Problem 1 (20 marks) Suppose we have two relations R(X, Y) and S(X, Y). Give an example of an SQL query over R and S so that:

- 1. If R and S contain no nulls, it correctly computes $\pi_X(R) \pi_X(S)$.
- 2. It does not use the IS NULL condition.
- 3. For some relations R and S so that $\pi_X(R) = \{1, 2\}$ and $\pi_X(S) = \{1\}$ (in particular, there are no nulls in attribute X), it produces an empty table.

Problem 2 (30 marks) Consider a data exchange setting where the source schema contains two relations $S_1(A, B)$ and $S_2(B, C)$ and the target schema contains two relations $T_1(A, D)$ and $T_2(D, C)$. The mapping is given by

$$T_1(x,z), T_2(z,y), T_2(z,z) := S_1(x,x'), S_2(x',y)$$

Assume that we have the following source:

	Α	В		В	С	
	1	2		2	2	ĺ
S_1 :	1	3	S_2 :	3	2	
	1	5		7	3	
	2	6		5	1	ļ

- (10 marks) Construct the canonical universal solution.
- (10 marks) Consider the query $Q = \pi_{AC}(T_1 \bowtie T_2)$. What are the certain answers to this query for the given source? Can you compute them using the canonical universal solution? If so, how?
- (10 marks) Answer the previous two questions when the source query in the mapping is changed to be $\exists x' \ S_1(x,x') \land S_2(x',y)$.

Problem 3 (10 marks) Using the schemas, the mapping, and the instances of Problem 1, give an example of a relational algebra (or calculus) query Q and two universal solutions U_1 and U_2 such that $Q(U_1) \neq Q(U_2)$.

You have to explain why U_1 and U_2 you constructed are universal, and compute both $Q(U_1)$ and $Q(U_2)$.

Problem 4 (40 marks) Consider mappings Σ and Δ shown on slide 14 from the "Schema mappings" set.

Let mapping Γ between σ and ω be given by

 $\forall n \exists y \forall c (Takes(n, c) \rightarrow Enroll(y, c))$

Does Γ define the composition of Σ and Δ ?

If it does, prove it; if it does not, give a counterexample.