

# Tractable XML Data Exchange via Relations

Rada Chirkova  
Department of Computer Science  
North Carolina State University  
chirkova@csc.ncsu.edu

Leonid Libkin  
School of Informatics  
University of Edinburgh  
libkin@inf.ed.ac.uk

Juan L. Reutter  
School of Informatics  
University of Edinburgh  
juan.reutter@ed.ac.uk

## ABSTRACT

We consider data exchange for XML documents: given source and target schemas, a mapping between them, and a document conforming to the source schema, construct a target document and answer target queries in a way that is consistent with the source information. The problem has primarily been studied in the relational context, in which data-exchange systems have also been built.

Since many XML documents are stored in relations, it is natural to consider using a relational system for XML data exchange. However, there is a complexity mismatch between query answering in relational and in XML data exchange. This indicates that to make the use of relational systems possible, restrictions have to be imposed on XML schemas and mappings, as well as on XML shredding schemes.

We isolate a set of five requirements that must be fulfilled in order to have a faithful representation of the XML data-exchange problem by a relational translation. We then demonstrate that these requirements naturally suggest the inlining technique for data-exchange tasks. Our key contribution is to provide shredding algorithms for schemas, documents, mappings and queries, and demonstrate that they enable us to correctly perform XML data-exchange tasks using a relational system.

### Categories and Subject Descriptors

H.2.5 [Heterogeneous Databases]: Data translation

### General Terms

Algorithms, Theory

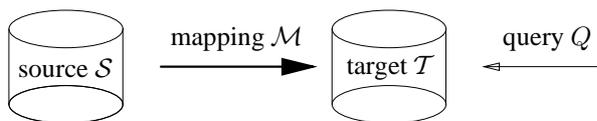
### Keywords

Data Exchange, XML, XML Shredding, Inlining

## 1. Introduction

In the problem of data exchange, given an instance of a source schema and a schema mapping, which is a specification of the relationship between the source and the target, the objective is to find an instance of a target schema. The target instance should correctly represent information from the source instance under the constraints imposed by the target schema, and should allow one to evaluate queries on the target instance in a way that is semantically consistent with the source data. The problem has received much attention in the past few years, with several surveys already available [21, 9, 8].

The general setting of data exchange is this:



We have fixed source and target schemas, an instance  $\mathcal{S}$  of the source schema, and a mapping  $\mathcal{M}$  that specifies the relationship between the source and the target schemas. The goal is to construct an instance  $\mathcal{T}$  of the target schema, based on the source and on the mapping, and to answer queries against the target data in a way consistent with the source data.

The mappings rarely specify the target instance completely. That is, for each source  $\mathcal{S}$  and mapping  $\mathcal{M}$ , there could be multiple target instances  $\mathcal{T}_1, \mathcal{T}_2, \dots$  that satisfy the conditions of the mapping. Such instances are called *solutions*. The notion of query answering has to account for their non-uniqueness. Typically, one tries to compute *certain answers*  $\text{CERTAIN}_{\mathcal{M}}(Q, \mathcal{S})$ , i.e., answers independent of a particular solution chosen. If  $Q$  produces relations, these are usually defined as  $\bigcap_i Q(\mathcal{T}_i)$ . Certain answers must be produced by evaluating some query – not necessarily  $Q$  but perhaps its *rewriting*  $Q_{\text{rewr}}$  over a particular solution  $\mathcal{T}$ , so that  $Q_{\text{rewr}}(\mathcal{T}) = \text{CERTAIN}_{\mathcal{M}}(Q, \mathcal{S})$ .

Thus, the key tasks in data exchange are: (a) choosing a particular solution  $\mathcal{T}$  among  $\{\mathcal{T}_1, \mathcal{T}_2, \dots\}$  to materialize, and (b) finding a way of producing query answers over that solution by running a rewritten query  $Q_{\text{rewr}}$  over it. Usually one builds a so-called *universal* solution [12, 8]; these solutions behave particularly nicely with respect to query answering.

These basics of data exchange are independent of a particular model of data. Most research on data exchange, however, has occurred in the relational context [12, 13, 21, 8] or slight extensions [32, 18]. The first paper that attempted to extend relational results to the XML context was [6], and a few followups have since appeared [4, 3]. They all concentrate on the algorithmic aspects of query answering and constructing solutions, with the main goal of isolating tractable cases. The problem these papers do not address is *how can XML data exchange be implemented?*

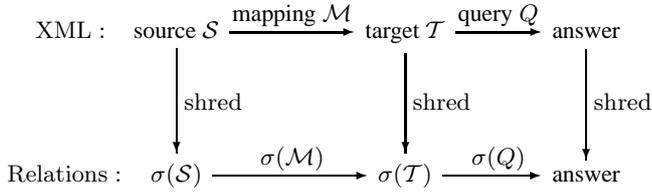
Previous work on algorithms for XML data exchange has tacitly assumed that one uses a native XML DBMS such as [19]. However, this is not the only (and perhaps not even the most common) route: XML documents are often stored in relational DBMSs. Note that it is natural and in many cases desirable to be able to use the established relational technology to solve the considerably more recent and not as well understood XML data-exchange task. In fact, many ETL products claim that they handle XML data simply by producing relational translations (known as *shredding* [22]). This leads to a two-step approach:

- first shred XML data into relations;
- then apply a relational data-exchange engine (and publish the result back as an XML document).

The approach seems very natural, but the key question is whether it will *work correctly*. That is, are we guaranteed to have the same

result as we would have gotten had we implemented a native XML data-exchange system? We answer this question in this paper.

To state more precisely the main question addressed in this paper, assume that we have a translation  $\sigma(\cdot)$  that can be applied to (a) XML schemas, (b) XML documents, (c) XML schema mappings, and (d) XML queries. Then the concept of *correctness* of such a translation is shown below:



That is, suppose we start with an XML document  $S$  and an XML schema mapping  $\mathcal{M}$ . In a native system, we would materialize some solution  $T$  over which we could answer queries  $Q$ .

But now we want a relational system to do the job. So we shred  $S$  into  $\sigma(S)$  and then apply to  $\sigma(S)$  the translation of the mapping  $\sigma(\mathcal{M})$  to get a solution – which itself is a shredding of an XML solution – so that the answer to  $Q$  could be reconstructed from the result of the query  $\sigma(Q)$  over that relational solution.

The idea seems simple and natural on the surface, but starts looking challenging once we look deeper into it. Before even attempting to show that the relational translation faithfully represents the XML data-exchange problem, we need to address the following.

*Complexity mismatch.* Without restrictions, there *cannot be a faithful representation* of XML data exchange by a relational system. Indeed, it is well known that positive relational-algebra queries can be efficiently evaluated in relational data exchange [12, 21, 8]. At the same time, finding query answers even for simple XML analogs of conjunctive queries can be coNP-hard [6]. So any claim that a relational data-exchange system correctly performs XML data exchange for arbitrary documents and queries is bound to be wrong. We thus need to identify the cases that can be handled by a relational system.

*Which shredding scheme to use?* There are several, which can roughly be divided into two groups: those that do not take the schema information into account (e.g., the edge representation [14], interval codings [33], and other numbering schemes [30]), and those that are based on schemas for XML, such as variants of the inlining technique [28, 22]. Since in data-exchange scenarios we start with two schemas, it seems more appropriate to apply schema-based techniques.

*Target constraints.* In relational data exchange, constraints in target schemas are required to satisfy certain acyclicity conditions; without them, the chase procedure that constructs a target instance does not terminate [12, 21, 8]. Constraints imposed by general XML schema specifications need not in general be even definable in relational calculus, let alone be acyclic [20]. We thus need to find a shredding technique that enables us to encode target schemas by means of constraints that guarantee chase termination.

As for the complexity issue, the work on the theory of XML data exchange has identified a class of mappings for which efficient query answering is possible [6, 4, 3]. The schemas (say, DTDs), have rules of the form  $db \rightarrow book^*$ ,  $book \rightarrow author^* subject$  (we shall give a formal definition later), and the mappings transform patterns satisfied over the source into patterns satisfied over targets. Observe that these mappings (just as nested-relational tgds

[15, 27]) are strictly more expressive than relational tgds; see the Related-Work section for a discussion.

This restriction suggests a relational representation to use. Going with the edge representation [14] is problematic: First, each edge in an XML pattern used in a mapping will result in a join in the relational translation, making it inefficient. Second, enforcing even a simple schema structure under that representation takes us out of the class of target constraints that relational data-exchange systems can handle. Verifiably correct translations based on numerical encodings [30, 33] will necessarily involve numerical and/or ordering constraints in relational translations of mappings, and this is something that relational data exchange cannot handle at the moment [21, 8], beyond simple ordering constraints [2].

One translation scheme however that fits in well with the restrictions identified in [6, 4, 3] is the *inlining* scheme. It works very well for DTDs of the “right” shape, and its output schemas involve only acyclic constraints, which is perfect for data-exchange scenarios.

**Desiderata for the translation** We now formulate some basic requirements for the translation  $\sigma$ , in order to be able to achieve our goals described in the diagram above.<sup>1</sup> We need the following:

**Requirement 1: translation of schemas** A translation  $\sigma(D)$  that, when applied to a DTD of a special form, produces a relational schema that has only acyclic constraints, which can be used in a relational data-exchange setting.

**Requirement 2: translation of documents** A translation  $\sigma_D(\cdot)$  for a DTD  $D$  that, when applied to document  $T$  conforming to  $D$ , produces relational database  $\sigma_D(T)$  of schema  $\sigma(D)$ .

**Requirement 3: translation of queries** For a DTD  $D$ , a translation  $\sigma_D(Q)$  of (analog of) conjunctive queries so that  $\sigma_D(Q)(\sigma_D(T)) = Q(T)$  (that is, the result of  $Q(T)$  can be computed by relational translations).

**Requirement 4: translation of mappings** For a mapping  $\mathcal{M}$  between a source DTD  $D_s$  and a target DTD  $D_t$ , its translation  $\sigma(\mathcal{M})$  is a mapping between  $\sigma(D_s)$  and  $\sigma(D_t)$  that preserves universal solutions. That is:

- (a) Each  $\sigma_{D_t}$ -translation of a universal solution for  $T$  under  $\mathcal{M}$  is a universal solution for  $\sigma_{D_s}(T)$  under  $\sigma(\mathcal{M})$ ; and
- (b) Each universal solution for  $\sigma_{D_s}(T)$  under  $\sigma(\mathcal{M})$  contains<sup>2</sup> a  $\sigma_{D_t}$ -translation of a universal solution of  $T$  under  $\mathcal{M}$ .

**Requirement 5: query answering** For (analog of) conjunctive queries over trees, computing the answer to  $Q$  under  $\mathcal{M}$  over a source tree  $T$  is the same as computing a  $\sigma(\mathcal{M})$ -solution of  $\sigma(T)$ , followed by evaluation of  $\sigma(Q)$  over that solution, as is normally done in a relational data-exchange system.

Satisfaction of these five requirements would guarantee that we have a *correct* relational translation of an XML data-exchange problem, which would guarantee correct evaluation of queries. The relational approach to XML data exchange, which we propose in this paper, satisfies all the five requirements.

For the choice of the query language, one has to be careful since the definition of certain answers depends on the output of the queries. We consider two classes of conjunctive queries over trees. The first is tree patterns that output tuples of attribute values. These are the queries most commonly considered in XML data exchange [6, 4, 3], because for them we can define certain answers as

<sup>1</sup>In the next sections we formalize each desideratum.

<sup>2</sup>We cannot require the equivalence, as relational solutions are open to adding new tuples and thus cannot always be translations of trees; we shall discuss this later.

the usual intersection  $\text{CERTAIN}_{\mathcal{M}}(Q, S) = \bigcap_i Q(\mathcal{T}_i)$ . The second is a simple XML-to-XML query language, in which queries output *trees*. It is essentially the positive fragment of FLWR expressions of XQuery [31]. For outputs which are XML trees, the intersection operator is no longer meaningful for defining certain answers. Instead, we use recent results of [11] that show how to define and compute certain answers for XML-to-XML queries.

**Contributions** We provide a relational approach to solve two of the most important problems of XML data-exchange settings: materializing solutions and answering queries. Our specific contributions are as follows. First, we introduce an architecture for XML data exchange using relational vehicles, with a focus on correct evaluation of (analogs of) conjunctive queries on XML data. Second, we identify a class of XML schema mappings and a shredding mechanism that allows us to overcome the complexity mismatch. Third, we provide algorithms for relational translation of schemas, XML documents, schema mappings, and queries in our proposed architecture. Finally, we prove the correctness of the translations: namely, we show that they satisfy the above five requirements, and thus enable us to use relational data-exchange systems for XML data-exchange tasks. Since the computational complexity of our proposed algorithms is quite low, and their correctness has been established, we believe this paper makes a case for using the relational technology for provably correct XML data exchange.

**Related work** In recent years, significant effort has been devoted to developing high-performance XML database systems, and to building tools for data exchange. One major direction of the XML effort is the “relational approach,” which uses relational DBMSs to store and query XML data. Documents could be translated into relational tuples using either a “DTD-aware” translation [29, 28] or a “schemaless” translation. The latter translations include the edge [14] and the node [33] representations of the data. Indexes could be prebuilt on the data to improve performance in relational query processing, see, e.g., [30, 33]. Constraints arising in the translation are sometimes dealt with explicitly [7, 23]. See [17] for a survey of the relational approach to answering XML queries.

The work on data exchange has concentrated primarily on relations, see [8, 21] for surveys and [26, 27] for system descriptions. Mappings for the XML data-exchange problem were studied in [6, 4]; it was noticed there that the complexity of many tasks in XML data exchange is higher than for their relational analogs, which suggests that restrictions must be imposed for a relational implementation. The problem of exchanging XML data was also studied in [15, 27], which give translations of documents and of DTDs into nested-relational schemas, and then show how to perform XML data exchange under this translation. Most RDBMSs, however, do not provide support for nested-relational schemas, and, thus, specific machinery has to be developed in order to implement this translation under a strictly relational setting. Moreover, XML mappings considered in this paper are strictly more expressive than nested-relational mappings, and every nested-relational data-exchange setting can be efficiently transformed into an equivalent XML data-exchange setting. Thus, the results of this paper may aid towards the development of a relational implementation for both XML and nested-relational data exchange.

**Outline** Key definitions are given in Section 2. Section 3 provides translations of schemas and documents, and shows that they fulfill our Requirements 1 and 2. Section 4 states the main concepts of relational and XML data exchange. Section 5 provides translations of mappings and queries, and shows that our Requirements 3, 4, and 5 are fulfilled. Section 6 studies queries that output XML trees.

