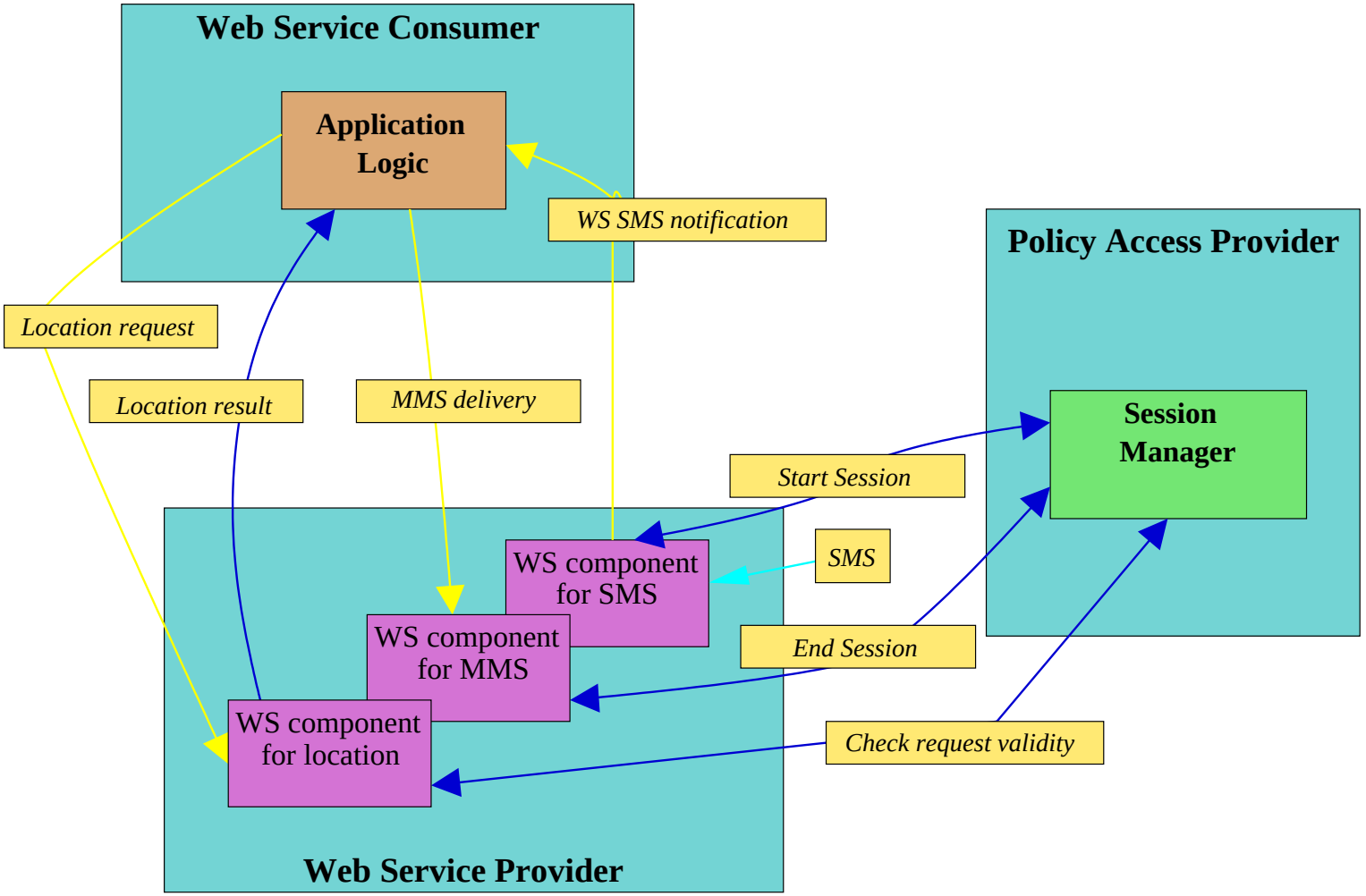

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Concurrency is introduced into the model by allowing multiple sessions rather than by representing the constituent web services separately.

Component *Customer*

The customer's behaviour is simply modelled with two local states.

