Probabilistic Sensor Models*

Beam-based, Scan-based, Landmarks

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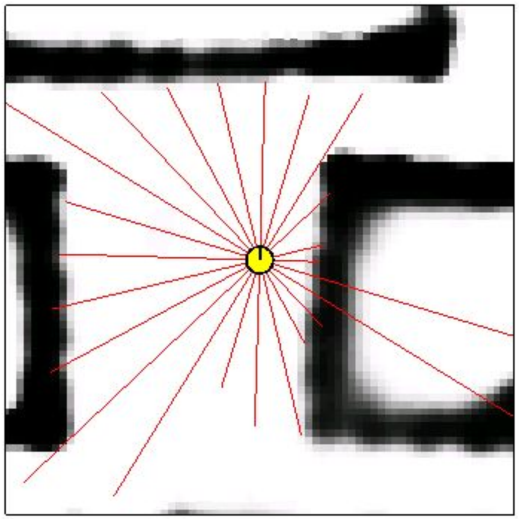
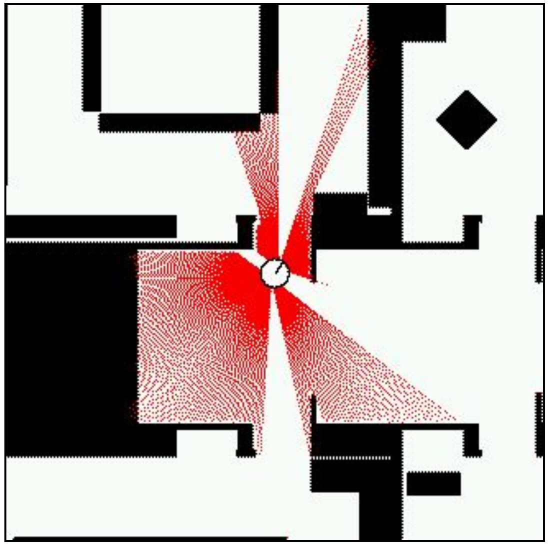
*Revised original slides that accompany the book: PR by Thrun, Burgard and Fox.

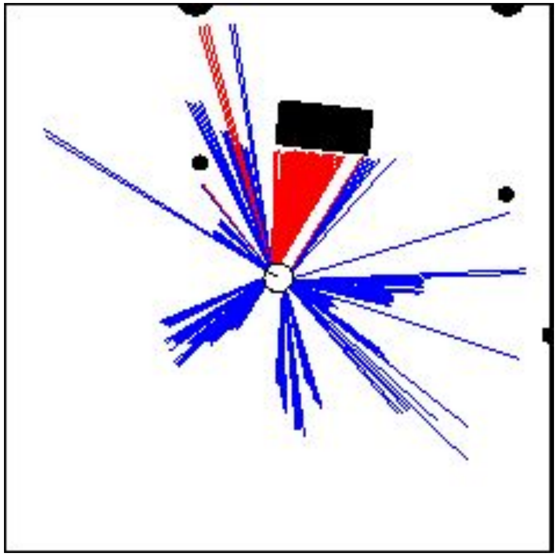
Sensors for Mobile Robots

- Contact sensors:
  - Bumpers
- Internal sensors:
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors:
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors:
  - Cameras
- Satellite-based sensors:
  - GPS

Proximity Sensors







- The central task is to determine *P(z|x)*, i.e., the probability of a measurement *z* given that the robot is at position *x*.

- **Question**: Where do the probabilities come from?

- **Approach**: Let us try to explain a measurement.

Beam-based Sensor Model

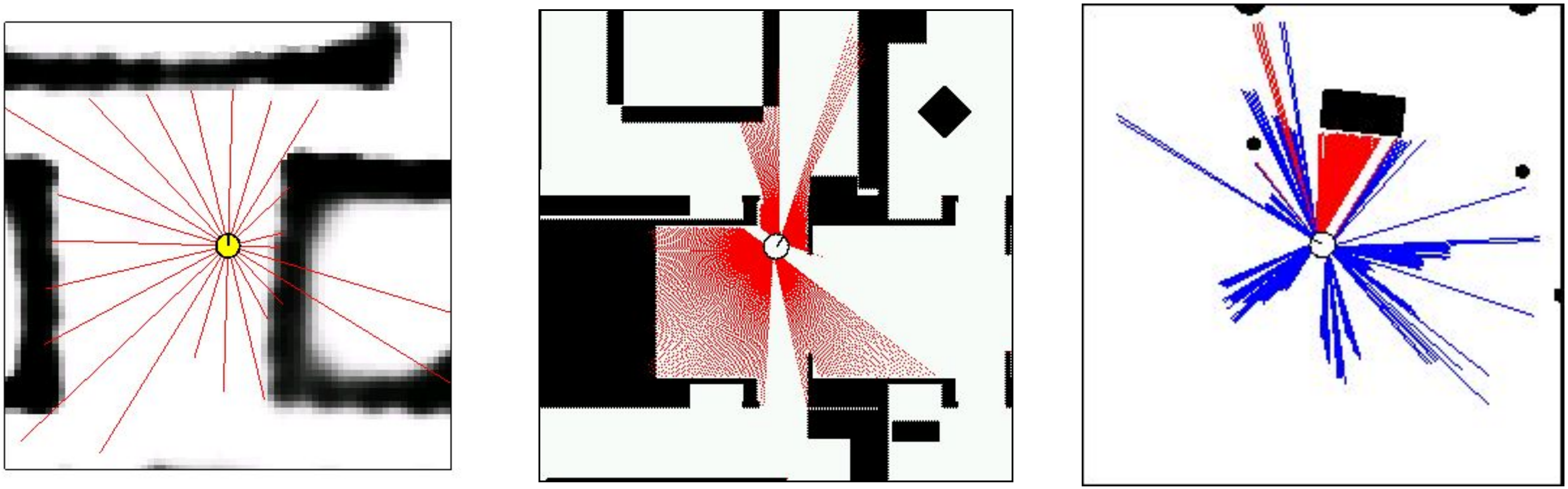
- Scan z consists of K measurements.

$$z = \{z_1, z_2, \ldots, z_K\}$$

- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

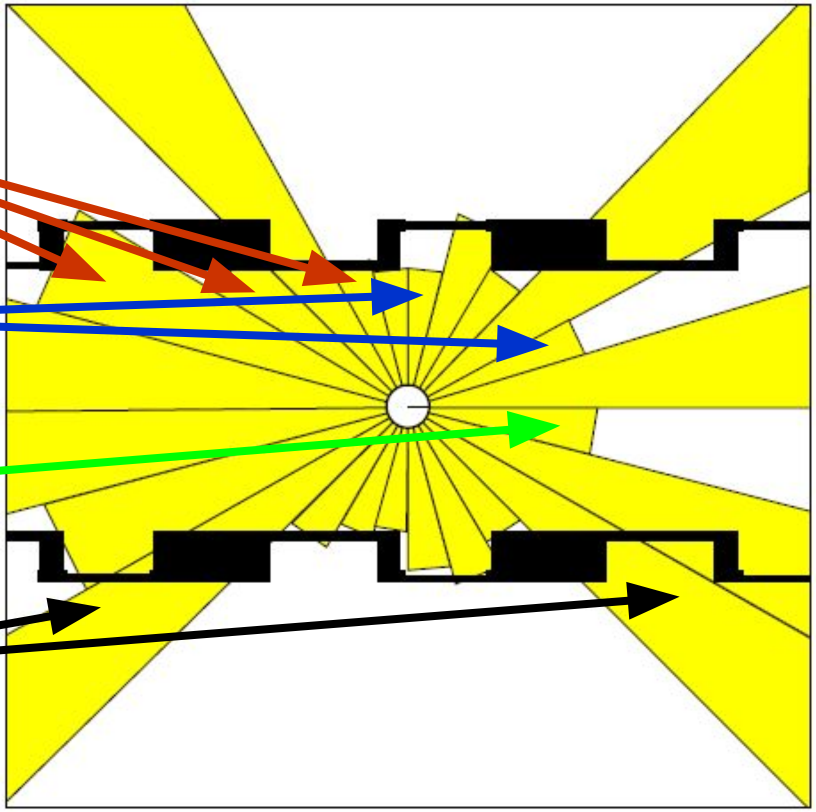
Beam-based Sensor Model



$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Typical Measurement Errors of Range Measurements

