Regular Path Queries on Graphs with Data*

Leonid Libkin Domagoj Vrgoč

ABSTRACT

Graph data models received much attention lately due to applications in social networks, semantic web, biological databases and other areas. Typical query languages for graph databases retrieve their topology, while actual data stored in them is usually queried using standard relational mechanisms.

Our goal is to develop techniques that combine these two modes of querying, and give us query languages that can ask questions about both data and topology. As the basic querying mechanism we consider regular path queries, with the key difference that conditions on paths between nodes now talk not only about labels but also specify how data changes along the path. Paths that combine edge labels with data values are closely related to data words, so for stating conditions in queries, we look at several data-word formalisms developed recently. We show that many of them immediately lead to intractable data complexity for graph queries, with the notable exception of register automata, which can specify many properties of interest, and have NLOGSPACE data and PSPACE combined complexity. As register automata themselves are not easy to use in querying, we define two types of extensions of regular expressions that are more user-friendly, and develop query evaluation techniques for them. For one class, regular expressions with memory, we achieve the same bounds as for automata, and for the other class, regular expressions with equality, we also obtain tractable combined complexity of query evaluation. In addition, we show that results extends to analogs of conjunctive regular path queries.

*Authors' address: School of Informatics, University of Edinburgh, email: libkin@inf.ed.ac.uk, s1058408@sms.ed.ac.uk.

ICDT 2011, March 21–23, 2011, Uppsala, Sweden.

Categories and Subject Descriptors

F.1.1 [**Computation by Abstract Devices**]: Models of Computation—*Automata*; F.4.3 [**Mathematical logic and formal languages**]: Formal Languages; H.2.3 [**Database management**]: Languages—*Query Languages*

General Terms

Theory, Languages, Algorithms

Keywords

Graph databases, regular path queries, data values, register automata, regular expressions

1. INTRODUCTION

Querying graph-structured data has been actively studied in recent years, due to numerous applications in areas including biological networks [31, 32, 36], social networks [38, 39], and the semantic Web [27, 37]. Such databases are represented as graphs in which nodes are objects, and edge labels specify relationships between them [1, 3]. Typical queries over such databases look for reachability patterns. A very common and well studied class of queries is that of *regular path queries*, or RPQs. An RPQ selects nodes connected by a path that belongs to a regular language over the labeling alphabet [13, 14, 15]. Their extensions have been studied extensively too; for example, conjunctive RPQs state the existence of several paths [12, 18, 22], and extended conjunctive RPQs add comparisons of paths [4].

These standard queries over graph databases talk about their topology, and do not mention data values. But graph databases do contain data. For example, in a social network, one would expect each node to correspond to a person, with his/her attributes such as name, age, city, email, etc.; labels can specify types of connections between people, e.g., like/dislike, professional, etc. The querying mechanisms one deals with are generally of one of these categories:

- queries about topology such as finding nodes connected by a path with a certain label (e.g., people who

are connected via professional links), or

- queries about data, i.e., essentially relational queries (e.g., finding pairs of people of the same age).

What these languages are incapable of doing is *combining* data and topology. As an example of a query that involves such a combination, consider a query looking for people who are connected via professional links and are of the same age. This query states the existence of a path with a certain property and then relates data values at the end of the path. Another example is a query that finds people who are connected via professional links restricted to people of the same age. In this case, comparison of data values (having the same age) is done for every node along the path.

Extending languages that handle structure to languages that handle both structure and data is not new in database theory. For very simple types of paths it was considered in graph object-oriented models [42], but most notably it happened in the study of XML [8, 40, 41]. For example, languages such as XPath exist in their structural variants as well as extensions that handle data comparisons [6, 9, 20, 34]. A standard abstraction one uses for extending from structure to data in the case of XML is *data trees*, in which data values are attached to tree nodes [9, 29, 40]. The focus of the study of such extensions has been both on querying, where one is concerned with efficient evaluation [7, 24], and on reasoning, where one is concerned with the decidability of the satisfiability problem [9, 10].

So likewise, we consider graph databases where nodes can carry data values. An example of such a graph database is shown in Fig. 1. It has five nodes, $v_1, \ldots, v_5$; data values are shown inside the nodes, and edge labels next to the edges. As an initial assumption, we assume that each node carries just one data value. This is not a real restriction for two reasons. First, if a node has a tuple of data values (e.g., person's name, age, email, etc., in a social network) this could be modeled by extra edges to nodes with those data values. And second, the way we design languages for querying graph databases with data values will make it very easy to extend them to such a setting.

An RPQ may ask for pairs of nodes connected by a path from the regular language $(ab)^*$. In the graph in Fig. 1, one possible answer is (v_1, v_3), another – (v_1, v_5). To combine this with data values, we may ask queries of the following kind:

- Find nodes connected by a path from $(ab)^*$ such that the data values at the beginning and at the end of the path are the same. In this case, (v_1, v_3) is still in the answer but (v_1, v_5) is not.
- We may extend comparisons to other nodes on the path, not only to the first and last nodes. For example, we may ask for nodes connected by paths along which the data value remains the same, or on which all data values are different from the first one. The pair (v_1, v_3) is in the answer to the first query (the path $v_1v_4v_3$ witnesses it), while the pair (v_1, v_5) is in the answer to the second, as witnessed by the path $v_1v_2v_5$.

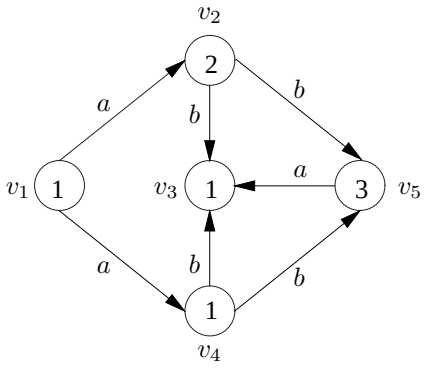


Figure 1: A graph database with data values

What kind of languages can we use in place of regular languages to specify paths with data? To answer this, consider, for example, a path $v_1v_2v_5v_3$ in the graph. If we traverse it by starting in v_1, reading its data value, then reading the label of (v_1, v_2), then the data value in v_2, etc., we end up with the following sequence: $1a2b3a1$. We shall refer to them as *data paths*. They are extremely close to an object that has been actively studied in the XML context – namely, *data words* [8, 10, 40, 41]. A data word is a word in which every position is labeled by both a letter from a finite alphabet (e.g., a or b) and a data value (e.g., a number). Data paths are essentially data words with an extra data value. We can represent the data path $1a2b3a1$ as a data word $\binom{\#}{1}\binom{a}{2}\binom{b}{3}\binom{a}{1}$, where $\#$ is a special symbol reserved for the extra data value.

We can thus use multiple formalisms developed for data words (with a minor adjustment for the extra value) to specify data paths. Such formalisms abound in the literature, and include first-order and monadic second-order logic with data comparisons [9, 10], LTL with freeze quantifiers [16], XPath fragments [8, 20], and various automata models such as pebble and register automata [11, 28, 29, 30, 33].

The question is then, which one to choose? To answer this, we look at data complexity of query answering for each of these formalisms. We show that as long as the formalism is capable of expressing what is perhaps the most primitive language with data value comparisons (two data values are equal) and is closed under complementation, then *data* complexity is NP-hard. Clearly one cannot tolerate such high data complexity, and this rules out most of the formalisms except *register automata*.

