Data Integration and Data Exchange

Traditional approach to databases

- A single large repository of data.
- Database administrator in charge of access to data.
- Users interact with the database through application programs.
- Programmers write those (embedded SQL, other ways of combining general purpose programming languages and DBMSs)
- Queries dominate; updates less common.
- DMBS takes care of lots of things for you such as

  query processing and optimisation

  concurrency control

  enforcing database integrity

Traditional approach to databases cont'd

- This model works very within a single organisation that either
  - does not interact much with the outside world, or
  - the interaction is heavily controlled by the DB administrators
- What do we expect from such a system?
  1. Data is relatively clean; little incompleteness
  2. Data is consistent (enforced by the DMBS)
  3. Data is there (resides on the disk)
  4. Well-defined semantics of query answering (if you ask a query, you know what you want to get)
  5. Access to data is controlled

The world is changing

- The traditional model still dominates, but the world is changing.
- Many huge repositories are publicly available
  - In fact many are well-organised databases, e.g., imdb.com, the CIA World Factbook, many genome databases, the DBLP server of CS publications, etc etc etc)
- Many queries cannot be answered using a single source.
- Often data from various sources needs to be combined, e.g.
  - company mergers
  - restructuring databases within a single organisation
  - combining data from several private and public sources

What industry offers now: ETL tools

- ETL stands for **E**xtract–**T**ransform–**L**oad
  - Extract data from multiple sources
  - Transform it so it is compatible with the schema
  - Load it into a database
- Many self-built tools in the 80s and the 90s; through acquisition fewer products exist now
- The big players – IBM, Microsoft, Oracle – all have their ETL products; Microsoft and Oracle offer them with their database products.
- A few independent vendors, e.g. Informatica PowerCenter.
- Several open source products exist, e.g. Clover ETL.

ETL tools

- Focus:
  - Data profiling
  - Data cleaning
  - Simple transformations
  - Bulk loading
  - Latency requirements
- What they don't do yet:
  - **nontrivial transformations**
  - **query answering**
- But techniques now exist for interesting data integration and for query answering – and we shall learn them.
- They soon will be reflected in products (IBM and Microsoft are particularly active in this area)

Data profiling/cleaning

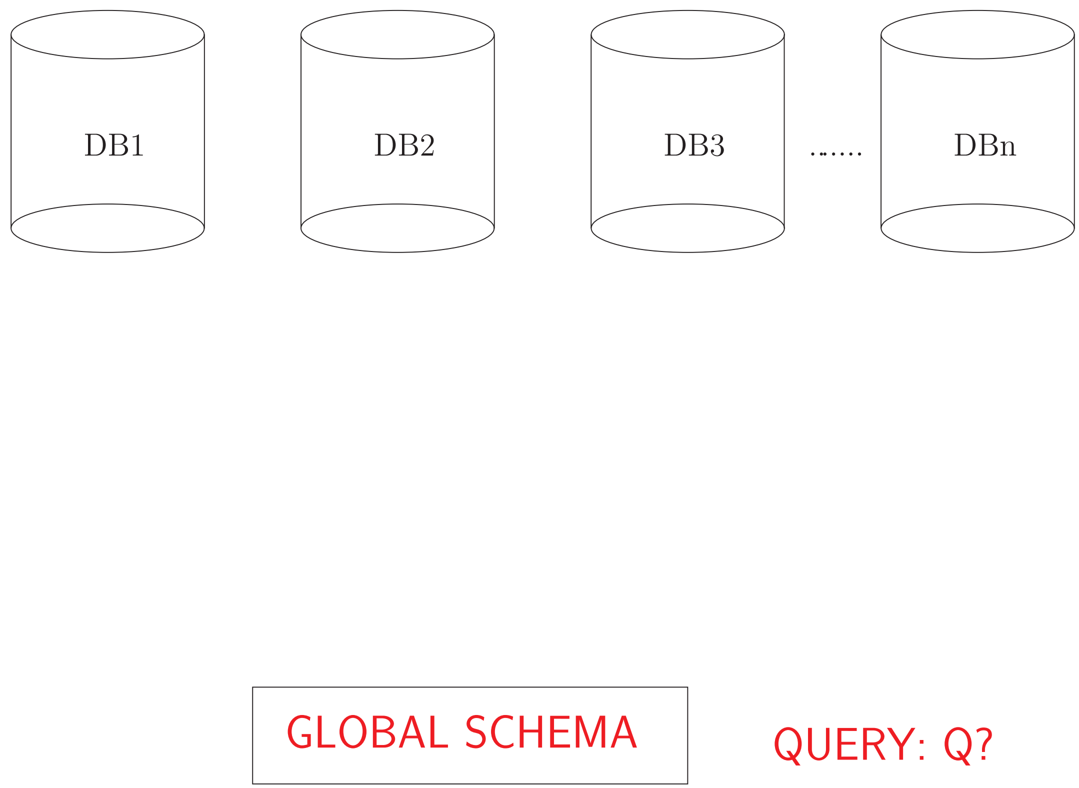
- Data profiling: gives the user a view of data:
  - Samples over large tables
  - statistics (how many different values etc)
  - Graphical tools for exploring the database
- Cleaning:
  - Same properties may have different names

    e.g. `Last_Name`, `L_Name`, `LastName`
  - Same data may have different representations
    - e.g. `(0131)555-1111` vs `01315551111`,
    - `George Str.` vs `George Street`
  - Some data may be just wrong

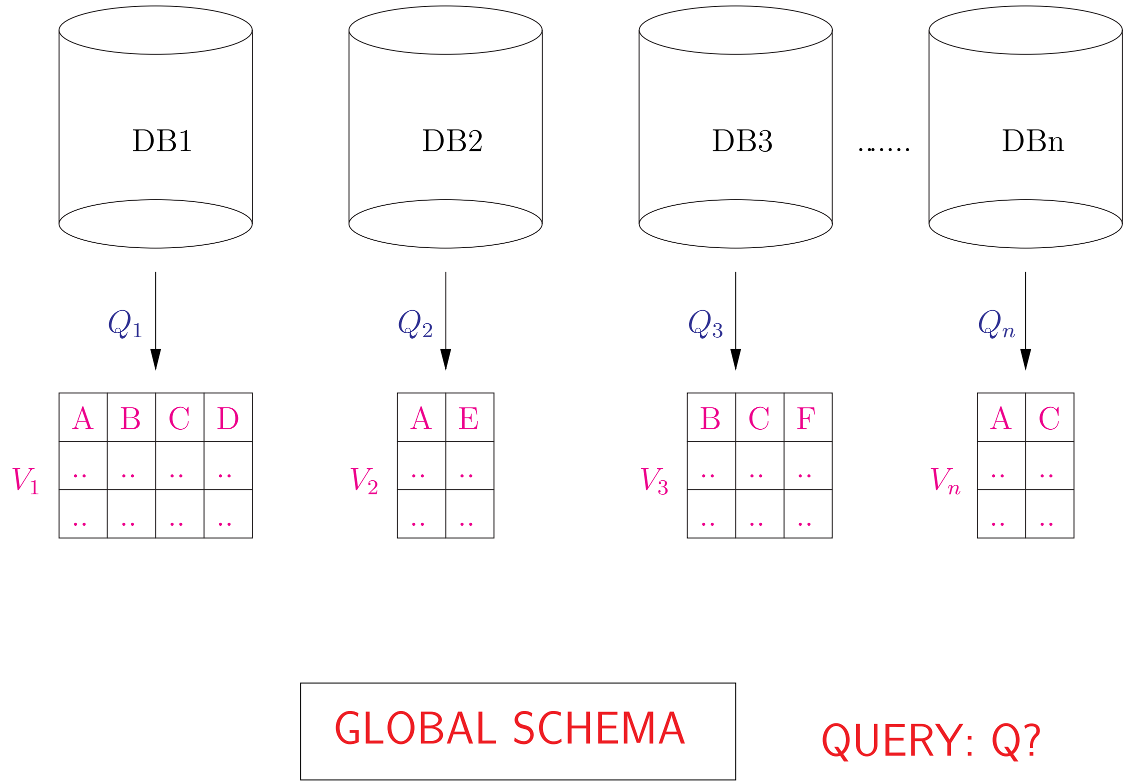
Data transformation

- Most transformation rules tend to be simple:
  - Copy attribute `LName` to `Last_Name`
  - Set age to be `current_year` – `DOB`
- Heavy emphasis on industry specific formats
- For example, Informatica B2B Data Exchange product offers versions for Healthcare and Financial services as well as specialised tools for formats including:
  - MS Word, Excel, PDF, UN/EDIFACT (Data Interchange For Administration, Commerce, and Transport), RosettaNet for B2B, and many specialised healthcare and financial form.
- These are format/industry specific and have little to do with the general tasks of data integration.