1. Beams reflected by obstacles
2. Beams reflected by persons / caused by crosstalk
3. Random measurements
4. Maximum range measurements



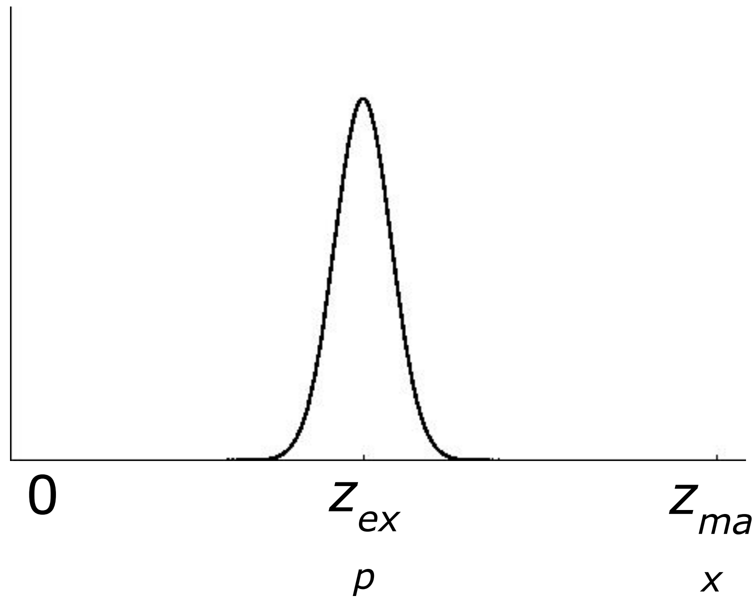
Proximity Measurement

- Measurement can be caused by:
  - a known obstacle.
  - cross-talk.
  - an unexpected obstacle (people, furniture, …).
  - missing all obstacles (total reflection, glass, …).

- Noise is due to uncertainty:
  - in measuring distance to known obstacle.
  - in position of known obstacles.
  - in position of additional obstacles.
  - whether obstacle is missed.

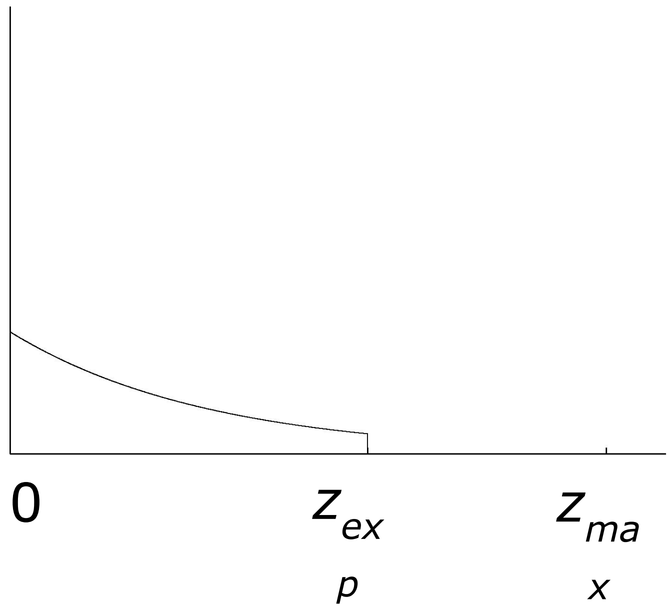
Beam-based Proximity Model

Measurement noise



$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z-z_{\exp})^2}{b}}$$

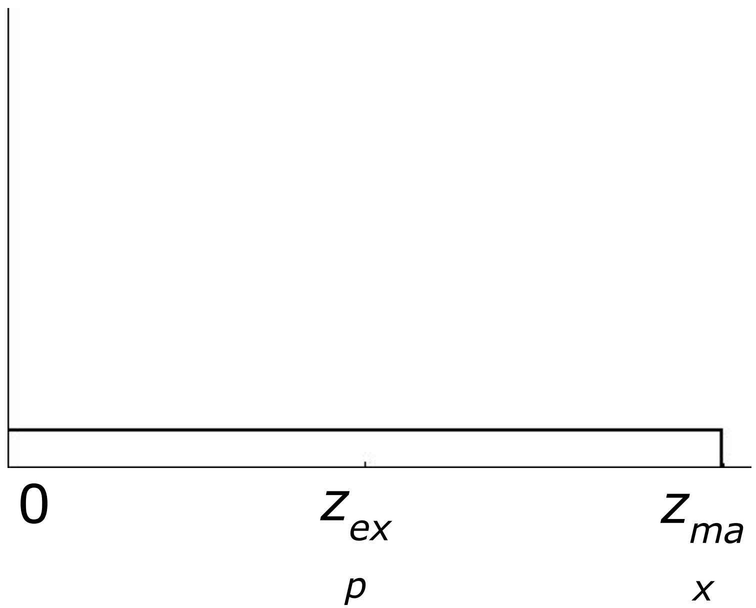
Unexpected obstacles



$$P_{\text{unexp}}(z \mid x, m) = \begin{cases} \eta\, \lambda\, e^{-\lambda z} & z < z_{\exp} \\ 0 & otherwise \end{cases}$$