We then study query answering with register automata (adjusted for data paths from data words). We present an algorithm that is based, as expected, on computing products of automata; with nonemptiness performed on-the-fly, this gives us an NLOGSPACE data complexity bound, and PSPACE-completeness for combined complexity. The bound for data complexity is good (it matches the usual RPQs) and the bound for combined complexity is tolerable (equivalent to that of FO, but higher than the NP bound for conjunctive RPQs or the PTIME bound for RPQs).

However, automata are not an ideal way of specifying con-

ditions in queries. In RPQs, we use regular expressions rather than NFAs. While some regular expressions have been considered for register automata [30], they are very far from intuitive. So we propose two types of regular expressions that can be used in queries.

The first, close in spirit to automata themselves, lets one bind a data value and use it later. For example, to express the query "connected by a path along which the data value remains the same", we would use the expression $\downarrow x.(\Sigma[x^{=}])^*$. This expression says: bind x in the beginning of the path (i.e., to the first data value), and then go along, if labels are arbitrary (Σ) and the condition $x^{=}$, meaning that the value is equal to x, holds. These expressions are much easier to write than the automata, and at the same time they can be translated into register automata; thus data complexity of queries remains in NLOGSPACE. We show that the combined complexity remains the same as for automata, i.e., PSPACE-complete (except in a rather limited case when the Kleene star is not used: then it drops to NP-complete).

This motivates a second class of expressions that restrict the ability to compare data values along the path; instead, one can only do comparisons for chosen subexpressions. A simple example of such an expression is $\Sigma^{+}_{=}$, which denotes nonempty data paths that have same data value at the beginning and at the end of the path: Σ^{+} indicates the label of the path, and the subscript $=$ states the condition for the first and the last data values. A slightly more elaborate example is $\Sigma^* \cdot \Sigma^{+}_{=} \cdot \Sigma^*$. It says that a subpath conforms to $\Sigma^{+}_{=}$, i.e., it denotes data paths on which two data values are equal. For expressions of this kind, we give a polynomial-time algorithm for combined complexity. The key idea is to translate expressions into push-down automata and then take the product with an automaton obtained efficiently from the graph database.

Finally, we show that our results extend to analogs of conjunctive regular path queries that use data comparisons. There is no penalty to pay in terms of complexity except one case, where we have to deal with the same increase of complexity as in going from the usual RPQs to their conjunctive analogs [12, 14].

Organization In Section 2 we define data graphs and generic queries over them. In Section 3 we rule out several formalisms for specifying data paths due to prohibitively high data complexity for them. In Section 4 we define register automata and study complexity of query evaluation for them. We do the same in Section 5 for regular expressions with memory and in Section 6 for regular expressions with equality. Finally in Section 7 we look at conjunctive queries based on the formalisms proposed in the previous sections. Due to space limitations, most proofs are only sketched, and complete proofs are given in the appendix.

2. PRELIMINARIES

Let Σ be a finite alphabet, and $\mathcal{D}$ a countably infinite set of data values. Data graphs will have edges labeled by letters from Σ and nodes that store data values from $\mathcal{D}$.

DEFINITION 2.1 (DATA GRAPHS). *A* data graph *(over Σ and $\mathcal{D}$) is a triple $G = \langle V, E, \rho \rangle$, where:*

- *V is a finite set of nodes;*
- *$E \subseteq V \times \Sigma \times V$ is a set of labeled edges; and*
- *$\rho : V \to \mathcal{D}$ is a function that assigns a data value to each node in V.*

A path between nodes v_1 and v_n in a graph is a sequence

$$\pi = v_1 a_1 v_2 a_2 v_3 \dots v_{n-1} a_{n-1} v_n \quad (1)$$

such that each (v_i, a_i, v_{i+1}), for $i < n$, is an edge in E. Corresponding to the path π (1) we have a *data path*

$$w_\pi = \rho(v_1) a_1 \rho(v_2) a_2 \rho(v_3) \dots \rho(v_{n-1}) a_{n-1} \rho(v_n) \quad (2)$$

which is a sequence of alternating data values and labels, starting and ending with data values. The set of all *data paths*, i.e., such alternating sequences over Σ and $\mathcal{D}$, will be denoted by $\Sigma[\mathcal{D}]^*$. For both paths and data paths, we use the notation $\lambda(\pi)$ or $\lambda(w_\pi)$ to denote their label, i.e. the word $a_1 \dots a_{n-1} \in \Sigma^*$.

Returning to Figure 1 from the Introduction, one example that we used was the path $\pi = v_1 a v_2 b v_5 a v_3$. The corresponding data path w_π is $1a2b3a1$ since data values of v_1, v_2, v_5, and v_3 are $1, 2, 3$, and 1, respectively. Its label is aba.

Recall that *regular path queries*, or *RPQs*, over usual labeled graphs are queries of the form $Q = x \xrightarrow{L} y$, where $L \subseteq \Sigma^*$ is a regular language. Given a graph G (the data part is irrelevant for RPQs), $Q(G)$ is the set of pairs of nodes (v, v') such that there is a path π from v to v' whose label $\lambda(\pi)$ is in L.

By analogy, we define *data path queries*. Syntactically they are expressions $Q = x \xrightarrow{L} y$, as before, but now $L \subseteq \Sigma[\mathcal{D}]^*$ is a set of data paths. If G is a data graph, then $Q(G)$ is the set of pairs of nodes (v, v') such that there is a path π from v to v' whose associated data path w_π is in L.

As with relational queries and RPQs, we will be interested in *data and combined complexity* of query evaluation problem, i.e. checking, for a data path query Q, a data graph G and a pair of nodes (v, v'), whether $(v, v') \in Q(G)$ (for data complexity, of course, the query Q will be fixed).

3. LANGUAGE FOR PATHS: RULING OUT BAD ALTERNATIVES

To talk about data path queries, as just defined, we need to express properties of paths with data. As we already mentioned, these are essentially data words, with an extra data value attached. Quite a few languages and automata models have been developed for data words over the past few years, mainly in connection with the study of XML, especially XPath. We now give a quick overview of them. A more extensive survey can be found in [40].

$\mathrm{FO}(\sim)$ **and** $\mathrm{MSO}(\sim)$ These are first-order logic and monadic second-order logic extended with the binary predicate $\sim$ saying that data values in two positions are the same. For example, $\exists x \exists y\, a(x) \wedge a(y) \wedge x \sim y$ says that there are two a-labeled positions with the same data value. Two-variable fragments of $\mathrm{FO}(\sim)$ and existential MSO with the $\sim$ predicate have been shown to have decidable satisfiability problem [9, 10].

Pebble automata These are basically finite state automata equipped with a finite set of pebbles. To ensure regular behavior pebbles are required to adhere to a stack discipline. The automata are modeled in such a way that the last placed pebble acts as the automaton head and we are allowed to drop and lift pebbles over the current position. In addition to this we can also compare the current data value to the one that already has a pebble placed over it. Algorithmic properties and connections with logics have been extensively studied in [33].

LTL$_\downarrow$ This is the standard LTL expanded with a freeze operator that allows us to store the current data value into a memory location and use it for future comparisons. The full logic has undecidable satisfiability problem, but various decidable restrictions are known [16, 17].

Register automata These are in essence finite state automata extended with a finite set of registers allowing us to store data values. Although first studied only on words over infinite alphabet [28, 33, 35] they are easily extended to handle data words, as illustrated in [16, 40]. They act as usual finite state automata in the sense that they move from one position to another by reading the appropriate letter from the finite alphabet, but are also allowed to compare the current data value with ones already stored in the registers.