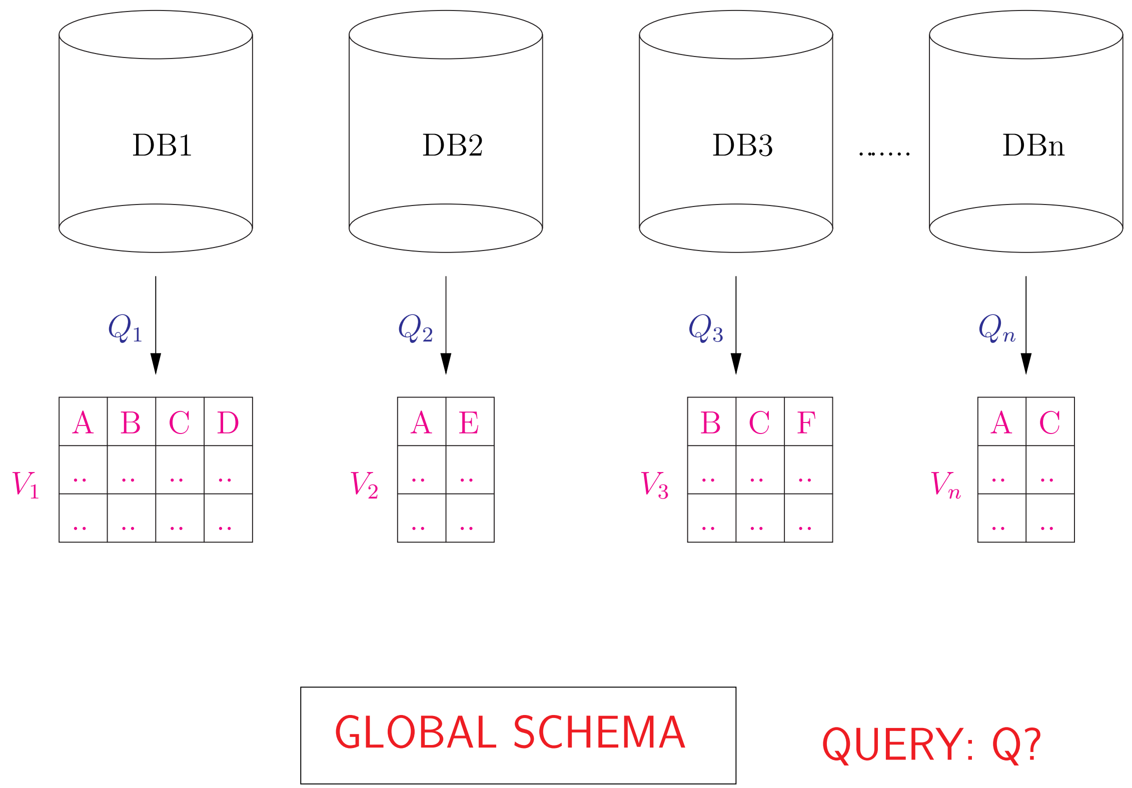
Data integration, scenario 1



Data integration



Data integration



Answer to Q is obtained by querying the views V_1 , ..., V_n

Data integration, query answering

- We have our view of the world (the Global Schema)
- We can access (parts of) databases $DB_1, \ldots, DB_n$ to get relevant data.
- It comes in the form of views, $V_1, \ldots, V_n$
- Our query against the global schema must be reformulated as a query against the views $V_1, \ldots, V_n$
- The approach is completely virtual: we never create a database the conforms to the global schema.

Data integration, query answering, a toy example

- List courses taught by permanent teaching staff during Winter 2007
- We have two databases:
  - D_1(name, age, salary) of permanent staff
  - D_2(teacher, course, semester, enrollment) of courses
- D_1 only publishes the value of the name attribute
- D_2 does not reveal enrollments
- The views:

$$\begin{aligned} V_1 &= \pi_{name}(D_1) \\ V_2 &= \pi_{teacher,course,semester}(D_2) \end{aligned}$$

- Next step: establish correspondence between attributes name of V_1 and teacher of V_2

Data integration, query answering, a toy example cont'd

- To answer query, we need to import the following data:

$$V_1$$

$$W_2 \;=\; \sigma_{semester='Winter\ 2007'}(V_2)$$

- Answering query:

$$\{course \mid \exists name, sem\ V_1(name) \wedge W_2(name, course, sem)\}$$

- Or, in relational algebra

$$\pi_{course}(V_1 \bowtie_{name=teacher} W_2)$$

Toy example, lessons learned

- We don't have access to all the data
- Some human intervention is essential (someone needs to tell us that teacher and name refer to the same entity)
- We don't run a query against a single database. Instead, we
  - run queries against different databases based on restrictions they impose
  - get results to use them locally
  - run another query against those results

Toy example, things getting more complicated

- Find informatics permanent staff who taught during the Winter 2007 semester, and their phone numbers
- We have additional personnel databases:
  - an informatics database $D_3(employee, phone, office)$, and
  - a university-wide database $D_4(employee, school, phone)$
  - for simplicity, assume all this information is public
- Now we have a choice:
  - use D_3 to get information about phones
  - use D_4 to get information about phones
  - use both D_3 and D_4 to get information about phones

Toy example cont'd

- First, we need some human involvement to see that employee, name, and teacher refer to the same category of objects
- If one uses D_3, then the query is

$$\{name,\ phone \mid \exists sem, course, office\ V_1(name) \wedge W_2(name, course, sem) \wedge D_3(name, phone, office)\}$$

- If one uses D_4, then the query is

$$\{name,\ phone \mid \exists sem, course, school\ V_1(name) \wedge W_2(name, course, sem) \wedge D_4(name, school, phone)\}$$

- But what if one uses both D_3 and D_4?

Toy example cont'd

- We could insist on the phone number being:
  - in either D_3 or D_4
  - in both D_3 and D_4, but not necessarily the same
  - in both D_3 and D_4, and the same in both databases
- One can write queries for all the cases, but which one should we use?
- New lessons:
  - databases that are being integrated are often inconsistent
  - query answering is by no means unique – there could be several ways to answer a query
  - different possibilities for answering queries are a result of inconsistencies and incomplete information

Toy example cont'd

- Suppose phone numbers in D_3 and D_4 are different.
- What is a sensible query answer then?
- A common approach is to use certain answers – these are guaranteed to be true.
- Another question: what if there is no record at all for the phone number in D_3 and D_4?
- Then we have an instance of incomplete information.

A different scenario

- So far we looked at virtual integration: no database of the global schema was created.
- Sometimes we need such a database to be created, for example, if many queries are expected to be asked against it.
- In general, this is a common problem with data integration: materialize vs federate.
- Materialize = create a new database based on integrating data from different sources.
- Federate = the virtual approach: obtain data from various sources and use them to answer queries.

Virtual vs Materialization

- A common situation for the materialization approach: merger of different organizations.
- A common situation for the federated approach: we don't have full access to the data, and the data changes often.

Common tasks in data integration

- How do we represent information?
  - Global schema, attributes, constraints
  - data formats of attributes
  - reconciling data from different sources
  - abbreviations, terminology, ontologies
- How do we deal with imperfect information?
  - resolve overlaps
  - handling missing data
  - handling inconsistencies

Common tasks in data integration cont'd

- How do we answer queries?
  - what information is available?
  - Can we get *the* answer?
  - if not, what is the semantics of query answering?
  - Is query answering feasible?
  - Is it possible to compute query answers at all?
  - If now, how do we approximate?
- Materialize or federate?

Common tasks in data integration cont'd

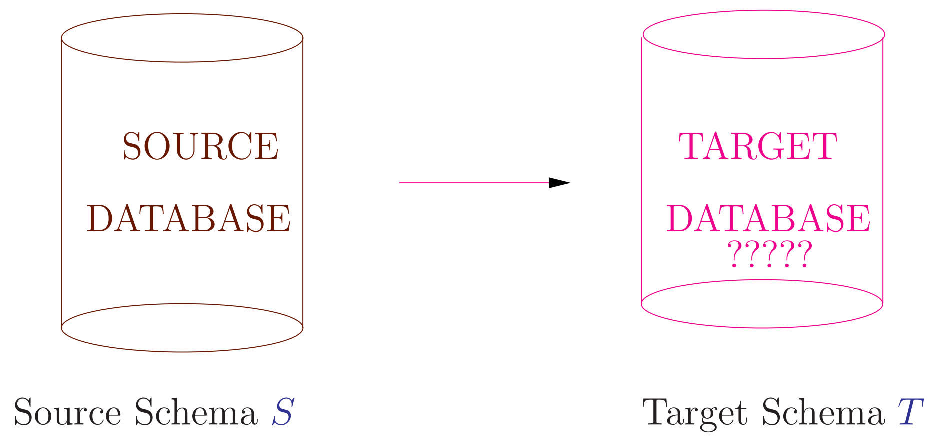
- Do it from scratch or use commercial tools?
  - many are available (just google for "data integration")
  - but do we fully understand them?
  - lots of them are very ad hoc, with poorly defined semantics
  - this is why it is so important to understand what really happens in data integration

Data Exchange

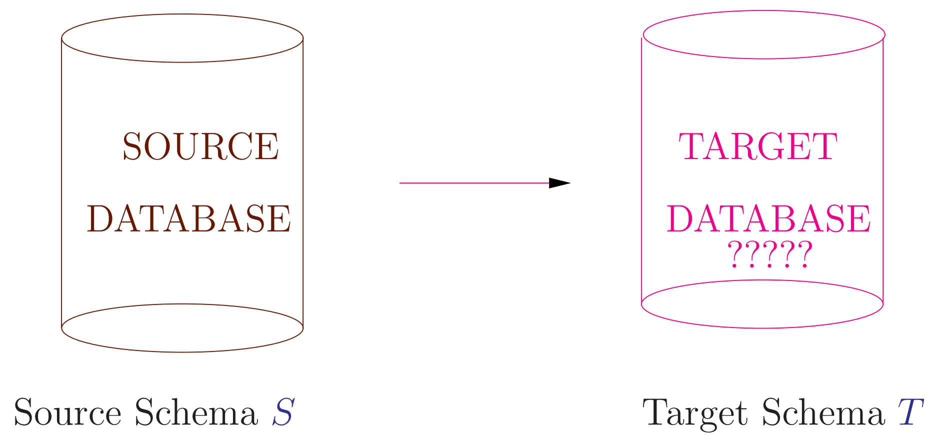
Source Schema S

Target Schema T

Data Exchange



Data Exchange



Query over the target schema: Q

How to answer Q so that the answer is consistent with the data in the source database?