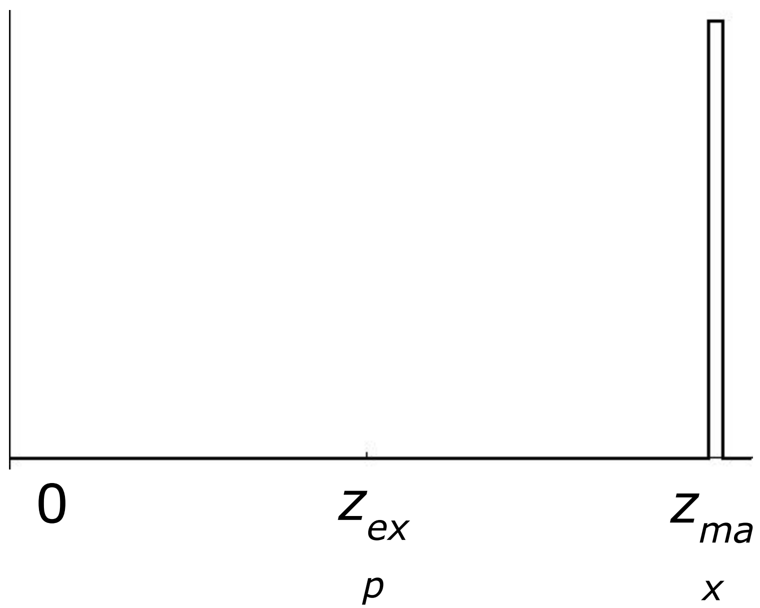
Beam-based Proximity Model

Random measurement



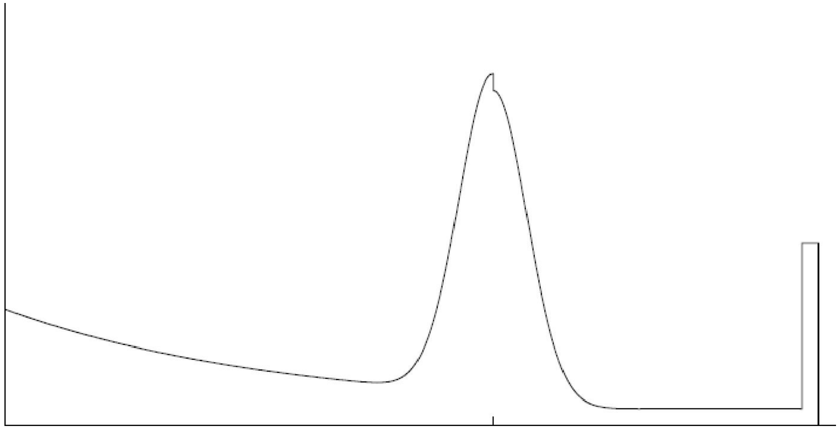
$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range



$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

Resulting Mixture Density



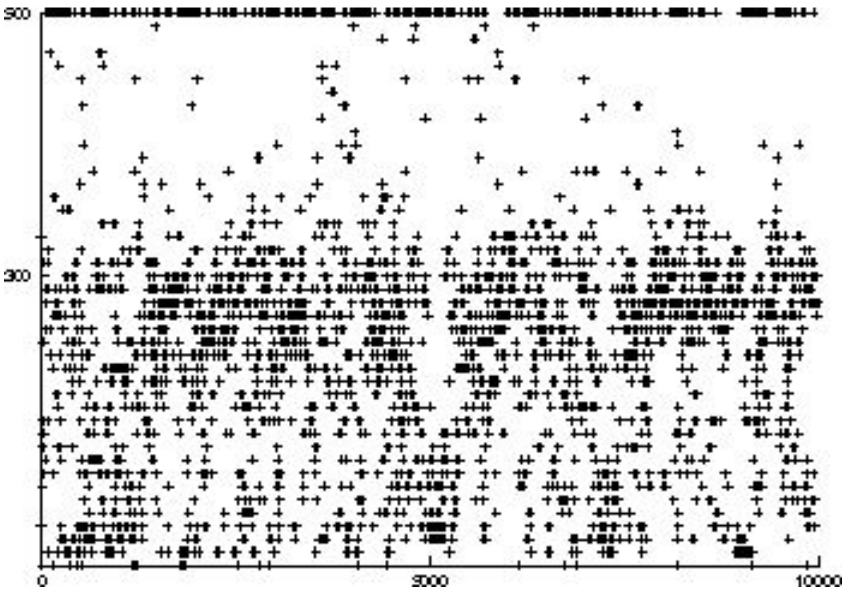
$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\max} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\max}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

How can we determine the model parameters?

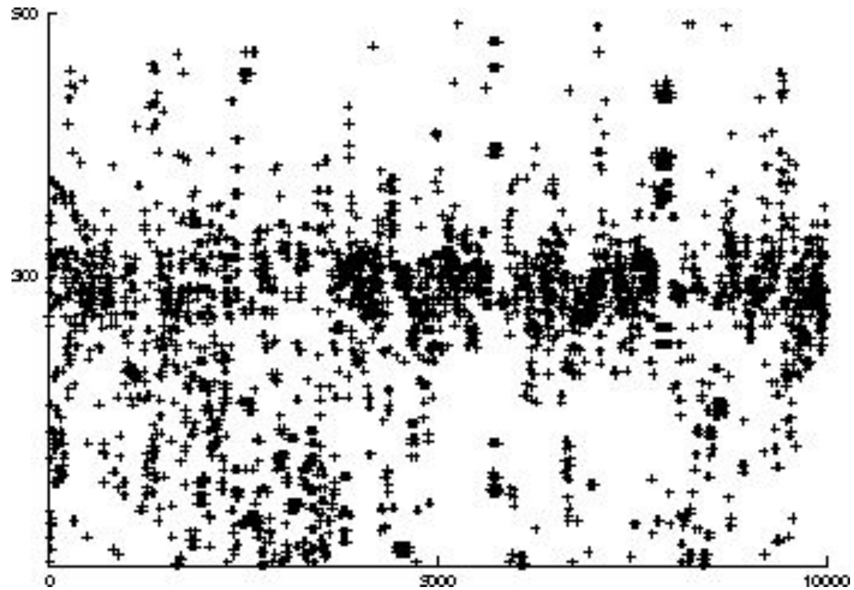
See Table 6.2.

Raw Sensor Data

Measured distances for expected distance of 300 cm.



Sonar



Laser

Approximation

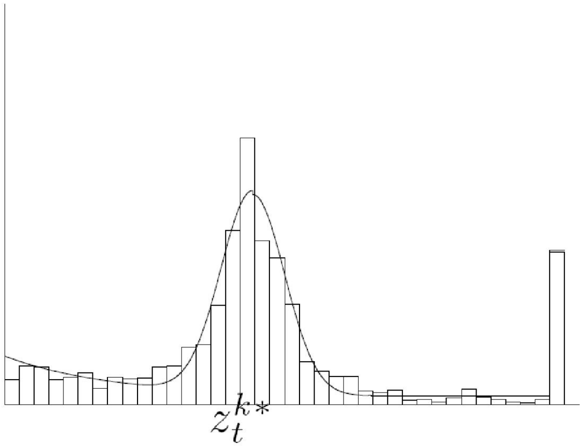
- Maximize log likelihood of the data:

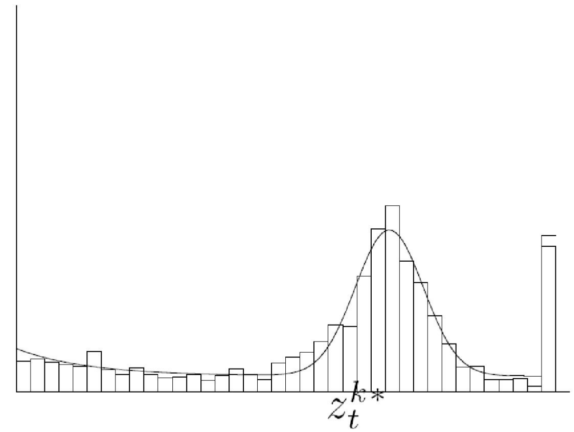
$$P(z \mid z_{\exp})$$

- Search space of n-1 parameters.
  - Hill climbing
  - Gradient descent
  - Genetic algorithms
  - …

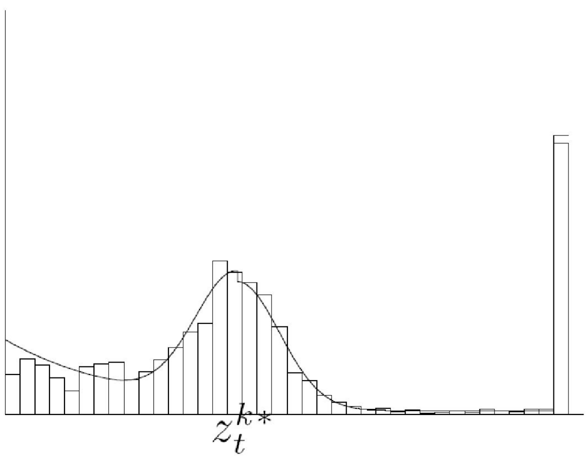
- Deterministically compute the n-th parameter to satisfy normalization constraint.