XPath fragments XPath is the standard language for navigating in XML documents, i.e., for describing paths in a way that may also include conditions on data values that occur in documents. Fragments of XPath (with and without data values) have been extensively studied, see, e.g., [6, 9]. While in general the satisfiability problem is undecidable, several decidable restrictions are known, e.g., [20, 21].

In deciding which formalism to choose, we look at the *data complexity* of evaluating data path queries, and try to rule out those for which data complexity is intractable. Technically, a formalism just defines a set of allowed languages $L \subseteq \Sigma[\mathcal{D}]^*$. It turns out that most of the formalisms for data words/paths are actually not suitable for graph querying. This is implied by the following result. Let L_{eq} be the language of data paths that contain two equal data values.

THEOREM 3.1. *Assume that we have a formalism for data paths that can define $\overline{L_{eq}}$. Then data complexity of evaluating data path queries is* NP*-hard.*

PROOF. The proof is by showing that with $\overline{L_{eq}}$, one can encode the 2-disjoint-paths problem which is NP-complete [23]. This problem is to check, for a graph G and four nodes s_1, t_1, s_2, t_2 in G, whether there exist two paths in G, one from s_1 to t_1 and the other from s_2 to t_2 that have no nodes in common.

Assume that $G = \langle V, E\rangle$ is a graph and s_1, t_1, s_2, t_2 are four nodes in G. Here we assume that all four nodes are distinct. It is easy to see that with this assumption the problem remains NP-complete, because we can always add two new nodes for each repeated node and connect them with all the nodes the repeated node was connected to, thus modifying our problem to have all source and target nodes different. We let our query be $Q = x \xrightarrow{\overline{L_{eq}}} y$. Since our query will disregard edge labels we can take $\Sigma = \{a\}$. We will construct a data graph G' and two nodes $s, t \in G'$ such that $(s,t) \in Q(G')$ if and only if there are two disjoint paths in G from s_1 to t_1 and from s_2 to t_2.

Let $V = \{v_1, \ldots, v_n\}$. The graph G' will contain two disjoint isomorphic copies of G (with data values and labels attached) connected by a single edge. We define the two isomorphic copies $G_1 = \langle V_1, E_1, \rho_1\rangle$ and $G_2 = \langle V_2, E_2, \rho_2\rangle$ by:

- $V_1 = \{v'_1, \ldots, v'_n\}$,
- $V_2 = \{v''_1, \ldots, v''_n\}$,
- $E_1 = \{(v'_i, a, v'_j) : (v_i, v_j) \in E\}$,
- $E_2 = \{(v''_i, a, v''_j) : (v_i, v_j) \in E\}$and
- $\rho_1(v'_i) = \rho_2(v''_i) = i$, for $i = 1 \ldots n$,

and then let $G' = \langle V', E', \rho'\rangle$, where

- $V' = V_1 \cup V_2$,
- $E' = E_1 \cup E_2 \cup \{(t'_1, a, s''_2)\}$ and
- $\rho' = \rho_1 \cup \rho_2$.

Note that ρ' is well defined since V_1 and V_2 are disjoint.

Finally we define $s = s'_1$ and $t = t''_2$.

We claim that $(s,t) \in Q(G')$ if and only if there are two disjoint paths in G from s_1 to t_1 and from s_2 to t_2 in G. To see this assume first that $(s,t) \in Q(G')$. This means that we have a path in G' which starts in s'_1 and ends in t''_2. In particular, it must pass the edge between t'_1 and s''_2, since this is the only edge connecting the two graphs. Also, since all data values on this path are different we know that no node can repeat. But then we simply split this path into two disjoint paths in G since the structure of edges in G' is the same as the one in G with the exception of edge between t'_1 and s''_2. Also, no node can be repeated, since the corresponding nodes in G_1 and G_2 have the same data values.

Conversely, if we have two disjoint paths from s_1 to t_1 and from s_2 to t_2 in G, we simply follow the corresponding path from s'_1 to t'_1 in G_1 (and thus in G'), traverse the edge between t'_1 and s''_2 and then follow the path in G_2 (and thus

in G') from s_2'' to t_2'' corresponding to the path from s_2 to t_2 in G.

This completes the proof. □

Note that L_{eq} is about the simplest property one can express about data paths/words; it would be hard to imagine a formalism that cannot check for the equality of data values. The corollary below effectively rules out closure under complement for such formalisms if they are to be used in graph querying.

COROLLARY 3.2. *Assume that we have a formalism for data paths that can define L_{eq} and that is closed under complement. Then data complexity of evaluating data path queries is* NP*-hard.*

This immediately rules out FO($\sim$) and its two-variable fragment, LTL with the freeze quantifier, XPath fragments closed under complement, and pebble automata.

The only hope we have among standard formalisms is *register automata*, since they are not closed under complementation [28]. In the next sections we show that we can achieve good query answering complexity with them, as well as sufficient expressivity.

4. DATA PATH QUERIES WITH REGISTER AUTOMATA

As stated in the previous section, register automata are the only standard formalism for defining classes of data words that does not immediately lead to NP-hard data complexity of queries on graphs with data. In this section we define them and study query evaluation for data path queries based on these automata. We will slightly alter the definition of register automata used in e.g. [16, 40] to work on data paths rather than data words, without affecting their desirable properties.

As mentioned earlier register automata move from one state to another by reading the appropriate letter from the finite alphabet and comparing the data value to one previously stored into the registers. Our version of register automata will use slightly more involved comparisons which will be boolean combinations of atomic $=, \neq$ comparisons of data values.

To define such conditions formally, assume that, for each $k > 0$, we have variables $x_1, \ldots, x_k$. Then conditions in $\mathcal{C}_k$ are given by the grammar:

$$c \;:=\; x_i^{=} \mid x_i^{\neq} \mid c \wedge c \mid c \vee c \mid \neg c, \quad 1 \leq i \leq k.$$

The satisfaction is defined with respect to a data value $d \in \mathcal{D}$ and a tuple $\tau = (d_1, \ldots, d_k) \in \mathcal{D}^k$ as follows:

- $d, \tau \models x_i^{=}$ iff $d = d_i$;
- $d, \tau \models x_i^{\neq}$ iff $d \neq d_i$;
- $d, \tau \models c_1 \wedge c_2$ iff $d, \tau \models c_1$ and $d, \tau \models c_2$ (and likewise for $c_1 \vee c_2$);
- $d, \tau \models \neg c$ iff $d, \tau \not\models c$.

In what follows, $[k]$ is a shorthand for $\{1, \ldots, k\}$.

DEFINITION 4.1 (REGISTER DATA PATH AUTOMATA). *Let Σ be a finite alphabet, and k a natural number. A* k-register data path automaton *is a tuple $\mathcal{A} = (Q, q_0, F, \tau_0, \delta)$, where:*

- *$Q = Q_w \cup Q_d$, where Q_w and Q_d are two finite disjoint sets of word states and data states;*
- *$q_0 \in Q_d$ is the initial state;*
- *$F \subseteq Q_w$ is the set of final states;*
- *$\tau_0 \in \mathcal{D}^k$ is the initial configuration of the registers;*
- *$\delta = (\delta_w, \delta_d)$ is a pair of transition relations:*
  - *$\delta_w \subseteq Q_w \times \Sigma \times Q_d$ is the word transition relation;*
  - *$\delta_d \subseteq Q_d \times \mathcal{C}_k \times 2^{[k]} \times Q_w$ is the data transition relation.*

The intuition behind this definition is that since we alternate between data values and word symbols in data paths, we also alternate between data states (which expect data value as the next symbol) and word states (which expect alphabet letters as the next symbol). We start with a data value, so q_0 is a data state, end with a data value, so final states, seen after reading that value, are word states.

