# GAV-sound with conjunctive queries

- Source and global schema as before:
  - source $R_1(A, B), R_2(B, C)$
  - Global schema: $T_1(A, C)$, $T_2(B, C)$
- GAV mappings become sound:
  - $T_1 \supseteq \{x, y, z | R_1(x, y) \wedge R_2(y, z)\}$
  - $T_2 \supseteq R_2$
- Let $D_{exact}$ be the unique database that arises from the *exact* setting (with $\supseteq$ replaced by $=$)
- Then every database $D_{sound}$ that satisfies the sound setting also satisfies

$$D_{exact} \quad \subseteq \quad D_{sound}$$

# GAV-sound with conjunctive queries cont'd

- Conjunctive queries are monotone:

$$D_1 \subseteq D_2 \quad \Rightarrow \quad Q(D_1) \subseteq Q(D_2)$$

- Exact solution is a sound solution too, and is contained in every sound solution.
- Hence certain answers for each conjunctive query

$$\text{certain}(D, Q) \;=\; \bigcap_{D_{sound}} Q(D_{sound}) \;=\; Q(D_{exact})$$

- The solution for GAV-exact gives us certain answers for GAV-sound, for conjunctive (and more generally, monotone) queries.

# Query answering using views

- General setting: database relations $R_1, \ldots, R_n$.
- Several views $V_1, \ldots, V_k$ are defined as results of queries over the $R_i$'s.
- We have a query $Q$ over $R_1, \ldots, R_n$.
- Question: Can $Q$ be answered in terms of the views?
  - In other words, can $Q$ be reformulated so it only refers to the data in $V_1, \ldots, V_k$?

# Query answering using views in data integration

- LAV:
  - ○ $R_1, \ldots, R_n$ are global schema relations; $Q$ is the global schema query
  - ○ $V_i$'s are the sources defined over the global schema
  - ○ We must answer $Q$ based on the sources (virtual integration)
- GAV:
  - ○ $R_1, \ldots, R_n$ are the sources that are not fully available.
  - ○ $Q$ is a query in terms of the source (or a query that was reformulated in terms of the sources)
  - ○ Must see if it is answerable from the available views $V_1, \ldots, V_k$.
- We know the problem is impossible to solve for full relational algebra, hence we concentrate on conjunctive queries.

# Conjunctive queries: rule-based notation

- We often write conjunctive queries as logical statements:

$$\{t, y, r \mid \exists d \, \big(\mathsf{Movie}(t, d, y) \wedge \mathsf{RV}(t, r) \wedge y > 2000\big)\}$$

- Rule-based:

$$Q(t, y, r) \;\; \text{:--} \;\; \mathsf{Movie}(t, d, y), \mathsf{RV}(t, r), y > 2000$$

  - $Q(t, y, r)$ is the head of the rule
  - $\mathsf{Movie}(t, d, y), \mathsf{RV}(t, r), y > 2000$ is its body
  - conjunctions are replaced by commas
  - variables that occur in the body but not in the head ($d$) are assumed to be existentially quantified
  - essentially logic programming notation (without functions)

# Query answering using views: example

- Two relations in the database: Cites(A,B) (if A cites B), and SameTopic(A,B) (if A, B work on the same topic)
- Query $Q(x, y) \ :\!- \ \text{SameTopic}(x, y), \text{Cites}(x, y), \text{Cites}(y, x)$
- Two views are given:
  - $V_1(x, y) \ :\!- \ \text{Cites}(x, y), \text{Cites}(y, x)$
  - $V_2(x, y) \ :\!- \ \text{SameTopic}(x, y), \text{Cites}(x, x'), \text{Cites}(y, y')$
- Suggested rewriting: $Q'(x, y) \ :\!- \ V_1(x, y), V_2(x, y)$
- Why? Unfold using the definitions:

  $Q'(x, y) \ :\!- \ \text{Cites}(x, y), \text{Cites}(y, x), \text{SameTopic}(x, y), \text{Cites}(x, x'), \text{Cites}(y, y')$
- Equivalent to $Q$