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\begin{aligned}
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We associate the user-perceived system performance with the throughput of the *getMap* action which can be calculated directly from the steady state probability distribution of the underlying Markov chain.

Component *WSConsumer*

Once a session has been started, it initiates a request for the user's current location and waits for a response.

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$$
\begin{aligned}
WSConsumer &\stackrel{def}{=} (notify, \top).WSConsumer_2 \\
WSConsumer_2 &\stackrel{def}{=} (locReq, r_4).WSConsumer_3 \\
WSConsumer_3 &\stackrel{def}{=} (locRes, \top).WSConsumer_4 \\
&\quad + \quad (locErr, \top).WSConsumer \\
WSConsumer_4 &\stackrel{def}{=} (compute, r_7).WSConsumer_5 \\
WSConsumer_5 &\stackrel{def}{=} (sendMMS, r_9).WSConsumer
\end{aligned}
$$

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The *checkValid* action is represented twice, to capture the two possible distinct outcomes of the action.

- If the check is successful the location must be returned to the Web Service Consumer in the form of a map (*getMap*).
- If the check revealed an invalid request (*locErr*) then an error must be returned to the Web Service Consumer (*get404*) and the session terminated (*stopSession*).

Component *WSProvider*

$$
\begin{aligned}
WSProvider &\stackrel{def}{=} (getSMS, \top).WSProvider_2 \\
WSProvider_2 &\stackrel{def}{=} (startSession, r_2).WSProvider_3 \\
WSProvider_3 &\stackrel{def}{=} (notify, r_3).WSProvider_4 \\
WSProvider_4 &\stackrel{def}{=} (locReq, \top).WSProvider_5 \\
WSProvider_5 &\stackrel{def}{=} (checkValid, 99 \cdot \top).WSProvider_6 \\
&\quad + \quad (checkValid, \top).WSProvider_{10}
\end{aligned}
$$

Component *WSProvider* cont.

$$
\begin{aligned}
WSProvider_6 &\stackrel{def}{=} (locRes, r_6).WSProvider_7 \\
WSProvider_7 &\stackrel{def}{=} (sendMMS, \top).WSProvider_8 \\
WSProvider_8 &\stackrel{def}{=} (getMap, r_8).WSProvider_9 \\
WSProvider_9 &\stackrel{def}{=} (stopSession, r_2).WSProvider \\
WSProvider_{10} &\stackrel{def}{=} (locErr, r_6).WSProvider_{11} \\
WSProvider_{11} &\stackrel{def}{=} (get404, r_8).WSProvider_9
\end{aligned}
$$

Component *PAProvider*

We consider a stateless implementation of the policy access provider.

$$
\begin{aligned}
PAProvider \quad &\stackrel{def}{=} \quad (startSession, \top).PAProvider \\
&+ \quad (checkValid, r_5).PAProvider \\
&+ \quad (stopSession, \top).PAProvider
\end{aligned}
$$

Model Component *WSComp*

The complete system is composed of some number of instances of the components interacting on their shared activities:

$$\begin{aligned} WSComp \stackrel{def}{=} & \big((Customer[N_C] \underset{L_1}{\bowtie} WSProvider[N_{WSP}]) \\ & \underset{L_2}{\bowtie} WSConsumer[N_{WSC}]\big) \\ & \underset{L_3}{\bowtie} PAProvider[N_{PAP}] \end{aligned}$$

where the cooperation sets are

$$\begin{aligned} L_1 &= \{getSMS, getMap, get404\} \\ L_2 &= \{notify, locReq, locRes, locErr, sendMMS\} \\ L_3 &= \{startSession, checkValid, stopSession\} \end{aligned}$$

Parameter Values

param.	*value*	*explanation*
r_1	0.0010	rate customers request maps
r_2	0.5	rate session can be started
r_3	0.1	notification exchange between consumer and provider
r_4	0.1	rate requests for location can be satisfied
r_5	0.05	rate the provider can check the validity of the request
r_6	0.1	rate location information can be returned to consumer
r_7	0.05	rate maps can be generated
r_8	0.02	rate MMS messages can be sent from provider to customer
r_9	$10.0 * r_8$	rate MMS messages can be sent via the Web Service