In a word state the automaton behaves like the usual NFA (but moves to a data state). In a data state, the automaton checks if the current data value and the configuration of the registers satisfy a condition, and if they do, moves to a word state and updates some of the registers with the read data value.

Given a data path $w = d_0 a_0 d_1 a_1 \ldots a_{n-1} d_n$, where each d_i is a data value and each a_l is a letter, a configuration of $\mathcal{A}$ on w is a tuple (j, q, τ), where j is the current position of the symbol in w that $\mathcal{A}$ reads, q is the current state and $\tau \in \mathcal{D}^k$ is the current state of the registers. The initial configuration is $(0, q_0, \tau_0)$ and any configuration (j, q, τ) with $q \in F$ is a final configuration.

From a configuration $C = (j, q, \tau)$ we can move to a configuration $C' = (j + 1, q', \tau')$ if one of the following holds:

- the jth symbol is a letter a, there is a transition $(q, a, q') \in \delta_w$, and $\tau' = \tau$; or
- the current symbol is a data value d, and there is a transition $(q, c, I, q') \in \delta_d$ such that $d, \tau \models c$ and τ' coincides with τ except that the ith component of τ' is set to d whenever $i \in I$.

A data path w is accepted by $\mathcal{A}$ if $\mathcal{A}$ can move from the initial configuration to a final configuration after reading w. The language of data paths accepted by $\mathcal{A}$ is denoted by $L(\mathcal{A})$.

Data paths vs data words

Register automata have been previously studied for data words [16, 40] and we now briefly explain the connection. A data word is a word in $(\Sigma \times \mathcal{D})^*$, i.e., each position carries a label from Σ and a data value from $\mathcal{D}$. A *k-register data word automaton* $\mathcal{A}$ is a tuple (Q, q_0, F, τ_0, T) where Q is a finite set of states (no longer split into two), $q_0 \in Q$ is the initial state, $F \subseteq Q$ is the set of final states, $\tau_0 \in \mathcal{D}^k$ is the initial register assignment, and T is a finite set of transitions of the form $(q, a, c) \to (I, q')$, where q, q' are states, a is a label, $I \subseteq [k]$, and c is a condition in $\mathcal{C}_k$.

The automaton traverses a data word from left to right, starting in q_0 with τ_0 as the register configuration. If it reads $\binom{a}{d}$ in state q with register configuration τ, it may apply a transition $(q, a, c) \to (I, q')$ if $d, \tau \models c$; it then enters state q' and changes contents of registers i, with $i \in I$, to d.

The relationship between automata models, as needed for our purposes, is described by the lemma below. With each data path $w = d_1 a_1 \dots a_{n-1} d_n \in \Sigma[\mathcal{D}]^*$ we associate a data word $s_w = \binom{\#}{d_1}\binom{a_1}{d_2}\dots\binom{a_{n-1}}{d_n}$ over $(\Sigma \cup \{\#\}) \times \mathcal{D}$, where $\# \notin \Sigma$ is a new alphabet symbol.

LEMMA 4.2. *Given a k-register data path automaton $\mathcal{A}$, one can construct, in* DLOGSPACE*, a k-register data word automaton $\mathcal{A}'$ such that a data path w is in $L(\mathcal{A})$ iff the data word s_w is in $L(\mathcal{A}')$.*

It is known [16] that nonemptiness problem for data word register automata is PSPACE-complete. The above lemma shows that the PSPACE upper bound applies to data path automata. Moreover, one can verify that the PSPACE-hardness reduction applies to such automata as well. Hence, we have

COROLLARY 4.3. *The nonemptiness problem for register data path automata is* PSPACE*-complete.*

4.1 Regular data path queries

Our basic class of regular path queries on graphs with data is based on register data path automata.

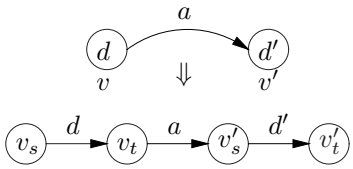
DEFINITION 4.4. *A* regular data path query (RDPQ) *is an expression $Q = x \xrightarrow{\mathcal{A}} y$ where $\mathcal{A}$ is a register data path automaton.*

Given a data graph G, the result of the query $Q(G)$ consists of pairs of nodes (v, v') such that there is a data path w from v to v' that belongs to $L(\mathcal{A})$.

To evaluate RDPQs, we transform both a data graph G and a k-register data path automaton $\mathcal{A}$ into NFAs over an extended alphabet and reduce query evaluation to NFA nonemptiness. More precisely, to evaluate $Q(G)$, we do the following:

1. Let D be the set of all data values in G.
2. Transform $G = \langle V, E, \rho\rangle$ into a graph $G' = \langle V', E'\rangle$ over the alphabet $\Sigma \cup D$ as follows:
   - $V' = \{v_s, v_t \mid v \in V\}$
   - $E' = \{(v_t, a, v'_s) \mid (v, a, v') \in E\} \bigcup \{(v_s, \rho(v), v_t) \mid v \in V\}$

   Basically, we split each node v with a data value d into a source node v_s and a target node v_t and add a d-labeled edge between them; after that we restore the edges from E so that they go from target to source nodes. This is illustrated below.



3. Transform the automaton $\mathcal{A} = (Q, q_0, F, \tau_0, (\delta_w, \delta_d))$ into an NFA $\mathcal{A}_D = (Q', q'_0, F', \delta')$ as follows:
   - $Q' = Q \times D^k$;
   - $q'_0 = (q_0, \tau_0)$;
   - $F' = F \times D^k$;
   - δ' includes two types of transitions.
     (a) Whenever we have a transition (q, a, q') in δ_w, we add transitions $((q, \tau), a, (q', \tau))$ to δ' for all $\tau \in D^k$.
     (b) Whenever we have a transition (q, c, I, q') in δ_d, we add transitions $((q, \tau), d, (q', \tau'))$ if $d, \tau \models c$ and τ' is obtained from τ by putting d in positions from the set I.

For two nodes v, v' of G, we turn G' into an NFA $\mathcal{A}_{G',v,v'}$ by letting v_s be its initial state and v'_t be its final state. Then we have the following.

PROPOSITION 4.5. *Let $Q = x \xrightarrow{\mathcal{A}} y$ be an RDPQ, and G a data graph whose data values form a set $D \subseteq \mathcal{D}$. Then*

$$(v, v') \in Q(G) \;\Leftrightarrow\; L(\mathcal{A}_{G',v,v'} \times \mathcal{A}_D) \neq \emptyset.$$

Thus, query evaluation, like in the case of the usual RPQs, is reduced to automata nonemptiness, although this time the automata are over larger alphabets. Since the construction is polynomial in the size of G and exponential in the size of $\mathcal{A}$ (as k gets into the exponent), we immediately get a PTIME upper bound for data complexity and an EXPTIME upper bound for combined complexity. By performing on-the-fly nonemptiness checking for the product, we can lower these bounds.