Data exchange vs Data integration

Data exchange appears to be an easier problem:

- there is only one source database;
- and one has complete access to the source data.

But there could be many different target instances.

Problem: which one to use for query answering?

When do we have the need for data exchange

- A typical scenario:
  - Two organizations have their legacy databases, schemas cannot be changed.
  - Data from one organization 1 needs to be transfered to data from organization 2.
  - Queries need to be answered against the transferred data.

Query answering using views

- General setting: database relations $R_1, \ldots, R_n$.
- Several views $V_1, \ldots, V_k$ are defined as results of queries over the R_i's.
- We have a query Q over $R_1, \ldots, R_n$.
- Question: Can Q be answered in terms of the views?
  - In other words, can Q be reformulated so it only refers to the data in $V_1, \ldots, V_k$?

Query answering using views in data integration

- LAV:
  - $R_1, \ldots, R_n$ are global schema relations; Q is the global schema query
  - V_i's are the sources defined over the global schema
  - We must answer Q based on the sources (virtual integration)
- GAV:
  - $R_1, \ldots, R_n$ are the sources that are not fully available.
  - Q is a query in terms of the source (or a query that was reformulated in terms of the sources)
  - Must see if it is answerable from the available views $V_1, \ldots, V_k$.
- We know the problem is impossible to solve for full relational algebra, hence we concentrate on conjunctive queries.

Query answering using views: example

- Two relations in the database: Cites(A,B) (if A cites B), and SameTopic(A,B) (if A, B work on the same topic)
- Query $Q(x,y)$:– $\mathsf{SameTopic}(x,y), \mathsf{Cites}(x,y), \mathsf{Cites}(y,x)$
- Two views are given:
  - $V_1(x,y)$:– $\mathsf{Cites}(x,y), \mathsf{Cites}(y,x)$
  - $V_2(x,y)$:– $\mathsf{SameTopic}(x,y), \mathsf{Cites}(x,x'), \mathsf{Cites}(y,y')$
- Suggested rewriting: $Q'(x,y)$:– $V_1(x,y), V_2(x,y)$
- Why? Unfold using the definitions:

  $Q'(x,y)$:– $\mathsf{Cites}(x,y), \mathsf{Cites}(y,x), \mathsf{SameTopic}(x,y), \mathsf{Cites}(x,x'), \mathsf{Cites}(y,y')$
- Equivalent to Q

Query answering using views

- Need a formal technique (algorithm): cannot rely on the semantics.
- Query Q:

```
SELECT R1.A
FROM R R1, R R2, S S1, S S2
WHERE R1.A=R2.A AND S1.A=S2.A AND R1.A=S1.A
      AND R1.B=1 and S2.B=1
```

- $Q(x)$:– $R(x,y), R(x,1), S(x,z), S(x,1)$
- Equivalent to $Q(x)$:– $R(x,1), S(x,1)$
- So if we have a view
  - $V(x,y)$:– $R(x,y), S(x,y)$ (i.e. $V = R \cap S$), then
  - $Q = \pi_A(\sigma_{B=1}(V))$
  - Q can be rewritten (as a conjunctive query) in terms of V

Query rewriting

- Setting:
  - Queries $V_1, \ldots, V_k$ over the same schema σ (assume to be conjunctive; they define the views)
  - Each Q_i is of arity n_i
  - A schema ω with relations of arities $n_1, \ldots, n_k$
- Given:
  - a query Q over σ
  - a query Q' over ω
- Q' is a rewriting of Q if for every σ-database D,

$$Q(D) \;=\; Q'\big(\; V_1(D), \ldots, V_k(D) \;\big)$$

Maximal rewriting

- Sometimes exact rewritings cannot be obtained
- Q' is a maximally-contained rewriting if:
  - it is contained in Q:
    $$Q'\big(\ V_1(D), \ldots, V_k(D)\ \big)\ \subseteq\ Q(D)$$
    for all D
  - it is maximal such: if
    $$Q''\big(\ V_1(D), \ldots, V_k(D)\ \big)\ \subseteq\ Q(D)$$
    for all D, then
    $$Q'' \subseteq Q'$$

Query rewriting: a naive algorithm

- Given:
  - ◦ conjunctive queries $V_1, \ldots, V_k$ over schema σ
  - ◦ a query Q over σ
- Algorithm:
  - ◦ guess a query Q' over the views
  - ◦ Unfold Q' in terms of the views
  - ◦ Check if the unfolding is contained in Q
- If one unfolding is equivalent to Q, then Q' is a rewriting
- Otherwise take the union of all unfoldings contained in Q
  - – it is a maximally contained rewriting

Why is it not an algorithm yet?

- Problem: the guess stage.
  - There are infinitely many conjunctive queries.
  - We cannot check them all.
  - Solution: we only need to check a few.

Guessing rewritings

- A basic fact:
  - If there is a rewriting of Q using $V_1, \ldots, V_k$, then there is a rewriting with no more conjuncts than in Q.
  - E.g., if $Q(x)\ \text{:-}\ R(x,y), R(x,1), S(x,z), S(x,1)$, we only need to check conjunctive queries over V with at most 4 conjuncts.
- Moreover, maximally contained rewriting is obtained as the union of all conjunctive rewritings of length of Q or less.
- Complexity: enumerate all candidates (exponentially many); for each an NP (or exponential) algorithm. Hence exponential time is required.
- Cannot lower this due to NP-completeness.

Query rewriting

- Recall the algorithm, for a given Q and view definitions $V_1, \ldots, V_k$:
  - Look at all rewritings that have as at most as many joins as Q
  - check if they are contained in Q
  - take the union of those that are
- This is the maximally contained rewriting
- There are algorithms that prune the search space and make looking for rewritings contained in Q more efficient
  - the bucket algorithm
  - MiniCon

How hard is it to answer queries using views?

- Setting: we now have an actual content of the views.
- As before, a query is Q posed against D, but must be answered using information in the views.
- Suppose $I_1, \ldots, I_k$ are view instances. Two possibilities:
  - Exact mappings: $I_j = V_j(D)$
  - Sound mappings: $I_j \subseteq V_j(D)$
- We need certain answers for given $\mathcal{I} = (I_1, \ldots, I_k)$:

$$\text{certain}_{exact}(Q, \mathcal{I}) \;=\; \bigcap_{D:\ I_j = V_j(D) \text{ for all } j} Q(D)$$

$$\text{certain}_{sound}(Q, \mathcal{I}) \;=\; \bigcap_{D:\ I_j \subseteq V_j(D) \text{ for all } j} Q(D)$$

How hard is it to answer queries using views?

- If $\mathsf{certain}_{exact}(Q, \mathcal{I})$ or $\mathsf{certain}_{sound}(Q, \mathcal{I})$ are impossible to obtain, we want maximally contained rewritings:
  - $Q'(\mathcal{I}) \subseteq \mathsf{certain}_{exact}(Q, \mathcal{I})$, and
  - if $Q''(\mathcal{I}) \subseteq \mathsf{certain}_{exact}(Q, \mathcal{I})$ then $Q''(\mathcal{I}) \subseteq Q'(\mathcal{I})$
  - (and likewise for $sound$)
- How hard is it to compute this from $\mathcal{I}$?

Complexity of query answering

- We want the complexity of finding

$$\text{certain}_{exact}(Q, \mathcal{I}) \quad \text{or} \quad \text{certain}_{sound}(Q, \mathcal{I})$$

  in terms of the size of $\mathcal{I}$

- If all view definitions are conjunctive queries and Q is a relational algebra or a SQL query, then the complexity is coNP.

- This is too high!

- If all view definitions are conjunctive queries and Q is a conjunctive query, then the complexity is PTIME.

  - Because: the maximally contained rewriting computes certain answers!

Complexity of query answering

	query language		
view language	CQ	$\mathrm{CQ}^{\neq}$	relational calculus
CQ	ptime	coNP	undecidable
$\mathrm{CQ}^{\neq}$	ptime	coNP	undecidable
relational calculus	undecidable	undecidable	undecidable

CQ – conjunctive queries

$\mathrm{CQ}^{\neq}$ – conjunctive queries with inequalities
(for example, $Q(x) \coloneq R(x, y), S(y, z), x \neq z$)

Data exchange

- Source schema, target schema; need to transfer data between them.
- A typical scenario:
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Source Schema S

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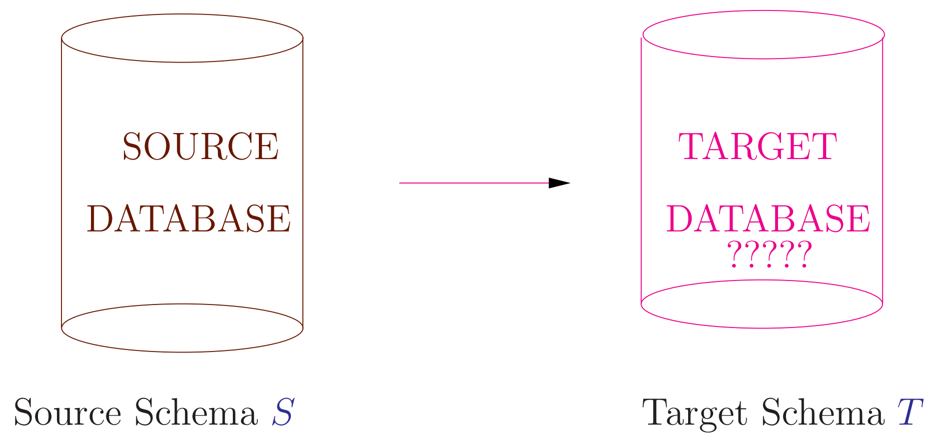