# Query answering using views

- Need a formal technique (algorithm): cannot rely on the semantics.
- Query $Q$:

```
SELECT R1.A
FROM R R1, R R2, S S1, S S2
WHERE R1.A=R2.A AND S1.A=S2.A AND R1.A=S1.A
      AND R1.B=1 and S2.B=1
```

- $Q(x)\ \text{:--}\ R(x,y), R(x,1), S(x,z), S(x,1)$
- Equivalent to $Q(x)$ :– $R(x,1), S(x,1)$
- So if we have a view
  - ○ $V(x,y)\ \text{:--}\ R(x,y), S(x,y)$ (i.e. $V = R \cap S$), then
  - ○ $Q = \pi_A(\sigma_{B=1}(V))$
  - ○ $Q$ can be rewritten (as a conjunctive query) in terms of $V$

# Query rewriting

- Setting:
  - ◦ Queries $V_1, \ldots, V_k$ over the same schema $\sigma$ (assume to be conjunctive; they define the views)
  - ◦ Each $Q_i$ is of arity $n_i$
  - ◦ A schema $\omega$ with relations of arities $n_1, \ldots, n_k$
- Given:
  - ◦ a query $Q$ over $\sigma$
  - ◦ a query $Q'$ over $\omega$
- $Q'$ is a rewriting of $Q$ if for every $\sigma$-database $D$,

$$Q(D) \;=\; Q'\big(\; V_1(D), \ldots, V_k(D) \;\big)$$

# Maximal rewriting

- Sometimes exact rewritings cannot be obtained
- $Q'$ is a maximally-contained rewriting if:
  - it is contained in $Q$:

    $$Q'(\ V_1(D), \ldots, V_k(D)\ )\ \subseteq\ Q(D)$$

    for all $D$
  - it is maximal such: if

    $$Q''(\ V_1(D), \ldots, V_k(D)\ )\ \subseteq\ Q(D)$$

    for all $D$, then

    $$Q'' \subseteq Q'$$

# Side remark: query rewriting and certain answers

- If we have sources $\mathbf{R} = (R_1, \ldots, R_k)$, we can view conditions

  $$V_1(D) = R_1, \ \ldots, \ V_k(D) = R_k$$

  as an incomplete specification of a database $D$

- To answer $Q$ over $D$, given $R_1, \ldots, R_k$, we want to compute certain answers:

  $$\text{certain}(Q, \mathbf{R}) \ = \ \bigcap \{Q(D) \mid V_1(D) = R_1, \ \ldots, \ V_k(D) = R_k\}$$

- If for every such $D$ we have $Q(D) = Q'(V_1(D), \ldots, V_k(D))$, then $\text{certain}(Q, \mathbf{R}) = Q'$.

- But we may even look at a more general way of query answering by finding a rewriting $Q'$ so that

  $$\text{certain}(Q, \mathbf{R}) = Q'(\mathbf{R})$$

# Query rewriting: a naive algorithm

- Given:
  - ○ conjunctive queries $V_1, \ldots, V_k$ over schema $\sigma$
  - ○ a query $Q$ over $\sigma$
- Algorithm:
  - ○ guess a query $Q'$ over the views
  - ○ Unfold $Q'$ in terms of the views
  - ○ Check if the unfolding is contained in $Q$
- If one unfolding is equivalent to $Q$, then $Q'$ is a rewriting
- Otherwise take the union of all unfoldings contained in $Q$
  - – it is a maximally contained rewriting

# Why is it not an algorithm yet?

- Problem 1: we do not yet know how to test containment and equivalence.
  - But we shall learn soon
- Problem 2: the guess stage.
  - There are infinitely many conjunctive queries.
  - We cannot check them all.
  - Solution: we only need to check a few.

# Guessing rewritings

- A basic fact:
  - If there is a rewriting of $Q$ using $V_1, \ldots, V_k$, then there is a rewriting with no more conjuncts than in $Q$.
  - E.g., if $Q(x)\ \text{:-}\ R(x,y), R(x,1), S(x,z), S(x,1)$, we only need to check conjunctive queries over $V$ with at most 4 conjuncts.
- Moreover, maximally contained rewriting is obtained as the union of all conjunctive rewritings of length of $Q$ or less.
- Complexity: enumerate all candidates (exponentially many); for each an NP (or exponential) algorithm. Hence exponential time is required.
- Cannot lower this due to NP-completeness.