## 2. Preliminaries

**Relational schemas and constraints.** A *relational schema*, or just *schema*, is a finite set  $\mathbf{R} = \{R_1, \dots, R_k\}$  of relation symbols, possibly with a set of integrity constraints (*dependencies*). Constraints used most often in data exchange are equality- and tuple-generating dependencies [12, 21, 8], but for our purposes it will suffice to consider only *keys* and *foreign keys*. If  $R$  is a relation over attributes  $U$ , and  $X$  is a set of attributes, then  $X$  is a key of  $R$  if no two tuples of  $R$  coincide on  $X$ -attributes (that is, for all tuples  $t_1, t_2 \in R$  with  $t_1 \neq t_2$  we have  $\pi_X(t_1) \neq \pi_X(t_2)$ ). If  $R_1$  and  $R_2$  are relations over sets of attributes  $U_1$  and  $U_2$ , respectively, then an inclusion constraint  $R_1[X] \subseteq R_2[Y]$ , where  $X \subseteq U_1$  and  $Y \subseteq U_2$  are of the same cardinality, holds when  $\pi_X(R_1) \subseteq \pi_Y(R_2)$ . We further say that a foreign key on the attributes of  $R_1[X] \subseteq_{FK} R_2[Y]$  holds if the inclusion constraint  $R_1[X] \subseteq R_2[Y]$  holds, and  $Y$  is a key of  $R_2$ .

With each set of keys and foreign keys, we associate a graph in which we put an edge between attributes  $A$  and  $B$  if there is a constraint  $R_1[X] \subseteq_{FK} R_2[Y]$  with  $A \in X$  and  $B \in Y$ . If this graph is acyclic, we say that the set of constraints is *acyclic*. A schema is acyclic if its constraints are acyclic. In data exchange, one often uses a more technical notion of weak acyclicity: it includes some cyclic schemas for which the chase procedure still terminates. For us, however, the simple concept of acyclicity will suffice, as our translations of schemas only produce acyclic constraints.

**XML documents and DTDs** Assume that we have the following disjoint countably infinite sets:  $El$  of element names,  $Att$  of attribute names, and  $Str$  of possible values of string-valued attributes. All attribute names start with the symbol  $@$ .

An *XML tree* is a finite rooted directed tree  $T = (N, G)$ , where  $N$  is the set of nodes and  $G$  is the set of edges, together with

1. a labeling function  $\lambda : N \rightarrow El$ ;
2. attribute-value assignments, which are partial functions  $\rho_{@a} : N \rightarrow Str$  for each  $@a \in Att$ ; and
3. an ordering on the children of every node.

A DTD  $D$  over  $El$  with a distinguished symbol  $r$  (for the root) and a set of attributes  $Att$  consists of a mapping  $P_D$  from  $El$  to regular expressions over  $El - \{r\}$ , usually written as productions  $\ell \rightarrow e$  if  $P_D(\ell) = e$ , and a mapping  $A_D$  from  $El$  to  $2^{Att}$  that assigns a (possibly empty) set of attributes to each element type. For notational convenience, we always assume that attributes come in some order, just like in the relational case: attributes in tuples come in some order, so we can write  $R(a_1, \dots, a_n)$ . Likewise, we shall describe an  $\ell$  labeled tree node with  $n$  attributes as  $\ell(a_1, \dots, a_n)$ .

A tree  $T$  conforms to a DTD  $D$  (written as  $T \models D$ ) if its root is labeled  $r$ , the set of attributes for a node labeled  $\ell$  is  $A_D(\ell)$ , and the labels of the children of such a node, read from left to right, form a string in the language of  $P_D(\ell)$ .

**A class of DTDs** In this paper we consider a restriction on DTDs called *nested-relational DTDs* [1, 6], a class of DTDs that naturally represent nested relational schemas such as the ones used by the Clio data-exchange system [26]. The reason for using them is that outside of this class, it is very easy to construct instances of XML data-exchange problems that will exhibit coNP-hardness of answering conjunctive queries (which are known to be tractable in practically all instances of relational data exchange), see [6].

A DTD  $D$  is *non-recursive* if the graph  $G(D)$  defined as  $\{(\ell, \ell') \mid \ell' \text{ is mentioned in } P(\ell)\}$  is acyclic. A non-recursive DTD  $D$  is *nested-relational* if all rules of  $D$  are of the form  $l \rightarrow \tilde{l}_0 \dots \tilde{l}_m$  where all the  $\tilde{l}_i$ 's are distinct, and each  $\tilde{l}_i$  is one of  $l_i$  and  $l_i^*$ .

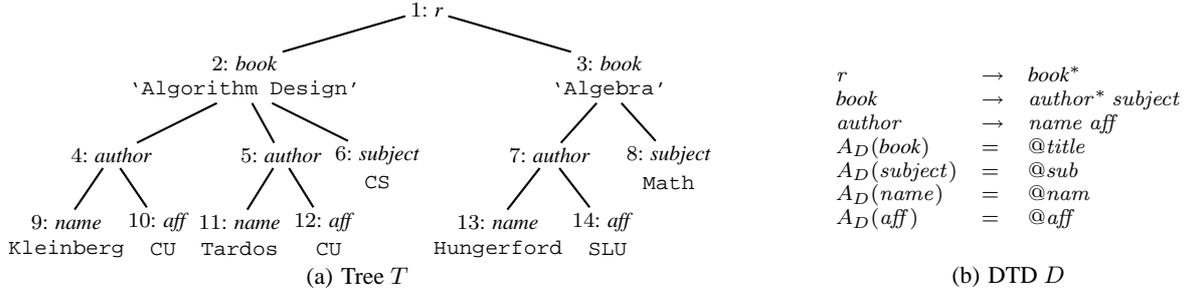


Figure 1: The XML tree  $T$  conforms to  $D$

From now on, unless otherwise noted, all DTDs are assumed to be nested-relational. We also assume, without loss of generality, that the graph  $G(D)$  is not a directed acyclic graph (DAG) but a tree. (One can always unfold a DAG into a tree by tagging occurrences of element types with the types of their predecessors.)

EXAMPLE 2.1. Figure 1(a) shows an example of an XML tree. In the Figure, the node identifiers precede the corresponding labels of each node in  $T$ ; we omit the attribute names and only show the attribute values of each node. In addition, Figure 1(b) shows an example of a nested-relational DTD. Moreover, it is easy to see that the tree  $T$  of Figure 1(a) conforms to  $D$ .  $\square$

### 3. Translations of schemas and documents

We now review the *inlining* technique [28], provide a precise definition of the translation, and show that it satisfies our **Requirements 1** and **2**. The main idea of inlining is that separate relations are created for the root and for each element type that appears under a star, and other element types are inlined in the relations corresponding to their “nearest appropriate ancestor”. Each relation for an element type has an ID attribute that is a key, as well as (for non-root) a “parent-ID” attribute that is a foreign key pointing to the “nearest appropriate ancestor” of that element in the document. All the attributes of a given element type in the DTD become attributes in the relation corresponding to that element type when such a relation exists, or otherwise become attributes in the relation for the “nearest appropriate ancestor” of the given element type.

We begin with a formal definition of the *nearest appropriate ancestor* for the element types used in  $D$ . Given a nested-relational DTD  $D = (P_D, A_D, r)$ , we “mark” in  $G(D)$  each element type that occurs under a star in  $P_D$ . In addition, we mark the root element type in  $G(D)$ . Then, for a given element type  $\ell$ , we define the *nearest appropriate ancestor* of  $\ell$ , denoted by  $\mu(\ell)$ , as the closest marked element type  $\ell'$  in the path from the root element to  $\ell$  in the graph  $G(D)$ . The inlining schema generation is formally captured by means of the procedure INLSHEMA below.

EXAMPLE 3.1. Consider again DTD  $D$  in Figure 1(b). The relational schema INLSHEMA( $D$ ) is as follows:

$$\begin{aligned}
 &R_r(\underline{rID}) \\
 &R_{book}(\underline{bookID}, @title, rID, subID, @sub) \\
 &R_{author}(\underline{authID}, bookID, nameID, affID, @nam, @aff)
 \end{aligned}$$

Keys are underlined; we also have the following foreign keys:  $R_{book}(rID) \subseteq_{FK} R_r(rID)$  and  $R_{author}(bookID) \subseteq_{FK} R_{book}(bookID)$ .  $\square$

The following shows that our **Requirement 1** is satisfied.

PROPOSITION 3.2. *For every nested relational DTD  $D$ , the output of INLSHEMA( $D$ ) is an acyclic relational schema.*

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#### Procedure INLSHEMA( $D$ )

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**Input** : A nested relational DTD  $D$ .

**Output**: A relational schema  $S_D$  and a set of integrity constraints  $\Delta_D$

Set  $S_D = \emptyset$  and  $\Delta_D = \emptyset$

**for each** marked element type  $\ell$  of  $D$ :

add to  $S_D$  a relation  $R_\ell$ , with attributes:

$$attr(R_\ell) = \begin{cases} id_\ell \\ A_D(\ell) \\ id_{\mu(\ell)} & | \text{ if } \ell \neq r. \\ id_{\ell'} & | \mu(\ell') = \ell, \ell' \text{ is not marked,} \\ A_D(\ell') & | \mu(\ell') = \ell, \ell' \text{ is not marked.} \end{cases}$$

**endfor**

**for each** relation  $R_\ell$  in  $S_D$ :

add to  $\Delta_D$  the constraint stating that  $id_\ell$  is key of  $R_\ell$  and, if  $\ell \neq r$ , the foreign key

$$R_\ell[id_{\mu(\ell)}] \subseteq_{FK} R_{\mu(\ell)}[id_{\mu(\ell)}].$$

**endfor**

add to  $\Delta_D$  the dependency (stating the uniqueness of the root)

$$\forall \bar{y} \forall \bar{z} R_r(x, \bar{y}) \wedge R_r(x', \bar{z}) \rightarrow x = x'.$$

**return** ( $S_D, \Delta_D$ )

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PROOF. Let  $D$  be a DTD over a set of element types  $El$ . Notice that all the foreign key constraints created with the procedure INLSHEMA( $D$ ) are of the form  $R_\ell[id_{\mu(\ell)}] \subseteq_{FK} R_{\mu(\ell)}[id_{\mu(\ell)}]$ , for some marked label  $\ell \in El$ ; that is, each relation  $R_\ell$  references the relation  $R_{\mu(\ell)}$  that corresponds to the *nearest appropriate ancestor* of  $\ell$ . Thus, the graph associated with the constraints of INLSHEMA( $D$ ) only contains edges from the attribute  $id_{\mu(\ell)}$  of relation  $R_\ell$  to attribute  $id_{\mu(\ell)}$  relation  $R_{\mu(\ell)}$ . The proof then follows from the fact that  $G(D)$  is acyclic, and thus the labels of  $D$  cannot form a cycle of nearest appropriate ancestors.  $\square$

**Shredding of XML documents.** We now move to the shredding procedure. Given the inlining INLSHEMA( $D$ ) = ( $S_D, \Delta_D$ ) of a DTD  $D$ , and an XML tree  $T$  conforming to  $D$ , we use the algorithm INLDOC to *shred*  $T$  into an instance of the relational schema  $S_D$  that satisfies the constraints in  $\Delta_D$ . Let us first explain this translation by means of an example.

EXAMPLE 3.3. Recall tree  $T$  from Figure 1(a) and DTD  $D$  from Figure 1(b). Figure 2 shows relations  $R_{book}$  and  $R_{author}$  in the shredding of  $T$ .  $\square$

To present the algorithm, we define the *nearest appropriate ancestor*  $\mu(n)$  of a node  $n$  of an XML document  $T$  that conforms to a DTD  $D$ , as follows. Mark each node  $n$  of  $T$  such that  $\lambda(n)$  is

bookID	@title	rID	subID	@sub
$id_2$	'Algorithm Design'	$id_1$	$id_6$	CS
$id_3$	'Algebra'	$id_1$	$id_8$	Math

(a) Relation  $R_{book}$  in  $\text{INLDOC}(T, D)$ 

authID	bookID	nameID	afID	@nam	@af
$id_4$	$id_2$	$id_9$	$id_{10}$	'Kleinberg'	CU
$id_5$	$id_2$	$id_{11}$	$id_{12}$	'Tardos'	CU
$id_7$	$id_3$	$id_{13}$	$id_{14}$	'Hungerford'	SLU

(b) Relation  $R_{author}$  in  $\text{INLDOC}(T, D)$ **Figure 2: Shredding of  $T$  into  $\text{INLSchema}(D)$** 

starred in  $D$ , as well as the root of  $T$ . Then  $\mu(n)$  is the closest marked node  $n'$  that belongs to the path from the root to  $n$ . In the following algorithm, and for the remainder of the paper, we denote by  $id_n$  the relational element representing the node  $n$  of a tree  $T$ .

**Procedure**  $\text{INLDOC}(T, D)$ 

**Input** : A nested relational DTD  $D$  and an XML tree  $T$  that conforms to  $D$ .

**Output**: A relational instance of the schema  $\text{INLSchema}(D)$ .

**for each** marked node  $n$  of  $T$ :

Let  $\ell$  be the label of  $n$ ; Add to the relation  $R_\ell$  of  $I$  a tuple that contains elements

$$\left\{ \begin{array}{ll} id_n & \\ \rho_{@a}(n) & | \quad @a \in A_D(\ell) \\ id_{\mu(n)} & | \quad \text{if } \ell \neq r \\ id_{n'} & | \quad \mu(n') = n, n' \text{ is not marked.} \\ \rho_{@a}(n') & | \quad \mu(n') = n, @a \in A_D(\lambda(n')) \text{ and} \\ & | \quad n' \text{ is not marked} \end{array} \right.$$

where the identifiers and attributes values for each of the elements  $id_{n'}$ ,  $id_{\mu(n)}$  and  $\rho_{@a}(n')$  coincide with the position of the attributes for  $id_{\lambda(n')}$ ,  $id_{\mu(\ell)}$  and  $A_D(\lambda(n'))$  of  $R_\ell$ .

**endfor**

**return**  $I$

The following proposition shows that our **Requirement 2** is satisfied.