THEOREM 4.6. *Data complexity of RDPQs over data graphs is in* NLOGSPACE*, and the combined complexity of RDPQs over data graphs is* PSPACE*-complete.*

The bound for data complexity cannot be lowered as there exist simple RPQs for which data complexity is NLOGSPACE-complete.

5. QUERIES BASED ON REGULAR EXPRESSIONS WITH MEMORY

Regular data path queries based on register automata have acceptable complexity bounds: data complexity is the same as for RPQs, and combined complexity, although exceeding the bounds on conjunctive queries and RPQs, is the same as for relational calculus or for RPQs extended with regular relations. Despite this, RDPQs as we defined them have no chance to lead to a practical language as it is inconceivable that the user will specify a register automaton over data words. Even for queries such as RPQs and their extensions, conditions are normally specified via regular expressions.

Our goal now is to introduce regular expressions that can be used in place of register automata in data path queries. Note that as long as they express languages accepted by register automata, we shall achieve an NLOGSPACE bound on data complexity by Theorem 4.6.

The first class of queries, studied in this section, is based on an extension of regular expressions with *memory* that lets us specify when data values are remembered and when they are used. The basic idea is this: we can write expressions like $\downarrow x.a^+[x^=]$ saying: store the current data value in x and check that after reading a word from a^+ we see the same data value (condition $x^=$ is true). This will define data words of the form $da \dots ad$. Such expressions are relatively easy to write and understand (much easier than automata), and the complexity of their query evaluation will not exceed that of register automata.

DEFINITION 5.1 (EXPRESSIONS WITH MEMORY). *Let Σ be a finite alphabet and $x_1, \dots, x_k$ a set of variables. Then* regular expressions with memory *are defined by the grammar:*

$$e \;:=\; \varepsilon \mid a \mid e + e \mid e \cdot e \mid e^+ \mid e[c] \mid \downarrow \bar{x}.e \qquad (3)$$

where a ranges over alphabet letters, c over conditions in $\mathcal{C}_k$, and $\bar{x}$ over tuples of variables from $x_1, \dots, x_k$.

A regular expression with memory e is well-formed if it satisfies two conditions:

- *Subexpressions e^+, $e[c]$, and $\downarrow \bar{x}.e$ are not allowed if e reduces to ε. Formally, e reduces to ε if it is ε, or it is $e_1 + e_2$ or $e_1 \cdot e_2$ or e_1^+ or $e_1[c]$ or $\downarrow \bar{x}.e_1$ where e_1 (and e_2) reduce to ε.*
- *No variable appears in a condition before it appears in $\downarrow \bar{x}$.*

The class of well-formed regular expressions with memory is denoted by $\mathrm{REG}(\Sigma[x_1, \dots, x_k])$.

The extra condition of being well-formed is to rule out pathological cases like $\varepsilon[c]$ for checking conditions over empty subexpressions, or $a[x^=]$ for checking equality with a variable that has not been defined. In what follows we always assume that regular expressions with memory are well-formed.

The intuition behind the expressions is that they process a data path in the same way that the register automaton would, by storing data values in variables, using these variables for comparisons and moving through the word by reading a letter from the finite alphabet. Note that when we bound a variable we do not specify the scope of this binding. This means that the variable can be used at any point after it was bounded till the end of the expression and is analogous to how register automata store and use data values.

EXAMPLE 5.2. We now give four examples of such expressions and languages they recognize, before formally defining their semantics.

1. The expression $\downarrow x.(a[x^{\neq}])^+$ defines the language of data paths where all edges are labeled a and the first data value is different from all other data values. It starts by binding x to the first data value; then it proceeds checking that the letter is a and condition $x^{\neq}$ is satisfied, which is expressed by $a[x^{\neq}]$; the expression is then put in the scope of $+$ to indicate that the number of such values is arbitrary.
2. The expression $\downarrow x.(ab)^+[x^{\neq}]$ denotes the language of data paths whose label is of the form $ab \dots ab$ and for which the first data value is different from the last. Note that the order of $+$ and condition is now different: the condition is checked after verifying that the label is in $(ab)^+$, i.e., at the end of the word.
3. The expression $\downarrow x.a^+[x^=] + \varepsilon$ denotes the language of data paths where all labels are a and the first data value is equal to the last. Note that one such data path is simply of the form d, for $d \in \mathcal{D}$, with label ε.
4. The language L_{eq} of data paths in which two data values are the same (see Section 3) is given by the expression $\Sigma^* \cdot \downarrow x.\Sigma^+[x^=] \cdot \Sigma^*$, where Σ is the shorthand for $a_1 + \ldots + a_l$, whenever $\Sigma = \{a_1, \dots, a_l\}$ and Σ^* is the shorthand for $\Sigma^+ + \varepsilon$. It says: at some point, bind x, and then check that after one or more edges, we have the same data value.

Semantics First, we define the *concatenation* of two data paths $w = d_1 a_1 \dots a_{n-1} d_n$ and $w' = d_n a_n \dots a_{m-1} d_m$ as $w \cdot w' = d_1 a_1 \dots a_{n-1} d_n a_n \dots a_{m-1} d_m$. Note that it is only defined if the last data value of w equals the first data value of w'. The definition naturally extends to concatenation of several data paths. If $w = w_1 \cdots w_l$, we shall refer to it as a *splitting* of a data path (into $w_1, \dots, w_l$).

The semantics is defined by means of a relation $(e, w, \sigma) \vdash \sigma'$, where $e \in \mathrm{REG}(\Sigma[x_1, \dots, x_k])$ is a regular expression with memory, w is a data path, and both σ and σ' are k-tuples over $\mathcal{D} \cup \{\perp\}$ (the symbol $\perp$ means that a register has not been assigned yet). The intuition is as follows: one can start with a memory configuration σ (i.e., values of $x_1, \dots, x_k$) and parse w according to e in such a way that at the end the memory configuration is σ'. The language of e is then defined as

$$L(e) \;=\; \{w \mid (e, w, \bar{\perp}) \vdash \sigma \text{ for some } \sigma\},$$

where $\bar{\perp}$ is the tuple of k values $\perp$.

The relation $\vdash$ is defined inductively on the structure of expressions. Recall that the empty word corresponds to a data path with a single data value d (i.e., a single node in a data graph). We use the notation $\sigma_{\bar{x}=d}$ for the valuation obtained from σ by setting all the variables in $\bar{x}$ to d.

- $(\varepsilon, w, \sigma) \vdash \sigma'$ iff $w = d$ for some $d \in \mathcal{D}$ and $\sigma' = \sigma$.
- $(a, w, \sigma) \vdash \sigma'$ iff $w = d_1 a d_2$ and $\sigma' = \sigma$.
- $(e_1 \cdot e_2, w, \sigma) \vdash \sigma'$ iff there is a splitting $w = w_1 \cdot w_2$ of w and a valuation σ'' such that $(e_1, w_1, \sigma) \vdash \sigma''$ and $(e_2, w_2, \sigma'') \vdash \sigma'$.
- $(e_1 + e_2, w, \sigma) \vdash \sigma'$ iff $(e_1, w, \sigma) \vdash \sigma'$ or $(e_2, w, \sigma) \vdash \sigma'$.
- $(e^+, w, \sigma) \vdash \sigma'$ iff there are a splitting $w = w_1 \cdots w_m$ of w and valuations $\sigma = \sigma_0, \sigma_1, \ldots, \sigma_m = \sigma'$ such that $(w, w_i, \sigma_{i-1}) \vdash \sigma_i$ for all $i \in [m]$.
- $(\downarrow \bar{x}.e, w, \sigma) \vdash \sigma'$ iff $(e, w, \sigma_{\bar{x}=d}) \vdash \sigma'$, where d is the first data value of w.
- $(e[c], w, \sigma) \vdash \sigma'$ iff $(e, w, \sigma) \vdash \sigma'$ and $\sigma', d \models c$, where d is the last data value of w.