# Containment and optimization of conjunctive queries

- Reminder:

  conjunctive queries<br>
  = SPJ queries<br>
  = rule-based queries<br>
  = simple SELECT-FROM-WHERE SQL queries<br>
  (only AND and equality in the WHERE clause)

- Extremely common, and thus special optimization techniques have been developed

- Reminder: for two relational algebra expressions $e_1, e_2$, $e_1 = e_2$ is undecidable.

- But for conjunctive queries, even $e_1 \subseteq e_2$ is decidable.

- Main goal of optimizing conjunctive queries: reduce the number of joins.

# Optimization of conjunctive queries: an example

- Given a relation $R$ with two attributes $A, B$
- Two SQL queries:

  Q1

  ```
  SELECT R1.B, R1.A
  FROM R R1, R R2
  WHERE R2.A=R1.B
  ```

  Q2

  ```
  SELECT R3.A, R1.A
  FROM R R1, R R2, R R3
  WHERE R1.B=R2.B AND R2.B=R3.A
  ```

- Are they equivalent?
- If they are, we saved one join operation.
- In relational algebra:

$$Q_1 = \pi_{2,1}(\sigma_{2=3}(R \times R))$$

$$Q_2 = \pi_{5,1}(\sigma_{2=4 \wedge 4=5}(R \times R \times R))$$

# Optimization of conjunctive queries cont'd

- Are $Q_1$ and $Q_2$ equivalent?
- If they are, we cannot show it by using equivalences for relational algebra expression.
- Because: they don't decrease the number of $\bowtie$ or $\times$ operators, but $Q_1$ has 1 join, and $Q_2$ has 2.
- But $Q_1$ and $Q_2$ are equivalent. How can we show this?
- But representing queries as databases. This representation is very close to rule-based queries.

$$\begin{aligned} Q_1(x,y) &\ :-\ R(y,x), R(x,z) \\ Q_2(x,y) &\ :-\ R(y,x), R(w,x), R(x,u) \end{aligned}$$

# Conjunctive queries into tableaux

- Tableau: representing of a conjunctive query as a database
- We first consider queries over a single relation
- $Q_1(x, y)$ :– $R(y, x), R(x, z)$
- $Q_2(x, y)$ :– $R(y, x), R(w, x), R(x, u)$
- Tableaux:

| A | B | |
|---|---|---|
| y | x | |
| x | z | |
| x | y | ← answer line |

| A | B | |
|---|---|---|
| y | x | |
| w | x | |
| x | u | |
| x | y | ← answer line |

- Variables in the answer line are called distinguished

# Tableau homomorphisms

- A homomorphism of two tableaux $f : T_1 \to T_2$ is a mapping

$$f : \{\text{variables of } T_1\} \to \{\text{variables of } T_2\} \ \bigcup \ \{\text{constants}\}$$

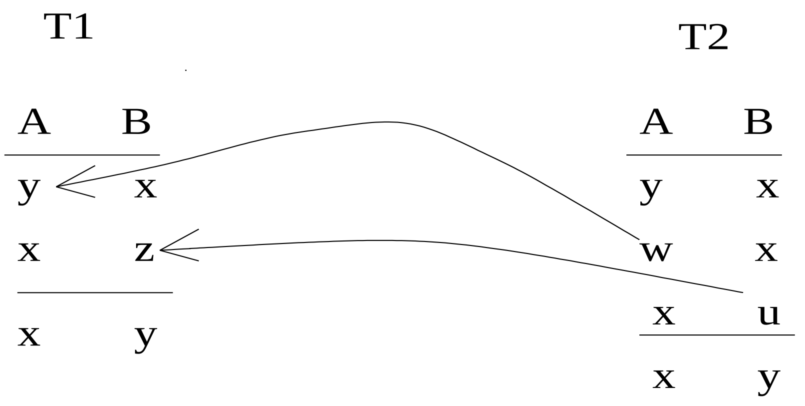
- For every distinguished $x$, $f(x) = x$
- For every row $x_1, \ldots, x_k$ in $T_1$, $f(x_1), \ldots, f(x_k)$ is a row of $T_2$
- Query containment: $Q \subseteq Q'$ if $Q(D) \subseteq Q'(D)$ for every database $D$
- Homomorphism Theorem: Let $Q, Q'$ be two conjunctive queries, and $T, T'$ their tableaux. Then

