GAV-sound with conjunctive queries

- Source and global schema as before:
  - source $R_1(A, B), R_2(B, C)$
  - Global schema: $T_1(A, C)$, $T_2(B, C)$
- GAV mappings become sound:
  - $T_1 \supseteq \{x, y, z | R_1(x, y) \wedge R_2(y, z)\}$
  - $T_2 \supseteq R_2$
- Let D_{exact} be the unique database that arises from the *exact* setting (with $\supseteq$ replaced by $=$)
- Then every database D_{sound} that satisfies the sound setting also satisfies

$$D_{exact} \quad \subseteq \quad D_{sound}$$

GAV-sound with conjunctive queries cont'd

- Conjunctive queries are monotone:

$$D_1 \subseteq D_2 \quad \Rightarrow \quad Q(D_1) \subseteq Q(D_2)$$

- Exact solution is a sound solution too, and is contained in every sound solution.
- Hence certain answers for each conjunctive query

$$\text{certain}(D, Q) \;=\; \bigcap_{D_{sound}} Q(D_{sound}) \;=\; Q(D_{exact})$$

- The solution for GAV-exact gives us certain answers for GAV-sound, for conjunctive (and more generally, monotone) queries.

Query answering using views

- General setting: database relations $R_1, \ldots, R_n$.
- Several views $V_1, \ldots, V_k$ are defined as results of queries over the R_i's.
- We have a query Q over $R_1, \ldots, R_n$.
- Question: Can Q be answered in terms of the views?
  - In other words, can Q be reformulated so it only refers to the data in $V_1, \ldots, V_k$?

Query answering using views in data integration

- LAV:
  - $R_1, \ldots, R_n$ are global schema relations; Q is the global schema query
  - V_i's are the sources defined over the global schema
  - We must answer Q based on the sources (virtual integration)
- GAV:
  - $R_1, \ldots, R_n$ are the sources that are not fully available.
  - Q is a query in terms of the source (or a query that was reformulated in terms of the sources)
  - Must see if it is answerable from the available views $V_1, \ldots, V_k$.
- We know the problem is impossible to solve for full relational algebra, hence we concentrate on conjunctive queries.

Conjunctive queries: rule-based notation

- We often write conjunctive queries as logical statements:

$$\{t, y, r \mid \exists d \ \big(\text{Movie}(t, d, y) \wedge \text{RV}(t, r) \wedge y > 2000\big)\}$$

- Rule-based:

$$Q(t, y, r) \ \text{:–}\ \text{Movie}(t, d, y), \text{RV}(t, r), y > 2000$$

  - $Q(t, y, r)$ is the head of the rule
  - $\text{Movie}(t, d, y), \text{RV}(t, r), y > 2000$ is its body
  - conjunctions are replaced by commas
  - variables that occur in the body but not in the head (d) are assumed to be existentially quantified
  - essentially logic programming notation (without functions)

Query answering using views: example

- Two relations in the database: Cites(A,B) (if A cites B), and SameTopic(A,B) (if A, B work on the same topic)
- Query $Q(x,y)$:– $\text{SameTopic}(x,y), \text{Cites}(x,y), \text{Cites}(y,x)$
- Two views are given:
  - $V_1(x,y)$:– $\text{Cites}(x,y), \text{Cites}(y,x)$
  - $V_2(x,y)$:– $\text{SameTopic}(x,y), \text{Cites}(x,x'), \text{Cites}(y,y')$
- Suggested rewriting: $Q'(x,y)$:– $V_1(x,y), V_2(x,y)$
- Why? Unfold using the definitions:

  $Q'(x,y)$:– $\text{Cites}(x,y), \text{Cites}(y,x), \text{SameTopic}(x,y), \text{Cites}(x,x'), \text{Cites}(y,y')$
- Equivalent to Q

Query answering using views

- Need a formal technique (algorithm): cannot rely on the semantics.
- Query Q:

```
SELECT R1.A
FROM R R1, R R2, S S1, S S2
WHERE R1.A=R2.A AND S1.A=S2.A AND R1.A=S1.A
      AND R1.B=1 and S2.B=1
```