Take note that in the last item we require that σ', and not σ, satisfies c. The reason for this is that our initial assignment might change before reaching the end of the expression and we want this change to be reflected when we check that condition c holds.

Translation into automata We now show that regular expressions with memory can be efficiently translated into register automata.

PROPOSITION 5.3. *For each regular expression with memory $e \in \mathrm{REG}(\Sigma[x_1, \ldots, x_k])$ one can construct, in* DLOGSPACE*, a k-register data path automaton $\mathcal{A}_e$ such that $L(e) = L(\mathcal{A}_e)$.*

More precisely, the automaton $\mathcal{A}_e = (Q, q_0, F, \bar{\perp}, \delta)$ (over data domain $\mathcal{D} \cup \{\perp\}$) has the property that for any two valuations σ, σ' and a data path w, we have $(e, w, \sigma) \vdash \sigma'$ iff the automaton $(Q, q_0, F, \sigma, \delta)$ has an accepting run on w that ends with the register configuration σ'.

5.1 Query evaluation

We now deal with the following queries.

DEFINITION 5.4. *A* regular data path query with memory *is an expression $Q = x \xrightarrow{e} y$, where e is regular expression with memory.*

Given a data graph G, the result of the query $Q(G)$ consists of pairs of nodes (v, v') such that there is a data path w from v to v' that belongs to $L(e)$.

The class of these queries is denoted by $\mathrm{RDPQ}_{\mathrm{mem}}$.

Using Proposition 5.3 combined with Theorem 4.6 we immediately obtain:

COROLLARY 5.5. *Data complexity of* $\mathrm{RDPQ}_{\mathrm{mem}}$ *queries is in* NLOGSPACE.

From the same connection we also get the upper bound (PSPACE) for combined complexity. It turns out that we can achieve PSPACE-hardness with expressions with memory (see the appendix for the proof). Thus, we have

THEOREM 5.6. *Combined complexity of evaluating* $\mathrm{RDPQ}_{\mathrm{mem}}$ *queries is* PSPACE*-complete.*

The question is whether we can reduce this complexity – ideally to PTIME, but at least to NP, to match the combined complexity of conjunctive queries. The following corollary (to the proof of Theorem 5.6) shows that many restrictions will not work.

COROLLARY 5.7. *Combined complexity of evaluating* $\mathrm{RDPQ}_{\mathrm{mem}}$ *queries remains* PSPACE*-hard for expressions that use at most one $^+$ and $\neq$ symbol, are specified over a singleton alphabet $\Sigma = \{a\}$, and are evaluated over a fixed graph.*

In one case, we can lower the complexity.

PROPOSITION 5.8. *Combined complexity of* $\mathrm{RDPQ}_{\mathrm{mem}}$ *queries whose regular expressions do not have subexpressions of the form e^+ is* NP*-complete.*

The restriction, while achieving better combined complexity, is too strong, as it effectively restricts one to languages of data paths whose projections on Σ^* are finite. All the examples we saw earlier use subexpressions e^+. So if we want to achieve tractability, we need to look at a very different way of restricting expressions. This is what we do in the next section.

6. QUERIES BASED ON REGULAR EXPRESSIONS WITH EQUALITY

The class of regular expressions for data paths that lets us lower the combined complexity of queries to PTIME permits testing for equality or inequality of data values at the beginning or the end of a data (sub)path. For example, $(\Sigma^+)_{\neq}$ denotes the set of all data paths having different first and last data values. The language L_{eq} of data paths on which two data values are the same is given by $\Sigma^* \cdot (\Sigma^+)_= \cdot \Sigma^*$: it checks for the existence of a nonempty subpath (with label in Σ^+) such that the nodes at the beginning and at the end of this subpath have the same data value, indicated by subscript $=$.

DEFINITION 6.1 (EXPRESSIONS WITH EQUALITY). *Let Σ be a finite alphabet. Then* regular expressions with equality *are defined by the grammar:*

$$e \;:=\; \varepsilon \mid a \mid e + e \mid e \cdot e \mid e^+ \mid e_= \mid e_{\neq} \qquad (4)$$

where a ranges over alphabet letters.

The language $L(e)$ of data paths denoted by a regular expression with equality e is defined as follows.

- $L(\varepsilon) = \{d \mid d \in \mathcal{D}\}$.
- $L(a) = \{dad' \mid d, d' \in \mathcal{D}\}$.
- $L(e \cdot e') = L(e) \cdot L(e')$.
- $L(e + e') = L(e) \cup L(e')$.
- $L(e^+) = \{w_1 \cdots w_k \mid k \geq 1 \text{ and each } w_i \in L(e)\}$.
- $L(e_=) = \{d_1 a_1 \dots a_{n-1} d_n \in L(e) \mid d_1 = d_n\}$.
- $L(e_{\neq}) = \{d_1 a_1 \dots a_{n-1} d_n \in L(e) \mid d_1 \neq d_n\}$.

These expressions sacrifice the ability to check conditions as one goes along the path, making it only possible to check conditions at the start and the end of chosen subexpressions. Looking at Example 5.2, all languages except the first can be defined by regular expressions with memory. We already saw how to do the language L_{eq}; the expression $\downarrow x.(ab)^+[x^{\neq}]$ is equivalent to $(ab)^+_{\neq}$. The expression $\downarrow x.(a[x^{\neq}])^+$ describing the language of data paths in which all data values are different from the first one, requires checking a condition multiple times. We now show that this goes beyond the power of expressions with equality, which are strictly weaker than expressions with memory.

PROPOSITION 6.2. *1. For each regular expression with equality, there is an equivalent regular expression with memory.*

2. For the regular expression with memory $\downarrow x.(a[x^{\neq}])^+$ there is no equivalent regular expression with equality.

6.1 Query evaluation

We now deal with the following queries.

DEFINITION 6.3. *A* regular data path query with equality *is an expression $Q = x \xrightarrow{e} y$, where e is regular expression with equality.*

Given a data graph G, the result of the query $Q(G)$ consists of pairs of nodes (v, v') such that there is a data path w from v to v' that belongs to $L(e)$.

The class of these queries is denoted by $\mathrm{RDPQ}_=$.

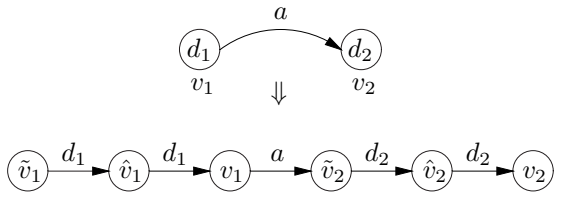
Combining Propositions 5.3 and 6.2 we see that the power of regular expressions with equality is subsumed by register automata; hence combined with Theorem 4.6 we immediately obtain:

COROLLARY 6.4. *Data complexity of* $\mathrm{RDPQ}_=$ *queries is in* NLOGSPACE.