$$Q \subseteq Q'$$

if and only if

there exists a homomorphism $f : T' \to T$

# Applying the Homomorphism Theorem: $Q_1 = Q_2$



T1

| A | B |
|---|---|
| y | x |
| x | z |
| x | y |

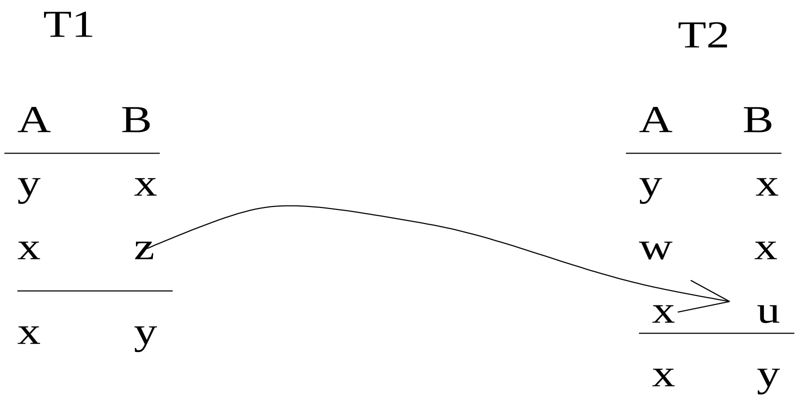
T2

| A | B |
|---|---|
| y | x |
| w | x |
| x | u |
| x | y |

f(x)=x, f(y)=y

f(u)=z, f(w)=y

Hence Q1 $\subseteq$ Q2



T1

| A | B |
|---|---|
| y | x |
| x | z |
| x | y |

T2

| A | B |
|---|---|
| y | x |
| w | x |
| x | u |
| x | y |

f(x)=x, f(y)=y

f(z)=u

Hence Q2 $\subseteq$ Q1

# Applying the Homomorphism Theorem: Complexity

- Given two conjunctive queries, how hard is it to test if $Q_1 = Q_2$?
- it is easy to transform them into tableaux, from either SPJ relational algebra queries, or SQL queries, or rule-based queries
- But testing the existence of a homomorphism between two tableaux is hard: NP-complete. Thus, a polynomial algorithm is unlikely to exists.
- However, queries are small, and conjunctive query optimization is possible in practice.

# Minimizing conjunctive queries

- Goal: given a conjunctive query $Q$, find an equivalent conjunctive query $Q'$ with the minimum number of joins.
- Assume $Q$ is
$$Q(\vec{x}) \ :\!-\ R_1(\vec{u}_1), \ldots, R_k(\vec{u}_k)$$
- Assume that there is an equivalent conjunctive query $Q'$ of the form
$$Q'(\vec{x}) \ :\!-\ S_1(\vec{v}_1), \ldots, S_l(\vec{v}_l)$$
with $l < k$
- Then $Q$ is equivalent to a query of the form
$$Q'(\vec{x}) \ :\!-\ R_{i_1}(\vec{u}_{i_1}), \ldots, R_{i_l}(\vec{u}_{i_l})$$
- In other words, to minimize a conjunctive query, one has to delete some subqueries on the right of :–

# Minimizing conjunctive queries cont'd

- Given a conjunctive query $Q$, transform it into a tableau $T$
- Let $Q'$ be a minimal conjunctive query equivalent to $Q$. Then its tableau $T'$ is a subset of $T$.
- Minimization algorithm:

```
$T' := T$
repeat until no change
    choose a row $t$ in $T'$
    if there is a homomorphism $f : T' \to T' - \{t\}$
        then $T' := T' - \{t\}$
end
```

- Note: if there exists a homomorphism $T' \to T' - \{t\}$, then the queries defined by $T'$ and $T' - \{t\}$ are equivalent. Because: there is always a homomorphism from $T' - \{t\}$ to $T'$. (Why?)