Data exchange: an example

- We want to create a target database with the schema

  Flight(city1,city2,aircraft,departure,arrival)
  Served(city,country,population,agency)

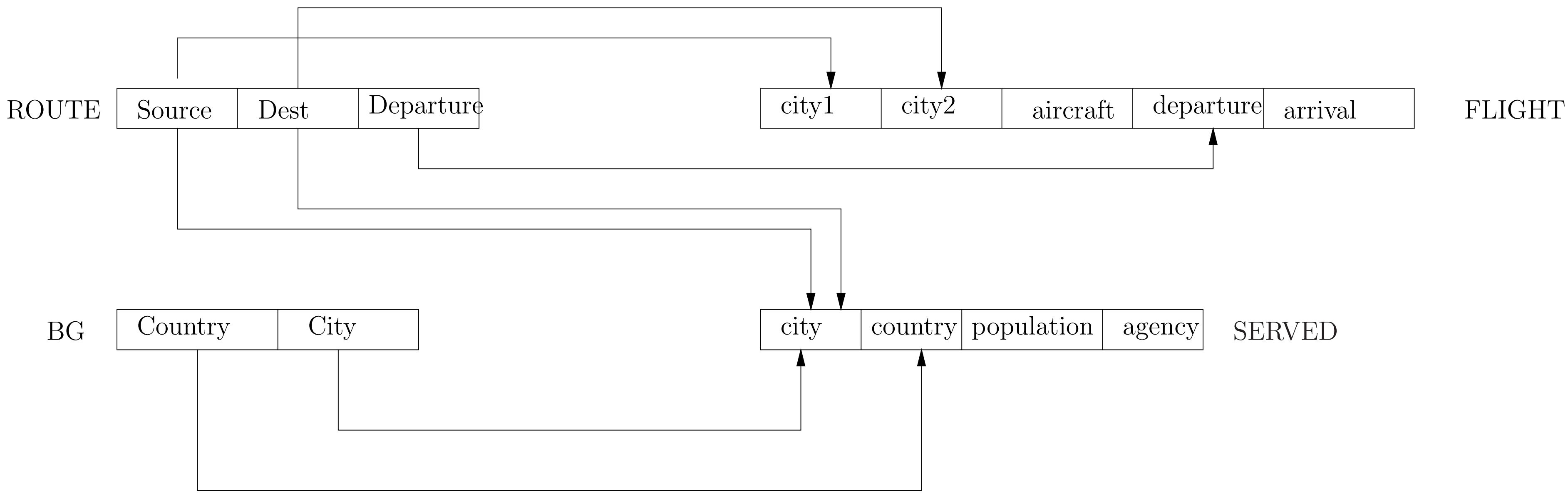
- We don't start from scratch: there is a source database containing relations

  Route(source,destination,departure)
  BG(country,city)

- We want to transfer data from the source to the target.

Data exchange – relationships between the source and the target

How to specify the relationship?



Relationships between the source and the target

- Formal specification: we have a *relational calculus query* over both the source and the target schema.
- The query is of a restricted form, and can be thought of as a sequence of rules:

$$Flight(c1, c2, \_\_, dept, \_\_) \text{ :– } Route(c1, c2, dept)$$

$$Served(city, country, \_\_, \_\_) \text{ :– } Route(city, \_\_, \_\_), BG(country, city)$$

$$Served(city, country, \_\_, \_\_) \text{ :– } Route(\_\_, city, \_\_), BG(country, city)$$

Data exchange – targets

- Target instances should satisfy the rules.
- What does it mean to satisfy a rule?
- Formally, if we take:

  Flight(c1, c2, __, dept, __) :– *Route(c1, c2, dept)*

  then it is satisfied by a source S and a target T if the constraint

$$\forall c_1, c_2, d \Big(\textit{Route}(c_1, c_2, d) \to \exists a_1, a_2 \ \big(\textit{Flight}(c_1, c_2, a_1, d, a_2) \big) \Big)$$

- This constraint is a relational calculus query that evaluates to *true* or *false*

Data exchange – targets

- What happens if there no values for some attributes, e.g. *aircraft*, *arrival*?
- We put in null values or some real values.
- But then we may have multiple solutions!

Data exchange – targets

Source Database:

ROUTE:

Source	Destination	Departure
Edinburgh	Amsterdam	0600
Edinburgh	London	0615
Edinburgh	Frankfurt	0700

BG:

Country	City
UK	London
UK	Edinburgh
NL	Amsterdam
GER	Frankfurt

Look at the rule

Flight(c1, c2, __, dept, __) :– *Route(c1, c2, dept)*

The right hand side is satisfied by

Route(Edinburgh, Amsterdam, 0600)

But what can we put in the target?

Data exchange – targets

Rule: *Flight(c1, c2, _, dept, _)* :– *Route(c1, c2, dept)*

Satisfied by: Route(Edinburgh, Amsterdam, 0600)

Possible targets:

- Flight(Edinburgh, Amsterdam, $\perp_1$, 0600, $\perp_2$)
- Flight(Edinburgh, Amsterdam, B737, 0600, $\perp$)
- Flight(Edinburgh, Amsterdam, $\perp$, 0600, 0845)
- Flight(Edinburgh, Amsterdam, $\perp$, 0600, $\perp$)
- Flight(Edinburgh, Amsterdam, B737, 0600, 0845)

They all satisfy the constraints!

Which target to choose

- One of them happens to be right:
  - Flight(Edinburgh, Amsterdam, B737, 0600, 0845)
- But in general we do not know this; it looks just as good as
  - Flight(Edinburgh, Amsterdam, 'The Spirit of St Louis', 0600, 1300),

  or
  - Flight(Edinburgh, Amsterdam, F16, 0600, 0620).
- Goal: look for the "most general" solution.
- How to define "most general": can be mapped into any other solution.
- It is not unique either, but the space of solution is greatly reduced.
- In our case Flight(Edinburgh, Amsterdam, $\bot_1$, 0600, $\bot_2$) is most general as it makes no additional assumptions about the nulls.

Towards good solutions

A solution is a database with nulls.
Reminder: every such database T represents many possible complete databases, without null values:

Example

A	B	C
1	2	$\perp_1$
$\perp_2$	$\perp_1$	3
$\perp_3$	5	1
2	$\perp_3$	3

$$v(\perp_1) = 4$$
$$v(\perp_2) = 3$$
$$v(\perp_3) = 5$$
$$\Longrightarrow$$

Semantics via valuations

A	B	C
1	2	4
3	4	3
5	5	1
2	5	3
3	7	8
4	2	1

$$[\![T]\!]_{\text{owa}} = \{R \mid v(T) \subseteq R \text{ for some valuation } v\}$$

Good solutions

- An optimistic view – A good solution represents ALL other solutions:

$$[\![T]\!]_{\text{owa}} = \{R \mid R \text{ is a solution without nulls}\}$$

- Shouldn't settle for less than – A good solution is at least as general as others:

$$[\![T]\!]_{\text{owa}} \supseteq [\![T']\!]_{\text{owa}} \quad \text{for every other solution } T'$$

- Good news: these two views are equivalent. Hence can take them as a definition of a good solutions.
- In data exchange, such solutions are called universal solutions.

Universal solutions: another description

- A homomorphism is a mapping h : Nulls $\longrightarrow$ Nulls $\cup$ Constants.
- For example, $h(\perp_1) = B737$, $h(\perp_2) = 0845$.
- If we have two solutions T_1 and T_2, then h is a homomorphism from T_1 into T_2 if for each tuple t in T_1, the tuple $h(t)$ is in T_2.
- For example, if we have a tuple

  $$t = \text{Flight(Edinburgh, Amsterdam}, \perp_1, \text{0600}, \perp_2)$$

  then

  $$h(t) = \text{Flight(Edinburgh, Amsterdam, B737, 0600, 0845).}$$

- A solution is universal if and only if there is a homomorphism from it into every other solution.

Universal solutions: still too many of them

- Take any $n > 0$ and consider the solution with n tuples:

$$\begin{array}{lllll}
\text{Flight(Edinburgh,} & \text{Amsterdam,} & \bot_1, & 0600, & \bot_2) \\
\text{Flight(Edinburgh,} & \text{Amsterdam,} & \bot_3, & 0600, & \bot_4) \\
\ldots & & & & \\
\text{Flight(Edinburgh,} & \text{Amsterdam,} & \bot_{2n-1}, & 0600, & \bot_{2n})
\end{array}$$

- It is universal too: take a homomorphism

$$h'(\bot_i) = \begin{cases} \bot_1 & \text{if } i \text{ is odd} \\ \bot_2 & \text{if } i \text{ is even} \end{cases}$$

- It sends this solution into

$$\text{Flight(Edinburgh, Amsterdam, } \bot_1, \text{ 0600, } \bot_2)$$

Universal solutions: cannot be distinguished by conjunctive queries

- There are queries that distinguish large and small universal solutions (e.g., does a relation have at least 2 tuples?)
- But these cannot be distinguished by conjunctive queries
- Because: if $\bot_{i_1}, \ldots, \bot_{i_k}$ witness a conjunctive query, so do $h(\bot_{i_1}), \ldots, h(\bot_{i_k})$ — hence, one tuple suffices
- In general, if we have
  - a homomorphism $h : T \to T'$,
  - a conjunctive query Q
  - a tuple t without nulls such that $t \in Q(T)$
- then $t \in Q(T')$

Universal solutions and conjunctive queries

- If
  - T and T' are two universal solutions
  - Q is a conjunctive query, and
  - t is a tuple without nulls,

  then
  $$t \in Q(T) \quad \Leftrightarrow \quad t \in Q(T')$$
  because we have homomorphisms $T \to T'$ and $T' \to T$.