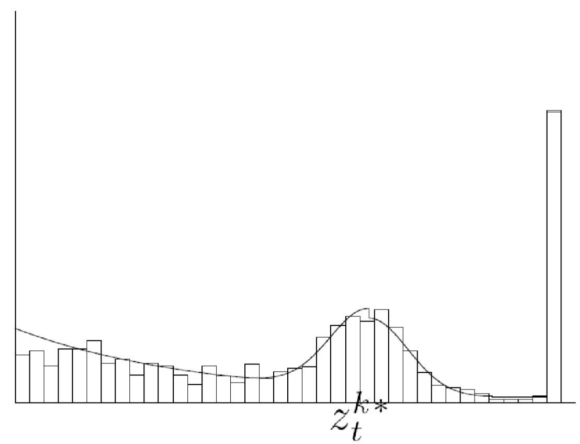
Approximation Results





Laser



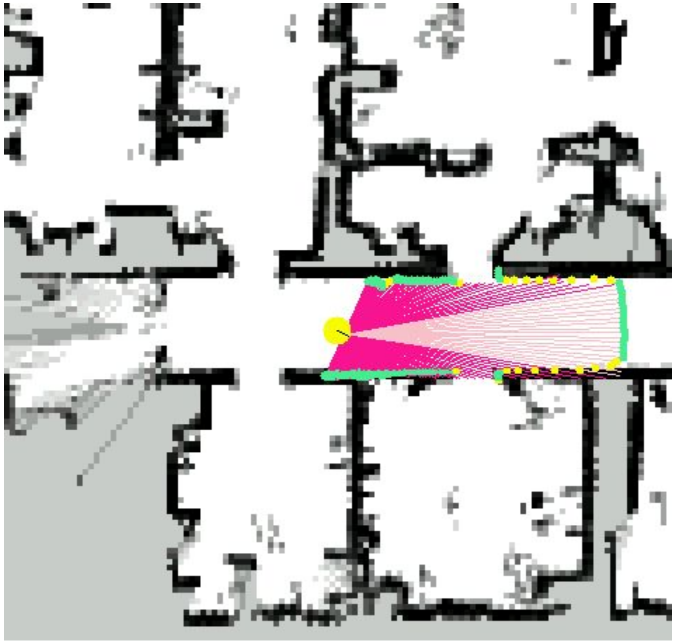


Sonar

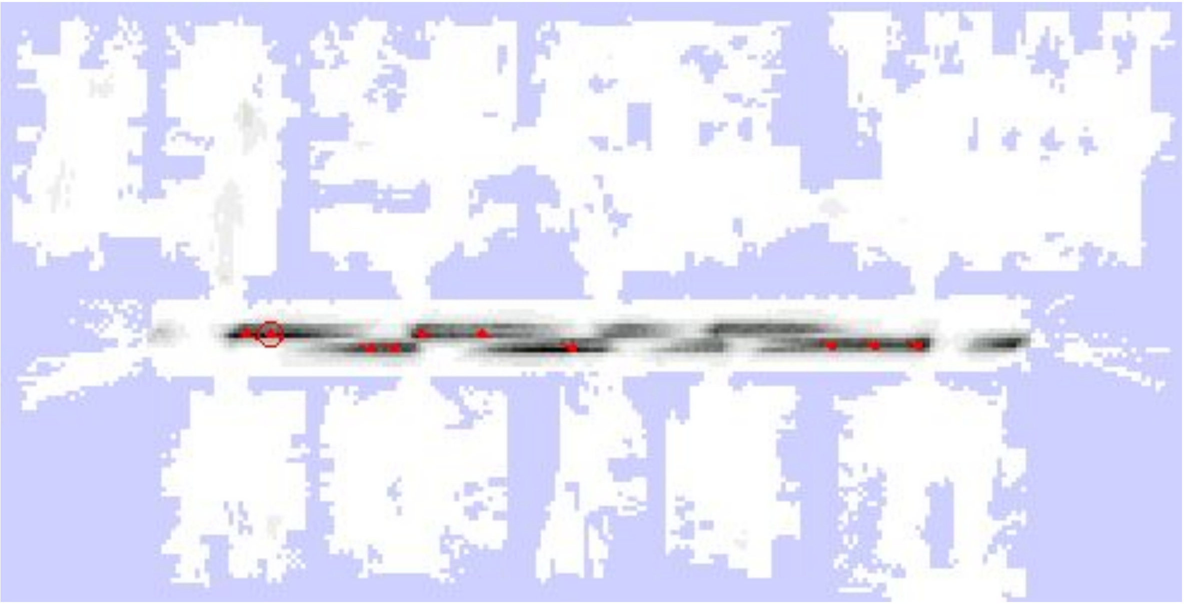
300cm

400cm

Example



z



$P(z|x,m)$

Summary Beam-based Model

- Assumes independence between beams.
- Models physical causes for measurements.
  - Mixture of densities for these causes.
  - Assumes independence between causes. Problem?

- Implementation:
  - Learn parameters based on real data. Different models for different angles at which the sensor beam hits the obstacle.
  - Expected distances by ray-tracing; distances precomputed.
  - Mathematical derivation: Section 6.3.3, PR.

- Limitations:
  - Lack of smoothness; multiple obstacles (clutter) in the beam region.
  - Incorrect belief of state, local minima in hill climbing approaches.
  - Computational expense of ray tracing; precomputation increases storage requirements.

Scan-based Model

- Beam-based model is:
  - not smooth for small obstacles and at edges.
  - not very efficient.

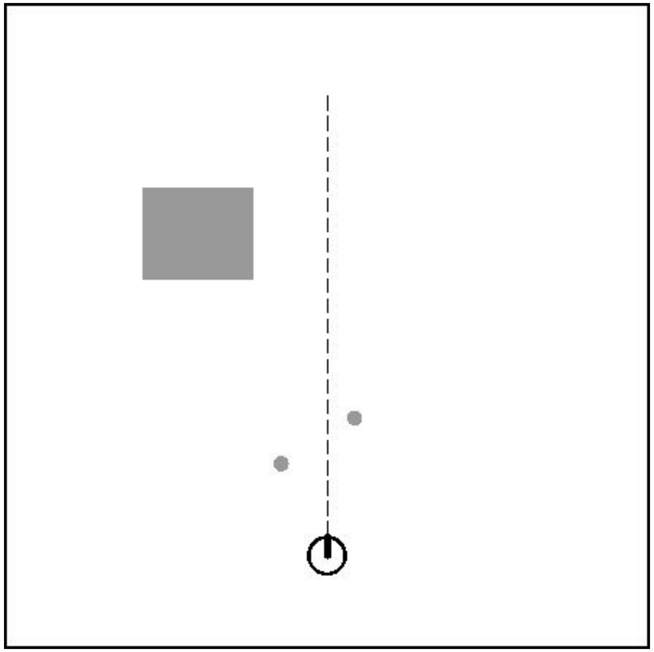
- Idea: Instead of following along the beam, just check the end point.

- Likelihood fields for range finders (Section 6.4, PR).