# Minimizing SPJ/conjunctive queries: example

- $R$ with three attributes $A, B, C$
- SPJ query

$$Q \quad = \quad \pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Equivalently, a SQL query:

```
SELECT R1.A, R2.B, R3.C
FROM R R1, R R2, R R3
WHERE R1.B=4 AND R2.A=R3.A AND
      R3.B=4 AND R2.B=R1.B
```

- Translate into a conjunctive query:

$$\exists x_1, z_1, z_2\, (R(x, 4, z_1) \wedge R(x_1, 4, z_2) \wedge R(x_1, 4, z) \wedge y = 4)$$

- Rule-based:

$$Q(x, y, z) \;\; \text{:--} R(x, 4, z_1), R(x_1, 4, z_2), R(x_1, 4, z), y = 4$$

# Minimizing SPJ/conjunctive queries cont'd

- Tableau $T$:

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x_1$ | $4$ | $z_2$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Minimization, step 1: is there a homomorphism from $T$ to

| A | B | C |
|---|---|---|
| $x_1$ | $4$ | $z_2$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Answer: No. For any homomorphism $f$, $f(x) = x$ (why?), thus the image of the first row is not in the small tableau.

# Minimizing SPJ/conjunctive queries cont'd

- Step 2: Is $T$ equivalent to

$$\begin{array}{ccc} \mathsf{A} & \mathsf{B} & \mathsf{C} \\ \hline x & 4 & z_1 \\ x_1 & 4 & z \\ \hline x & 4 & z \end{array}$$

- Answer: Yes. Homomorphism $f$: $f(z_2) = z$, all other variables stay the same.

- The new tableau is not equivalent to

$$\begin{array}{ccc} \mathsf{A} & \mathsf{B} & \mathsf{C} \\ \hline x & 4 & z_1 \\ \hline x & 4 & z \end{array} \qquad \text{or} \qquad \begin{array}{ccc} \mathsf{A} & \mathsf{B} & \mathsf{C} \\ \hline x_1 & 4 & z \\ \hline x & 4 & z \end{array}$$

- Because $f(x) = x, f(z) = z$, and the image of one of the rows is not present.

# Minimizing SPJ/conjunctive queries cont'd

- Minimal tableau:

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Back to conjunctive query:

$$Q'(x, y, z) \ :- \ R(x, y, z_1), R(x_1, y, z), y = 4$$

- An SPJ query:

$$\pi_{AB}(\sigma_{B=4}(R)) \ \bowtie \ \pi_{BC}(\sigma_{B=4}(R))$$

-
  ```
  SELECT R1.A, R1.B, R2.C
  FROM R R1, R R2
  WHERE R1.B=R2.B AND R1.B=4
  ```

# Review of the journey

- We started with

$$\pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Translated into a conjunctive query
- Built a tableau and minimized it
- Translated back into conjunctive query and SPJ query
- Applied algebraic equivalences and obtained

$$\pi_{AB}(\sigma_{B=4}(R)) \;\bowtie\; \pi_{BC}(\sigma_{B=4}(R))$$

- Savings: one join.

# All minimizations are equivalent

- Let $Q$ be a conjunctive query, and $Q_1$, $Q_2$ two conjunctive queries equivalent to $Q$
- Assume that $Q_1$ and $Q_2$ are both minimal, and let $T_1$ and $T_2$ be their tableaux.
- Then $T_1$ and $T_2$ are isomorphic; that is, $T_2$ can be obtained from $T_1$ by renaming of variables.
- That is, all minimizations are equivalent.
- In particular, in the minimization algorithm, the order in which rows are considered, is irrelevant.

# Equivalence of conjunctive queries: the general case

- So far we assumed that there is only one relation $R$, but what if there are many?
- Construct tableaux as before:

$$Q(x,y) \text{:–} B(x,y), R(y,z), R(y,w), R(w,y)$$

- Tableau:

B:

| A | B |
|---|---|
| x | y |

R:

| A | B |
|---|---|
| y | z |
| y | w |
| w | y |

x y

- Formally, a tableau is just a database where variables can appear in tuples, plus a set of distinguished variables.