**PROPOSITION 3.4.** *Let  $D$  be a DTD, and  $T$  an XML tree such that  $T \models D$ . Then  $\text{INLDOC}(T, D)$  is an instance of the schema computed by  $\text{INLSchema}(D)$ .*

**PROOF.** Let  $D$  and  $T$  as stated in the Proposition, and  $(S_D, \Delta_D)$  be the output of  $\text{INLSchema}(D)$ . That  $\text{INLDOC}(T, D)$  satisfies the key constraints of  $\Delta_D$  is trivial, since the identifier of each node in  $T$  is unique. The same applies for the dependency stating the uniqueness of the root; since  $T$  conforms to  $D$ , the root of  $T$  (and only the root) must be labelled  $r$ . Moreover, for each foreign key in  $\Delta$  of the form  $R_\ell[id_{\mu(\ell)}] \subseteq R_{\mu(\ell)}[id_{\mu(\ell)}]$ , notice that, since  $G(D)$  is a tree, for each  $\ell \in E_I - \{r\}$ , there is exactly one element  $\ell'$  such that  $\ell' = \mu(\ell)$ . Since  $T$  conforms to  $D$ , every  $\ell$ -labelled node in  $T$  must be a descendant of an  $\ell'$ -labelled node. This guarantees that the interpretation of relations  $R_\ell$  and  $R_{\ell'}$  in  $\text{INLDOC}(T, D)$  satisfy the constraint  $R_\ell[id_{\mu(\ell)}] \subseteq R_{\mu(\ell)}[id_{\mu(\ell)}]$ ; each tuple in the interpretation of  $R_\ell$  over  $\text{INLDOC}(T, D)$  corresponds to a node  $n$  in  $T$  that must be a descendant of an  $\ell'$  labelled node  $n'$  in  $T$ , and thus there must be a tuple in the interpretation of  $R_{\ell'}$  identified with the element  $id_{n'}$ .  $\square$

## 4. Relational and XML Data Exchange

We now quickly review the basics of relational data exchange and introduce XML schema mappings that guarantee tractable query answering.

**Relational Data Exchange** A schema mapping  $\mathcal{M}$  is a triple  $(\mathbb{S}, \mathbb{T}, \Sigma)$ , where  $\mathbb{S}$  is a source schema,  $\mathbb{T} = (\mathbf{T}, \Delta_{\mathbf{T}})$  is a target schema with a set of constraints  $\Delta_{\mathbf{T}}$ , and  $\Sigma$  is a set of *source-to-target dependencies* that specify how the source and the target are related. Most commonly these are given as source-to-target tuple generating dependencies (st-tgds):

$$\varphi(\bar{x}) \rightarrow \exists \bar{z} \psi(\bar{x}, \bar{z}), \quad (1)$$

where  $\varphi$  and  $\psi$  are conjunctions of relational atoms over  $\mathbb{S}$  and  $\mathbb{T}$ , respectively.

In data-exchange literature, one normally considers instances with two types of values: constants and nulls. Instances  $\mathcal{S}$  of the source schema  $\mathbb{S}$  consist only of constant values, and nulls are used to populate target instances  $\mathcal{T}$  when some values are unknown.

An instance  $\mathcal{T}$  of  $\mathbb{T}$  (which may contain both constants and nulls) is called a *solution* for an instance  $\mathcal{S}$  of  $\mathbb{S}$  under  $\mathcal{M}$ , or an  $\mathcal{M}$ -*solution*, if every st-tgd (1) from  $\Sigma$  is satisfied by  $(\mathcal{S}, \mathcal{T})$  (that is, for each tuple  $\bar{a}$  such that  $\varphi(\bar{a})$  is true in  $\mathcal{S}$ , there is a tuple  $\bar{b}$  such that  $\psi(\bar{a}, \bar{b})$  is true in  $\mathcal{T}$ ). The set of all  $\mathcal{M}$ -solutions for  $\mathcal{S}$  is denoted by  $\text{SOL}_{\mathcal{M}}(\mathcal{S})$  (or  $\text{SOL}(\mathcal{S})$  if  $\mathcal{M}$  is understood).

**Certain answers and canonical universal solution** The main difficulty in answering a query  $Q$  against the target schema is that there could be many possible solutions for a given source. Thus, for query answering in data exchange one normally uses the notion of certain answers, that is, answers that do not depend on a particular solution. Formally, for a source  $\mathcal{S}$  and a mapping  $\mathcal{M}$ , we define  $\text{CERTAIN}_{\mathcal{M}}(Q, \mathcal{S})$  as  $\bigcap \{Q(\mathcal{T}) \mid \mathcal{T} \in \text{SOL}_{\mathcal{M}}(\mathcal{S})\}$ .

Building all solutions is impractical (or even impossible), so it is important to find a particular solution  $\mathcal{T}_0 \in \text{SOL}_{\mathcal{M}}(\mathcal{S})$ , and a rewriting  $Q_{\text{rewr}}$  of  $Q$ , so that  $\text{CERTAIN}_{\mathcal{M}}(Q, \mathcal{S}) = Q_{\text{rewr}}(\mathcal{T}_0)$ .

*Universal solutions* were identified in [12] as the preferred solutions in data exchange. (We provide a precise definition later in this section.) Over them, every positive query can be answered, with a particularly simple rewriting: after  $Q$  is evaluated on a universal solution  $\mathcal{T}_0$ , tuples containing null values are discarded. Even among universal solutions there are ones that are most commonly materialized in data-exchange systems, such as the *canonical solution*  $\text{CANSOL}_{\mathcal{M}}(\mathcal{S})$ , computed by applying the chase procedure with constraints  $\Sigma$  and  $\Delta_{\mathbf{T}}$  to the source instance  $\mathcal{S}$ . If all the constraints in  $\Delta_{\mathbf{T}}$  are acyclic (in fact, even a weaker notion suffices), such a chase terminates and computes  $\text{CANSOL}_{\mathcal{M}}(\mathcal{S})$  in polynomial time [12].

Note that our **Requirement 4** relates universal solutions in relational and XML data exchange. In particular, we do not insist on working with the canonical solutions; others, such as the core [13] or the algorithmic constructions of [25], can be used as well.

**Towards XML schema mappings: patterns** To define XML schema mappings, we need the notions of schemas and source-to-target dependencies. The notion of schema is well understood in the XML context. Our dependencies, as in [6, 4, 3], will be based on *tree patterns*. Patterns are defined inductively as follows:

- $\ell(\bar{x})$  is a pattern, where  $\ell$  is a label, and  $\bar{x}$  is a (possibly empty) tuple of variables (listing attributes of a node);

- $\ell(\bar{x})[\pi_1, \dots, \pi_k]$  is a pattern, where  $\pi_1, \dots, \pi_k$  are patterns, and  $\ell$  and  $\bar{x}$  are as above.

We write  $\pi(\bar{x})$  to indicate that  $\bar{x}$  is the tuple of all the variables used in a pattern. The semantics is defined with respect to a node of a tree and to a valuation of all the variables of a pattern as attribute values. Formally,  $(T, v) \models \pi(\bar{a})$  means that  $\pi$  is satisfied in node  $v$  when  $\bar{x}$  is interpreted as  $\bar{a}$ . It is defined as follows:

- $(T, v) \models \ell(\bar{a})$  if  $v$  is labeled  $\ell$  and its tuple of attributes is  $\bar{a}$ ;
- $(T, v) \models \ell(\bar{a})[\pi_1(\bar{a}_1), \dots, \pi_k(\bar{a}_k)]$  if
  1.  $(T, v) \models \ell(\bar{a})$  and
  2. there exist children  $v_1, \dots, v_k$  of  $v$  (not necessarily distinct) so that  $(T, v_i) \models \pi_i(\bar{a}_i)$  for every  $i \leq k$ .

We write  $T \models \pi(\bar{a})$  if  $(T, r) \models \pi(\bar{a})$ , that is, the pattern is witnessed at the root.

EXAMPLE 4.1. Consider tree  $T$  from Figure 1(a), and the tree pattern  $\pi(x, y) = r[\text{book}(x)[\text{author}[\text{name}(y)]]]$ , which finds books together with the names of their authors. Then it is easy to see that  $T \models \pi(\text{'Algorithm Design', Tardos})$ . In fact, evaluation of  $\pi(x, y)$  over  $T$  returns the tuples ( $\text{'Algorithm Design', Tardos}$ ), ( $\text{'Algorithm Design', Kleinberg}$ ), and ( $\text{'Algebra', Hungerford}$ ).  $\square$

Given a DTD  $D$  and a tree pattern  $\pi$ , we say that  $\pi$  is *compatible* with  $D$  if there exists a tree  $T$  that conforms to  $D$  and a tuple of attribute values  $\bar{a}$  such that  $T \models \pi(\bar{a})$ . In general, checking compatibility of patterns with DTDs is NP-complete [10], but for the DTDs we consider here it can be easily done in polynomial time.

EXAMPLE 4.2.[Example 4.1 continued] The pattern  $\pi(x, y)$  is compatible with the DTD  $D$  of Figure 1(b). On the other hand, the pattern  $\pi'(x) = r[\text{author}(x)]$  is not, because no tree consistent with  $D$  can have a child of  $r$  labeled as *author*, or an *author*-labeled node with an attribute.  $\square$

*Remark* More general patterns have been considered in the literature [5, 24, 10, 4, 3]; in particular, they may involve descendant navigation, wild cards for labels, and sibling order. However, [6, 4, 3] showed that with these features added, query answering in data exchange becomes intractable even for very simple queries. In fact, the restrictions we use in our definition were identified in [6] as essential for tractability of query answering. Note that the same restriction was imposed on queries when transforming XML data into nested-relational schemas [15, 27].

**XML schema mappings** As our descriptions of XML schemas we shall use DTDs. Indeed, for complex schemas, query answering in data exchange is known to be intractable [6], and DTDs will suffice to capture all the known tractable cases. Source-to-target constraints will be given via patterns.

Formally, an *XML schema mapping* is a triple  $\mathcal{M} = (D_S, D_T, \Sigma)$ , where  $D_S$  is the source (nested relational) DTD,  $D_T$  is the target (nested relational) DTD, and  $\Sigma$  is a set of *XML source-to-target dependencies* [6], or XML stds, of form

$$\pi(\bar{x}) \rightarrow \pi'(\bar{x}, \bar{z}), \quad (2)$$

where  $\pi$  and  $\pi'$  are tree patterns compatible with  $D_S$  and  $D_T$ , respectively.

As in the relational case, target trees may contain nulls to account for values not specified by mappings. Given a tree  $T$  that conforms to  $D_S$ , a tree  $T'$  (over constants and nulls) is an  $\mathcal{M}$ -solution for  $T$  if  $T'$  conforms to  $D_T$ , and the pair  $(T, T')$  satisfies all the dependencies of the form (2) from  $\Sigma$ . The latter means that for every tuple  $\bar{a}$  of attribute values from  $T$ , if  $T$  satisfies  $\pi(\bar{a})$ , then there exists

a tuple  $\bar{b}$  of attribute values from  $T'$  such that  $T'$  satisfies  $\pi'(\bar{a}, \bar{b})$ . The set of all  $\mathcal{M}$ -solutions for  $T$  is denoted by  $\text{SOL}_{\mathcal{M}}(T)$ .

EXAMPLE 4.3. Consider the data-exchange scenario  $(D, D_T, \mathcal{M})$  given by the DTDs  $D$  and  $D_T$  of Figures 1(b) and 3(b), respectively, and where  $\mathcal{M}$  is specified by the dependency

$$r[\text{book}(x)[\text{author}[\text{name}(y)]]] \rightarrow r[\text{writer}[\text{name}(y), \text{work}(x)]],$$

that restructures book-author pairs as writer-work. It can be shown that the XML tree  $T'$  in Figure 3(a) is an  $\mathcal{M}$ -solution for  $T$ .  $\square$

We now formally define universal solutions. While building up auxiliary definitions that are needed to define the term, we also introduce some technical notions that will be used through the remainder of the paper.

**Homomorphisms and tree homomorphisms.** Let  $K_1$  and  $K_2$  be instances of the same schema  $\mathbf{R}$ . A *homomorphism*  $h$  from  $K_1$  to  $K_2$  is a function  $h$  defined from the domain of  $K_1$  to the domain of  $K_2$  such that: (1)  $h(c) = c$  for every constant element  $c$  in  $K_1$ , and (2) for every  $R \in \mathbf{R}$  and every tuple  $\bar{a} = (a_1, \dots, a_k)$  in the relation  $R$  in  $K_1$ , it holds that  $h(\bar{a}) = (h(a_1), \dots, h(a_k))$  belongs to the relation  $R$  in  $K_2$ . Notice that this definition of homomorphism slightly differs from the usual one, as the additional constraint that homomorphisms are the identity on the constants is imposed.

Given a conjunctive query  $Q(\bar{x})$  over a schema  $\mathbf{R}$ , we denote by  $I_{Q(\bar{x})}$  the instance of  $\mathbf{R}$  constructed as follows: for every relational symbol  $R \in \mathbf{R}$  and relational atom  $R(\bar{b})$  occurring in  $Q(\bar{x})$ , we include tuple  $\bar{b}$  in the relation  $R$  of  $I_{Q(\bar{x})}$ . We define all variables in  $\bar{x}$  to be constant elements in  $I_{Q(\bar{x})}$ , whereas every existentially quantified variable of  $Q$  is a null element.

It is now straightforward to prove the following lemma:

LEMMA 4.4. *Let  $I$  be an instance of schema  $\mathbf{R}$ , and  $Q$  a conjunctive query. Then, a tuple  $\bar{a}$  of constant elements belongs to the evaluation of  $Q$  over  $I$  if and only if there is a homomorphism from  $I_{Q(\bar{a})}$  to  $I$ .*

We also need to introduce the equivalent definition of homomorphisms for XML trees, or *tree homomorphism* [6]. Let  $T = (N, G)$  and  $T' = (N', G')$  be XML trees, let  $n_r$  and  $n'_r$  be the roots of  $T$  and  $T'$ , respectively, and let  $\text{Str}(T) = \{s \in \text{Str} \mid \text{there exists } n \in N \text{ and } @a \in \text{Att} \text{ such that } \rho_{@a}(n) = s\}$ ,  $\text{Str}(T')$  defined correspondingly. Then,  $h : N \cup \text{Str}(T) \rightarrow N' \cup \text{Str}(T')$  is a homomorphism from  $T$  to  $T'$ , if:

- for every  $n \in N$ ,  $h(n) \in N'$ ;
- for every constant element  $s \in \text{Str}(T)$ ,  $h(s) = s$ , and for every null  $s \in \text{Str}(T)$ ,  $h(s) \in \text{Str}(T')$ ;
- $h(n_r) = n'_r$ ;
- for every  $n_1, n_2 \in N$ , if  $G(n_1, n_2)$ , then  $G'(h(n_1), h(n_2))$ ;
- for every  $n \in N$ ,  $\lambda_T(n) = \lambda_{T'}(h(n))$ ; and
- for every  $n \in N$  and  $@a \in \text{Att}$  such that  $\rho_{@a}(n)$  is defined,  $h(\rho_{@a}(n)) = \rho_{@a}(h(n))$ .