Steady State Analysis for System Tuning

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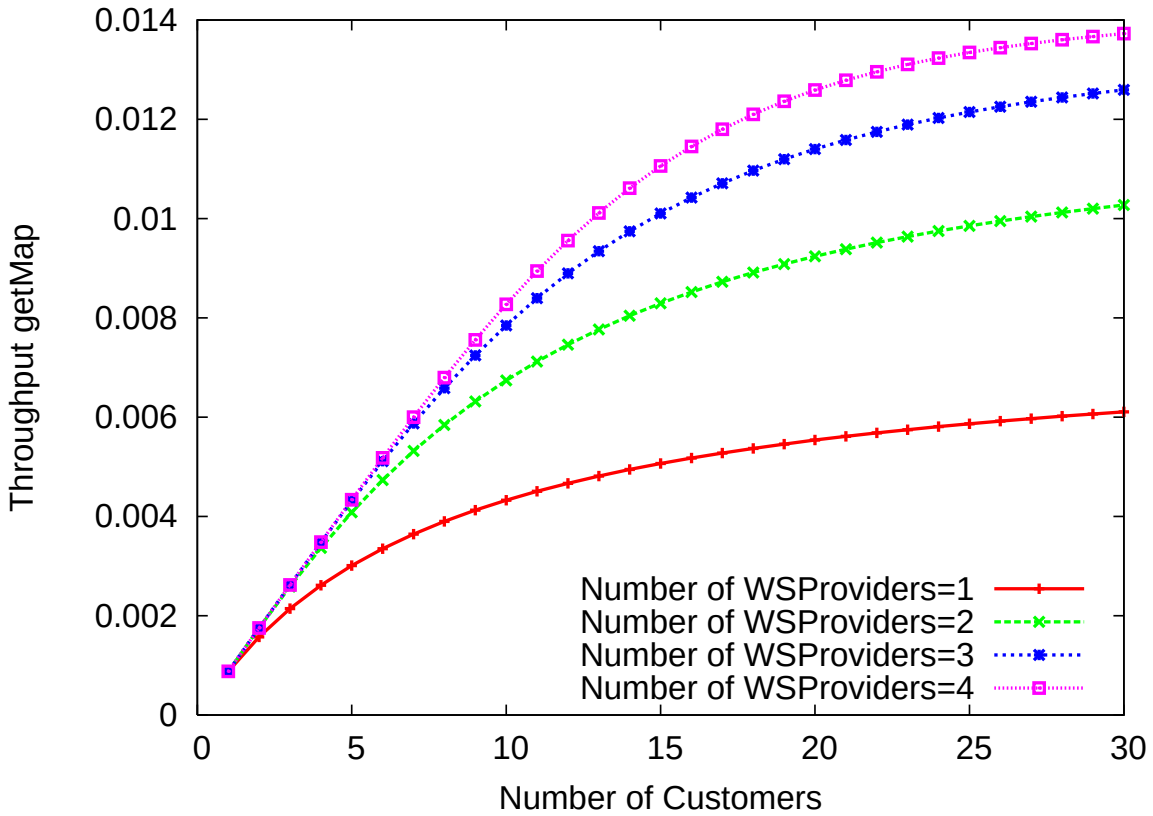
- However, there are a number of degrees of freedom which let us vary, for example, the number of threads of control of the components of the system.
- The aim of the analysis is to deliver a satisfactory service in a cost-effective way.
- The simplest example of a cost function may be a linearly dependency on the number of copies of a component or the rate at which an activity is performed.