- Furthermore, if
  - T is a universal solution, and T'' is an arbitrary solution,

  then
  $$t \in Q(T) \quad \Rightarrow \quad t \in Q(T'')$$

Universal solutions and conjunctive queries cont'd

- Now recall what we learned about answering conjunctive queries over databases with nulls:
  - T is a naive table
  - the set of tuples without nulls in $Q(T)$ is precisely certain(Q,T) – certain answers over T
- Hence if T is an arbitrary universal solution

$$\text{certain}(Q,T) \quad = \quad \bigcap\{Q(T') \mid T' \text{ is a solution}\}$$

- $\bigcap\{Q(T') \mid T' \text{ is a solution}\}$ is the set of certain answers in data exchange under mapping M: certain$_M(Q,S)$. Thus

$$\text{certain}_M(Q,S) \quad = \quad \text{certain}(Q,T)$$

  for every universal solution T for S under M.

Universal solutions cont'd

- To answer conjunctive queries, one needs an arbitrary universal solution.
- We saw some; intuitively, it is better to have:

$$\text{Flight(Edinburgh, Amsterdam, } \perp_1, \text{ 0600, } \perp_2)$$

than

$$\begin{array}{llll}\text{Flight(Edinburgh, Amsterdam,} & \perp_1, & \text{0600,} & \perp_2) \\ \text{Flight(Edinburgh, Amsterdam,} & \perp_3, & \text{0600,} & \perp_4) \\ \ldots & & & \\ \text{Flight(Edinburgh, Amsterdam,} & \perp_{2n-1}, & \text{0600,} & \perp_{2n}) \end{array}$$

- We now define a canonical universal solution.

Canonical universal solution

- Convert each rule into a rule of the form:

$$\psi(x_1, \ldots, x_n,\ z_1, \ldots, z_k) \quad \text{:–} \quad \varphi(x_1, \ldots, x_n,\ y_1, \ldots, y_m)$$

  (for example,
  Flight(c1, c2, __, dept, __) :– *Route(c1, c2, dept)*

  becomes

  Flight(x_1, x_2, z_1, x_3, z_2) :– *Route*(x_1, x_2, x_3))

- Evaluate $\varphi(x_1, \ldots, x_n,\ y_1, \ldots, y_m)$ in S.
- For each tuple $(a_1, \ldots, a_n,\ b_1, \ldots, b_m)$ that belongs to the result (i.e.

$$\varphi(a_1, \ldots, a_n,\ b_1, \ldots, b_m) \text{ holds in } S,$$

  do the following:

Canonical universal solution cont'd

- ... do the following:
  - Create new (not previously used) null values $\perp_1, \ldots, \perp_k$
  - Put tuples in target relations so that

    $$\psi(a_1, \ldots, a_n,\ \perp_1, \ldots, \perp_k)$$

    holds.
- What is ψ?
- It is normally assumed that ψ is a conjunction of atomic formulae, i.e.

  $$R_1(\bar{x}_1, \bar{z}_1) \wedge \ldots \wedge R_l(\bar{x}_l, \bar{z}_l)$$

- Tuples are put in the target to satisfy these formulae

Canonical universal solution cont'd

- Example: no-direct-route airline:

$$\text{Newroute}(x_1, z) \wedge \text{Newroute}(z, x_2) \;\; \text{:–} \;\; \text{Oldroute}(x_1, x_2)$$

- If $(a_1, a_2) \in \text{Oldroute}(a_1, a_2)$, then create a new null $\perp$ and put:

  $\text{Newroute}(a_1, \perp)$
  $\text{Newroute}(\perp, a_2)$

  into the target.

- Complexity of finding this solution: polynomial in the size of the source S:

$$O(\sum_{\text{rules } \psi \text{ :- } \varphi} \text{Evaluation of } \varphi \text{ on } S)$$

Canonical universal solution and conjunctive queries

- Canonical solution: $\textsc{CanSol}_M(S)$.
- We know that if Q is a conjunctive query, then $\text{certain}_M(Q, S) = \text{certain}(Q, T)$ for every universal solution T for S under M.
- Hence
$$\text{certain}_M(Q, S) \quad = \quad \text{certain}(Q, \textsc{CanSol}_M(S))$$
- Algorithm for answering Q:
  - Construct $\textsc{CanSol}_M(S)$
  - Apply naive evaluation to Q over $\textsc{CanSol}_M(S)$

Beyond conjunctive queries

- Everything still works the same way for $\sigma, \pi, \bowtie, \cup$ queries of relational algebra. Adding union is harmless.
- Adding difference (i.e. going to the full relational algebra) is not.
- Reason: same as before, can encode validity problem in logic.
- Single rule, saying "copy the source into the target"

$$T(x,y) \quad \text{:--} \quad S(x,y)$$

- If the source is empty, what can a target be? Anything!
- The meaning of $T(x,y) \;\text{:--}\; S(x,y)$ is

$$\forall x \forall y \; \big(S(x,y) \rightarrow T(x,y)\big)$$

Beyond conjunctive queries cont'd

- Look at $\varphi = \forall x \forall y \, \big(S(x,y) \rightarrow T(x,y)\big)$
- $S(x,y)$ is always false (S is empty), hence $S(x,y) \rightarrow T(x,y)$ is true ($p \rightarrow q$ is $\neg p \vee q$)
- Hence φ is true.
- Even if T is empty, φ is true: universal quantification over the empty set evaluates to true:
  - Remember SQL's `ALL`:

    ```
    SELECT * FROM R
    WHERE R.A > ALL (SELECT S.B FROM S)
    ```

  - The condition is true if `SELECT S.B FROM S` is empty.

Beyond conjunctive queries cont'd

- Thus if S is empty and we have a rule $T(x, y) \quad {:\!-} \quad S(x, y)$, then all T's are solutions.
- Let Q be a Boolean (yes/no) query. Then

$$\text{certain}_M(Q, S) \;=\; \text{true} \quad \Leftrightarrow \quad Q \text{ is valid}$$

- Valid = always true.
- Validity problem in logic: given a logical statement, is it:
  - valid, or
  - valid over finite databases
- Both are undecidable.

Beyond conjunctive queries cont'd

- If we want to answer queries by rewritings, i.e. find a query Q' so that

$$\text{certain}_M(Q, S) \quad = \quad Q'(\text{CanSol}_M(S))$$

  then there is no algorithm that can construct Q' from Q!

- Hence a different approach is needed.

Key problem

- Our main problem:

  Solutions are open to adding new facts

- How to close them?
- By applying the CWA (Closed World Assumption) instead of the OWA (Open World Assumption)

More flexible query answering: dealing with incomplete information

- Key issue in dealing with incomplete information:
  - Closed vs Open World Assumption (CWA vs OWA)
- CWA: database is closed to adding new facts except those consistent with one of the incomplete tuples in it.
- OWA opens databases to such facts.
- In data exchange:
  - we move data from source to target;
  - query answering should be based on that data and not on tuples that might be added later.
- Hence in data exchange CWA seems more reasonable.

Solutions under CWA – informally

- Each null introduced in the target must be justified:
  - there must be a constraint $\ldots T(\ldots, z, \ldots) \ldots$:– $\varphi(\ldots)$ with φ satisfied in the source.
- The same justification shouldn't generate multiple nulls:
  - for $T(\ldots, z, \ldots)$:– $\varphi(\bar{a})$ only one new null $\perp$ is generated in the target.
- No unjustified facts about targets should be invented:
  - assume we have $T(x, z)$:– $\varphi(x)$, $T(z', x)$:– $\psi(x)$ and $\varphi(a)$, $\psi(b)$ are true in the source.
  - Then we put $T(a, \perp)$ and $T(\perp', b)$ in the target but not $T(a, \perp), T(\perp, b)$ which would invent a new "fact": a and b are connected by a path of length 2.

Solutions under the CWA: summary

- There are homomorphisms

$$h : \textsc{CanSol}(S) \to T \qquad h' : T \to \textsc{CanSol}(S)$$

  - so that $T = h(\textsc{CanSol}(S))$
- T contains the core of $\textsc{CanSol}(S)$
- Core: the smallest $C \subseteq \textsc{CanSol}(S)$ such that there is a homomorphism from $\textsc{CanSol}(S)$ to C.
- Often saves space, but takes time to compute.
- Data exchange systems try to move from $\textsc{CanSol}(S)$ to the core but usually stop half-way due to the complexity of computation.

Query answering under the CWA

- Given
  - a source S,
  - a set of rules M,
  - a target query Q,

  a tuple t is in

$$\mathsf{certain}_M^{\mathrm{CWA}}(Q, S)$$

  if it is in $Q(R)$ for every
  - solution T under the CWA, and
  - $R \in [\![T]\!]_{\mathsf{owa}}$
- (i.e. no matter which solution we choose and how we interpret the nulls)

Query answering under the CWA – characterization

- Given a source S, a set of rules M, and a target query Q:

$$\text{certain}_M^{\text{CWA}}(Q, S) \quad = \quad \text{certain}(Q, \textsc{CanSol}(S))$$

- That is, to compute the answer to query one needs to:
  - Compute the canonical solution $\textsc{CanSol}(S)$ – which has nulls in it
  - Find certain answers to Q over $\textsc{CanSol}(S)$
- If Q is a conjunctive query, this is exactly what we had before
- Under the CWA, the same evaluation strategy applies to all queries!