Given a tree pattern  $\pi(\bar{x})$ , we construct the tree  $T_{\pi(\bar{x})}$  inductively: if  $\pi(\bar{x}) = \ell(\bar{x})[\pi_1(\bar{x}_1), \dots, \pi_k(\bar{x}_k)]$ , then the root of  $T_{\pi(\bar{x})}$  is a node labelled  $\ell$ , with attributes  $\bar{x}$ , and  $k$  children corresponding to  $T_{\pi_1(\bar{x}_1)}, \dots, T_{\pi_k(\bar{x}_k)}$ . As for the relational case, it is easy to prove the following lemma:

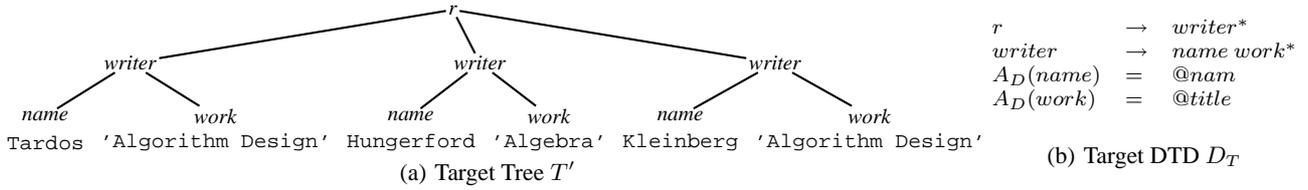


Figure 3: Tree  $T'$  is an  $\mathcal{M}$ -solution for  $T$

LEMMA 4.5. *Let  $T$  be an XML tree,  $\pi(\bar{x})$  a tree pattern, and  $s$  a tuple of values in  $Str$ . Then,  $\bar{s} \in \pi(T)$  if and only if there is a homomorphism from  $T_{\pi(\bar{s})}$  to  $T$ .*

**Universal Solutions.** By means of homomorphisms, we give a precise definition of universal solutions in relational or XML data exchange settings. Formally, let  $(\mathbb{S}, \mathbb{T}, \mathcal{M})$  be a relational data exchange setting. Then, given an instance  $I$  of  $\mathbb{S}$ , we say that an  $\mathcal{M}$ -solution  $J$  for  $I$  is an  $\mathcal{M}$ -universal solution for  $I$  if for every other  $\mathcal{M}$ -solution  $J'$  for  $I$ , there exists an homomorphism from  $J$  to  $J'$  [12]. The definition for the case of XML data exchange setting is analogously formulated using the notion of tree homomorphism [6].

## 5. XML data exchange using relations

We now provide algorithms for implementing XML data exchange via relational translations. Since we have already shown how to translate DTDs and documents, we need to present translations of stds of mappings and queries. Both of them are based on translating patterns into relational conjunctive queries. We first concentrate on that translation. Then we show how to extend it easily to mappings and queries, and prove the correctness of the translations. This will complete our program of using a relational system for XML data exchange in a semantically correct way.

**Inlining tree patterns.** The key ingredient in our algorithms is a translation of patterns  $\pi$  compatible with a DTD  $D$  into a conjunctive query  $\text{INLPATTERN}(\pi, D)$  over the relational schema  $\text{INLSHEMA}(D)$ . Very roughly, it can be viewed as this:

1. View a pattern  $\pi(\bar{x})$  as a tree  $T_\pi$  in which some attribute values could be variables;
2. Compute the relational database  $\text{INLDOC}(T_\pi, D)$  (which may have variables as attribute values);
3. View  $\text{INLDOC}(T_\pi, D)$  as a tableau of a conjunctive query; the resulting query is  $\text{INLPATTERN}(\pi, D)$ .

The algorithm is actually more complicated because  $\text{INLDOC}$  cannot be used in Step 2; we shall explain shortly why.

Towards defining  $\text{INLPATTERN}$ , observe that each tree pattern  $\pi(\bar{x})$  can be viewed as an XML document  $T_{\pi(\bar{x})}$ , in which both values and variables can be used as attribute values. It is defined inductively as follows:  $T_{\ell(\bar{x})}$  is a single-node tree labeled  $\ell$ , with  $\bar{x}$  as attribute values, and if  $\pi$  is  $\ell(\bar{x})[\pi_1(\bar{x}_1), \dots, \pi_k(\bar{x}_k)]$ , then the root of  $T_\pi$  is labeled  $\ell$  and has  $\bar{x}$  as attribute values. It also has  $k$  children, with the subtrees rooted at them being  $T_{\pi_1(\bar{x}_1)}, \dots, T_{\pi_k(\bar{x}_k)}$ .

However, even for a pattern  $\pi(\bar{x})$  compatible with a DTD  $D$ , we may not be able to define its inlining as the inlining of  $T_{\pi(\bar{x})}$ , because  $T_{\pi(\bar{x})}$  need not conform to  $D$ . For example, if a DTD has a rule  $r \rightarrow ab$  and we have a pattern  $r[a]$ , it is compatible with  $D$ , but  $T_{r[a]}$  does not conform to  $D$ , as it is missing a  $b$ -node. Hence, the procedure  $\text{INLDOC}$  cannot be used ‘as-is’ in our algorithm.

Nevertheless, we can still mark the nodes of  $T_{\pi(\bar{x})}$  with respect to  $D$  and define the nearest appropriate ancestor exactly as it has been done previously. Intuitively, the procedure  $\text{INLPATTERN}$  shreds each node of  $T_{\pi(\bar{x})}$  into a different predicate, and then joins these predicates using the nearest appropriate ancestor.

---

### Procedure $\text{INLPATTERN}(\pi, D)$

---

**Input** : A DTD  $D$ , a tree pattern  $\pi(\bar{x})$  compatible with  $D$ .

**Output**: Conjunctive query over  $\text{INLSHEMA}(D)$ .

**for each** node  $v$  of  $T_{\pi(\bar{x})}$  of form  $\ell(\bar{x}_v)$ :

Construct a query  $Q_v(\bar{x}_v)$  as follows:

**if**  $v$  is marked **then**

$$Q_v(\bar{x}_v) := \exists id_v \exists id_{\mu(v)} \exists \bar{z} R_\ell(id_v, \bar{x}_v, id_{\mu(v)}, \bar{z}),$$

where  $\bar{z}$  is a tuple of fresh variables, and the positions of variables  $id_v$ ,  $\bar{x}_v$  and  $id_{\mu(v)}$  are consistent with the attributes  $id_\ell$ ,  $A_D(\ell)$  and  $id_{\mu(\ell)}$  respectively in  $\text{attr}(R_\ell)$ .

If  $\ell = r$ , then  $Q_v$  does not use  $id_{\mu(v)}$ .

**else** ( $v$  is not marked):

set  $v' := \mu(v)$ ,  $\ell' := \lambda(v')$ , and let  $Q_v(\bar{x}_v)$  be

$$\exists id_{v'} \exists id_{\mu(v')} \exists id_v \exists \bar{z} R_{\ell'}(id_{v'}, id_{\mu(v')}, id_v, \bar{x}_v, \bar{z}),$$

where  $\bar{z}$  is a tuple of fresh variables, and the positions of the variables  $id_{v'}$ ,  $id_{\mu(v')}$ ,  $id_v$  and  $\bar{x}_v$  are consistent with the attributes  $id_{\ell'}$ ,  $id_{\mu(\ell')}$ ,  $id_\ell$  and  $A_D(\ell)$  respectively in  $\text{attr}(R_{\ell'})$ . If  $\ell' = r$ , then  $Q_v$  does not use  $id_{\mu(v')}$ .

**endfor**

**return**  $\bigwedge_{v \in T_{\pi(\bar{x})}} Q_v(\bar{x}_v)$ .

---

Note that the compatibility of  $\pi$  with  $D$  ensures that  $\text{INLPATTERN}$  is well defined. That is, (1) every attribute formula of the form  $\ell(\bar{x})$  only mentions attributes in  $A_D(\ell)$ , and (2) for all nodes  $v, v' \in T_{\pi(\bar{x})}$ , if  $v'$  is a child of  $v$ , then  $\lambda(v') \in P_D(\lambda(v))$ .

**Correctness.** Given a pattern  $\pi(\bar{x})$ , the evaluation of  $\pi$  on a tree  $T$  is  $\pi(T) = \{\bar{a} \mid T \models \pi(\bar{a})\}$ . The following proposition shows the correctness of  $\text{INLPATTERN}$ .

PROPOSITION 5.1. *Given a nested relational DTD  $D$ , a pattern  $\pi$  compatible with  $D$ , and a tree  $T$  that conforms to  $D$ , we have  $\pi(T) = \text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D))$ .*

That is, the inlining of  $\pi$ , applied to the inlining of  $T$ , returns  $\pi(T)$ .

PROOF. The proof has two parts: First, we show (1) that

$$\pi(T) \subseteq \text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D))$$

holds, and then complete the proof by showing (2)

$$\text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D)) \subseteq \pi(T).$$

Part (1): To prove that

$$\pi(T) \subseteq \text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D)),$$

let  $\pi(\bar{x})$ ,  $D$  and  $T$  be as defined, so that  $T$  conforms to  $D$ . Assume now that  $\bar{a}$  is a tuple of attribute values such that  $\bar{a} \in \pi(T)$ , and let  $h$  be the homomorphism from  $T_{\pi(\bar{a})}$  to  $T$ . (By Lemma 4.5,  $h$  is guaranteed to exist.)

In order to show that  $\bar{a}$  belongs to  $\text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D))$ , we show how to construct a homomorphism  $g$  from  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  to  $\text{INLDOC}(T, D)$  (this, by Lemma 4.4, suffices for the proof). Recall that the elements of  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  corresponds precisely to the variables of  $\text{INLPATTERN}(\pi, D)(\bar{a})$ . Define  $g$  as follows:

- For each variable of the form  $id_v$  in  $\text{INLPATTERN}(\pi, D)(\bar{a})$ , where  $v$  is a node of  $T_{\pi(\bar{a})}$ , define  $g(id_v) = id_{h(v)}$ ,
- for each  $a \in \bar{a}$ , let  $g(a) = h(a)$ , and
- for each other existentially quantified variable  $z$  in  $\text{INLPATTERN}(\pi, D)(\bar{a})$  not of form  $id_v$ , assume that  $z$  belongs to a predicate  $R_\ell(\bar{z})$  in  $\text{INLPATTERN}(\pi, D)(\bar{a})$ . Let  $id_v$  be the variable in predicate  $R_\ell(\bar{z})$  that corresponds to the position of the attribute  $id_\ell$  of relation  $R_\ell$ , and assume that  $h(v) = n$ , for some node  $n \in T$ . Then, as defined in the previous item,  $g(id_v) = id_n$ . From the definition of the inlining procedure, we know that  $\text{INLDOC}(T, D)$  contains a fact (and only one, since the attribute  $id_\ell$  is a key for the relation  $R_\ell$ ) of the form  $R_\ell(id_n, \bar{b})$ , for some tuple  $\bar{b}$  of elements. Define  $g$  so that it maps the variable  $z$  to the element in the position of  $(id_n, \bar{b})$  that corresponds to the position that  $z$  occupies in the predicate  $R_\ell(\bar{z})$  in  $\text{INLPATTERN}(\pi, D)(\bar{a})$ .

We first show that  $g$  is well defined. First, it is easy to see that  $g$  is defined for every element of  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$ . We now prove that there is no element in  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  that is mapped by  $g$  to two different values in  $\text{INLDOC}(T, D)$ . To see this, assume for the sake of contradiction that there is an element  $x$  in  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  such that  $g$  is defined to map  $x$  to two elements of  $\text{INLDOC}(T)$ . Then, there are three facts to consider:

- $x$  cannot be a variable in  $\text{INLPATTERN}(\pi, D)(\bar{a})$  of the form  $id_v$  for some node  $v$  of  $T_{\pi(\bar{a})}$ , since we have defined  $x$  to be mapped to  $id_{h(v)}$  only;
- $x$  cannot belong to  $\bar{a}$ , since we have defined every  $a \in \bar{a}$  to be mapped only to  $h(a)$ ;
- then,  $x$  is an existentially quantified variable in  $\text{INLPATTERN}(\pi, D)(\bar{a})$  that is not of form  $id_v$  (that is, it is a fresh variable generated by the procedure  $\text{INLPATTERN}$ ). But notice then that  $x$  belongs to only one predicate of  $\text{INLPATTERN}(\pi, D)(\bar{a})$ . Moreover, as explained in the definition of  $g$ , there is only one tuple in  $\text{INLDOC}(T, D)$  to which  $x$  is being mapped.

We now prove that  $g$  is indeed a valid homomorphism. First, it is easy to see that for every  $a \in \bar{a}$ ,  $g(a) = a$ . This follows (i) from two facts: (i) we have defined  $g(a)$  as  $h(a)$ , and (ii) by construction of  $T_{\pi(\bar{a})}$ , every  $a \in \bar{a}$  is a constant, and thus  $h(a) = a$ .

Consider now a fact of the form  $R_\ell(\bar{w})$  in  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$ . We need to show that  $R_\ell(g(\bar{w}))$  belongs to  $\text{INLDOC}(T, D)$ . We will assume for the sake of readability that  $\ell \neq r$ . The proof can be easily adapted for the case when  $\ell = r$ . From the inlining procedure for queries, there must be a node  $v$  of  $T_{\pi(\bar{a})}$  such

that  $\text{INLPATTERN}$  adds to  $\text{INLPATTERN}(\pi, D)(\bar{a})$  some existential quantification of the predicate  $R_\ell(\bar{w})$  in the step that corresponds to  $v$  (that is,  $R_\ell(\bar{w})$  is part of  $Q(\bar{a}_v)$ ). We have two cases. Assume first that  $v$  is marked. Then,

$$Q_v(\bar{a}_v) = \exists id_v \exists id_{\mu(v)} \exists \bar{z} R_\ell(id_v, \bar{a}_v, id_{\mu(v)}, \bar{z}),$$

where  $\bar{z}$  is a tuple of fresh variables not used elsewhere in  $\text{INLPATTERN}(\pi, D)(\bar{a})$  and the position of the variables  $id_v$ ,  $\bar{a}_v$  and  $id_{\mu(v)}$  coincide with the attributes  $id_\ell$ ,  $A_D(\ell)$  and  $id_{\mu(\ell)}$  in  $\text{attr}(R_\ell)$ .

Further, we now have that the homomorphism  $h$  maps the node  $v$  of  $T_{\pi(\bar{a})}$  to some node  $h(v)$  in  $T$ . Thus, from the properties of tree homomorphisms, we also know that  $h(v)$  has the element type  $\ell$ , and that for every  $a \in \bar{a}_v$  and  $@_a \in \text{Att}$ , if  $\rho_{@_a}(v) = a$ , then  $\rho_{@_a}(h(v)) = a$ . Moreover, since homomorphisms must preserve the child relation, it is easy to see that the nearest appropriate ancestor of  $h(v)$  in  $T$  must be  $h(\mu(v))$ . Then, it is clear that  $\text{INLDOC}(T, D)$  must contain a tuple of the form  $R_\ell(id_{h(v)}, \bar{a}_v, id_{h(\mu(v))}, \bar{b})$ , for some tuple  $\bar{b}$  of elements, and where the positions of  $\bar{a}_v$  correspond to the attributes in  $A_D(\ell)$  of  $\text{attr}(R_\ell)$  where  $\rho(v)$  is defined. From the definition of  $g$ , it is clear that  $g(id_v, \bar{a}_v, id_{\mu(v)}, \bar{z})$  is the tuple  $(id_{h(v)}, id_{h(\mu(v))}, \bar{a}_v, g(\bar{z}))$ . The proof then follows since  $g(\bar{z})$  is defined to be  $\bar{b}$ .