Throughput of the *getMap* action

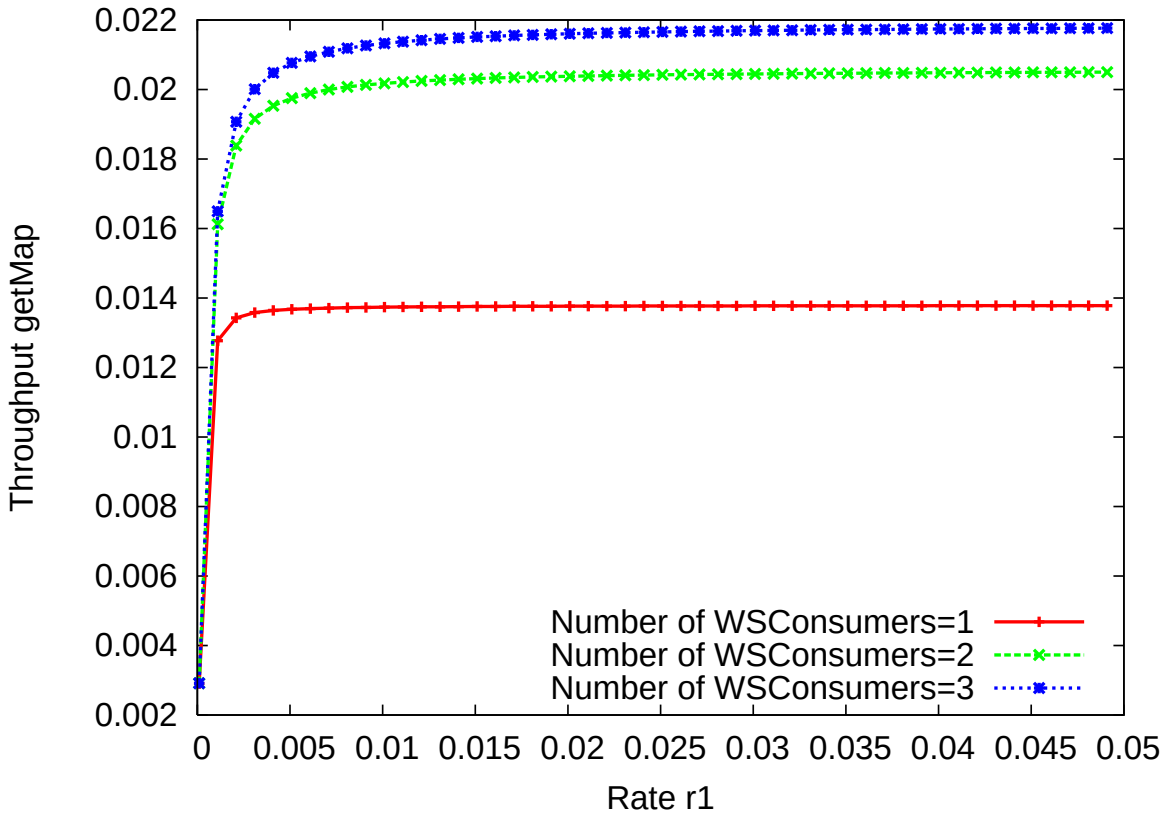


as the number of customers varies between 1 and 30 for various numbers of copies of the *WSProvider* component.

Throughput of the *getMap* action

- Under heavy load increasing the number of providers initially leads to a sharp increase in the throughput. However the gain deteriorates so that the system with four copies is just 8.7% faster than the system with three.
- In the following we settle on three copies of *WSProvider*.

Throughput of *getMap* action



as the request arrival rate (r_1) varies for differing numbers of *WSConsumer*.