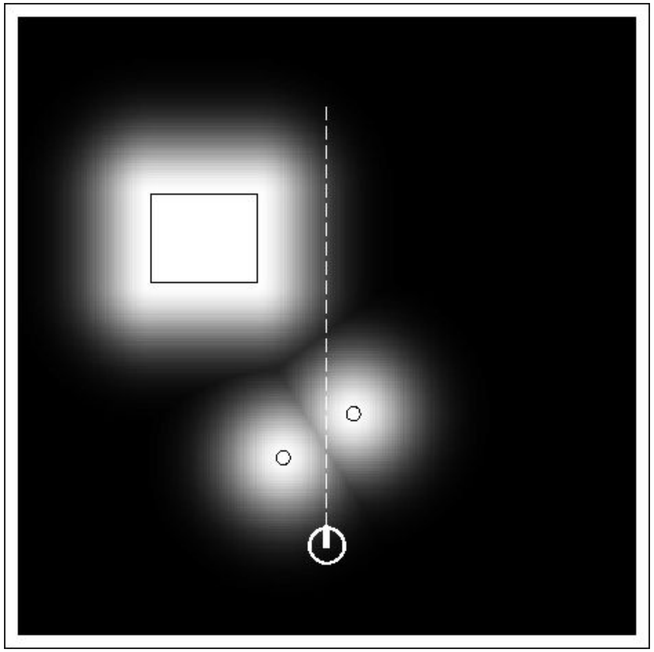
Scan-based Model

- Probability of a range finder scan given the location and the map $p(z_t \mid x_t, m)$ is based on:
  - Measurement noise: Gaussian distribution with mean at distance to closest obstacle.
  - Unexplained measurements: uniform distribution for random measurements.
  - Failures: a point mass distribution for max range measurements.
- Desired probability integrates three distributions assuming independence between the components.
- Likelihood field: darker a location, less likely it is to contain an obstacle.
- See algorithm in Table 6.3 and figures in Section 6.4.