# Tableaux and multiple relations

- Given two tableaux $T_1$ and $T_2$ over the same set of relations, and the same distinguished variables, a homomorphism $h : T_1 \rightarrow T_2$ is a mapping

  $$f : \{\text{variables of } T_1\} \;\rightarrow\; \{\text{variables of } T_2\}$$

  such that

  - $f(x) = x$ for every distinguished variable, and
  - for each row $\vec{t}$ in $R$ in $T_1$, $f(\vec{t})$ is in $R$ in $T_2$.

- **Homomorphism theorem**: let $Q_1$ and $Q_2$ be conjunctive queries, and $T_1, T_2$ their tableaux. Then

  $$Q_2 \subseteq Q_1$$
  if and only if
  there exists a homomorphism $f : T_1 \rightarrow T_2$

# Minimization with multiple relations

- The algorithm is the same as before, but one has to try rows in different relations. Consider homomorphism $f(z) = w$, and $f$ is the identity for other variables. Applying this to the tableau for $Q$ yields

| B: |  | R: |  |
|---|---|---|---|
| A | B | A | B |
| x | y | y | w |
|  |  | w | y |
| x | y |  |  |

- This cannot be further reduced, as for any homomorphism $f$, $f(x) = x$, $f(y) = y$.

- Thus $Q$ is equivalent to

$$Q'(x,y) \;\text{:-}\; B(x,y), R(y,w), R(w,y)$$

- One join is eliminated.

# Query rewriting

- Recall the algorithm, for a given $Q$ and view definitions $V_1, \ldots, V_k$:
  - ○ Look at all rewritings that have as at most as many joins as $Q$
  - ○ check if they are contained in $Q$
  - ○ take the union of those that are
- This is the maximally contained rewriting
- There are algorithms that prune the search space and make looking for rewritings contained in $Q$ more efficient
  - ○ the bucket algorithm
  - ○ MiniCon

# How hard is it to answer queries using views?

- Setting: we now have an actual content of the views.
- As before, a query is $Q$ posed against $D$, but must be answered using information in the views.
- Suppose $I_1, \ldots, I_k$ are view instances. Two possibilities:
  - Exact mappings: $I_j = V_j(D)$
  - Sound mappings: $I_j \subseteq V_j(D)$
- We need certain answers for given $\mathcal{I} = (I_1, \ldots, I_k)$:

$$\text{certain}_{exact}(Q, \mathcal{I}) = \bigcap_{D:\ I_j = V_j(D) \text{ for all } j} Q(D)$$

$$\text{certain}_{sound}(Q, \mathcal{I}) = \bigcap_{D:\ I_j \subseteq V_j(D) \text{ for all } j} Q(D)$$

# How hard is it to answer queries using views?

- If certain$_{exact}(Q, \mathcal{I})$ or certain$_{sound}(Q, \mathcal{I})$ are impossible to obtain, we want maximally contained rewritings:
  - $Q'(\mathcal{I}) \subseteq$ certain$_{exact}(Q, \mathcal{I})$, and
  - if $Q''(\mathcal{I}) \subseteq$ certain$_{exact}(Q, \mathcal{I})$ then $Q''(\mathcal{I}) \subseteq Q'(\mathcal{I})$
  - (and likewise for $sound$)
- How hard is it to compute this from $\mathcal{I}$?
- In databases, we reason about complexity in two ways:
  - The big-O notation ($O(n \log n)$ vs $O(n^2)$ vs $O(2^n)$)
  - Complexity-theoretic notions: PTIME, NP, DLOGSPACE, etc
- Advantage of complexity-theoretic notions: if you have a $O(2^n)$ algorithm, is it because the problem is inherently hard, or because we are not smart enough to come up with a better algorithm (or both)?