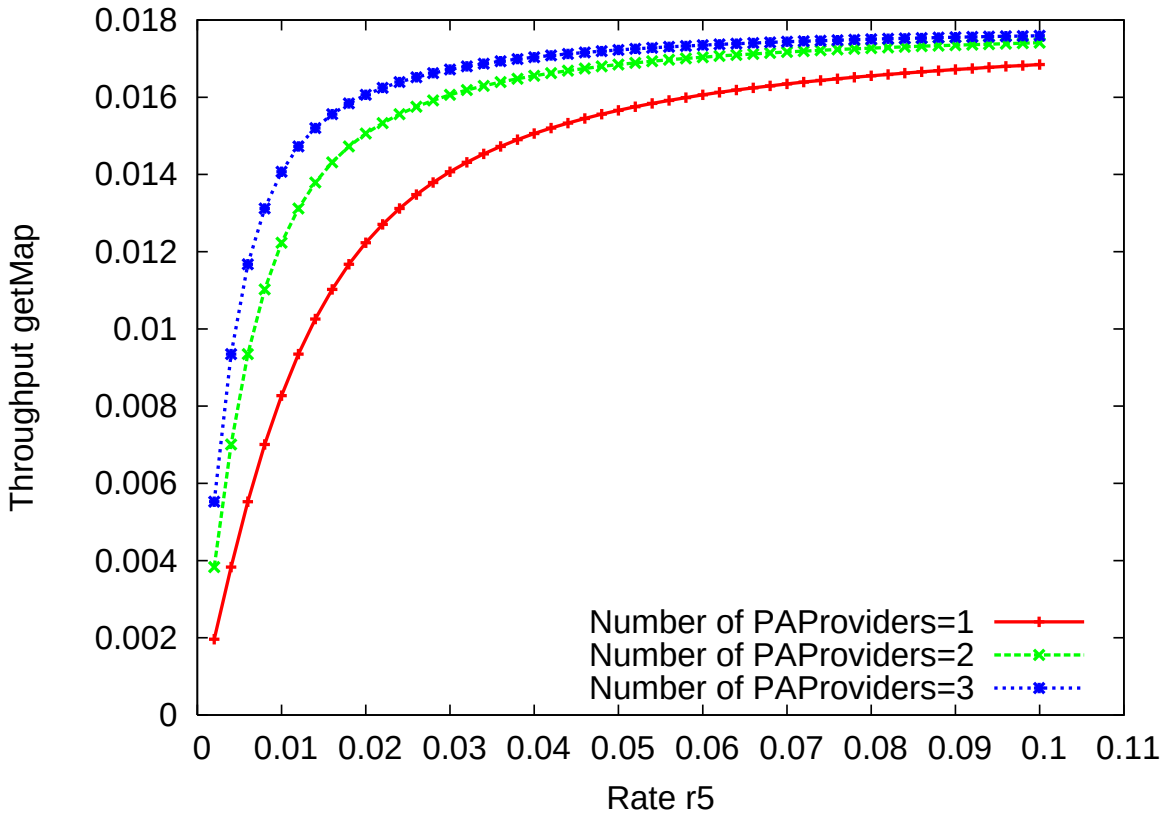
Throughput of *getMap* action

- Every line starts to plateau at approximately $r_1 = 0.010$ following an initial sharp increase. This suggests that the user is the bottle next in the system when the arrival rate is lower. Conversely, at high rates the system becomes congested.
- Whilst having two copies of *WSConsumer*, corresponding to two operating threads of control, improves performance significantly, the subsequent increase with three copies is less pronounced.
- So we set the number of copies of *WSConsumer* to 2.

Optimising the number of copies of *PAProvider*

- Here we are particularly interested in the overall impact of the rate at which the validity check is performed.
- Slower rates may mean more computationally expensive validation.
- Faster rates may involve less accuracy and lower security of the system.

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider*.

Throughput of *getMap* action

- A sharp increase followed by a constant levelling off suggests that optimal rate values lie on the left of the plateau, as faster rates do not improve the system considerably.
- As for the optimal number of copies of *PAProvider*, deploying two copies rather than one dramatically increases the quality of service of the overall system.
- With a similar approach as previously discussed, the modeller may want to consider the trade-off between the cost of adding a third copy and the throughput increase.

An alternative design for *PAProvider*

- The original design of *PAProvider* is stateless.
- Any of its services can be called at any point, the correctness of the system being guaranteed by implementation-specific constraints such as session identifiers being uniquely assigned to the clients and passed as parameters of the method calls.