Data exchange and integrity constraints

- Integrity constraints are often specified over target schemas
- In SQL's data definition language one uses keys and foreign keys most often, but other constraints can be specified too.
- Adding integrity constraints in data exchange is often problematic, as some natural solutions – e.g., the canonical solution – may fail them.

Target constraints cause problems

- The simplest example:
  - Copy source to target
  - Impose a constraint on target not satisfied in the source
- Data exchange setting:
  - $T(x, y) \ :- \ S(x, y)$ and
  - Constraint: the first attribute is a key
- Instance S:

1	2
1	3

- Every target T must include these tuples and hence violates the key.

Target constraints: more problems

- A common problem: an attempt to repair violations of constraints leads to an sequence of adding tuples.
- Example:
  - Source DeptEmpl(dept_id,manager_name,empl_id)
  - Target
    - Dept(dept_id,manager_id,manager_name),
    - Empl(empl_id,dept_id)
  - Rule Dept(d, z, n), Empl(e, d) :– DeptEmpl(d, n, e)
  - Target constraints:
    - Dept[manager_id] $\subseteq$ Empl[empl_id]
    - Empl[dept_id] $\subseteq$ Dept[dept_id]

Target constraints: more problems cont'd

- Start with (CS, John, 001) in DeptEmpl.
- Put Dept(CS, $\perp_1$, John) and Empl(001, CS) in the target
- Use the first constraint and add a tuple Empl($\perp_1$, $\perp_2$) in the target
- Use the second constraint and put Dept($\perp_2$, $\perp_3$, $\perp_3$') into the target
- Use the first constraint and add a tuple Empl($\perp_3$, $\perp_4$) in the target
- Use the second constraint and put Dept($\perp_4$, $\perp_5$, $\perp_5$') into the target
- this never stops....

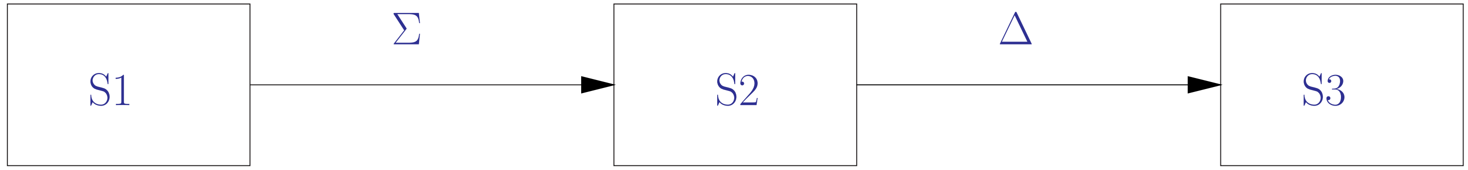
Target constraints: avoiding this problem

- Change the target constraints slightly:
  - Target constraints:
    - Dept[dept_id,manager_id] $\subseteq$ Empl[empl_id, dept_id]
    - Empl[dept_id] $\subseteq$ Dept[dept_id]
- Again start with (CS, John, 001) in DeptEmpl.
- Put Dept(CS, $\perp_1$, John) and Empl(001, CS) in the target
- Use the first constraint and add a tuple Empl($\perp_1$, CS)
- Now constraints are satisfied – we have a target instance!
- What's the difference? In our first example constraints are very cyclic causing an infinite loop. There is less cyclicity in the second example.
- Bottom line: avoid cyclic constraints.

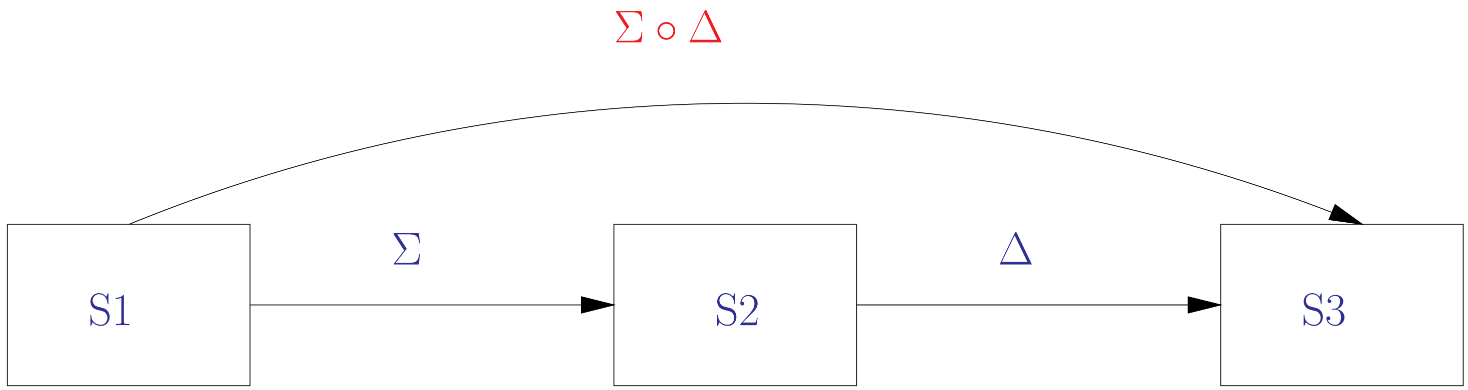
Schema mappings

- Rules used in data exchange specify mappings between schemas.
- To understand the evolution of data one needs to study operations on schema mappings.
- Most commonly we need to deal with two operations:
  - composition
  - inverse