Second, assume that  $v$  is not marked, and that  $\lambda(v) = \ell$ ,  $\mu(v)$  in  $T_{\pi(\bar{a})}$  is the node  $v'$ , and  $\lambda(v') = \ell'$ . Then, as defined, the query  $Q_v(\bar{a}_v)$  is of form:

$$Q_v(\bar{a}_v) = \exists id_{v'} \exists id_{\mu(v')} \exists id_v \exists \bar{z} R_{\ell'}(id_{v'}, id_{\mu(v')}, id_v, \bar{a}_v, \bar{z}),$$

where  $\bar{z}$  is a tuple of fresh variables not used elsewhere in  $\text{INLPATTERN}(\pi, D)(\bar{a})$ , and the position of the variables  $id_{v'}$ ,  $id_{\mu(v')}$  and  $\bar{a}_v$  is consistent with the attributes  $id_{\ell'}$ ,  $id_\ell$ ,  $id_{\mu(\ell')}$  and  $A_D(\ell)$  in  $\text{attr}(R_{\ell'})$ .

Further, we know that the homomorphism  $h$  maps the nodes  $v$  and  $v'$  of  $T_{\pi(\bar{a})}$  to some nodes  $h(v)$  and  $h(v')$  in  $T$ . Then, from the properties of tree homomorphisms, we obtain that  $\lambda$  assigns the types  $\ell$  and  $\ell'$  to  $h(v)$  and  $h(v')$ , respectively, and that for every  $a \in \bar{a}_v$  and  $@_a \in \text{Att}$ , if  $\rho_{@_a}(v) = a$ , then  $\rho_{@_a}(h(v)) = a$ . Moreover, since homomorphisms preserve the child relation, it is easy to see that  $h(v')$  must be the nearest appropriate ancestor of  $h(v)$  in  $T$ , and that the nearest appropriate ancestor of  $h(v')$  must be  $h(\mu(v'))$ . Then, it is clear that the inlining of  $T$  must contain a tuple of the form  $R_{\ell'}(id_{h(v')}, id_{h(\mu(v'))}, id_{h(v)}, \bar{a}_v, \bar{b})$  for some tuple  $\bar{b}$  of elements, where the positions of  $\bar{a}_v$  correspond to the attributes in  $A_D(\ell)$  such that  $\rho(v)$  is defined. Again, the proof follows since we have defined  $g(\bar{z})$  as  $\bar{b}$ .

Part (2): For the proof that

$$\text{INLPATTERN}(\pi, D)(\text{INLDOC}(T, D)) \subseteq \pi(T),$$

assume that for a tuple  $\bar{a}$  of constants there is a homomorphism  $h$  from  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  to  $\text{INLDOC}(T, D)$ . We construct a homomorphism  $g$  from  $T_{\pi(\bar{a})}$  to  $T$ . By Lemma 4.5, this suffices for the proof.

Define  $g$  as follows:

- For every node  $v$  of  $T_{\pi(\bar{a})}$ , consider the variable  $id_v$  defined in the procedure  $\text{INLDOC}$ , and assume that  $h(id_v) = id_n$ , for some element  $id_n$  of  $\text{INLDOC}(T, D)$ . Define  $g(v) = n$ . Notice that this is well defined: from the definition of  $\text{INLDOC}$ , and the properties of homomorphisms, we know that  $n$  must be a node of  $T$ . (Both  $id_v$  and  $id_n$  occur in a position of the predicates that corresponds to the identifiers of the nodes in the schema  $\text{INLSHEMA}(D)$ .)

- For every  $s \in \text{Str}(T_{\pi(\bar{a})})$ , let  $v$  be the node of  $T_{\pi(\bar{a})}$  such that  $s = \rho_{@a}(v)$ . Then, notice that from the definition of the translation of patterns,  $s$  must be a free variable of the query  $Q_v$  in  $\text{INLPATTERN}(\pi, D)$ , and thus  $I_{\text{INLPATTERN}(\pi, D)}$  contains the variable  $s$ . Define  $g(s) = h(s)$ .

We now prove that  $g$  is a valid homomorphism from  $T_{\pi(\bar{a})}$  to  $T$ . First, as mentioned in the definition of  $g$ , it is clear that  $g(v) \in N$ , for every  $v \in T_{\pi(\bar{a})}$ .

Second, we prove that, if  $v$  is the root of  $T_{\pi(\bar{a})}$ , then  $g(v) = n_r$ , where  $n_r$  is the root of  $T$ . This follows from the fact that, since  $\pi$  is fully specified,  $\pi$  must be of form  $r(\bar{a})[\pi']$ . Then, the variable  $id_v$  must be mentioned in a predicate of  $R_r$  of  $\text{INLPATTERN}(\pi, D)$ . Since  $h$  is a homomorphism,  $h(id_v)$  must belong to a tuple in  $R_r$ . It follows from the construction of  $\text{INLSHEMA}(D)$  and from Proposition 3.4 that it must be the (unique) identifier of  $R_r$ , and thus the identifier of the root node of  $T$ .

Next, we prove that for every node  $v$  of  $T_{\pi(\bar{a})}$ ,  $\lambda_{T_{\pi(\bar{a})}}(v) = \lambda_T(g(v))$ . Assume that for a node  $v$  in  $T_{\pi(\bar{a})}$  it is the case that  $\lambda_{T_{\pi(\bar{a})}}(v) = \ell$ . There are two cases. The claim for the case when  $v$  is marked follows from the fact that there must be a tuple in the interpretation of the relation  $R_\ell$  in  $\text{INLDOC}(T, D)$  that contains  $h(id_v)$  in its  $id_\ell$ -attribute. Then, since  $g$  maps  $v$  to the node in  $T$  that corresponds to  $h(id_v)$  in  $\text{INLDOC}(T, D)$ , it must be the case that  $\lambda_T(g(v)) = \ell$ . If  $v$  is not marked, let  $\ell'$  be the nearest appropriate ancestor of  $\ell$ , and consider the tuple in the interpretation of relation  $R_{\ell'}$  in  $\text{INLDOC}(T, D)$  that contains the element  $id_v$  in the position that corresponds to the attribute  $id_{\ell'}$ . The proof follows easily using the same argument as for the other case.

Assume now that two nodes  $v_1, v_2$  of  $T_{\pi(\bar{a})}$  are such that  $v_2$  is a child of  $v_1$  in  $T_{\pi(\bar{a})}$ . For the sake of readability, we shall write  $\lambda$  instead of  $\lambda_{T_{\pi(\bar{a})}}$ , since it will always be clear from the context. Let then  $\ell_1 = \lambda(v_1)$  and  $\ell_2 = \lambda(v_2)$ , and assume that  $h(id_{v_1}) = id_{n_1}$  and  $h(id_{v_2}) = id_{n_2}$ , for some nodes  $n_1, n_2$  of  $T$ . Thus,  $g(v_1) = n_1$ , and  $g(v_2) = n_2$ . The proof that  $g(v_2)$  is a child of  $g(v_1)$  follows easily from the fact that  $g$  preserves the labelling of the nodes, the graph  $G(D)$  is a tree,  $\pi$  is compatible with  $D$  and  $T$  conforms to  $D$ : If  $v_2$  is a child of  $v_1$  in  $T_{\pi(\bar{a})}$ , then it must be that  $\ell_1 \in P_D(\ell_2)$ , and that  $\ell_1$  does not appear in the production of any other label in  $D$ . Then, since  $\lambda_T(n_2) = \ell_2$  and  $\lambda_T(n_1) = \ell_1$  and  $T$  conforms to  $D$ , it must be that  $n_2$  is a child of  $n_1$ .

Next, it is easy to see that for every  $s \in \text{Str}(T_{\pi(\bar{a})})$ ,  $g(s) \in \text{Str}(T)$ . Moreover, since we have defined  $g(s) = h(s)$ , we also have that  $g(s) = s$  for every constant  $s$ .

Finally, we prove that for every node  $v$  of  $T_{\pi(\bar{a})}$  and  $@a \in \text{Att}$  such that  $\rho_{@a}(v)$  is defined,  $g(\rho_{@a}(v)) = \rho_{@a}(g(v))$ . Assume that for a node  $v$  of  $T_{\pi(\bar{a})}$  and for an attribute  $@a \in \text{Att}$ , it is the case that  $\rho_{@a}(v) = s$ . We must prove that  $g(s) = \rho_{@a}(g(v))$ . But we have defined  $g(s) = h(s)$ , and thus, we need to prove that  $h(s) = \rho_{@a}(g(v))$ . Assume first that  $v$  is marked. Then, notice that  $s$  is the variable in the position corresponding to  $@a$  in  $\text{attr}(R_{\lambda(v)})$  in the predicate of  $\text{INLPATTERN}(\pi, D)$  added in the step corresponding to  $Q_v$ . Thus, from the properties of relational homomorphisms,  $s$  must belong to the tuple in  $R_{\lambda(v)}$  in  $\text{INLDOC}(T, D)$  that contains  $h(id_v)$  in its first position. Since  $g$  maps  $v$  to the node in  $T$  identified by  $h(id_v)$ , it must be the case that  $\rho_{@a}(g(v)) = h(s)$ . For the case where  $v$  is not marked, consider the nearest appropriate ancestor of  $v$  in  $T_{\pi(\bar{a})}$ , and let  $v'$  be such node. Notice that since  $g$  preserves the child relation,  $g(v')$  is the nearest appropriate ancestor of  $g(v)$ . The proof then follows by considering the attribute corresponding to  $@a$  in  $A_D(\ell)$  in the relation  $R_{\ell'}$ , where  $\ell' = \lambda(v')$  and then using the same argument as in the previous case.  $\square$

By combining this result with Lemmas 4.4 and 4.5, it is not difficult to obtain the following corollary:

**COROLLARY 5.2.** *Let  $D$  be a DTD,  $T$  an XML document that conforms to  $D$ , and  $\pi$  a pattern compatible with  $D$ . In addition, let  $\bar{a}$  be a tuple of elements and variables. Then, there exists a homomorphism from  $T_{\pi(\bar{a})}$  to  $T$  if and only if there is a homomorphism from  $I_{\text{INLPATTERN}(\pi, D)(\bar{a})}$  to  $\text{INLDOC}(T, D)$ .*

Moreover, it is not difficult to adapt this proof to show the following:

**LEMMA 5.3.** *Let  $D$  be a DTD, and  $T_1, T_2$  two trees that conform to  $D$ . There is a tree homomorphism from  $T_1$  and  $T_2$  if and only if there is a homomorphism from  $\text{INLDOC}(T_1, D)$  to  $\text{INLDOC}(T_2, D)$ .*

**Conjunctive queries over trees.** We use the language that is essentially conjunctive queries over trees [6, 16, 10] with navigation along the child axis.

The language  $\mathcal{CTQ}$  is obtained by closing patterns under conjunction and existential quantification:

$$Q := \pi \mid Q \wedge Q \mid \exists x Q,$$

where  $\pi$  is a fully specified tree-pattern formula. The semantics is straightforward, given the semantics of patterns defined above:  $Q(\bar{a}) \wedge Q'(\bar{b})$  is true iff both  $Q(\bar{a})$  and  $Q'(\bar{b})$  are true, and  $\exists x Q(\bar{a}, x)$  is true iff  $Q(\bar{a}, c)$  is true for some value  $c$ . The output of  $Q$  on a tree  $T$  is denoted by  $Q(T)$ .

We say that a query  $Q$  is compatible with the DTD  $D$  if every pattern used in it is compatible with  $D$ .

The inlining of queries  $Q$  compatible with  $D$  is given by the recursive algorithm  $\text{INLQUERY}$  below.

---

**Procedure**  $\text{INLQUERY}(Q, D)$

---

**Input** : A DTD  $D$ , a query  $Q$  compatible with  $D$ .

**Output**: A conjunctive query over  $\text{INLSHEMA}(D)$ .

**if**  $Q = \pi$  **then**

**return**  $\text{INLPATTERN}(\pi, D)$

**else if**  $Q = Q_1 \wedge Q_2$  **then**

**return**  $\text{INLQUERY}(Q_1, D) \wedge \text{INLQUERY}(Q_2, D)$

**else if**  $Q = \exists x Q_1$  **then**

**return**  $\exists x \text{INLQUERY}(Q_1, D)$

---

Now we show that every query  $Q$  in  $\mathcal{CTQ}$  can be computed by its inlining on the inlining of its input (assuming, of course, compatibility with a DTD). In other words, **Requirement 3** is satisfied.

**THEOREM 5.4.** *Given a DTD  $D$ , a tree  $T$  that conforms to it, and a compatible query  $Q$ , we have*

$$Q(T) = \text{INLQUERY}(Q, D)(\text{INLDOC}(T, D)).$$

**PROOF.** Fix a DTD  $D$  and a tree  $T$ . The proof is done by induction. We have already proved the base case with the proof of Proposition 5.1.

For the induction step, assume first that  $Q$  is of form  $\exists z Q_1(\bar{x}, \bar{z})$ , and that  $Q_1(T) = \text{INLQUERY}(Q_1, D)(\text{INLDOC}(T, D))$ . It is now easy to see that  $Q(T) = \text{INLQUERY}(Q, D)(\text{INLDOC}(T, D))$ : Assume first that a tuple  $\bar{a}$  belongs to  $Q(T)$ . Then, there must be a tuple  $\bar{z}$  of variables such that  $(\bar{a}, \bar{z})$  belongs to  $Q_1(T)$ . Thus, from the inductive hypothesis, we obtain that  $(\bar{a}, \bar{z})$  belong to the evaluation of  $\text{INLQUERY}(Q_1, D)(\bar{a}, \bar{z})$  over  $\text{INLDOC}(T, D)$ . It follows that  $(\bar{a}, \bar{z})$  belong to the evaluation of  $\text{INLQUERY}(Q, D)(\bar{a}, \bar{z})$  over

$\text{INLDOC}(T, D)$ , since the algorithm defines  $\text{INLQUERY}(Q, D) = \exists \bar{z} \text{INLQUERY}(Q_1, D)$ . The other direction is analogous.