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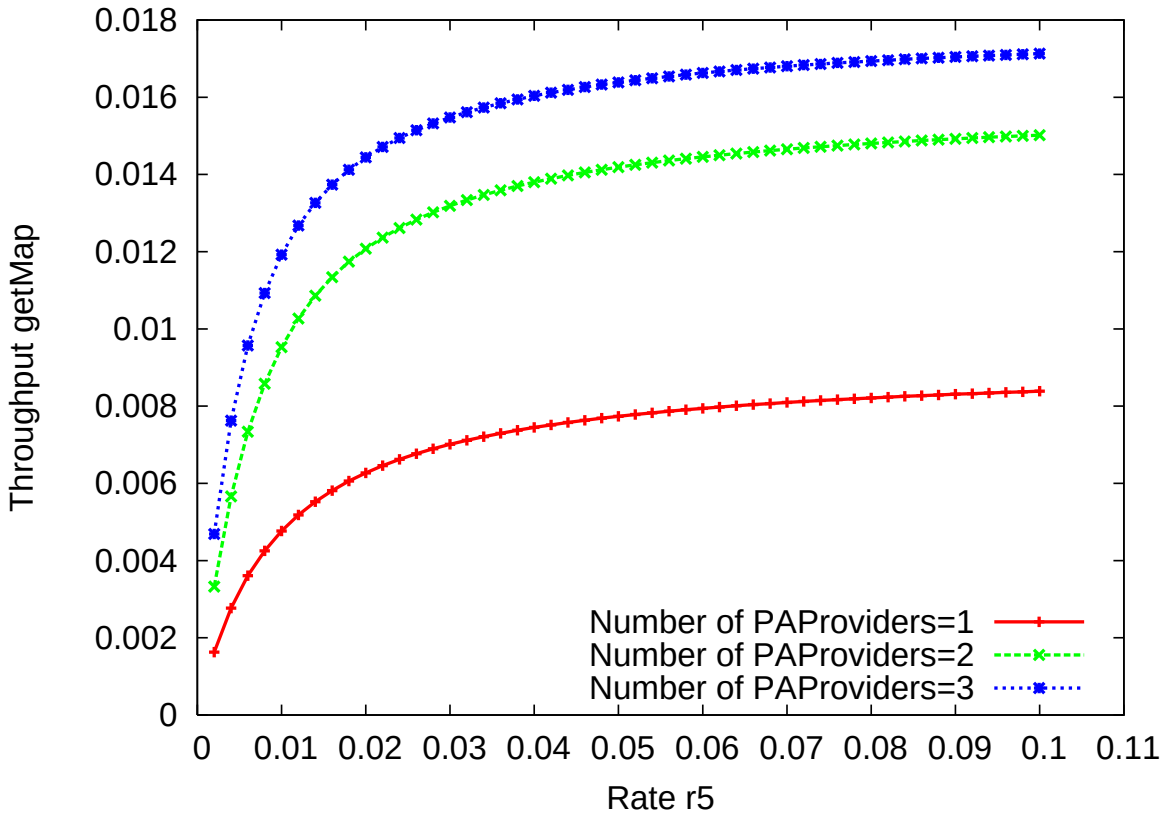
- The original design of *PAProvider* is stateless.
- Any of its services can be called at any point, the correctness of the system being guaranteed by implementation-specific constraints such as session identifiers being uniquely assigned to the clients and passed as parameters of the method calls.
- Alternatively we may consider a stateful implementation, modelled as a sequential component with three local states.
- This implementation has the consequence that there can never be more than N_{PAP} *WSProvider* which have started a session with a *PAProvider*

Component *PAProvider* — Stateful Version

It maintains a thread for each session and carries out the validity check on behalf of the Web Service Provider.

$$
\begin{aligned}
PAProvider &\stackrel{def}{=} (startSession, \top).PAProvider_2 \\
PAProvider_2 &\stackrel{def}{=} (checkValid, r_5).PAProvider_3 \\
PAProvider_3 &\stackrel{def}{=} (stopSession, \top).PAProvider
\end{aligned}
$$

Throughput of *getMap* action



as the validity check rate (r_5) varies for differing numbers of *PAProvider* (stateful version).

Throughput of *getMap* action

- In this case the incremental gain in adding more copies has become more marked.
- However, the modeller may want to prefer the original version, as three copies of the stateful provider deliver about as much as the throughput of only one copy of the stateless implementation.

Thank you

The models of Roland the Gunslinger are due to Allan Clark and the model of the web service composition is joint work with Mirco Tribastone.