Example

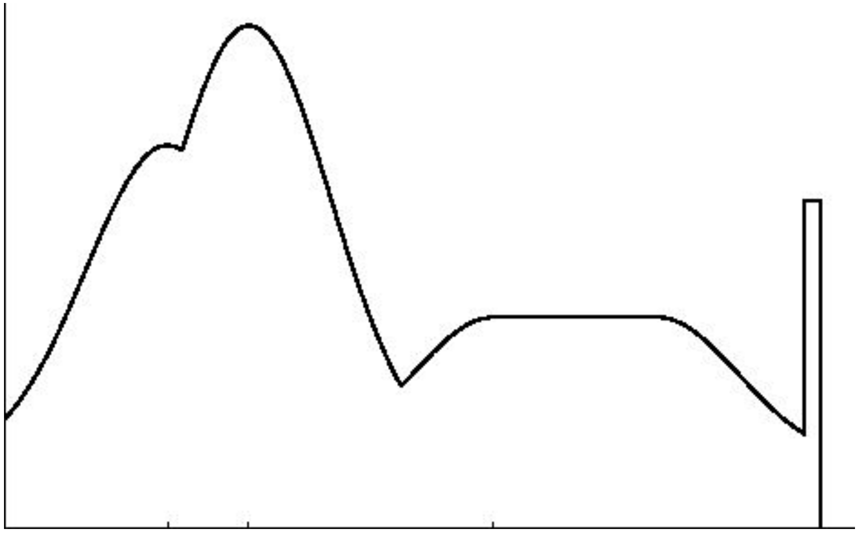


Map *m*

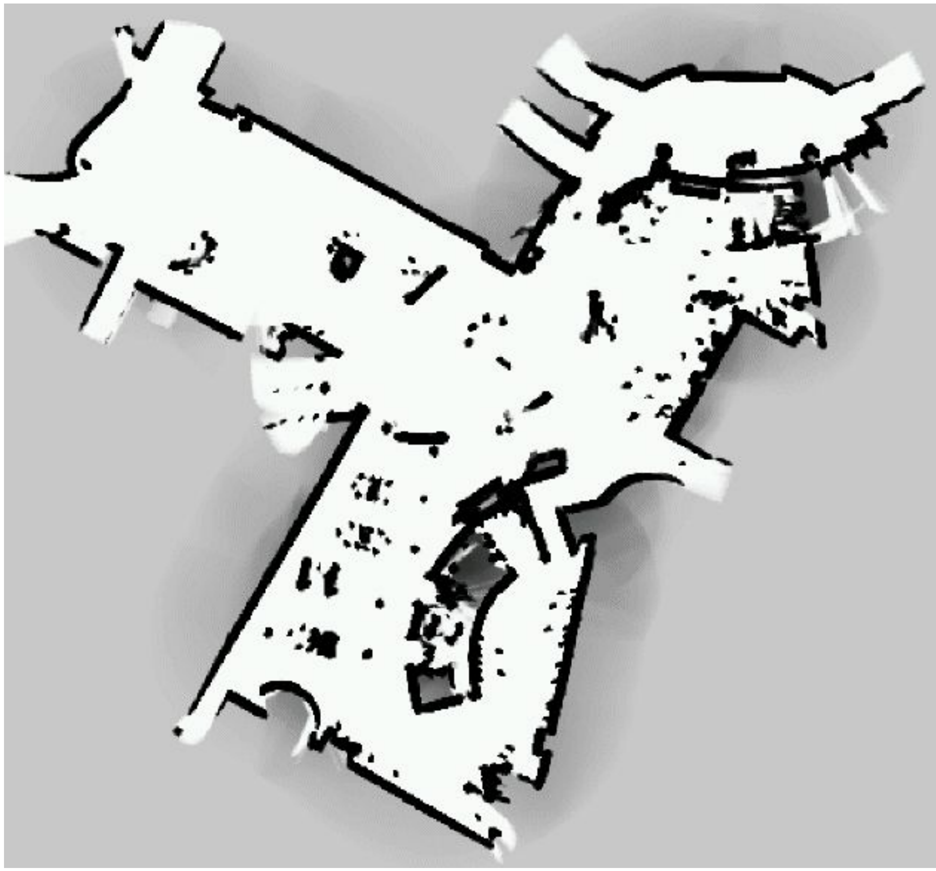


Likelihood field

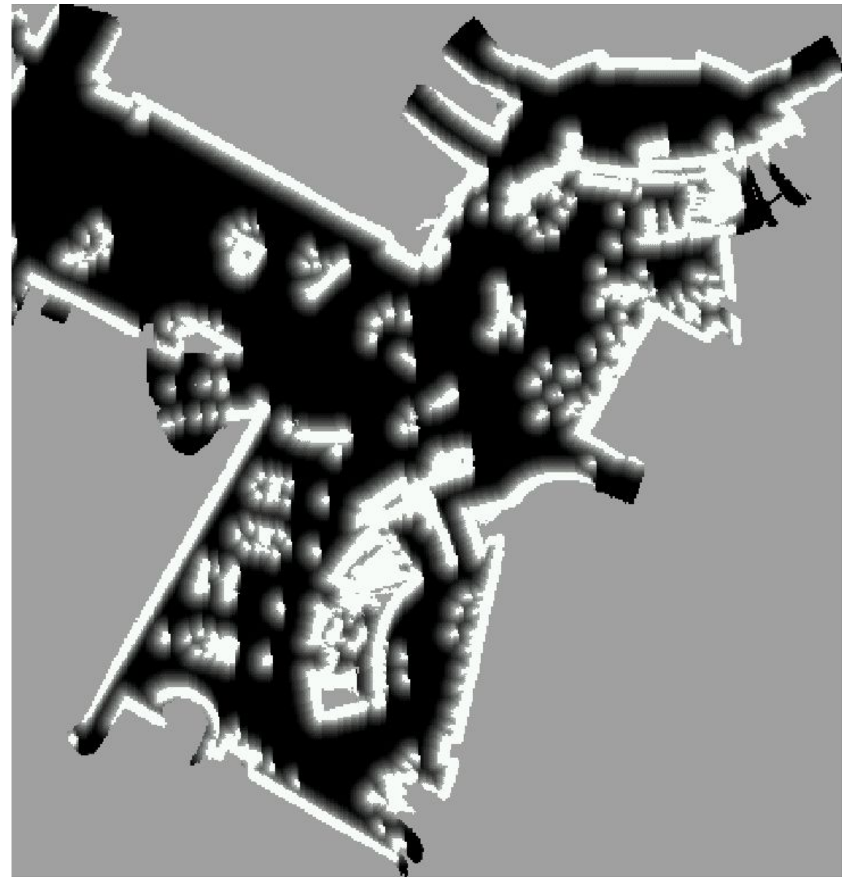
$P(z|x,m)$



San Jose Tech Museum



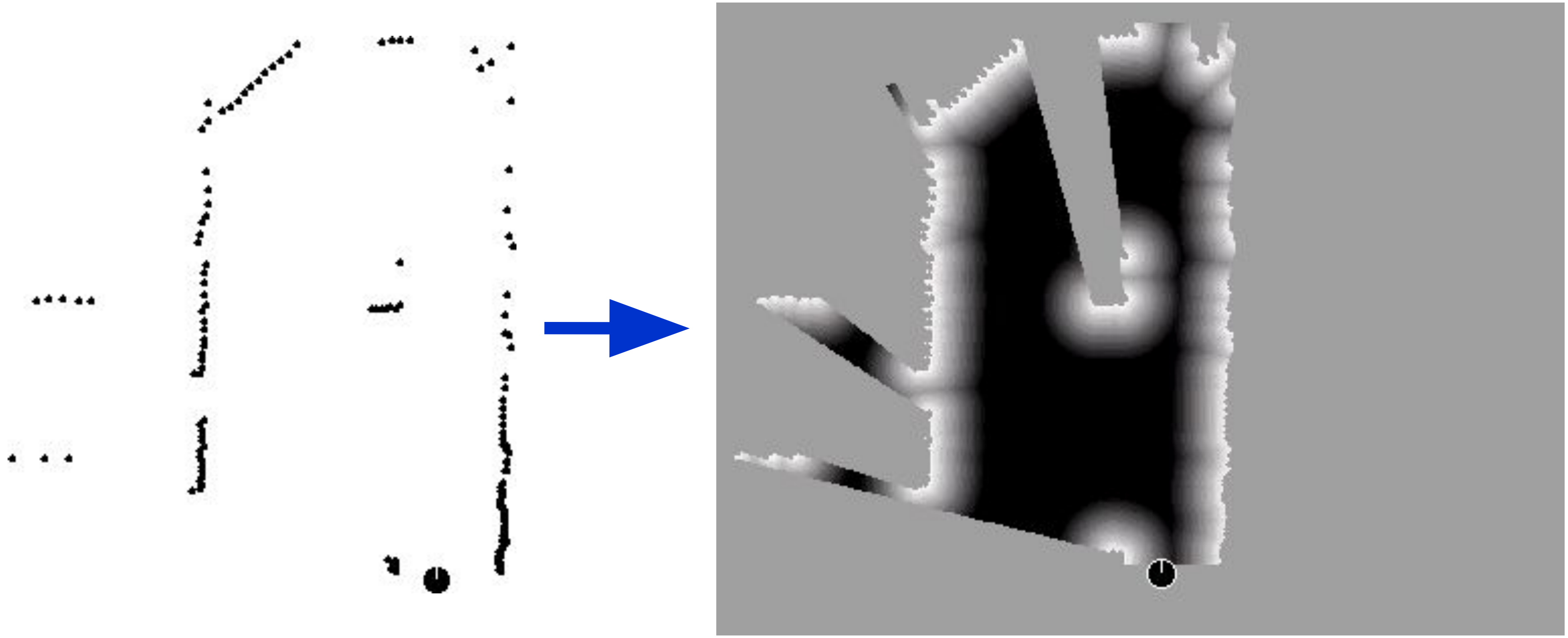
Occupancy grid map



Likelihood field

Scan Matching

- Extract likelihood field from scan and use it to match different scans.
- Correlation-based measurement models (Section 6.5).



Scan Matching

- Extract likelihood field from first scan and use it to match second scan.
- Can formulate scan matching as the task of matching or comparing two histograms.
- Many established ways to accomplish this comparison.

Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Smooth with regard to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.
- Question: Will it work for ultrasound sensors?

Additional Models of Proximity Sensors

- Map matching (sonar,laser): generate small, local maps from sensor data and match local maps against global model.

- Scan matching (laser): map is represented by scan endpoints, match scan into this map.

- Features (sonar, laser, vision): Extract features such as doors, hallways from sensor data.

- Challenge: data association, especially when landmarks or features are not unique.

Landmarks

- Active beacons (*e.g.*, radio, GPS).
- Passive (*e.g.*, visual, retro-reflective).
- Standard approach is triangulation.
- Sensor provides:
  - Distance.
  - Bearing.
  - Distance and bearing.

Distance and Bearing

Probabilistic Model

(correspondence known)

1. Algorithm **landmark_detection_model**(z,x,m):

   $$z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$$

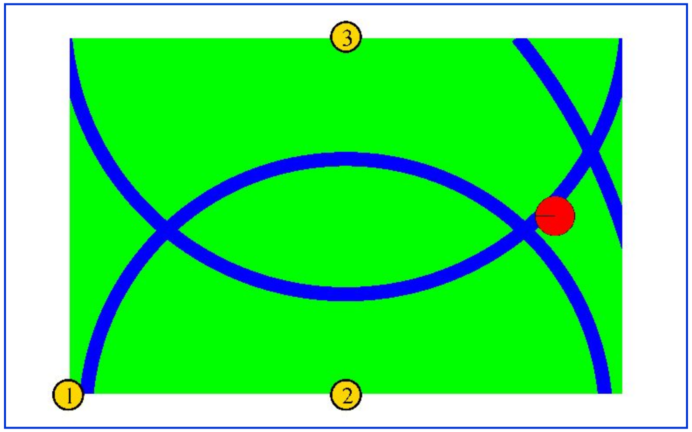
2. $$\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$$

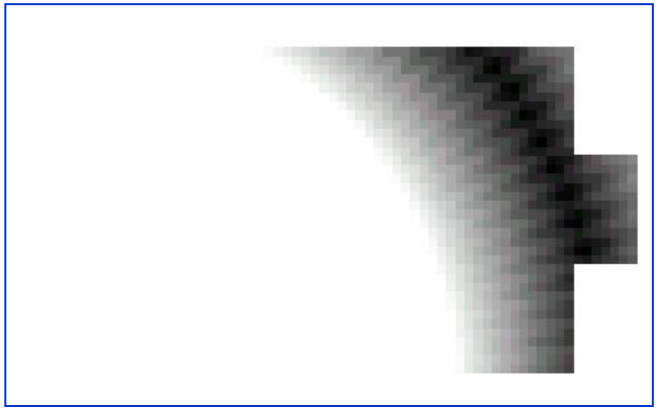
3. $$\hat{a} = \mathrm{atan2}(m_y(i) - y, m_x(i) - x) - \theta$$