Next, assume that  $Q = Q_1(\bar{x}_1) \wedge Q_2(\bar{x}_2)$ , and that  $Q_1(T) = \text{INLQUERY}(Q_1, D)(\text{INLDOC}(T, D))$  and  $Q_2(T) = \text{INLQUERY}(Q_2, D)(\text{INLDOC}(T, D))$ . The argument is similar to the previous case: assume first that a tuple  $\bar{a}$  belongs to  $Q(T)$ . Then, there must be subtuples  $\bar{a}_1, \bar{a}_2$  of  $\bar{a}$  such that  $(\bar{a}_1)$  and  $(\bar{a}_2)$  belong to  $Q_1(T)$  and  $Q_2(T)$ , respectively. We obtain that  $(\bar{a}_1)$  and  $(\bar{a}_2)$  belong to the evaluation of  $\text{INLQUERY}(Q_1, D)$  and  $\text{INLQUERY}(Q_2, D)$  over  $\text{INLDOC}(T, D)$ , and thus, since  $\text{INLQUERY}(Q, D) = \text{INLQUERY}(Q_1, D) \wedge \text{INLQUERY}(Q_2, D)$ ,  $\bar{a}$  belongs to the evaluation of  $\text{INLQUERY}(Q, D)$  over  $T$ . The other direction is also analogous.  $\square$

**Inlining XML schema mappings** We use our transformation of tree patterns to define the procedure  $\text{INLMAP}$ , that, given source and target DTDs  $D_S$  and  $D_T$ , transforms an XML mapping  $\mathcal{M}$  into a relational mapping  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$  specified with a set of source-to-target tuple generating dependencies.

---

**Procedure**  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$

---

**Input** : An XML mapping  $\mathcal{M}$  from a source DTD  $D_S$  to a target DTD  $D_T$ .

**Output**: A relational mapping from  $\text{INLSHEMA}(D_S)$  to  $\text{INLSHEMA}(D_T)$ .

Set  $\text{INLMAP}(\mathcal{M}, D_S, D_T) := \emptyset$

**for** dependency  $\pi(\bar{x}) \rightarrow \exists \bar{z} \pi'(\bar{x}, \bar{z})$  in  $\mathcal{M}$  **do**

$\text{INLMAP}(\mathcal{M}, D_S, D_T) := \text{INLMAP}(\mathcal{M}, D_S, D_T) \cup \{\text{INLQUERY}(\pi, D_S)(\bar{x}) \rightarrow \exists \bar{z} \text{INLQUERY}(\pi', D_T)(\bar{x}, \bar{z})\}$

**end**

**return**  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$

---

**Correctness** While one could be tempted to ask for a translation that preserves all solutions, such a result need not hold. The relational mapping  $\text{INLMAP}$  uses null values to represent the shredded nodes of XML trees, and thus we should only consider solutions whose null values have not been renamed. However, relational solutions are open to renaming of nulls. This intuition can be formalized by means of the universal solutions, which are the most general among all solutions, and thus do not permit null renaming. Furthermore, one typically materializes a universal solution, as these solutions contain all the information needed to compute certain answers of conjunctive queries. This motivates the restriction of our **Requirement 4** to universal solutions.

The theorem below shows that parts (a) and (b) of **Requirement 4** hold. Note that in part (b), relational universal solutions are only required to contain a shredding of an XML universal solution. This is because relational solutions are also open to adding arbitrary tuples, which need not reflect a tree structure of an XML document.

**THEOREM 5.5.** *a) Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping and  $T$  an XML document that conforms to  $D_S$ . If  $T'$  is an  $\mathcal{M}$ -universal solution for  $T$ , then its inlining  $\text{INLDOC}(T', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for  $\text{INLDOC}(T, D_S)$ .*

*b) Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping, and  $T$  an XML document that conforms to  $D_S$ . Then for every  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution  $R$  for  $\text{INLDOC}(T, D_S)$  there exists an  $\mathcal{M}$ -universal solution  $T'$  such that  $\text{INLDOC}(T', D_T)$  is contained in  $R$ .*

To prove Theorem 5.5, we first provide a key lemma. Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping,  $T$  be an XML tree

that conforms to  $D_S$ , and  $J$  an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D)$ . For a relation  $R_\ell$  of  $\text{INLSHEMA}(D_T)$ , we denote all the positions that correspond to an attribute  $id_\ell$  or  $id_{\mu(\ell)}$  of  $R_\ell$  as the *identifier positions* of  $R_\ell$ . Moreover, an element  $a$  in a tuple  $t$  in the interpretation of  $R_\ell$  in  $J$  is an *identifier element* if it occupies an identifier position in  $t$ . We also define the *attribute positions* of a relation  $R_\ell$  as the positions that correspond to attributes of  $\ell$  or of  $\ell' \mid \mu(\ell') = \ell$  in  $D$ , and define the notion of an *attribute element* as expected. We now present the lemma:

**LEMMA 5.6.** *Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping, and  $T$  be an XML tree that conforms to  $D_S$ . Moreover, let  $J$  be an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D)$  such that (1) every identifier element in  $J$  does not appear in two identifier positions in two (not necessarily different) tuples, and (2) no identifier element is also an attribute element. Then, there exists a tree  $T'$  such that  $\text{INLDOC}(T', D_T) \subseteq J$ , and such that  $T'$  is an  $\mathcal{M}$ -solution for  $T$ .*

Lemma 5.6 formalizes the intuition that this class of "well behaved"  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solutions contains the correct representation of a shredded tree. The proof of this Lemma constructs from  $J$  a correct tree representation, in which each identifier element in  $J$  represents a node of the tree  $T'$  such that  $\text{INLDOC}(T', D_T) \subseteq J$ . We leave out the details, since the proof is lengthy and straightforward.

We now prove Theorem 5.5.

**PROOF. Part a:** Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping, and  $T$  an XML document that conforms to  $D_S$ . Consider an arbitrary  $\mathcal{M}$ -universal solution  $T'$  for  $T$ . We need to show that  $\text{INLDOC}(T', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for  $T$ . The proof is split into two parts, proving first that  $\text{INLDOC}(T', D_T)$  is a solution, and then that it is universal.

As stated, we first prove that  $\text{INLDOC}(T', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D_S)$ . From Proposition 3.4, it is clear that  $\text{INLDOC}(T', D_T)$  satisfies the dependencies in  $\Delta_D$ . We now show that the pair  $(\text{INLDOC}(D_S, T), \text{INLDOC}(D_T, T'))$  satisfies all the dependencies of  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ . Assume that for a dependency of the form

$$\text{INLPATTERN}(\pi(\bar{x}), D_S) \rightarrow \exists \bar{z} \text{INLPATTERN}(\pi'(\bar{x}, \bar{z}), D_T)$$

there is a tuple  $\bar{t}_x$  such that  $\text{INLDOC}(D_S, T) \models \text{INLPATTERN}(\pi(\bar{t}_x), D)$ . From Proposition 5.1, it must be the case that  $T \models \pi(\bar{t}_x)$ . Thus, since  $T'$  is a solution for  $T$ , there must be a tuple  $\bar{t}_z$  of constant and/or null elements such that  $T' \models \pi'(\bar{t}_x, \bar{t}_z)$ . Again, from Proposition 5.1, we obtain that  $\text{INLDOC}(D_T, T') \models \text{INLPATTERN}(\pi'(\bar{t}_x, \bar{t}_z), D)$ . This finishes the proof that  $\text{INLDOC}(T', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D_S)$ .

We now prove that  $\text{INLDOC}(T', D_T)$  is indeed universal. Assume for the sake of contradiction that it is not an universal solution, that is, there exists a solution  $J$  such that there does not exist a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $J$ . Construct from  $J$  a solution  $J'$  as follows: For each identifier position of every relation  $R_\ell$  in  $\text{INLSHEMA}(D_T)$ , and for each tuple in the interpretation of  $R_\ell$ , replace each identifier element  $a$  of  $t$  with a fresh null element  $z_a$ . In addition, replace each occurrence of  $a$  in the position  $id_{\mu(\ell')}$  of tuples in the interpretation of relations  $R_{\ell'}$  that reference  $R_\ell$  in a constraint in  $\text{INLSHEMA}(D_T)$  for  $z_a$ , and replace each other occurrence of  $a$  with a different, fresh null element. It is easy to see that  $J'$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D_S)$

as well. In fact, since we have replaced each of those elements  $a$  with nulls in a "cascade" fashion,  $J'$  clearly satisfies all dependencies in  $\text{INLSHEMA}(D_T)$ . The same argument can be used to show that  $(\text{INLDOC}(T, D_S), J')$  satisfies as well the dependencies in  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ . Finally, there is a homomorphism from  $J'$  to  $J$ : map each  $z_a$  and each fresh null replacing  $a$  as explained above back to the element  $a$ , and map each other element to itself. Thus, by composition of homomorphisms, there cannot exist a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $J'$ , as this would imply the existence of a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $J$ . However, note that solution  $J'$  satisfies the property of Lemma 5.6, since all identifying elements not satisfying it have been replaced by fresh new null elements. Let then  $T_{J'}$  be the  $\mathcal{M}$ -solution for  $T$  such that  $\text{INLDOC}(T_{J'}, D_T) \subseteq J'$ . (Lemma 5.6 proves the existence of  $T_{J'}$ .) Notice that, since  $\text{INLDOC}(T_{J'}, D_T) \subseteq J'$ , there also exists a homomorphism from  $\text{INLDOC}(T_{J'}, D_T)$  to  $J'$ . Yet again, by composition of homomorphisms, we conclude that there cannot exist a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $\text{INLDOC}(T_{J'}, D_T)$ .

On the other hand, the XML tree  $T'$  is an  $\mathcal{M}$ -universal solution, and thus there is an homomorphisms from  $T'$  to  $T_{J'}$ . But then, by Lemma 5.3, there exists a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $\text{INLDOC}(T_{J'}, D_T)$ . This is a contradiction.

**Part b:** Assume that  $R$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for  $\text{INLDOC}(T, D_S)$ . By inspecting the form of the dependencies of  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ , one notes that  $R$  needs to satisfy the conditions of Lemma 5.6, that is, every identifier element in  $R$  does not appear in two tuples in two different identifier positions; this can be easily using simple tools from relational data exchange (see [12]). Then, from Lemma 5.6, let  $T'$  be an  $\mathcal{M}$ -solution for  $T$  such that  $\text{INLDOC}(T', D_T) \subseteq R$ .

To prove that  $T'$  is an  $\mathcal{M}$ -universal solution for  $T$ , let  $T''$  be an  $\mathcal{M}$ -solution for  $T$ ; we need to prove that there is a homomorphism from  $T'$  to  $T''$ . From the part a) of this Theorem,  $\text{INLDOC}(T'', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -solution for  $\text{INLDOC}(T, D_S)$ , and, since  $R$  is universal, there is a homomorphism  $h$  from  $R$  to  $\text{INLDOC}(T'', D_T)$ . Moreover, since  $\text{INLDOC}(T', D_T) \subseteq R$ ,  $h$  is also a homomorphism from  $\text{INLDOC}(T', D_T)$  to  $\text{INLDOC}(T'', D_T)$ . Thus, from Lemma 5.3, there is a homomorphism from  $T'$  to  $T''$ . This concludes the proof.  $\square$

**Answering XML queries using relational data exchange.** The semantics of query answering in data exchange, both relational and XML [12, 21, 8, 6, 4], is defined by means of certain answers. That is, given a schema mapping  $\mathcal{M} = (D_S, D_T, \Sigma)$ , a tree  $T$  that conforms to  $D_S$ , and a conjunctive tree query  $Q$  that is compatible with  $D_T$ , the *certain answers of  $Q$  for  $T$  under  $\mathcal{M}$* , denoted by  $\text{CERTAIN}_{\mathcal{M}}(Q, T)$ , is the set of tuples that belong to the evaluation of  $Q$  over every possible  $\mathcal{M}$ -solution for  $T$ , that is,  $\bigcap \{Q(T') \mid T' \text{ is an } \mathcal{M}\text{-solution for } T\}$ . Note that our queries return sets of tuples, so we can talk about the intersection operator.

It was shown in [6, 4] that, for conjunctive tree queries and mappings using nested-relational DTDs, computing certain answers for a given source tree  $T$  is solvable in polynomial time. Thus, for the classes of mappings and queries we consider, there is no complexity mismatch between relational and XML data exchange. The next theorem shows that our translation is correct with respect to query answering, that is, our **Requirement 5** is satisfied.

**THEOREM 5.7.** *Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping. Then, for every XML tree  $T$  that satisfies  $D_S$  and for every conjunctive tree query  $Q$ , the certain answers of  $Q$  for*

*$T$  under  $\mathcal{M}$  and the certain answers of  $\text{INLQUERY}(Q, D_T)$  for  $\text{INLDOC}(T, D_S)$  over  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$  coincide:*

$$\text{CERTAIN}_{\mathcal{M}}(Q, T) = \text{CERTAIN}_{\text{INLMAP}(\mathcal{M})}(\text{INLQUERY}(Q, D_T), \text{INLDOC}(T, D_S)).$$

**PROOF.** Assume first that a tuple  $\bar{t}$  belongs to the certain answers of a query  $Q$  over a tree  $T$  under a mapping  $\mathcal{M} = (D_S, D_T, \Sigma)$ . Then, clearly,  $\bar{t}$  belongs to the evaluation of  $Q$  over the canonical solution  $\text{CANSOL}(T)$  for  $T$  (which, in this case, is guaranteed to exist [6]) under  $\mathcal{M}$ . Then, by Proposition 5.4,  $\bar{t}$  belongs to the evaluation of  $\text{INLQUERY}(Q, D_T)$  over  $\text{INLDOC}(\text{CANSOL}(T), D_T)$ . Moreover, from Proposition 5.5,  $\text{INLDOC}(\text{CANSOL}(T), D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for  $\text{INLDOC}(T, D_S)$ . From results in [12], we obtain that  $\bar{t}$  belongs to the certain answers of  $\text{INLQUERY}(Q, D_T)$  over  $\text{INLDOC}(T, D_S)$  under  $\mathcal{M}$ . The other direction is symmetric.  $\square$

The result of Theorem 5.7, combined with the standard procedure for evaluating conjunctive queries in relational data exchange, also gives us an algorithm for computing certain answers.

**COROLLARY 5.8.** *Under the conditions of Theorem 5.7,  $\text{CERTAIN}_{\mathcal{M}}(Q, T)$  can be obtained by the following procedure:*

1. *run  $\text{INLQUERY}(Q, D_T)$  on an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for  $\text{INLDOC}(T, D_S)$ ;*
2. *discard all tuples that contain null values.*

## 6. XML-to-XML Queries

Up to now, we have only considered queries that output tuples of attribute values. In this section we shall focus on proper XML-to-XML query languages, that is, on queries that output XML trees.

Some immediate questions arise when dealing with these formalisms in the data-exchange context. Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping,  $T$  be a tree conforming to  $D_S$ , and  $Q$  be an XML-to-XML query. Since the evaluation of  $Q$  over  $T$  returns an XML tree, we cannot define certain answers as  $\bigcap \{Q(T') \mid T' \text{ is a solution for } T\}$ , since the meaning of the intersection operator for XML documents is not clear.