- $Q(x)\ \text{:-}\ R(x,y), R(x,1), S(x,z), S(x,1)$
- Equivalent to $Q(x)\ \text{:-}\ R(x,1), S(x,1)$
- So if we have a view
  - $V(x,y)\ \text{:-}\ R(x,y), S(x,y)$ (i.e. $V = R \cap S$), then
  - $Q = \pi_A(\sigma_{B=1}(V))$
  - Q can be rewritten (as a conjunctive query) in terms of V

Query rewriting

- Setting:
  - Queries $V_1, \ldots, V_k$ over the same schema σ (assume to be conjunctive; they define the views)
  - Each Q_i is of arity n_i
  - A schema ω with relations of arities $n_1, \ldots, n_k$
- Given:
  - a query Q over σ
  - a query Q' over ω
- Q' is a rewriting of Q if for every σ-database D,

$$Q(D) \; = \; Q'\big(\; V_1(D), \ldots, V_k(D) \;\big)$$

Maximal rewriting

- Sometimes exact rewritings cannot be obtained
- Q' is a maximally-contained rewriting if:
  - it is contained in Q:

    $$Q'(\ V_1(D), \ldots, V_k(D)\ ) \ \subseteq \ Q(D)$$

    for all D
  - it is maximal such: if

    $$Q''(\ V_1(D), \ldots, V_k(D)\ ) \ \subseteq \ Q(D)$$

    for all D, then

    $$Q'' \subseteq Q'$$

Side remark: query rewriting and certain answers

- If we have sources $\mathbf{R} = (R_1, \ldots, R_k)$, we can view conditions

$$V_1(D) = R_1, \; \ldots, \; V_k(D) = R_k$$

  as an incomplete specification of a database D

- To answer Q over D, given $R_1, \ldots, R_k$, we want to compute certain answers:

$$\text{certain}(Q, \mathbf{R}) \; = \; \bigcap \{Q(D) \; \mid \; V_1(D) = R_1, \; \ldots, \; V_k(D) = R_k\}$$

- If for every such D we have $Q(D) = Q'(V_1(D), \ldots, V_k(D))$, then $\text{certain}(Q, \mathbf{R}) = Q'$.

- But we may even look at a more general way of query answering by finding a rewriting Q' so that

$$\text{certain}(Q, \mathbf{R}) = Q'(\mathbf{R})$$

Query rewriting: a naive algorithm

- Given:
  - ◦ conjunctive queries $V_1, \ldots, V_k$ over schema σ
  - ◦ a query Q over σ
- Algorithm:
  - ◦ guess a query Q' over the views
  - ◦ Unfold Q' in terms of the views
  - ◦ Check if the unfolding is contained in Q
- If one unfolding is equivalent to Q, then Q' is a rewriting
- Otherwise take the union of all unfoldings contained in Q
  - – it is a maximally contained rewriting

Why is it not an algorithm yet?

- Problem 1: we do not yet know how to test containment and equivalence.
  - But we shall learn soon
- Problem 2: the guess stage.
  - There are infinitely many conjunctive queries.
  - We cannot check them all.
  - Solution: we only need to check a few.

Guessing rewritings

- A basic fact:
  - If there is a rewriting of Q using $V_1, \ldots, V_k$, then there is a rewriting with no more conjuncts than in Q.
  - E.g., if $Q(x) \;:\!-\; R(x,y), R(x,1), S(x,z), S(x,1)$, we only need to check conjunctive queries over V with at most 4 conjuncts.
- Moreover, maximally contained rewriting is obtained as the union of all conjunctive rewritings of length of Q or less.
- Complexity: enumerate all candidates (exponentially many); for each an NP (or exponential) algorithm. Hence exponential time is required.
- Cannot lower this due to NP-completeness.

Containment and optimization of conjunctive queries

- Reminder:

$$\begin{aligned} & \text{conjunctive queries} \\ = \ & \text{SPJ queries} \\ = \ & \text{rule-based queries} \\ = \ & \text{simple SELECT-FROM-WHERE SQL queries} \\ & \text{(only AND and equality in the WHERE clause)} \end{aligned}$$

- Extremely common, and thus special optimization techniques have been developed
- Reminder: for two relational algebra expressions e_1, e_2, $e_1 = e_2$ is undecidable.
- But for conjunctive queries, even $e_1 \subseteq e_2$ is decidable.
- Main goal of optimizing conjunctive queries: reduce the number of joins.