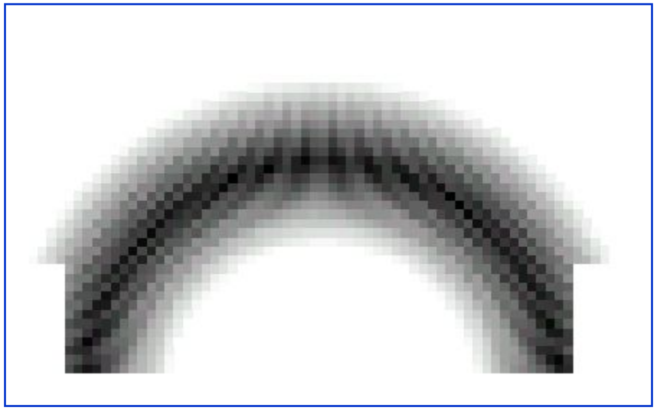
4. $$p_{\mathrm{det}} = \mathrm{prob}(\hat{d} - d, \varepsilon_d) \cdot \mathrm{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$$

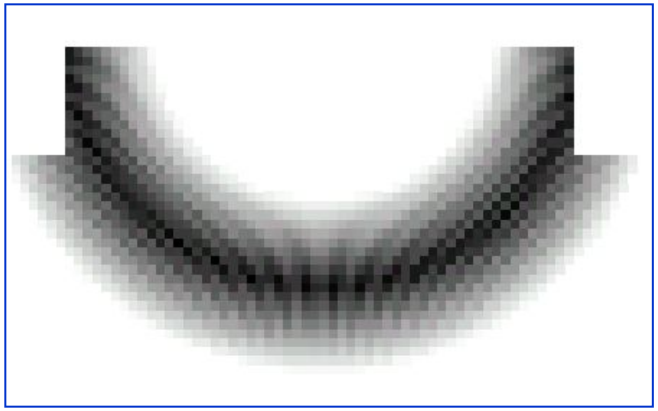
5. Return $z_{\mathrm{det}} p_{\mathrm{det}} + z_{\mathrm{fp}} P_{\mathrm{uniform}}(z \mid x, m)$

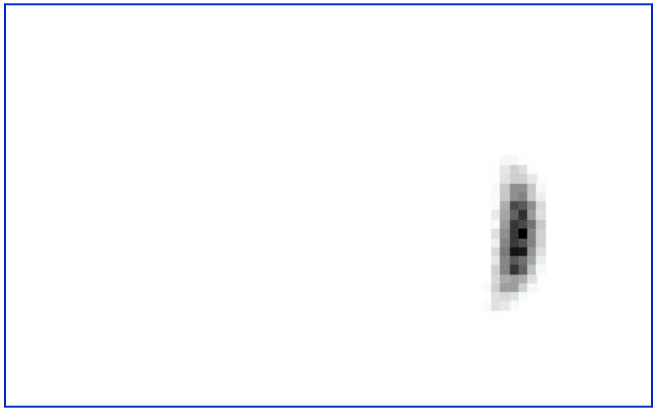
Distributions





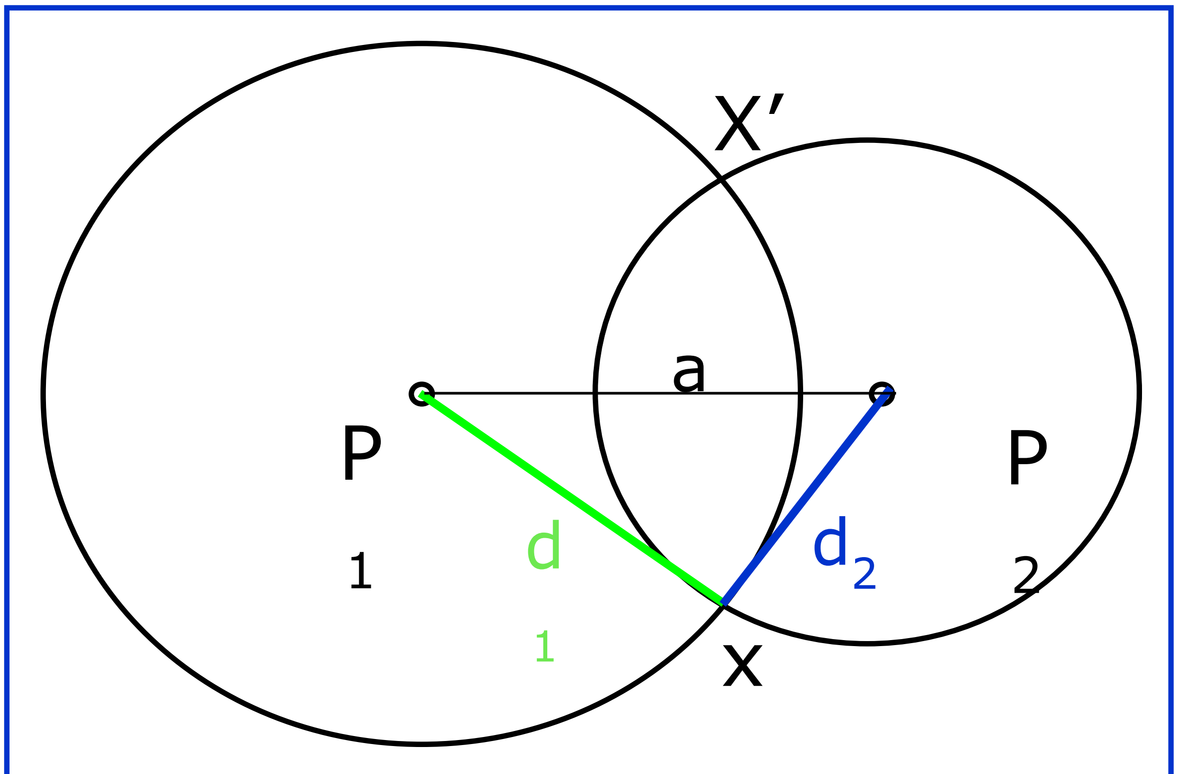






Distances Only
No Uncertainty

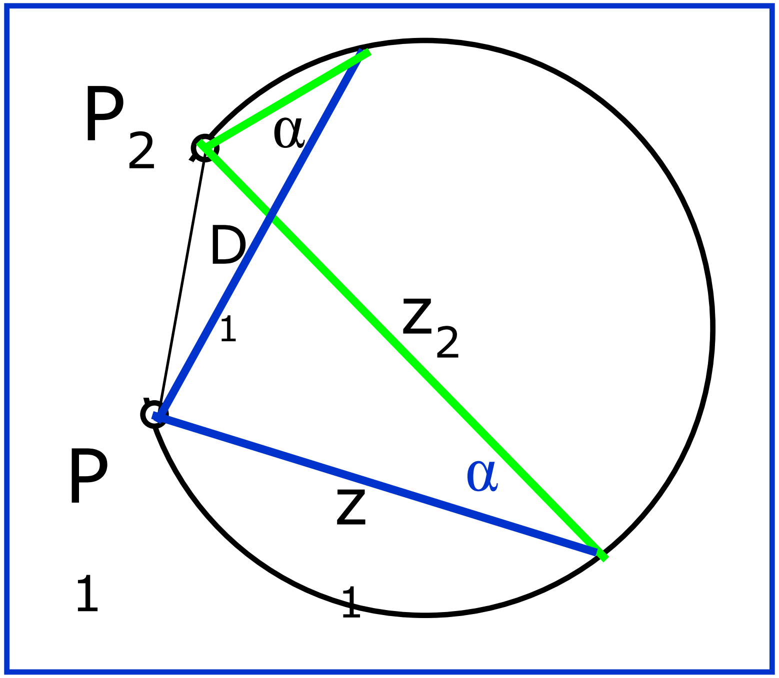
$$x = (a^2 + d_1^2 - d_2^2)/2a$$
$$y = \pm\sqrt{(d_1^2 - x^2)}$$