To overcome this problem, we use recent results from [11], which showed how to define certain answers for queries returning XML trees, and how to use them in the data-exchange context. The key idea of [11] is to use tree patterns to define information contained in documents, and to use them to represent compactly the certain knowledge from the collection  $\{Q(T') \mid T' \text{ is a solution for } T\}$ . More precisely, if  $\Pi$  is a set of tree patterns which are matched by every tree  $Q(T')$ , we look for a small set  $\Pi_0$  of patterns that is equivalent to  $\Pi$  as a description of certain answers. By equivalence we mean that a tree matches every pattern in  $\Pi$  iff it matches every pattern in  $\Pi_0$ . If the set  $\Pi_0$  is finite, then its patterns can be put together to create a tree with nulls, which we then view as the certain answer.

We shall not need here additional details of the construction; instead, we shall use a result from [11] that tells us how certain answers can be computed for a specific XML-to-XML query language. The language, which is called TQL (to be defined shortly), is inspired by XQuery's FLWR (for-let-where-return) expressions, and is restricted to positive features (i.e., no negation). The key result from [11] is the following:

**PROPOSITION 6.1** ([11]). *Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping,  $Q$  a TQL query, and  $T$  a tree that conforms to  $D_S$ . If  $T'$  is an  $\mathcal{M}$ -universal solution for  $T$ , then  $\text{CERTAIN}_{\mathcal{M}}(Q, T) = Q(T')$ .*

Given this result, we now do the following. We provide a formal definition of the TQL language of [11], which can express XML-to-XML analogs of relational conjunctive queries. We then show how to adapt the machinery that we have previously developed for evaluating certain answers over a universal solution. Note that for this new translation, a TQL query  $Q$  returning trees needs to be translated into a *set* of relational queries generating views that define the shredding of the tree  $Q(T)$ .

## 6.1 TQL queries

TQL queries [11] are inspired by the FLWR (for-let-where-return) expressions of XQuery [31], but use only positive features. The key construct is **for**  $\pi(\bar{x})$  **return**  $q(\bar{x})$ , where  $\pi(\bar{x})$  is a pattern and  $q(\bar{x})$  is a query that defines a “forest expression.” Formally, the syntax of forest expressions is

$$q(\bar{x}) ::= \epsilon \\ \quad | \ell(\bar{a}, \bar{x}') [q'(\bar{x}'')] \\ \quad | q'(\bar{x}'), q''(\bar{x}'') \\ \quad | \text{for } \pi(\bar{a}, \bar{x}, \bar{y}) \text{ return } q'(\bar{x}, \bar{y})$$

where  $\ell$  ranges over node labels,  $\bar{a}$  over constant attribute values, and  $\bar{x}$  etc are tuples of variables.

A TQL query  $Q$  is an expression of the form  $r[q]$ , where  $q$  is a forest expression without variables. To define the semantics of this language, we first define inductively the forest  $\llbracket q(\bar{x}) \rrbracket_{T,v}$ , for a valuation  $v$  of all variables in  $\bar{x}$  as attribute values. We use the notation  $\ell(\bar{a})[f]$  for a tree whose root is labeled  $\ell$  and carries a tuple of attributes  $\bar{a}$ ; further,  $f$  is the forest of subtrees below the root.

$$\begin{aligned} \llbracket \epsilon \rrbracket_{T,v} &= \epsilon \text{ (empty forest)} \\ \llbracket \ell(\bar{a}, \bar{x}') [q'(\bar{x}'')] \rrbracket_{T,v} &= \ell(\bar{a}, v(\bar{x}')) [\llbracket q' \rrbracket_{T,v}] \\ \llbracket q'(\bar{x}), q''(\bar{x}'') \rrbracket_{T,v} &= \llbracket q' \rrbracket_{T,v} \cup \llbracket q'' \rrbracket_{T,v} \end{aligned}$$

$$\llbracket \text{for } \pi(\bar{a}, \bar{x}, \bar{y}) \text{ return } q'(\bar{x}, \bar{y}) \rrbracket_{T,v} =$$

$$\bigcup \{ \llbracket q' \rrbracket_{T,v'} \mid v' \text{ extends } v \text{ and } T \models \pi(\bar{a}, v'(\bar{x}), v'(\bar{y})) \}$$

For a tree  $T$  and a query  $Q = r[q]$ , the evaluation  $Q(T)$  of  $Q$  over  $T$  is defined as the tree  $r[\llbracket q \rrbracket_T]$ , i.e., the forest  $\llbracket q \rrbracket_T$  under root  $r$ .

EXAMPLE 6.2. Recall the tree  $T$  from Figure 1(a). The tree  $T'$  from Figure 3(a) can also be obtained as the transformation  $Q(T)$  resulting from the evaluation of a TQL query  $Q$  over  $T$ , where  $Q = r[q]$ , and  $q$  is defined as

$$\text{for } r/\text{book}(x)/\text{author}/\text{name}(y) \text{ return} \\ \text{writer}[\text{name}(y), \text{work}(x)] \quad (3)$$

For the sake of readability, we use the / operator to denote the child axis in tree patterns.  $\square$

## 6.2 Inlining TQL queries

If  $Q$  is a TQL query, then, to be able to define its inlining translation, we need to specify a DTD for trees  $Q(T)$ . Note that TQL queries define the shape of their outputs, and at the same time do not put restrictions on the number of appearances of labels. Hence it is natural to define the DTD for outputs of  $Q$  as a *starred* DTD  $D_Q$ , whose shape is determined by  $Q$ , and where each element type except the root occurs under the Kleene star.

More precisely, for a forest expression  $q$ , we define a forest  $F_q$  inductively as follows:  $F_\epsilon$  is the empty forest;  $F_{\ell[q']}$  is  $\ell[F_{q'}]$ ;  $F_{q' \cup q''} = F_{q'} \cup F_{q''}$ , and  $F_{\text{for } \pi \text{ return } q'} = F_{q'}$ . For  $Q = r[q]$  we let  $T_Q = r[F_q]$ .

Then  $D_Q$  is a non-recursive DTD that has a rule  $p \rightarrow c_1^* \cdots c_n^*$  for each node  $p$  in  $T_Q$  with children labelled  $c_1, \dots, c_n$ . As usual, we require that  $D_Q$  be acyclic and we assume without loss of generality that  $G(D_Q)$  is a tree.

EXAMPLE 6.3. (Example 6.2 continued) Recall query  $Q = r[q]$ . Then,  $T_Q$  is the XML tree given by  $r[\text{writer}[\text{name}, \text{work}]]$ , and thus  $D_Q$  contains productions  $r \rightarrow \text{writer}^*$ ,  $\text{writer} \rightarrow \text{name}^* \text{work}^*$ ,  $\text{name} \rightarrow \epsilon$  and  $\text{work} \rightarrow \epsilon$ .  $\square$

Before showing the algorithm INLTQL, we need to introduce some features that will be used in the algorithm. Consider again query (3) and DTD  $D_Q$  in Examples 6.2 and 6.3. For each pair of attributes that satisfy  $r/\text{book}(x)/\text{author}/\text{name}(y)$ , the query  $Q$  creates a subtree  $\text{writer}[\text{name}(y), \text{work}(x)]$  in the tree  $Q(T)$ . Thus, the relational translation would need to create one tuple in the relations corresponding to  $\text{writer}$ ,  $\text{name}$  and  $\text{work}$  for each pair of attributes  $x, y$  that satisfy the relational translation of the pattern  $r/\text{book}(x)/\text{author}/\text{name}(y)$  in the instance  $\text{INLDOC}(T)$ .

In the relational translation we need a way to associate each particular  $\text{writer}$  with a particular  $\text{name}$  and  $\text{work}$ . One possible way of doing this is by creating a (Skolem) function  $f$  that associates with each pair  $(\text{name}, \text{work})$  a unique identifier for the corresponding  $\text{writer}$ . The function  $f$  must be defined in such a way that  $f(\text{book}, \text{name})$  is different for each different pair  $(\text{name}, \text{work})$ . We enforce this requirement by letting each term  $f(\bar{a})$  represent a distinct constant  $c_{f(\bar{a})}$ .

We will define our translation algorithm inductively. The key procedure TQLSTEP for the inductive step is described below. Its inputs, in addition to a query and a DTD, include a conjunctive query corresponding to the conjunction of patterns in the query, and a function term corresponding to the parent in the tree  $Q(T)$  (for example, when creating views for relation  $R_{\text{work}}$ , we would input the identifier  $f(x, y)$  of the parent node labelled  $\text{writer}$ ). This is illustrated by the example below.

EXAMPLE 6.4. (Example 6.3 continued) Assume that query  $Q = r[q]$  of Examples 6.2 and 6.3 is posed over  $T$  under schema  $D$ . The following views define the translation for  $Q$ :

$$\begin{aligned} R_r(f_r) &:= \text{true} \\ R_{\text{writer}}(f_{\text{writer}}(x, y), f_r) &:= \\ &\quad \text{INLQUERY}(r/\text{book}(x)/\text{author}/\text{name}(y), D) \\ R_{\text{name}}(f_{\text{name}}(x, y), f_{\text{writer}}(x, y), y) &:= \\ &\quad \text{INLQUERY}(r/\text{book}(x)/\text{author}/\text{name}(y), D) \\ R_{\text{work}}(f_{\text{work}}(x, y), f_{\text{writer}}(x, y), x) &:= \\ &\quad \text{INLQUERY}(r/\text{book}(x)/\text{author}/\text{name}(y), D) \end{aligned}$$

Notice how each tuple in relations  $R_{\text{name}}$  and  $R_{\text{work}}$  is set to reference the correct tuple in relation  $R_{\text{writer}}$ .  $\square$

To define the inlining translation INLTQL, we simply need a Skolem term for the root of the tree, as the basis for the inductive procedure TQLSTEP.

A TQL query  $Q$  is compatible with a DTD  $D$  if all the patterns used in  $Q$  are compatible with  $D$ . The following proposition shows that INLTQL satisfies an analog of **Requirement 3** for queries that output trees.

PROPOSITION 6.5. *Given a DTD  $D$ , a TQL query  $Q$  compatible with  $D$ , and a tree  $T$  that conforms to  $D$ , we have that  $\text{INLDOC}(Q(T), D_Q) = \text{INLTQL}(Q, D)(\text{INLDOC}(T))$ , up to renaming of nulls.*

That is, the set of views  $\text{INLTQL}(Q, D)$  applied to the inlining of  $T$  yields the same answer as the inlining of  $Q(T)$ .

---

**Procedure** TQLSTEP( $Q, D, \varphi, t$ )

---

**Input** : A forest expression  $q(\bar{x})$ , a DTD  $D$ , a conjunctive query  $\varphi(\bar{x})$  and a Skolem term  $t$ .

**Output**: A set of views over INLSHEMA( $D_Q$ ).

**if**  $q(\bar{x}) ::= \epsilon$  **then**

**return**  $\emptyset$

**else if**  $q(\bar{x}) ::= q'(\bar{x}'), q''(\bar{x}'')$  **then**

**return** TQLSTEP( $q', D, \varphi, t$ )  $\cup$  TQLSTEP( $q'', D, \varphi, t$ )

**else if**  $q(\bar{x}) ::= \ell(\bar{a}, \bar{x}') [q'(\bar{x}'')]$  **then**

    Let  $f$  be a fresh Skolem function. Define view  $V$  as

$R_\ell(f(\bar{x}), t, \bar{a}, \bar{x}') := \text{INLQUERY}(\varphi, D)$ , or just

$R_\ell(f(), t, \bar{a}) := \text{true}$  if  $\varphi = \emptyset$ .

**return**  $\{V\} \cup \text{TQLSTEP}(q', D, \varphi, f(\bar{x}))$

**else if**  $q(\bar{x}) ::= \text{for } \pi(\bar{a}, \bar{x}, \bar{y}) \text{ return } q'(\bar{x}, \bar{y})$  **then**

    Let  $\varphi'(\bar{a}, \bar{x}, \bar{y}) := \varphi(\bar{x}) \wedge \pi(\bar{a}, \bar{x}, \bar{y})$ .

**return** TQLSTEP( $q', D, \varphi', t$ )

---

---

**Procedure** INLTQL( $Q, D$ )

---

**Input** : A TQL query  $Q = r[q]$  and a DTD  $D$ .

**Output**: A set of views over INLSHEMA( $D_Q$ ).

Create a 0-ary function  $f_r$ .

**return** TQLSTEP( $Q, D, \emptyset, f_r()$ )

---

PROOF. We begin by proving that  $\text{INLTQL}(Q, D)(\text{INLDOC}(T, D)) \subseteq \text{INLDOC}(Q(T), D_Q)$ . Let  $D_Q$  be the DTD corresponding to  $Q$ . Assume that there exists a tuple  $t$  that is part of the evaluation of a view  $V$  in  $\text{INLTQL}(Q, D)(\text{INLDOC}(T, D))$ , with view  $V$  of form  $R_\ell(f(\bar{x}), g(\bar{x}''), \bar{a}, \bar{x}') := \text{INLQUERY}(\varphi(\bar{x}), D)$  (we do not prove the case when  $\ell = r$  since it is very similar). Let  $v$  be a homomorphism so that  $v(f(\bar{x}), g(\bar{x}''), \bar{a}, \bar{x}') = t$ . For the sake of readability, we let  $v(\bar{x}) = \bar{b}$ . Notice that, from the definition of INLTQL, we have that  $\bar{b}$  belongs to  $\text{INLQUERY}(\varphi, D)(\text{INLDOC}(T, D))$ . By Theorem 5.4,  $\bar{b}$  belongs to  $\varphi(T)$ . Assume that the forest query that created view  $V$  in the inlining of  $Q$  is of the form  $\ell(\bar{a}, \bar{x}') [q'(\bar{x}'')]$ . It can be proved by induction that  $\llbracket q(\bar{x}) \rrbracket_{T, v}$  must contain a node of the form  $\ell(\bar{a}, v(\bar{x}')) [\llbracket q'(\bar{x}'') \rrbracket_{T, v}]$ . Thus, the inlining of  $Q(T)$  must contain a tuple in  $R_\ell$  of the form  $(id_n, id_{\mu(n)}, \bar{a}, v(\bar{x}'))$ ; the proof follows by renaming nulls  $id_n$  and  $id_{\mu(n)}$  into  $v(f(\bar{x}))$  and  $v(g(\bar{x}''))$ , respectively. We only need to show that no null value has to be renamed as two different constants. This follows since the attributes  $id_\ell$  and  $id_{\mu(\ell)}$  correspond respectively to a key and foreign key of relation  $R_\ell$ , and the algorithm INLDOC chooses fresh null symbols for each value in the position corresponding to the attribute  $id_\ell$ .

Next, we show that  $\text{INLDOC}(Q(T), D_Q) \subseteq \text{INLTQL}(Q, D)(\text{INLDOC}(T, D))$ , up to renaming of nulls.