Optimization of conjunctive queries: an example

- Given a relation R with two attributes A, B
- Two SQL queries:

Q1

```
SELECT R1.B, R1.A
FROM R R1, R R2
WHERE R2.A=R1.B
```

Q2

```
SELECT R3.A, R1.A
FROM R R1, R R2, R R3
WHERE R1.B=R2.B AND R2.B=R3.A
```

- Are they equivalent?
- If they are, we saved one join operation.
- In relational algebra:

$$Q_1 = \pi_{2,1}(\sigma_{2=3}(R \times R))$$

$$Q_2 = \pi_{5,1}(\sigma_{2=4 \wedge 4=5}(R \times R \times R))$$

Optimization of conjunctive queries cont'd

- Are Q_1 and Q_2 equivalent?
- If they are, we cannot show it by using equivalences for relational algebra expression.
- Because: they don't decrease the number of $\bowtie$ or $\times$ operators, but Q_1 has 1 join, and Q_2 has 2.
- But Q_1 and Q_2 are equivalent. How can we show this?
- But representing queries as databases. This representation is very close to rule-based queries.

$$Q_1(x, y) \ :\!-\ R(y, x), R(x, z)$$

$$Q_2(x, y) \ :\!-\ R(y, x), R(w, x), R(x, u)$$

Conjunctive queries into tableaux

- Tableau: representing of a conjunctive query as a database
- We first consider queries over a single relation
- $Q_1(x, y) \;\;\text{:-}\;\; R(y, x), R(x, z)$
- $Q_2(x, y) \;\;\text{:-}\;\; R(y, x), R(w, x), R(x, u)$
- Tableaux:

A	B	
y	x	
x	z	
x	y	← answer line

A	B	
y	x	
w	x	
x	u	
x	y	← answer line

- Variables in the answer line are called distinguished

Tableau homomorphisms

- A homomorphism of two tableaux $f : T_1 \to T_2$ is a mapping

$$f : \{\text{variables of } T_1\} \to \{\text{variables of } T_2\} \ \bigcup \ \{\text{constants}\}$$

- For every distinguished x, $f(x) = x$
- For every row $x_1, \ldots, x_k$ in T_1, $f(x_1), \ldots, f(x_k)$ is a row of T_2
- Query containment: $Q \subseteq Q'$ if $Q(D) \subseteq Q'(D)$ for every database D
- Homomorphism Theorem: Let Q, Q' be two conjunctive queries, and T, T' their tableaux. Then

$$Q \subseteq Q'$$
if and only if
there exists a homomorphism $f : T' \to T$

Applying the Homomorphism Theorem: $Q_1 = Q_2$

T1

A	B
y	x
x	z
x	y

T2

A	B
y	x
w	x
x	u
x	y

f(x)=x, f(y)=y

f(u)=z, f(w)=y

Hence Q1 $\subseteq$ Q2

T1

A	B
y	x
x	z
x	y

T2

A	B
y	x
w	x
x	u
x	y

f(x)=x, f(y)=y

f(z)=u

Hence Q2 $\subseteq$ Q1

Applying the Homomorphism Theorem: Complexity

- Given two conjunctive queries, how hard is it to test if $Q_1 = Q_2$?
- it is easy to transform them into tableaux, from either SPJ relational algebra queries, or SQL queries, or rule-based queries
- But testing the existence of a homomorphism between two tableaux is hard: NP-complete. Thus, a polynomial algorithm is unlikely to exists.
- However, queries are small, and conjunctive query optimization is possible in practice.