We now show that combined complexity for $\mathrm{RDPQ}_=$ queries is tractable, i.e., is even better than the combined complexity of conjunctive queries. Our outline of the polynomial-time algorithm is as follows. We start with a data graph $G = \langle V, E, \rho \rangle$ whose data values form a (finite) set $D \subset \mathcal{D}$ and a regular expression with equality e.

1. We first show that we can efficiently generate a context-free grammar $\mathcal{G}_{e,D}$ whose language corresponds to the set of all data paths from $L(e)$ whose data values are in D. More precisely, every word in $L(\mathcal{G}_{e,D})$ will be of the form $d_1 a_1 d_2 d_2 a_2 d_3 d_3 \dots d_{n-1} d_{n-1} a_{n-1} d_n$, where $d_i \in D$ and $a_i \in \Sigma$. We say that this word, in which each data value, except the first and the last, appears twice, corresponds to the data path $d_1 a_1 d_2 a_2 d_3 \dots a_{n-1} d_n$.
2. We then convert $\mathcal{G}_{e,D}$, in polynomial time, into an equivalent PDA $\mathcal{A}(\mathcal{G}_{e,D})$.
3. Given two nodes v, v' in G, we construct an NFA $\mathcal{A}_{G,v,v'}$. To do so we first define a graph $G' = \langle V', E' \rangle$ that will reflect the fact that all data values from G have to be doubled if they appear on a path as intermediate nodes. We define $G' = \langle V', E' \rangle$ as follows:
   - $V' = V \cup \{\tilde{u}, \hat{u} \mid u \in V\} \cup \{s, t\}$
   - $E' = \{(v_1, a, \tilde{v}_2) \mid (v_1, a, v_2) \in E\} \bigcup \{(\tilde{u}, \rho(u), \hat{u}), (\hat{u}, \rho(u), u) \mid u \in V\}$

   Similarly as when dealing with register automata we triple each node and add an edge between new nodes that will reflect the fact that every intermediate data value will have to be doubled. This is illustrated below.



   In addition, we also add edges $(s, \rho(v), v)$ and $(\tilde{v}', \rho(v'), t)$ to E'. We now get the automaton $\mathcal{A}_{G,v,v'}$ as the automaton obtained from G' by setting s as the initial and t as the final state. Note that the construction of the automaton $\mathcal{A}_{G,v,v'}$ is polynomial.
4. Finally, for $Q = x \xrightarrow{e} y$ we have $(v, v') \in Q(G)$ iff the language $\mathcal{A}_{G,v,v'}$ has nonempty intersection with the language generated by the grammar $\mathcal{G}_{e,D}$. This follows by an argument similar to the proof of Proposition 4.5.

   Since the intersection of a context-free language and a regular language is context-free and can be obtained by the product construction of a PDA and an NFA, this means that $(v, v') \in Q(G)$ iff the product $\mathcal{A}(\mathcal{G}_{e,D}) \times \mathcal{A}_{G,v,v'}$ defines a nonempty language. This product is a PDA, so we can check its nonemptiness in polynomial time, giving us a polynomial algorithm for query evaluation.

Steps 2, 3, and 4 above use the standard constructions of converting CFGs into PDAs, taking products, and checking PDAs for nonemptiness. So what is missing is the construction of the CFG $\mathcal{G}_{e,D}$, which we show next.

Regular expressions with equality into CFGs Assume that we have a finite set D of data values. We now inductively construct CFGs $\mathcal{G}_{e,D}$ for all regular expressions with equality. The terminal symbols of these CFGs will be Σ plus all elements of D. All nonterminals in $\mathcal{G}_{e,D}$ will be of the

form $A_{e'}$ and $A_{e'}^{dd'}$, where e' ranges over subexpressions of e and $d, d' \in D$. Intuitively, words derived from $A_{e'}^{dd'}$ will correspond to (in a way previously described) data paths in $L(e')$ with data values from D that start with d and end with d'; words derived from $A_{e'}$ will correspond to data paths in $L(e')$ with data values from D. The start symbol for the grammar corresponding to the expression e will be A_e.

The productions of the grammars $\mathcal{G}_{e,D}$ are now defined inductively as follows.

- If $e = \varepsilon$, we have productions $A_\varepsilon \to \bigvee_{d \in D} A_\varepsilon^{dd}$ and $A_\varepsilon^{dd} \to d$ for each $d \in D$.
- If $e = a$, for $a \in \Sigma$, we have productions $A_e \to \bigvee_{d,d' \in D} A_e^{dd'}$ and $A_e^{dd'} \to dad'$ for all $d, d' \in D$.
- If $e = e_1 \cdot e_2$, we have productions $A_e \to \bigvee_{d,d' \in D} A_e^{dd'}$ and $A_e^{dd'} \to \bigvee_{d'' \in D} A_{e_1}^{dd''} A_{e_2}^{d''d'}$ for all $d, d' \in D$ together with all the productions of the grammars $\mathcal{G}_{e_1,D}$ and $\mathcal{G}_{e_2,D}$.
- If $e = e_1 + e_2$, we have productions $A_e \to \bigvee_{d,d' \in D} A_e^{dd'}$ and $A_e^{dd'} \to A_{e_1}^{dd'} | A_{e_2}^{dd'}$ for all $d, d' \in D$ together with all the productions of the grammars $\mathcal{G}_{e_1,D}$ and $\mathcal{G}_{e_2,D}$.
- If $e = (e_1)^+$, we have productions $A_e \to \bigvee_{d,d' \in D} A_e^{dd'}$ and $A_e^{dd'} \to A_{e_1}^{dd'} | \bigvee_{d'' \in D} A_{e_1}^{dd''} A_e^{d''d'}$ for all $d, d' \in D$ together with all the productions of the grammar $\mathcal{G}_{e_1,D}$.
- If $e = (e_1)_=$, we have productions $A_e \to \bigvee_{d \in D} A_e^{dd}$ and $A_e^{dd} \to A_{e_1}^{dd}$ for all $d \in D$ together with all the productions of the grammar $\mathcal{G}_{e_1,D}$.
- If $e = (e_1)_\neq$, we have productions $A_e \to \bigvee_{d,d' \in D,\ d \neq d'} A_e^{dd'}$ and $A_e^{dd'} \to A_{e_1}^{dd'}$ for all $d, d' \in D$ with $d \neq d'$, together with all the productions of the grammar $\mathcal{G}_{e_1,D}$.

It is clear from the construction that all words generated by this grammar(with the sole exception of the empty word) have all of their intermediate data values (i.e. letters corresponding to values in D) doubled, except the first and the last one.

Note that with these expressions we assume that ε can appear only when denoting the empty word and will be removed otherwise. We require this, so that we would not get productions that produce objects that are not data paths, such as e.g. ddd for the expression $\varepsilon \cdot \varepsilon \cdot \varepsilon$. Note that this is not a problem, since all expressions can be rewritten to be of this form in DLOGSPACE.

The main result connecting these CFGs with languages of regular expressions with equality is this. Recall that when we say that a word over Σ and D corresponds to a data path with values in D, we mean that it equals the data path with all the data values, except the first and the last, doubled.

PROPOSITION 6.5. *The language of words derived by each CFG $\mathcal{G}_{e,D}$ corresponds to the set of data paths in $L(e)$ whose data values come from D. Furthermore, the set of words derived from each nonterminal $A_e^{dd'}$ corresponds to the set of data paths in $L(e)$ which start with d, end with d', and whose data values come from D.*

Moreover, the CFG $\mathcal{G}_{e,D}$ can be constructed in polynomial time from e and D.