Since every element of  $D_Q$  is under a star, it is easy to see that relation  $R_\ell$  will contain only attributes  $id_\ell, id_{\mu(\ell)}$  and  $A_{D_Q}(\ell)$ . We first rename all elements that are in a position corresponding to attributes  $id_\ell$  as follows:

Let  $\bar{t}$  be a tuple of relation  $R_\ell$  in  $\text{INLDOC}(Q(T), D_Q)$ , and assume that  $id_n$  is the element that corresponds to attribute  $id_\ell$  of  $R_\ell$ . If  $\ell = r$ , rename  $id_n$  by the 0-ary term  $f_r()$  used in procedure INLTQL. For the case when  $\ell \neq r$ , it is easy to see from the definition of the procedure INLDOC that  $Q(T)$  must contain an  $\ell$ -labelled node  $n$ .

Thus, from the semantics of TQL queries, there must be a sub-forest  $q$  of  $Q$  of the form  $q(\bar{x}) = \ell(\bar{a}, \bar{x}') [q'(\bar{x}'')]$  and a valuation

$v$  such that  $n$  is the top node of forest  $\llbracket q(\bar{x}) \rrbracket_{T, v}$ . Let  $f$  be the function created by procedure INLTQL in the step corresponding to  $q$ . Finally, let  $\pi_1(\bar{z}_1), \dots, \pi_k(\bar{z}_k)$  be the sequence of patterns present in **for-return** constructs in  $Q$  from the root until  $q$ , and let  $\bar{z}$  be the union of  $\bar{z}_1, \dots, \bar{z}_k$ . Then, rename  $id_n$  as  $c_{f(v(\bar{z}))}$ . Notice that this procedure is well defined, since  $v$  must apply to each variable of  $\bar{z}$ .

Let us denote by  $J$  the instance resulting from renaming all elements of  $\text{INLDOC}(Q(T), D_Q)$  accordingly. We show that  $J \subseteq \text{INLTQL}(Q, D)(\text{INLDOC}(T, D))$ , up to renaming of nulls in attribute positions, that is, nulls in positions  $A_D(\ell)$  in tuples on  $R_\ell$ .

Let  $\bar{t}$  be a tuple of relation  $R_\ell$  in  $J$ , and assume that the elements in  $t$  corresponding to attributes  $id_\ell, id_{\mu(\ell)}$  and  $A_{D_Q}(\ell)$  are  $c_{f(\bar{b})}, c_{g(\bar{b}')} , \bar{a}$ , respectively.

We need to show that such tuple is in fact in  $\text{INLTQL}(Q, D)(\text{INLDOC}(T, D))$ . Let  $n$  and  $n'$  be the nodes in  $Q(T)$  such that  $id_n$  and  $id_{n'}$  were replaced by  $c_{f(\bar{b})}$  and  $c_{g(\bar{b}'')}$ , respectively,  $v$  and  $v'$  the valuations witnessing the membership of  $n$  and  $n'$  in  $Q(T)$ , as explained above; and  $q(\bar{x}), q'(\bar{x}'')$  the forest queries that give rise to the creation of  $f$  and respectively  $g$  by procedure INLTQL. Moreover, let  $\varphi(\bar{z}) = \pi_1(\bar{z}_1), \dots, \pi_k(\bar{z}_k)$  be the sequence of patterns present in **for-return** constructs in  $Q$  from the root until  $q$ , where  $\bar{z}$  is the union of  $\bar{z}_1, \dots, \bar{z}_k$ . Then notice that valuation  $v$  is such that  $(T, v) \models \varphi(\bar{z})$ .

In the same fashion, we select  $\varphi'(\bar{z}') = \pi'_1(\bar{z}'_1), \dots, \pi'_k(\bar{z}'_k)$  and  $\bar{z}'$  for forest query  $q'$ . As a remark, since  $n'$  is the parent of  $n$ , observe that each pattern  $\pi'_i$  corresponds to a pattern  $\pi_j$ , for some  $j \leq k$ . Finally, it is easy to see that there is no other query of the form  $\ell(\bar{y}, \bar{a}) [q''(\bar{y}'')]$  in between  $q$  and  $q'$ . Thus, the step of INLTQL corresponding to  $q(\bar{x})$  must have received the term  $g(\bar{z}')$  as input.

By following these remarks, one notices that procedure INLTQL creates the following view  $V$  for the step of  $q(\bar{x})$ :  $R_\ell(f(\bar{z}), g(\bar{z}'), \bar{a}, \bar{x}) := \text{INLQUERY}(\varphi, D)$ .

All that remains to see is that, since  $(T, v) \models \varphi(\bar{z})$ , it must be that  $\text{INLDOC}(T, D) \models \text{INLQUERY}(\varphi(v(\bar{z})), D)$ . This ensures the existence of a fact of the form  $R_\ell(c_{f(v(\bar{z}))}, c_{g(v(\bar{z}''))}, \bar{a}, v(\bar{x})) = R_\ell(c_{f(\bar{b})}, c_{g(\bar{b}'')} , \bar{a})$  in  $\text{INLTQL}(Q, D)(\text{INLDOC}(T, D))$ .  $\square$

### Translating relations back into XML

To complete the translation, we need an algorithm to publish back the relational data as an XML document. This is done by means of the algorithm PUBREL. We say that an instance  $I$  of  $\text{INLSHEMA}(D)$   $D$ -represents a tree  $T$  that conforms to  $D$  if  $I = \text{INLDOC}(T, D)$ .

This algorithm will only work for relational instances that represent shredded documents. The following proposition shows its correctness.

**PROPOSITION 6.6.** *Given a DTD  $D$  and a relational instance  $I$  of  $\text{INLSHEMA}(D)$ , it is the case that  $\text{INLDOC}(\text{PUBREL}(D, I)) = I$ .*

**PROOF.** Let  $T$  be a tree such that  $\text{INLDOC}(T, D) = I$ . We construct a mapping  $h$  between  $T$  and  $\text{PUBREL}(I)$  as follows:

- For each node  $n$  of  $T$  that is marked, let  $\ell$  be its label, and  $id_n$  be the identifier of  $I = \text{INLDOC}(T, D)$  that belongs to the attribute  $id_\ell$  of the tuple  $t$  created by procedure INLDOC from node  $n$ . Then, define  $h$  so that it maps  $n$  to the node of  $\text{PUBREL}(I)$  created by procedure PUBREL from tuple  $t$  of  $R_\ell$ .
- For each node  $n$  that is not marked, let  $n' = \mu(n)$ , and  $t$  the corresponding tuple in INLDOC. Let  $\ell$  and  $\ell'$  be the labels of  $n$  and  $n'$ , respectively, and assume that  $id_n, id_{n'}$  are

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**Procedure** PUBREL( $D, I$ )

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**Input** : A DTD  $D$  and an instance  $I$  that  $D$ -represents some tree.

**Output**: An XML tree  $T$  that is  $D$ -represented by  $I$ .

**for each** node  $\ell$  of  $G(D)$ , traversed as Depth-first-search **do**

**for each** tuple  $t$  of  $R_\ell$  in  $I$  with elements  $n, \bar{a}$  and  $n'$  corresponding to attributes  $id_n, A_D(\ell)$  and  $id_{\mu(n)}$  **do**

**for every** non-starred node  $\ell'$  of  $G(D)$  such that  $\mu(\ell') = \ell$ , and elements  $n''$  and  $\bar{b}$  in  $t$  corresponding to attributes  $id_{\ell'}$  and  $A_D(\ell')$  **do**

Create a node  $n''$  in  $T$  labelled  $\ell'$ , with attributes  $\bar{b}$ , in a parent-child scheme that resembles  $G(D)$ .

**endfor**

Add to  $T$  a node  $n$  labelled  $\ell$ , with attributes  $\bar{a}$ , with  $n'$  as ancestor, according to the parent-child sequence defined by  $G(D)$  (no parent if  $\ell = r$ ).

**endfor**

**endfor**

**return**  $T$

---

the identifiers of  $t$  in positions  $id_\ell$  and  $id_{\ell'}$  of tuple  $t$  in  $R'_\ell$ . Then, procedure PUBREL will create from  $t$  a node  $n'_t$  labelled  $\ell'$  and a node  $n_t$  labelled with  $\ell$ , such that  $\mu(n_t) = n'_t$  in PUBREL( $I$ ). Define  $h$  so that it maps  $n$  to  $n_t$ .

It is clear that this mapping is one to one, since  $I = \text{INLDOC}(T, D)$ . Furthermore, since  $G(D)$  is a tree, it is also clear that this mapping preserves the relation  $\mu$  of nearest appropriate ancestors, as the way in which procedure PUBREL creates the parent-child relation of nodes is always unique. Finally, from the definition of procedures PUBREL and INLDOC it must be the case that for every  $n$  in  $T$  labelled  $\ell$ , the set  $\{\rho_{@a}(n) \mid @a \in A_D(\ell)\}$  is the same as  $\{\rho_{@a}(h(n)) \mid @a \in A_D(\ell)\}$  in PUBREL( $I$ ).

It is now an easy exercise to prove that INLDOC creates the same relations (up to renaming of nulls) for PUBREL( $I$ ) and ( $T$ ), since for every marked node  $n$  of  $T$  the procedure creates exactly the same tuple as marked node  $h(n)$  of PUBREL( $I$ ).  $\square$

### 6.3 TQL queries in XML data exchange

Combining the previously mentioned result in [11] with the correctness of the algorithms we presented we conclude that **Requirements 1-5** are satisfied for data exchange with XML-to-XML queries:

**THEOREM 6.7.** *Let  $\mathcal{M} = (D_S, D_T, \Sigma)$  be an XML schema mapping. Then, for every XML tree  $T$  that satisfies  $D_S$  and for every TQL query  $Q$ , the certain answers of  $Q$  for  $T$  under  $\mathcal{M}$  coincide with the certain answers of INLTQL( $Q, D_T$ ) for INLDOC( $T, D_S$ ) over INLMAP( $\mathcal{M}, D_S, D_T$ ):*

$$\text{INLDOC}(\text{CERTAIN}_{\mathcal{M}}(Q, T), D_Q) = \text{CERTAIN}_{\text{INLMAP}(\mathcal{M})}(\text{INLTQL}(Q, D_T), \text{INLDOC}(T, D_S)).$$

*Remark:* The notion of certain answers naturally (component-wise) extends to queries computing multiple relations.

**PROOF.** Fix an  $\mathcal{M}$ -universal solution  $T'$  for  $T$ . By Proposition 6.1,  $\text{CERTAIN}_{\mathcal{M}}(Q, T) = Q(T')$ , where  $T'$  is a universal solution. Furthermore, by Proposition 6.5,  $\text{INLDOC}(Q(T'), D_Q) = \text{INLTQL}(Q, D_T)(\text{INLDOC}(T', D_T))$ .

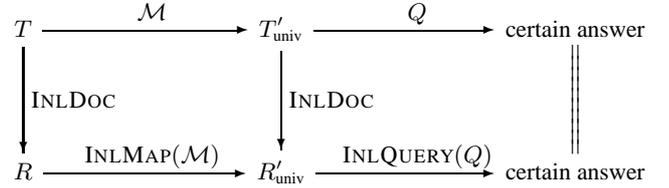
Finally, since the views created by the procedure INLTQL are essentially conjunctive queries using Skolem

terms, and (by Theorem 5.5)  $\text{INLDOC}(T', D_T)$  is an  $\text{INLMAP}(\mathcal{M}, D_S, D_T)$ -universal solution for INLDOC, it can be proved that  $\text{INLTQL}(Q, D_T)(\text{INLDOC}(T', D_T)) = \text{CERTAIN}_{\text{INLMAP}(\mathcal{M})}(\text{INLTQL}(Q, D_T), \text{INLDOC}(T, D_S))$ , using standard tools from the data-exchange literature (see [12, 8]).  $\square$

Theorem 6.7 and Proposition 6.6 give us a way of computing  $\text{CERTAIN}_{\mathcal{M}}(Q, T)$ . First, compute  $\text{CERTAIN}_{\text{INLMAP}(\mathcal{M})}(\text{INLTQL}(Q, D_T), \text{INLDOC}(T, D_S))$  by materializing views  $\text{INLTQL}(Q, D_T)$  over the canonical solution for  $\text{INLDOC}(T, D_S)$ , and then use the procedure PUBREL to output it as the tree  $\text{CERTAIN}_{\mathcal{M}}(Q, T)$ .

## 7. Concluding Remarks

Our technique provides a relational approach to solve two of the most important problems of XML data-exchange settings: materializing solutions and answering queries. The diagram below summarizes this. In a pure XML setting, we can start with a document  $T$  and use a mapping  $\mathcal{M}$  to find a (universal) solution  $T'_{\text{univ}}$ , over which we can then answer a query  $Q$  to produce certain answers.



Using the translation INLDOC of documents, we generate a relational instance  $R$ , on which the translation of the mapping  $\text{INLMAP}(\mathcal{M})$  generates a universal solution  $R'_{\text{univ}}$ . This solution is a shredding, via INLDOC, of a universal XML solution, and also conforms to the shredding of source DTD. Finally, we apply the standard technique [12] for evaluating queries in relational data exchange to the query translation  $\text{INLQUERY}(Q)$  or  $\text{INLTQL}(Q)$  to produce the correct answers, in the latter case with the possibility of using PUBREL to publish back the results into XML.

Implementing our proposed algorithms for use in practical systems would be straightforward using the specifications given in this paper. A natural next step is to evaluate XML data-exchange systems using relational data storage and implementations of our algorithms. We are currently working in this direction.

We finish with a remark about the possibility of allowing operators  $?$  and  $+$  in DTDs, as well as a choice operator for representing multiple choices. We say that a non-recursive DTD  $D$  is an *extended nested relational* DTD if all rules of  $D$  are of the form  $\ell \rightarrow \tilde{\ell}_0 \dots \tilde{\ell}_m$ , or  $\ell \rightarrow \ell_0 + \dots + \ell_m$ , where all the  $\ell_i$ 's and  $\tilde{\ell}_i$ 's are distinct, and each  $\tilde{\ell}_i$  is one of  $\ell_i, \ell_i?, \ell_i^*$  or  $\ell_i^+$  (as usual,  $\ell?$  stands for  $\ell|\epsilon$  and  $\ell^+$  for  $\ell\ell^*$ ).

The procedure INLSchema can be extended to these DTDs. For each element  $\ell$  that is under the operator  $?$ , the transformation creates a special relation  $\ell$  that references the relation of the nearest appropriate ancestor of  $\ell$ . Furthermore, the transformation for a rule of the form  $\ell_1 \rightarrow \ell_2^+$  can be defined by including a dependency that ensures that there is at least one tuple in the relation  $R_{\ell_2}$  for each tuple in  $R_{\ell_1}$ . Finally, for the choice operator  $\ell \rightarrow \ell_0 + \dots + \ell_m$  the transformation would create one relation  $R_\ell$  for each possible choice of  $\ell_0, \dots, \ell_m$ . Then, it is possible to extend all the procedures in a way that still satisfies **Requirements 1-5** under extended nested relational DTDs.

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