Minimizing conjunctive queries

- Goal: given a conjunctive query Q, find an equivalent conjunctive query Q' with the minimum number of joins.
- Assume Q is

$$Q(\vec{x}) \ \text{:–}\ R_1(\vec{u}_1), \ldots, R_k(\vec{u}_k)$$

- Assume that there is an equivalent conjunctive query Q' of the form

$$Q'(\vec{x}) \ \text{:–}\ S_1(\vec{v}_1), \ldots, S_l(\vec{v}_l)$$

  with $l < k$
- Then Q is equivalent to a query of the form

$$Q'(\vec{x}) \ \text{:–}\ R_{i_1}(\vec{u}_{i_1}), \ldots, R_{i_l}(\vec{u}_{i_l})$$

- In other words, to minimize a conjunctive query, one has to delete some subqueries on the right of :–

Minimizing conjunctive queries cont'd

- Given a conjunctive query Q, transform it into a tableau T
- Let Q' be a minimal conjunctive query equivalent to Q. Then its tableau T' is a subset of T.
- Minimization algorithm:

  $T' := T$

  repeat until no change

      choose a row t in T'

      if there is a homomorphism $f : T' \to T' - \{t\}$

          then $T' := T' - \{t\}$

  end
- Note: if there exists a homomorphism $T' \to T' - \{t\}$, then the queries defined by T' and $T' - \{t\}$ are equivalent. Because: there is always a homomorphism from $T' - \{t\}$ to T'. (Why?)

Minimizing SPJ/conjunctive queries: example

- R with three attributes A, B, C
- SPJ query

$$Q \quad = \quad \pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Equivalently, a SQL query:

```
SELECT R1.A, R2.B, R3.C
FROM R R1, R R2, R R3
WHERE R1.B=4 AND R2.A=R3.A AND
      R3.B=4 AND R2.B=R1.B
```

- Translate into a conjunctive query:

$$\exists x_1, z_1, z_2 \, (R(x, 4, z_1) \wedge R(x_1, 4, z_2) \wedge R(x_1, 4, z) \wedge y = 4)$$

- Rule-based:

$$Q(x, y, z) \;\; :\!- R(x, 4, z_1), R(x_1, 4, z_2), R(x_1, 4, z), y = 4$$

Minimizing SPJ/conjunctive queries cont'd

- Tableau T:

A	B	C
x	4	z_1
x_1	4	z_2
x_1	4	z
x	4	z

- Minimization, step 1: is there a homomorphism from T to

A	B	C
x_1	4	z_2
x_1	4	z
x	4	z

- Answer: No. For any homomorphism f, $f(x) = x$ (why?), thus the image of the first row is not in the small tableau.

Minimizing SPJ/conjunctive queries cont'd

- Step 2: Is T equivalent to

A	B	C
x	4	z_1
x_1	4	z
x	4	z

- Answer: Yes. Homomorphism f: $f(z_2) = z$, all other variables stay the same.
- The new tableau is not equivalent to

A	B	C
x	4	z_1
x	4	z

or

A	B	C
x_1	4	z
x	4	z

- Because $f(x) = x, f(z) = z$, and the image of one of the rows is not present.

Minimizing SPJ/conjunctive queries cont'd

- Minimal tableau: $\begin{array}{ccc} \text{A} & \text{B} & \text{C} \\ \hline x & 4 & z_1 \\ x_1 & 4 & z \\ \hline x & 4 & z \end{array}$

- Back to conjunctive query:

$$Q'(x, y, z) \;\; \text{:-} \;\; R(x, y, z_1), R(x_1, y, z), y = 4$$

- An SPJ query:

$$\pi_{AB}(\sigma_{B=4}(R)) \;\; \bowtie \;\; \pi_{BC}(\sigma_{B=4}(R))$$

- ```
  SELECT R1.A, R1.B, R2.C
  FROM R R1, R R2
  WHERE R1.B=R2.B AND R1.B=4
  ```
  ```

Review of the journey

- We started with

$$\pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Translated into a conjunctive query
- Built a tableau and minimized it
- Translated back into conjunctive query and SPJ query
- Applied algebraic equivalences and obtained

$$\pi_{AB}(\sigma_{B=4}(R)) \;\bowtie\; \pi_{BC}(\sigma_{B=4}(R))$$

- Savings: one join.

All minimizations are equivalent

- Let Q be a conjunctive query, and Q_1, Q_2 two conjunctive queries equivalent to Q
- Assume that Q_1 and Q_2 are both minimal, and let T_1 and T_2 be their tableaux.
- Then T_1 and T_2 are isomorphic; that is, T_2 can be obtained from T_1 by renaming of variables.
- That is, all minimizations are equivalent.
- In particular, in the minimization algorithm, the order in which rows are considered, is irrelevant.

Equivalence of conjunctive queries: the general case

- So far we assumed that there is only one relation R, but what if there are many?
- Construct tableaux as before:

$$Q(x,y) \text{:–} B(x,y), R(y,z), R(y,w), R(w,y)$$

- Tableau:

B:

A	B
x	y

R:

A	B
y	z
y	w
w	y

x	y

- Formally, a tableau is just a database where variables can appear in tuples, plus a set of distinguished variables.

Tableaux and multiple relations

- Given two tableaux T_1 and T_2 over the same set of relations, and the same distinguished variables, a homomorphism $h : T_1 \to T_2$ is a mapping
$$f : \{\text{variables of } T_1\} \;\to\; \{\text{variables of } T_2\}$$
such that
  - $f(x) = x$ for every distinguished variable, and
  - for each row $\vec{t}$ in R in T_1, $f(\vec{t})$ is in R in T_2.

- **Homomorphism theorem**: let Q_1 and Q_2 be conjunctive queries, and T_1, T_2 their tableaux. Then

$$Q_2 \subseteq Q_1$$
if and only if
there exists a homomorphism $f : T_1 \to T_2$

Minimization with multiple relations

- The algorithm is the same as before, but one has to try rows in different relations. Consider homomorphism $f(z) = w$, and f is the identity for other variables. Applying this to the tableau for Q yields

$$
\begin{array}{ll}
\text{B:}\ \begin{array}{cc} \text{A} & \text{B} \\ \hline \text{x} & \text{y} \end{array} & \qquad\qquad \text{R:}\ \begin{array}{cc} \text{A} & \text{B} \\ \hline \text{y} & \text{w} \\ \text{w} & \text{y} \end{array} \\
\hline
\quad\text{x} \qquad\qquad \text{y} &
\end{array}
$$

- This cannot be further reduced, as for any homomorphism f, $f(x) = x$, $f(y) = y$.

- Thus Q is equivalent to

$$Q'(x, y) \;:\!-\; B(x, y), R(y, w), R(w, y)$$

- One join is eliminated.

Query rewriting

- Recall the algorithm, for a given Q and view definitions $V_1, \ldots, V_k$:
  - Look at all rewritings that have as at most as many joins as Q
  - check if they are contained in Q
  - take the union of those that are
- This is the maximally contained rewriting
- There are algorithms that prune the search space and make looking for rewritings contained in Q more efficient
  - the bucket algorithm
  - MiniCon
- May see of them later

How hard is it to answer queries using views?

- Setting: we now have an actual content of the views.
- As before, a query is Q posed against D, but must be answered using information in the views.
- Suppose $I_1, \ldots, I_k$ are view instances. Two possibilities:
  - Exact mappings: $I_j = V_j(D)$
  - Sound mappings: $I_j \subseteq V_j(D)$
- We need certain answers for given $\mathcal{I} = (I_1, \ldots, I_k)$:

$$\text{certain}_{exact}(Q, \mathcal{I}) = \bigcap_{D:\ I_j = V_j(D) \text{ for all } j} Q(D)$$

$$\text{certain}_{sound}(Q, \mathcal{I}) = \bigcap_{D:\ I_j \subseteq V_j(D) \text{ for all } j} Q(D)$$

How hard is it to answer queries using views?

- If $\text{certain}_{exact}(Q, \mathcal{I})$ or $\text{certain}_{sound}(Q, \mathcal{I})$ are impossible to obtain, we want maximally contained rewritings:
  - $Q'(\mathcal{I}) \subseteq \text{certain}_{exact}(Q, \mathcal{I})$, and
  - if $Q''(\mathcal{I}) \subseteq \text{certain}_{exact}(Q, \mathcal{I})$ then $Q''(\mathcal{I}) \subseteq Q'(\mathcal{I})$
  - (and likewise for $sound$)
- How hard is it to compute this from $\mathcal{I}$?
- In databases, we reason about complexity in two ways:
  - The big-O notation ($O(n \log n)$ vs $O(n^2)$ vs $O(2^n)$)
  - Complexity-theoretic notions: PTIME, NP, DLOGSPACE, etc
- Advantage of complexity-theoretic notions: if you have a $O(2^n)$ algorithm, is it because the problem is inherently hard, or because we are not smart enough to come up with a better algorithm (or both)?

Complexity classes: what you always wanted to know but never dared to ask

- Or a $5/5$-introduction: a five minute review that tells you what are likely to remember 5 years after taking a complexity theory course.
- The big divide: PTIME (computable in polynomial time, i.e. $O(n^k)$ for some fixed k)
- Inside PTIME: tractable queries (although high-degree polynomial are intractable)
- Outside PTIME: intractable queries (efficient algorithms are unlikely)
- Way outside PTIME: provably intractable queries (efficient algorithms do not exist)
  - EXPTIME: c^n-algorithms for a constant c. Could still be ok for not very large inputs
  - Even further – 2-EXPTIME: c^{c^n}. Cannot be ok even for small inputs (compare 2^{10} and $2^{2^{10}}$).

Inside PTIME

$$\mathsf{AC}^0 \subsetneq \mathsf{TC}^0 \subseteq \mathsf{NC}^1 \subseteq \mathsf{DLOG} \subseteq \mathsf{NLOG} \subseteq \mathsf{PTIME}$$

- AC^0: very efficient parallel algorithms (constant time, simple circuits)
  - relational calculus
- TC^0: very efficient parallel algorithms in a more powerful computational model with counting gates
  - basic SQL (relational calculus/grouping/aggregation)
- NC^1: efficient parallel algorithms
  - regular languages
- DLOG: very little – $O(\log n)$ – space is required
  - SQL + (restricted) transitive closure
- NLOG: $O(\log n)$ space is required if nondeterminism is allowed
  - SQL + transitive closure (as in the SQL3 standard)

Beyond PTIME

$$\text{PTIME} \subseteq \left\{ \begin{array}{c} \text{NP} \\ \text{coNP} \end{array} \right\} \subseteq \text{PSPACE}$$

- PTIME: can solve a problem in polynomial time
- NP: can check a given candidate solution in polynomial time
  - another way of looking at it: guess a solution, and then verify if you guessed it right in polynomial time
- coNP: complement of NP – verify that all "reasonable" candidates are solutions to a given problem.
  - Appears to be harder than NP but the precise relationship isn't known
- PSPACE: can be solved using memory of polynomial size (but perhaps an exponential-time algorithm)

Complete problems

- These are the hardest problems in a class.
- If our problem is as hard as a complete problem, it is very unlikely it can be done with lower complexity.
- For NP:
  - SAT (satisfiability of Boolean formulae)
  - many graph problems (e.g. 3-colourability)
  - Integer linear programming etc
- For PSPACE:
  - Quantified SAT
  - Two XML DTDs are equivalent

Complexity of query answering

- We want the complexity of finding

$$\text{certain}_{exact}(Q, \mathcal{I}) \quad \text{or} \quad \text{certain}_{sound}(Q, \mathcal{I})$$

  in terms of the size of $\mathcal{I}$

- If all view definitions are conjunctive queries and Q is a relational algebra or a SQL query, then the complexity is coNP.
- (blackboard)
- This is too high!
- If all view definitions are conjunctive queries and Q is a conjunctive query, then the complexity is PTIME.
  - Because: the maximally contained rewriting computes certain answers!

Complexity of query answering

	query language		
view language	CQ	$\mathrm{CQ}^{\neq}$	relational calculus
CQ	ptime	coNP	undecidable
$\mathrm{CQ}^{\neq}$	ptime	coNP	undecidable
relational calculus	undecidable	undecidable	undecidable

CQ – conjunctive queries

$\mathrm{CQ}^{\neq}$ – conjunctive queries with inequalities
(for example, $Q(x)$:– $R(x, y), S(y, z), x \neq z$)

Complexity of query answering: coNP-completeness idea

- Start with a graph G – this is our instance
- D is G together with a colouring, with 3 colours; each node is assigned one colour.
- Q asks if we have an edge (a, b) with $a \neq b$ and a, b of the same colour.
- If G is not 3-colourable, then every instance D would satisfy Q
- Otherwise, if G is 3-colourable, we can find extensions that are and that are not 3-colourable – hence certain answers are empty.
- Thus if we can compute certain answers, we can test non-3-colourability $\Rightarrow$ coNP-completeness.