# Complexity classes: what you always wanted to know but never dared to ask

- Or a $5/5$-introduction: a five minute review that tells you what are likely to remember 5 years after taking a complexity theory course.
- The big divide: PTIME (computable in polynomial time, i.e. $O(n^k)$ for some fixed $k$)
- Inside PTIME: tractable queries (although high-degree polynomial are intractable)
- Outside PTIME: intractable queries (efficient algorithms are unlikely)
- Way outside PTIME: provably intractable queries (efficient algorithms do not exist)
  - EXPTIME: $c^n$-algorithms for a constant $c$. Could still be ok for not very large inputs
  - Even further – 2-EXPTIME: $c^{c^n}$. Cannot be ok even for small inputs (compare $2^{10}$ and $2^{2^{10}}$).

# Inside PTIME

$$\mathsf{AC}^0 \subsetneq \mathsf{TC}^0 \subseteq \mathsf{NC}^1 \subseteq \mathsf{DLOG} \subseteq \mathsf{NLOG} \subseteq \mathsf{PTIME}$$

- $\mathsf{AC}^0$: very efficient parallel algorithms (constant time, simple circuits)
  - relational calculus
- $\mathsf{TC}^0$: very efficient parallel algorithms in a more powerful computational model with counting gates
  - basic SQL (relational calculus/grouping/aggregation)
- $\mathsf{NC}^1$: efficient parallel algorithms
  - regular languages
- DLOG: very little – $O(\log n)$ – space is required
  - SQL + (restricted) transitive closure
- NLOG: $O(\log n)$ space is required if nondeterminism is allowed
  - SQL + transitive closure (as in the SQL3 standard)

# Beyond PTIME

$$\mathrm{PTIME} \subseteq \left\{ \begin{array}{l} \mathrm{NP} \\ \mathrm{coNP} \end{array} \right\} \subseteq \mathrm{PSPACE}$$

- PTIME: can solve a problem in polynomial time
- NP: can check a given candidate solution in polynomial time
  - another way of looking at it: guess a solution, and then verify if you guessed it right in polynomial time
- coNP: complement of NP – verify that all "reasonable" candidates are solutions to a given problem.
  - Appears to be harder than NP but the precise relationship isn't known
- PSPACE: can be solved using memory of polynomial size (but perhaps an exponential-time algorithm)

# Complete problems

- These are the hardest problems in a class.
- If our problem is as hard as a complete problem, it is very unlikely it can be done with lower complexity.
- For NP:
  - SAT (satisfiability of Boolean formulae)
  - many graph problems (e.g. 3-colourability)
  - Integer linear programming etc
- For PSPACE:
  - Quantified SAT
  - Two XML DTDs are equivalent

# Complexity of query answering

- We want the complexity of finding

$$\text{certain}_{exact}(Q, \mathcal{I}) \quad \text{or} \quad \text{certain}_{sound}(Q, \mathcal{I})$$

  in terms of the size of $\mathcal{I}$

- If all view definitions are conjunctive queries and $Q$ is a relational algebra or a SQL query, then the complexity is coNP.
- (blackboard)
- This is too high!
- If all view definitions are conjunctive queries and $Q$ is a conjunctive query, then the complexity is PTIME.
  - Because: the maximally contained rewriting computes certain answers!

# Complexity of query answering

|  | query language |  |  |
|---|---|---|---|
| view language | CQ | CQ$^{\neq}$ | relational calculus |
| CQ | ptime | coNP | undecidable |
| CQ$^{\neq}$ | ptime | coNP | undecidable |
| relational calculus | undecidable | undecidable | undecidable |

CQ – conjunctive queries

CQ$^{\neq}$ – conjunctive queries with inequalities
(for example, $Q(x) \text{ :- } R(x, y), S(y, z), x \neq z$ )

# Complexity of query answering: coNP-completeness idea

- Start with a graph $G$ – this is our instance
- $D$ is $G$ together with a colouring, with 3 colours; each node is assigned one colour.
- $Q$ asks if we have an edge $(a, b)$ with $a \neq b$ and $a, b$ of the same colour.
- If $G$ is not 3-colourable, then every instance $D$ would satisfy $Q$
- Otherwise, if $G$ is 3-colourable, we can find extensions that are and that are not 3-colourable – hence certain answers are empty.
- Thus if we can compute certain answers, we can test non-3-colourability $\Rightarrow$ coNP-completeness.