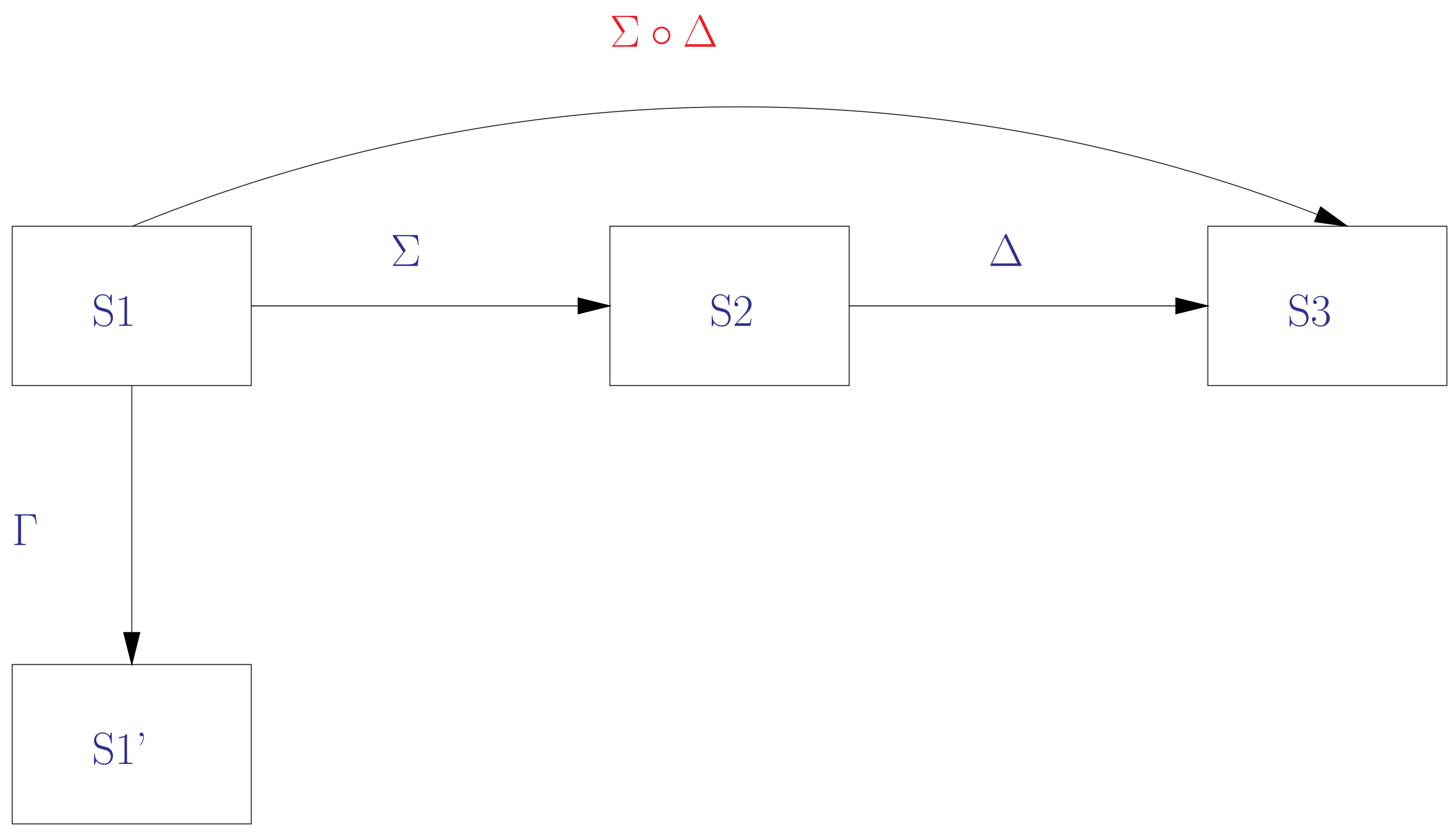
Composition and inverse



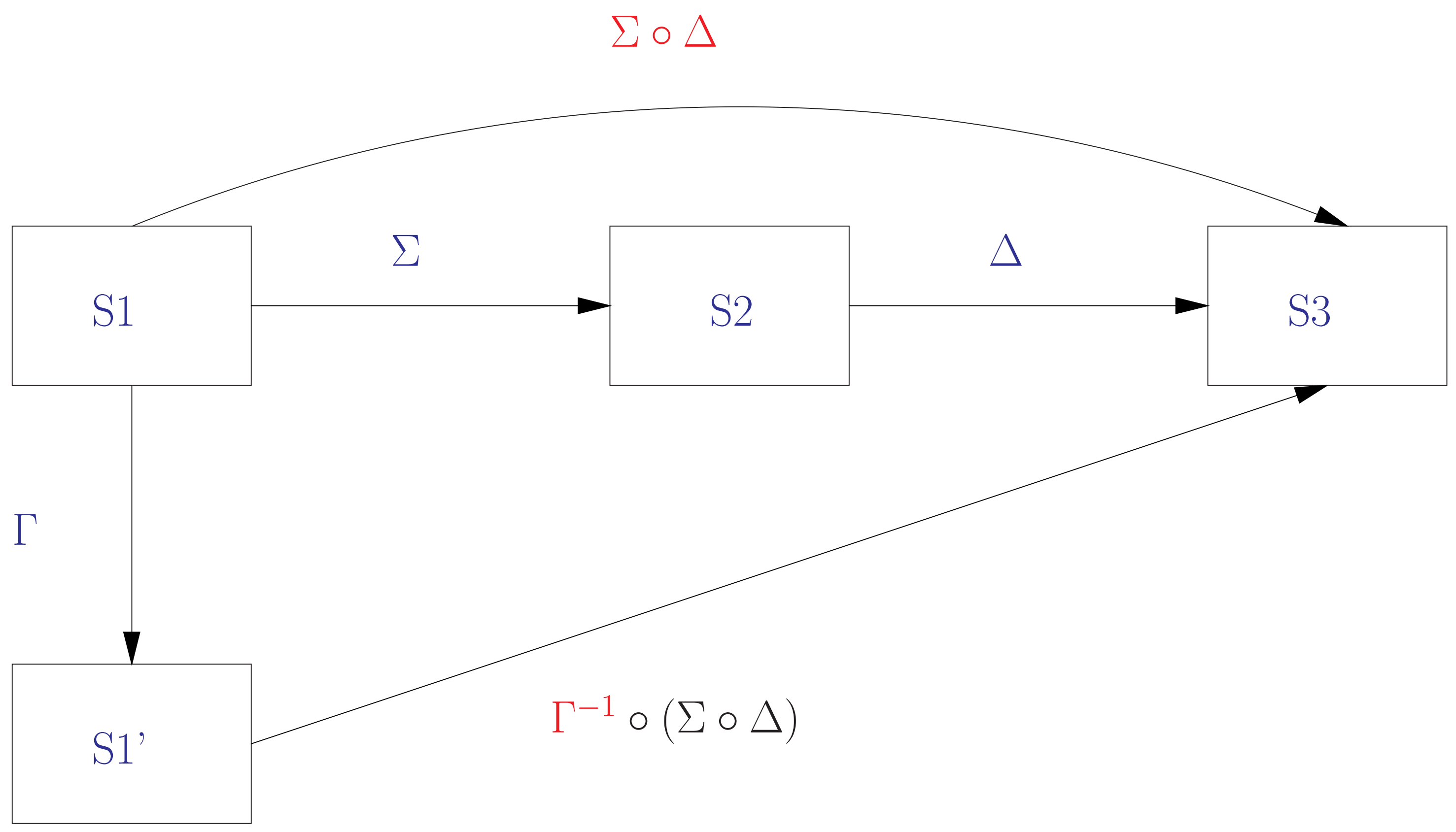
Composition and inverse



Composition and inverse



Composition and inverse



Mappings

- Schema mappings are typically given by rules

$$\psi(\bar{x},\bar{z}) \quad :\!- \quad \exists \bar{u}\ \varphi(\bar{x},\bar{y},\bar{u})$$

where

  - ψ is a conjunction of atoms over the target:

$$T_1(\bar{x}_1,\bar{z}_1) \wedge \ldots \wedge T_m(\bar{x}_m,\bar{z}_m)$$

  - φ is a conjunction of atoms over the source:

$$S_1(\bar{x}'_1,\bar{y}_1,\bar{u}_1) \wedge \ldots \wedge S_k(\bar{x}'_k,\bar{y}_k,\bar{u}_k)$$

- Example: *Served*(x_1,x_2,z_1,z_2) :– $\exists u_1,u_2$ *Route*$(x_1,u_1,u_2)\wedge$*BG*(x_1,x_2)

The closure problem

- Are mappings closed under
  - composition?
  - inverse?
- If not, what needs to be added?
- It turns out that mappings are not closed under inverses and composition.
- We next see what might need to be added to them.

Skolem functions

- Source: EP(empl_name,dept,project);
  Target: EDPH(empl_id,dept,phone), DP(dept,project)
- A natural mapping is:

$$\text{EDPH}(z_1, x_2, z_3) \wedge \text{DP}(x_2, x_3) \ \text{:--}\ \text{EP}(x_1, x_2, x_3)$$

- This is problematic: if we have tuples

$$(\text{John, CS}, P_1) \qquad (\text{John, CS}, P_2)$$

  in EP, the canonical solution would have

EDPH

$\perp_1$	CS	$\perp_1'$
$\perp_2$	CS	$\perp_2'$

  corresponding to two projects P_1 and P_2.
- So empl_id is hardly an id!

Skolem functions cont'd

- Solution: make empl_id a function of empl_name.
- Such "invented" functions are called Skolem functions (see Logic 001 for a proper definition)
- Source: EP(empl_name,dept,project);
  Target: EDPH(empl_id,dept,phone), DP(dept,project)
- A new mapping is:

$$\mathsf{EDPH}(f(x_1), x_2, z_3) \wedge \mathsf{DP}(x_2, x_3) \ \ \text{:--}\ \ \mathsf{EP}(x_1, x_2, x_3)$$

- f assigns a unique id to every name.

Other possible additions

- One can look at more general queries used in mappings.
- Most generally, relational algebra queries, but to be more modest, one can start with just adding inequalities.
- One may also disjunctions: for example, if we want to invert

$$\begin{array}{lcl} T(x) & \text{:-} & S_1(x) \\ T(x) & \text{:-} & S_2(x) \end{array}$$

  it seems natural to introduce a rule

$$S_1(x) \vee S_2(x) \ \text{:-}\ T(x)$$

Composition: definition

- Recall the definition of composition of binary relations R and R':

$$(x, z) \in R \circ R' \quad \Leftrightarrow \quad \exists y: \ (x, y) \in R \text{ and } (y, z) \in R'$$

- A schema mapping Σ for two schemas σ and τ is viewed as a binary relation

$$\mathbf{\Sigma} \quad = \quad \left\{ (S, T) \;\middle|\; \begin{array}{l} S \text{ is a } \sigma\text{-instance} \\ T \text{ is a } \tau\text{-instance} \\ T \text{ is a solution for } S \end{array} \right\}$$

- The composition of mappings Σ from σ to τ and Δ from τ to ω is now

$$\mathbf{\Sigma} \circ \mathbf{\Delta}$$

- Question (closure): is there a mapping Γ between σ and ω so that

$$\mathbf{\Gamma} = \mathbf{\Sigma} \circ \mathbf{\Delta}$$

Composition: when it works

- If Σ
  - does not generate any nulls, and
  - no variables $\bar{u}$ for source formulas
- Example:

$$\begin{array}{lrcl} \Sigma: & T(x_1,x_2) \wedge T(x_2,x_3) & :- & S(x_1,x_2,x_3) \\ \Delta: & W(x_1,x_2,z) & :- & T(x_1,x_2) \end{array}$$

- First modify into:

$$\begin{array}{lrcl} \Sigma: & T(x_1,x_2) & :- & S(x_1,x_2,x_3) \\ \Sigma: & T(x_2,x_3) & :- & S(x_1,x_2,x_3) \\ \Delta: & W(x_1,x_2,z) & :- & T(x_1,x_2) \end{array}$$

- Then substitute in the definition of W:

Composition: when it cont'd

$$
\begin{array}{rcl}
W(x_1, x_2, z) & \text{:-} & S(x_1, x_2, y) \\
W(x_1, x_2, z) & \text{:-} & S(y, x_1, x_2)
\end{array}
$$

to get $\mathbf{\Sigma} \circ \mathbf{\Delta}$.

Explaining the second rule:

$$
\begin{array}{ll}
& W(x_1, x_2, z) \\
\longrightarrow & T(x_1, x_2) \qquad \text{using } T(var_1, var_2) \ \text{:-}\ S(var_3, var_1, var_2) \\
\longrightarrow & S(y, x_1, x_2)
\end{array}
$$

Composition: when it doesn't work

- Schema σ: Takes(st_name, course)
- Schema τ: Takes'(st_name, course), NameId(st_name, st_id)
- Schema ω: Enroll(st_id, course)
- Mapping Σ from σ to τ:

$$\begin{array}{rcl} \text{Takes}'(s,c) & \text{:–} & \text{Takes}(s,c) \\ \text{NameId}(s,i) & \text{:–} & \exists c\ \text{Takes}(s,c) \end{array}$$

- Mapping Δ from τ to ω:

$$\text{Enroll}(i,c) \ \text{:–}\ \text{NameId}(s,i) \wedge \text{Takes}'(s,c)$$

- A first attempt at the composition: $\text{Enroll}(i,c) \ \text{:–}\ \text{Takes}(s,c)$

Composition: when it doesn't work cont'd

- What's wrong with Γ: Enroll(i, c) :– Takes(s, c)?
- Student id i depends on both name and course!