This, together with the algorithm shown above, finally gives us tractability of combined complexity.

THEOREM 6.6. *Combined complexity of* $\mathrm{RDPQ}_=$ *queries is in* PTIME.

The correctness of the procedure shown in this section is proved in the appendix.

7. CONJUNCTIVE REGULAR PATH QUERIES WITH DATA

A standard extension of RPQs is that to *conjunctive RPQs*, or CRPQs [12, 18, 22]. These add conjunctions of RPQs and existential quantification over variables, in the same way as conjunctive queries extend atomic formulae of relational calculus. We now look at similar extensions of RPQs with data.

Formally, a *conjunctive regular data path query (CRDPQ)* is an expression of the form

$$Ans(\bar{z}) \ := \bigwedge_{1 \leq i \leq m} x_i \xrightarrow{L_i} y_i, \tag{5}$$

where $m > 0$, each $x_i \xrightarrow{L_i} y_i$ is a regular data path query (in one of the formalisms studied here), and $\bar{z}$ is a tuple of variables among $\bar{x}$ and $\bar{y}$. A query with the head $Ans()$ (i.e., no variables in the output) is called a *Boolean* query. Depending on which RDPQs are used in (5) we shall be referring to CRDPQs, or CRDPQs with memory, or CRDPQs with equality.

These queries extend RDPQs with conjunction, as well as existential quantification: variables that appear in the body but not in the head (i.e., variables in $\bar{x}$ and $\bar{y}$ but not $\bar{z}$) are assumed to be existentially quantified.

The semantics of a CRDPQ Q of the form (5) over a data graph $G = \langle V, E, \rho \rangle$ is defined as follows. Given a valuation $\nu : \bigcup_{1 \leq i \leq m} \{x_i, y_i\} \to V$, we write $(G, \nu) \models Q$ if $(\nu(x_i), \nu(y_i))$ is in the answer of $x_i \xrightarrow{L_i} y_i$ on G, for each $i = 1, \ldots, m$. Then $Q(G)$ is defined as the set of all tuples $\nu(\bar{z})$ such that $(G, \nu) \models Q$. If Q is Boolean, we let $Q(G)$ be true if $(G, \nu) \models Q$ for some ν (that is, as usual, the empty tuple models the Boolean constant true, and the empty set models the Boolean constant false).

As with RDPQs, we study data and combined complexity of the query evaluation problem, i.e. checking, for a CRDPQ

Query answering	RDPQ	RDPQ_{mem}	$\mathrm{RDPQ}_{\mathrm{mem}}$ over finite words	$\mathrm{RDPQ}_{=}$
data complexity	NLOGSPACE-c.	NLOGSPACE-c.	NLOGSPACE-c.	NLOGSPACE-c.
combined complexity	PSPACE-c.	PSPACE-c.	NP-c.	PTIME

(a) for single data path query

Query answering	CRDPQ	$\mathrm{CRDPQ}_{\mathrm{mem}}$	$\mathrm{CRDPQ}_{=}$
data complexity	NLOGSPACE-c.	NLOGSPACE-c.	NLOGSPACE-c.
combined complexity	PSPACE-c.	PSPACE-c.	NP-c.

(b) for conjunctive queries

Figure 2: Summary of complexity results for classes of queries

Q, a data graph G and a tuple of nodes $\bar{v}$, whether $\bar{v} \in Q(G)$ (for data complexity the query Q is fixed).

First, we show that for all the three formalisms based on register automata and regular expressions for them, no cost is incurred by going from RDPQs to CRDPQs as far as data complexity is concerned.

THEOREM 7.1. *Data complexity of conjunctive regular data path queries remains* NLOGSPACE*-complete if they are specified using register automata, regular expressions with memory, or regular expressions with equality.*

PROOF. Consider a query of the form (5) and let $\bar{z}'$ be the tuple of variables from $\bar{x}$ and $\bar{y}$ that is not present in $\bar{z}$. To check whether $\bar{v} \in Q(G)$, we need to check whether there exists a valuation $\bar{v}'$ for $\bar{z}'$ so that under that valuation each of the RDPQs in the conjunction in (5) is true.

We know from the previous sections that checking whether $v \xrightarrow{L} v'$ evaluates to true for some nodes v, v' can be done with NLOGSPACE data complexity for all the formalisms mentioned in the theorem. Thus, given a data graph $G = \langle V, E, \rho \rangle$, we can enumerate all the tuples from $V^{|\bar{z}'|}$, and for each of them check the truth of all the RDPQs in conjunction (5). Since we deal with data complexity, $|\bar{z}'|$ is fixed, and thus such an enumeration can be done in logarithmic space, showing that query evaluation remains in NLOGSPACE. □

For combined complexity, we have the same bounds for CRDPQs given by register automata and expressions with memory as in the case of a single RDPQ. For regular expressions with equality we get NP-completeness, which matches the combined complexity of conjunctive queries and CRPQs.

THEOREM 7.2. *Combined complexity of conjunctive regular data path queries remains* PSPACE*-complete if they are specified using register automata or regular expressions with memory. It is* NP*-complete if they are specified using regular expressions with equality.*

PROOF. PSPACE-hardness follows from the corresponding results for RDPQs and RDPQs with memory, and NP-hardness follows from NP-hardness of relational conjunctive queries. Thus we show upper bounds. The algorithm (using notations from the proof of Theorem 7.1) is the same in all three cases: guess a tuple $\bar{v}'$ of nodes for $\bar{z}'$, and check whether all the RDPQs in conjunction (5) are true. We know that for register automata and regular expressions with memory the latter can be done in PSPACE; since PSPACE is closed under nondeterministic guesses we have the PSPACE upper bound for combined complexity. For regular expressions with equality, an NP upper bound for the algorithm follows from the PTIME bound for combined complexity for RDPQs with equality. □

8. SUMMARY AND FUTURE WORK

The tables in Figure 2 give the summary of data and combined complexity for various query languages studied in this paper. As we introduced models that expand the usual RPQs and CRPQs that handle only edge labels and can now manipulate data in the nodes, we get, as expected, a slightly higher complexity bounds for combined complexity. However, using a large class of regular expressions that can express many properties of interest, we can match the usual bound of RPQs. For CRPQs with data, the bounds are only slightly higher than those for data-free CRPQs; in some cases they coincide with bounds for CRPQs extended with comparisons of paths, and for some, there is no price to pay for incorporating data comparisons into queries.

This is an initial investigation on combining data and topology in graph query languages, and we plan to extend this work in several directions. One of them has to do with optimizing queries, in particular, with studying containment and equivalence as in [18, 25]. We are also interested in handling constraints in graph query languages [2, 26]. Another direction is to study extensions with path comparisons as in [4], combined with querying data. We also plan to study incomplete data, by extending patterns in [5] with data, potentially incomplete.

Yet another direction we intend to pursue is to define our expressions over data words, a setting usually treated in the literature, and try to study their classical language theoretic properties, such as membership testing, nonemptiness, containment, etc. To lower complexity we might even consider restricting regular expressions with memory in such a way that equality tests are more explicit, while still allowing them to be far more expressive than expressions with equality. We

would also like to specify a class of expressions that precisely capture register automata in the same manner that regular expressions capture finite state automata. We have strong indications that we will be able to do so with regular expressions with memory.

Acknowledgment Work partially supported by EPSRC grant G049165 and FET-Open Project FoX, grant agreement 233599.

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