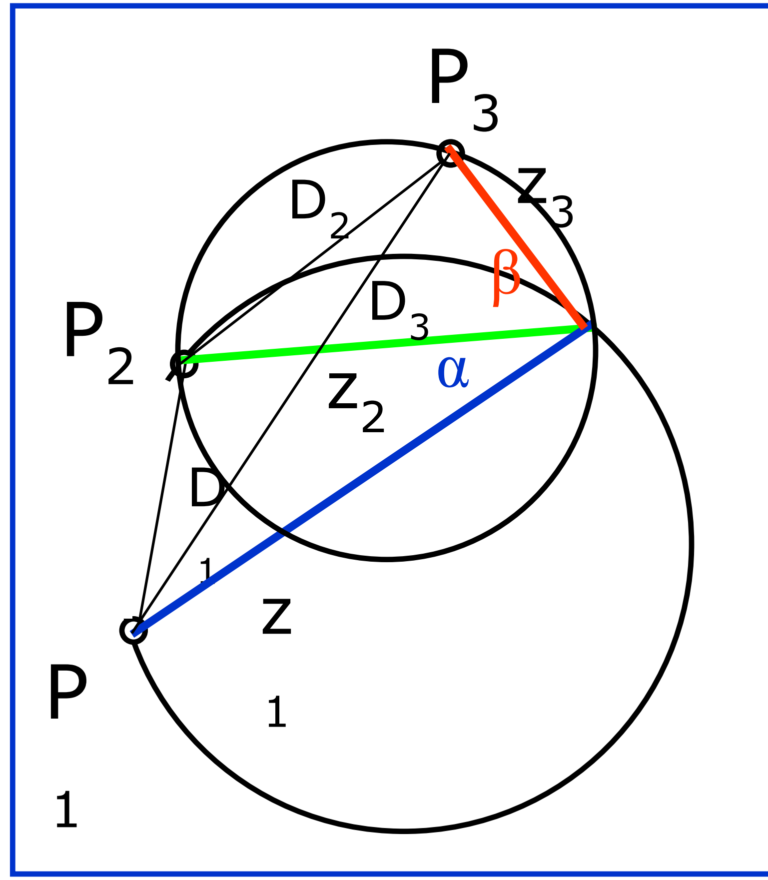


$P_1=(0,0)$

$P_2=(a,0)$

Bearings Only
No Uncertainty





Law of cosine

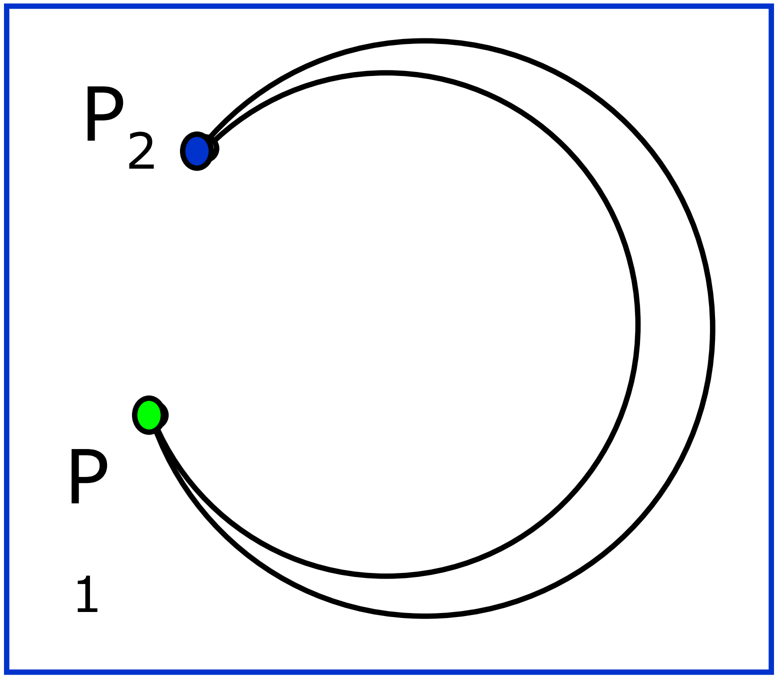
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos\alpha$$

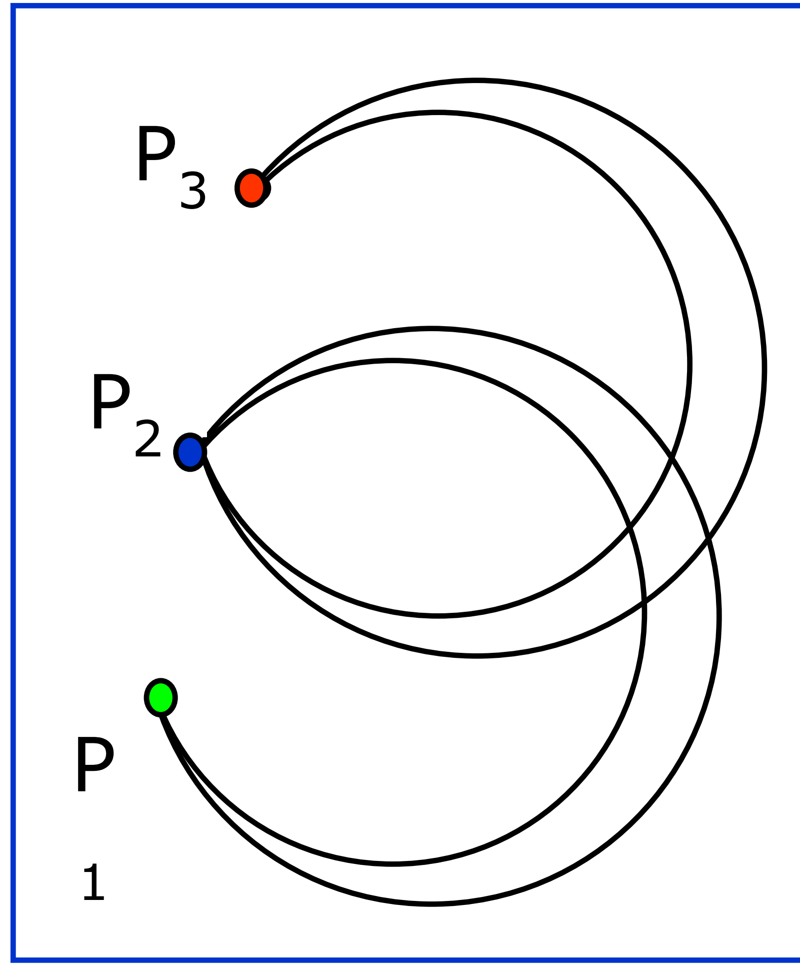
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos(\alpha)$$

$$D_2^2 = z_2^2 + z_3^2 - 2\, z_1 z_2 \cos(\beta)$$

$$D_3^2 = z_1^2 + z_3^2 - 2\, z_1 z_2 \cos(\alpha + \beta)$$

Bearings Only With Uncertainty





Most approaches attempt to find estimation mean.

Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness.
- Good models can typically be found by using the approach:
  1. Determine parametric model of noise free measurement.
  2. Analyze sources of noise.
  3. Add noise to parameters.
  4. Learn (and verify) parameters by fitting model to data.
  5. Likelihood of measurement is given by “probabilistically comparing” the actual with the expected measurement.
- This holds for motion models as well.
- Very important to be aware of the underlying assumptions!