Takes:

John	CS1
John	CS2

$\overset{\Sigma}{\Rightarrow}$

Takes':

John	CS1
John	CS2

NameId:

John	$\perp$

$\overset{\Delta}{\Rightarrow}$ Enroll:

$\perp$	CS1
$\perp$	CS2

But:

Takes:

John	CS1
John	CS2

$\overset{\Gamma}{\Rightarrow}$ Enroll:

$\perp_1$	CS1
$\perp_2$	CS2

Composition: when it doesn't work cont'd

- Solution: Skolem functions.
- Γ': Enroll$(f(s), c)$:– Takes(s, c)
- Then:

Takes:

John	CS1
John	CS2

$\stackrel{\Gamma}{\Rightarrow}$ Enroll:

$\perp$	CS1
$\perp$	CS2

- where $\perp = f(\text{John})$

Composition: another example

- Schema σ: Empl(eid)
- Schema τ: Mngr(eid,mngid)
- Schema ω: Mngr'(eid,mngid), SelfMng(id)
- Mapping Σ from σ to τ:

$$\text{Mngr}(e, m) \quad :- \quad \text{Empl}(e)$$

- Mapping Δ from τ to ω:

$$\begin{array}{rcl} \text{Mngr'}(e, m) & :- & \text{Mngr}(e, m) \\ \text{SelfMng}(e) & :- & \text{Mngr}(e, e) \end{array}$$

- Composition:

$$\begin{array}{rcl} \text{Mngr'}(e, f(e)) & :- & \text{Empl}(e) \\ \text{SelfMng}(e) & :- & \text{Empl}(e) \wedge e = f(e) \end{array}$$

Composition and Skolem functions

- Schema mappings with Skolem functions compose!
- Algorithm:
  - replace all nulls by Skolem functions
    - Mngr$(e, f(e))$:– Empl(e)
    - Δ stays as before
  - Use substitution:
    - Mngr'(e, m) :– Mngr(e, m) becomes

      Mngr'$(e, f(e))$:– Empl(e)
    - SelfMng(e) :– Mngr(e, e) becomes

      SelfMng(e) :– Empl$(e) \wedge e = f(e)$

Inverting mappings

- Harder than composition.
- Intuition: $\mathbf{\Sigma} \circ \mathbf{\Sigma}^{-1} = \mathbf{ID}$.
- But even what $\mathbf{ID}$ should be is not entirely clear.
- Some intuitive examples will follow.

Examples of inversion

- The inverse of projection is null invention:
  - $T(x)$:– $S(x, y)$
  - $S(x, y)$:– $T(x)$
- Inverse of union requires disjunction:
  - $T(x)$:– $S(x)$ $\quad$ $T(x)$:– $S'(x)$
  - $S(x) \vee S'(x)$:– $T(x)$
- So reversing the rules doesn't always work.

Examples of inversion cont'd

- Inverse of decomposition is join:
  - $T(x_1, x_2) \wedge T'(x_2, x_3) \ :\!-\ S(x_1, x_2, x_3)$
  - $S(x_1, x_2, x_3) \ :\!-\ T(x_1, x_2) \wedge T'(x_2, x_3)$
- But this is also an inverse of $T(x_1, x_2) \wedge T'(x_2, x_3) \ :\!-\ S(x_1, x_2, x_3)$:
  - $S(x_1, x_2, z) \ :\!-\ T(x_1, x_2)$
  - $S(z, x_2, x_3) \ :\!-\ T'(x_2, x_3)$

Examples of inversion cont'd

- One may need to distinguish nulls from values in inverses.
- Σ given by

$$\begin{array}{rcl} T_1(x) & :\!- & S(x,x) \\ T_2(x,z) & :\!- & S(x,y) \wedge S(y,x) \\ T_3(x_1,x_2,z) & :\!- & S(x_1,x_2) \end{array}$$

- Its inverse Σ^{-1} requires:
  - a predicate NotNull and
  - inequalities:

$$\begin{array}{rcl} S(x,x) & :\!- & T_1(x) \wedge T_2(x,y_1) \wedge T_3(x,x,y_2) \wedge \mathsf{NotNull}(x) \\ S(x_1,x_2) & :\!- & T_3(x_1,x_2,y) \wedge (x_1 \neq x_2) \wedge \mathsf{NotNull}(x_1) \wedge \mathsf{NotNull}(x_2) \end{array}$$

Integrating preferences/rankings

Problem statement

- Each object has m grades, one for each of m criteria.
- The grade of an object for field i is x_i.
- Normally assume $0 \leq x_i \leq 1$.
  - Typically evaluations based on different criteria
  - The higher the value of x_i, the better the object is according to the ith criterion
- The objects are given in m sorted lists
  - the ith list is sorted by x_i value
  - These lists correspond to different sources or to different criteria.
- Goal: find the top k objects.

Example

Grade 1
(17, 0.9936)
(1352,0.9916)
(702,0.9826)
...
(12, 0.3256)
...

Grade 2
(235, 0.9996)
(12, 0.9966)
(8201, 0.9926)
...
(17, 0.406)
...

Aggregation Functions

- Have an aggregation function F.
- Let $x_1, \ldots, x_m$ be the grades of object R under the m criteria.
- Then $F(x_1, \ldots, x_m)$ is the overall grade of object R.
- Common choices for F:
  - min
  - average or sum
- An aggregation function F is monotone if

$$F(x_1, \ldots, x_m) \leq F(x'_1, \ldots, x'_m)$$

whenever $x_i \leq x'_i$ for all i.

Other Applications

- Information retrieval
- Objects R are documents.
- The m criteria are search terms $s_1, \ldots, s_m$.
- The grade x_i: how relevant document R is for search term s_i.
- Common to take the aggregation function F to be the sum

$$F(x_1, \ldots, x_m) = x_1 + \cdots + x_m.$$

Modes of Access

- Sorted access
  - Can obtain the next object with its grade in list L_i
  - cost c_S.
- Random access
  - Can obtain the grade of object R in list L_i
  - cost c_R.
- Middleware cost:

$$c_S \cdot \ (\text{\# of sorted accesses}) + c_R \cdot \ (\text{\# of random accesses}).$$

Algorithms

- Want an algorithm for finding the top k objects.
- Naive algorithm:
  - compute the overall grade of every object;
  - return the top k answers.
- Too expensive.

Fagin's Algorithm (FA)

1. Do sorted access in parallel to each of the m sorted lists L_i.
   - Stop when there are at least k objects, each of which have been seen in all the lists.
2. For each object R that has been seen:
   - Retrieve all of its fields $x_1, \ldots, x_m$ by random access.
   - Compute $F(R) = F(x_1, \ldots, x_m)$.
3. Return the top k answers.

Fagin's algorithm is correct

- Assume object R was not seen
  - its grades are $x_1, \ldots, x_m$.
- Assume object S is one of the answers returned by FA
  - its grades are $y_1, \ldots, y_m$.
- Then $x_i \leq y_i$ for each i
- Hence

$$F(R) = F(x_1, \ldots, x_m) \leq F(y_1, \ldots, y_m) = F(S).$$

Fagin's algorithm: performance guarantees

- Typically probabilistic guarantees
- Orderings are independent
- Then with high probability the middleware cost is

$$O\left(N \cdot \sqrt[m]{\frac{k}{N}}\right)$$

- i.e., sublinear
- But may perform poorly
  - e.g., if F is constant:
  - still takes $O\left(N \cdot \sqrt[m]{k/N}\right)$ instead of a constant time algorithm

Optimal algorithm: The Threshold Algorithm

1. Do sorted access in parallel to each of the m sorted lists L_i. As each object R is seen under sorted access:
   - Retrieve all of its fields $x_1, \ldots, x_m$ by random access.
   - Compute $F(R) = F(x_1, \ldots, x_m)$.
   - If this is one of the top k answers so far, remember it.
   - Note: buffer of bounded size.
2. For each list L_i, let $\hat{x}_i$ be the grade of the last object seen under sorted access.
3. Define the *threshold value* t to be $F(\hat{x}_1, \ldots, \hat{x}_m)$.
4. When k objects have been seen whose grade is at least t, then stop.
5. Return the top k answers.

Threshold Algorithm: correctness and optimality

- The Threshold Algorithm is correct for every monotone aggregate function F.
- Optimal in a very strong sense:
  - it is as good as any other algorithm on every instance
  - any other algorithm means: except pathological algorithms
  - as good means: within a constant factor
  - pathological means: making wild guesses.

Wild guesses can help

- An algorithm "makes a wild guess" if it performs random access on an object not previously encountered by sorted access.
- Neither FA nor TA make wild guesses, nor does any "natural" algorithm.
- Example: The aggregation function is min; $k = 1$.

LIST L_1
(1, 1)
(2, 1)
(3, 1)
...
(n+1, 1)
(n+2, 0)
(n+3, 0)
...
(2n+1, 0)

LIST L_2
(2n+1, 1)
(2n, 1)
(2n-1, 1)
...
(n+1, 1)
(n, 0)
(n-1, 0)
...
(1, 0)

Threshold Algorithm as an approximation algorithm

- Approximately finding top k answers.
- For $\varepsilon > 0$, an ε-approximation of top k answers is a collection of k objects $R_1, \ldots, R_k$ so that
  - for each R not among them,

$$(1+\varepsilon) \cdot F(R_i) \;\geq\; F(R)$$

- Turning TA into an approximation algorithm:
- Simply change the stopping rule into:
  - When k objects have been seen whose grade is at least

$$\frac{t}{1+\varepsilon},$$

  then stop.