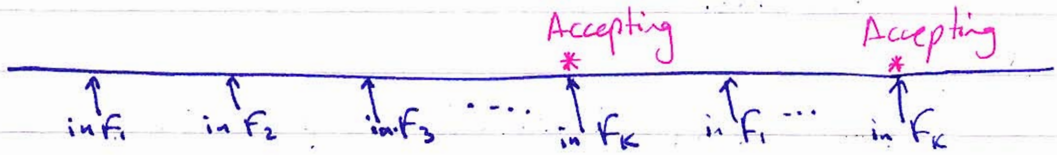
Multi - Büchi Automaton

$(Q, Q_0, \delta, F_1 \dots F_k)$

a run ρ is accepting iff $Inf(\rho) \cap F_i \neq \phi \ \forall i$.



(Like before: product construction) $\Downarrow$

Büchi Automaton

$\#$ states $\leq 2^{|\varphi|}$

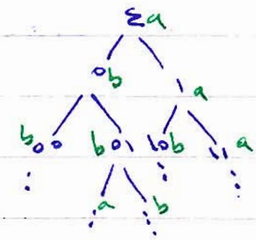
$\#$ states $\leq 2^{|\varphi|} \cdot \underbrace{O(|\varphi|)}$ $\rightarrow$ the counter each subformula of φ.

$= 2^{O(|\varphi|)}$

Infinite Trees (Binary)

11/3/08

Tree Domain: $\{0,1\}^*$



Infinite tree over Σ

$\rightarrow$ Given by a labelling $\lambda : \{0,1\}^* \rightarrow \Sigma$

(Goal: Look at S2S: Theory of two successors)

$\star$ $S2S = Th_{MSO}(\{0,1\}^*, Succ_0, Succ_1)$

$Succ_0 = \{(s, s0) \mid s \in \{0,1\}^*\}$

$Succ_1 = \{(s, s1) \mid s \in \{0,1\}^*\}$

$S1S = Th_{MSO}(\mathbb{N}, Succ) = Th_{MSO}(0^*, Succ_0)$

Decidable

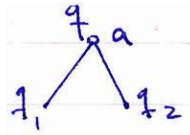
$f : 0^* \rightarrow \mathbb{N}$ $\quad f(0^i) = i$ $\quad f(0^i 0) = i+1$

$\star$ Rabin Theorem $\rightarrow$ S2S is Decidable.

Automata:

Top_down => non_deterministic

$\mathcal{A}\,(Q,\ Q_0,\ \delta : Q \times \Sigma \rightarrow 2^{Q\times Q},\ \underset{\text{acceptance condition}}{Acc})$



$(q_1, q_2) \in \delta(q, a)$

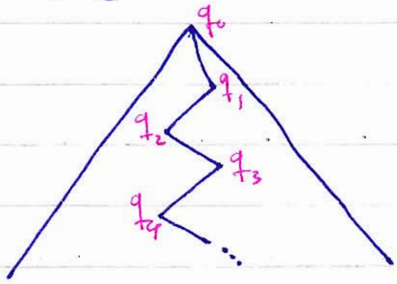
<u>Run</u> $\rho : \{0,1\}^* \rightarrow Q$

1) $\rho(\varepsilon) \in Q_0$

2) $(\rho(s.0), \rho(s.1)) \in \delta(\rho(s), \lambda(s))$

Let t be an infinite branch of a tree. T. Then

ρ_t can be viewed as an <u>ω-word</u> over Q.



$\rho_t = q_0 q_1 q_2 \ldots$

ρ is <u>accepting</u> if for <u>every</u> infinite branch t,

ρ_t satisfies ~~Acc~~ Acc.

1. Acc : Muller Condition $\mathcal{F} \subseteq 2^Q$

$\forall t \ \ Inf(\rho_t) \in \mathcal{F}$

ω-regular tree languages are those accepted by a tree automaton with a Muller acceptance condition.

2. Acc : Büchi Condition $F \subseteq Q$

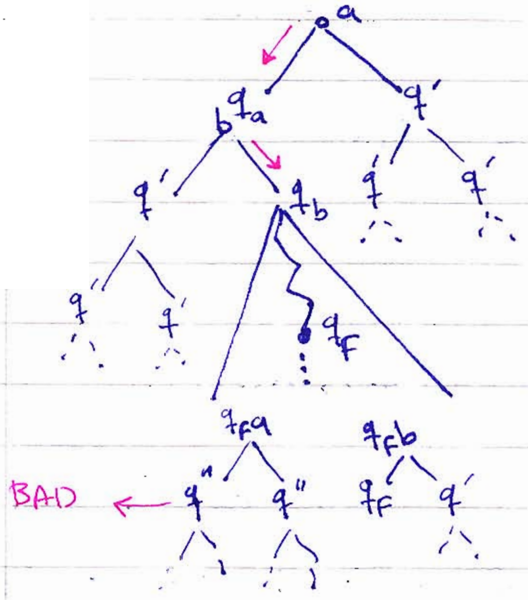
$\forall t \ \ Inf(\rho_t) \cap F \neq \emptyset$

Büchi-tree languages are those accepted by a tree automaton with a Büchi condition.

Are strictly weaker than ω-regular tree languages.

Example

$L = \{ T \mid \exists \text{ branch } t \text{ with finitely many } a\text{'s} \}$ * Same: can't be accepted by det. Büchi.



non-det choices. (guess)

guess: no more a's!

$F = \{ q', q_F \}$

Büchi tree automaton.

$L' = \{ T \mid \forall \text{ branch } t \text{ there are only finitely many } a\text{'s} \}$

Not accepted by any non-deterministic Tree automaton with Büchi acceptance condition.
(So we work with Muller).

Closure Properties

- Union ✓
- Intersection Product Construction (just like ω-strings)
- Complementation Very Hard!

(Rabin) For every regular-tree language L accepted by an automaton $\mathcal{A}$, one can construct an automaton $\bar{\mathcal{A}}$ which accepts $\bar{L}$.

- different automata (parity acceptance condition)
- games (memoryless ⇒ finite)
- Complement: Path finder wins.

Theorem

Every MSO sentence φ over $(\{0,1\}^*, Succ_0, Succ_1)$ can be converted (by an algorithm) to a tree automaton $\mathcal{A}_\varphi$ with a Muller acceptance condition such that:

$$T \models \varphi \iff T \text{ is accepted by } \mathcal{A}_\varphi.$$

($\Leftarrow$) Code the run

$<^*$ – desendent relation

$\alpha(X) = \forall x \forall y (X(x) \wedge X(y) \to x <^* y \vee y <^* x) \wedge$

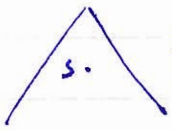
X a branch $\quad \forall x (X(x) \to \exists y (X(y) \wedge (Succ_0(x,y) \vee Succ_1(x,y)))$

($\Rightarrow$) Induction

$\cup, \cap, \neg, \exists \to$ non-det guess

φ MSO sentence over $(\{0,1\}^*, Succ_0, Succ_1)$

$\mathcal{A}_\varphi$. over $\{0,1\}^*, Succ_0, Succ_1$ $\quad$ Boolean Combination $\exists X \Psi(X)$



$\begin{cases} a & s \in X \\ b & s \notin X \end{cases}$ $\quad T_X$ $\qquad A_\Psi$ accepts T_X $\Updownarrow$ $T_X \models \Psi(X)$

$L(\mathcal{A}_\varphi) \neq \emptyset$?

Is decidable. and NP-complete.