

POLYNOM_3

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*C polynom_3_begin          ***** POLYNOM_3 *****
*C omral_com               =====
                           ORDERED-MONOID RING A-LISTS
                           =====
                           omral = o(rdered) m(onooid) a-l(ist)
*C omral_com_1            =====
                           DEFINITION OF OMRAL TYPE
                           =====

*D omralist_df            omral(<g:g:*>;<r:r:*>)== omralist{<g>; <r>}
*A omralist               omral(g;r) == oal(g↓oSet;r↓+gp)
*T omralist_wf            0 2 ∀g:OCMon. ∀r:CRng. omral(g;r) ∈ DSet
*M omralist_ml            note_reduction_strength 'omralist' '8';;
*D omral_dom_df          dom{<g:g:*>;<r:r:*>}<ps:ps:*>== omral_dom{<g>; <r>; <ps>}
                           dom{<ps:ps:*>}== omral_dom{<g>; <r>; <ps>}
*A omral_dom              dom(ps) == dom(ps)
*T omral_dom_wf          0 2 ∀g:OCMon. ∀r:CRng. ∀ps:(|g| × |r|) List. dom(ps) ∈ MSet{g↓oSet}
*T omral_dom_wf2         0 3 ∀g:OCMon. ∀r:CRng. ∀ps:|omral(g;r)|. dom(ps) ∈ FSet{g↓oSet}
*M omral_dom_eval        let omral_dom_nilC =
                           MacroC 'omral_dom_nilC'
                           (EvalC 'omral_dom'
                            [dom([])]
                            IdC
                            [0{g↓oSet}])
                           ;;
                           let omral_dom_cons_prC =
                           MacroC 'omral_cons_prC'
                           (EvalC 'omral_dom'
                            [dom(<k, v>::ps)]
                            (UnfoldC 'omral_dom'
                             [mset_inj{g↓oSet}(k) + dom(ps)]))
                           ;;
                           add_AbReduce_conv 'omral_dom'
                           (omral_dom_nilC ORELSEC omral_dom_cons_prC)
                           ;;
*T omralist_car_properties 2 2
                           ∀g:OCMon. ∀r:CRng. ∀ws:|omral(g;r)|.
                           ↑sd_ordered(map(λx.x.1;ws)) ∧ ¬↑(0 ∈b map(λx.x.2;ws))
*M oal_to_omral          % Lifting Theorems from oalists to omralists %
                           let omral_opids =
                           'omralist omral_plus omral_dom grp_lt grp_leq grp_blt
                           omral_zero omral_minus omral_inj'
                           ;;
                           let OmRalC =
                           ForceReduceC '5' ANDTHENC TryC (FoldsC omral_opids) ;;
                           let OmRalCStr =
                           "ForceReduceC '5' ANDTHENC TryC (FoldsC '"
                           J
                           concatenate_strings
                           (map (\id.tok_to_string id J " ") omral_opids)
                           J
                           "'')
                           ;;

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let mk_omral_thm old_name new_name new_pos =
  add_specialized_theorem
    old_name
    ['g',「OCMon」; 'r',「CRng」] % New outer context %
    ['「parm{i}」;「g↓oset」;「r↓+gp」] % Bindings for outer context of old thm %
    OmRalC
    OmRalCStr
    new_name
    new_pos
    ; refresh()
;;
*T rng_before_imp_before_all 2 2
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀ps:|omral(g;r)|.
  ↑before(k;map(λz.z.1;ps)) ⇒ ↑(∀bx:(|g|) ∈ map(λz.z.1;ps). x <b k)
*T rng_before_all_imp_before 2 2
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀ps:(|g| × |r|) List.
  ↑(∀bx:(|g|) ∈ map(λz.z.1;ps). x <b k) ⇒ ↑before(k;map(λz.z.1;ps))
*T omralist_cases 2 2
  ∀g:OCMon. ∀r:CRng. ∀Q:|omral(g;r)| → ℙ.
  Q[[]]
  ⇒ (∀ws:|omral(g;r)|. ∀x:|g|. ∀y:|r|.
    ↑before(x;map(λx.x.1;ws)) ⇒ ¬(y = 0) ⇒ Q[<x, y>:ws])
  ⇒ {∀ws:|omral(g;r)|. Q[ws]}
*T omralist_ind_a 2 2
  ∀g:OCMon. ∀r:CRng. ∀Q:|omral(g;r)| → ℙ.
  Q[[]]
  ⇒ (∀ws:|omral(g;r)|
    Q[ws]
    ⇒ (∀x:|g|. ∀y:|r|.
      ↑before(x;map(λx.x.1;ws)) ⇒ ¬(y = 0) ⇒ Q[<x, y>:ws]))
  ⇒ {∀ws:|omral(g;r)|. Q[ws]}
*T omral_lookups_same_a 2 2
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. (∀u:|g|. ps[u] = qs[u]) ⇒ ps = qs
*T rng_lookup_before_start 2 2
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀ps:|omral(g;r)|.
  ↑before(k;map(λz.z.1;ps)) ⇒ ps[k] = 0
*T lookup_omral_eq_zero 2 2
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀ps:|omral(g;r)|. ¬↑(k ∈b dom(ps)) ⇒ ps[k] = 0
*C omral_plus_com
  =====
  OMRAL PLUS FUNCTION
  =====
  Lifting of oal merge function
*D omral_plus_df
  Parens ::Prec(inop)::
  <ps:ps:L> ++<g:g:L>, <r:r:L> <qs:qs:L>
  == omral_plus{<g>; <r>; <ps>; <qs>}
  Parens ::Prec(inop)::
  <ps:ps:L> ++ <qs:qs:L>
  == omral_plus{<g>; <r>; <ps>; <qs>}
*A omral_plus
  ps ++ qs == ps ++ qs
*T omral_plus_wf 1 3
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List. ps ++ qs ∈ (|g| × |r|) List
*T omral_plus_sd_ordered 2 2
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List.
  ↑sd_ordered(map(λx.x.1;ps))
  ⇒ ↑sd_ordered(map(λx.x.1;qs))

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      ⇒ ↑sd_ordered(map(λx.x.1;ps ++ qs))
*T omral_plus_non_zero_vals 2 2
      ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List.
      ¬↑(0 ∈b map(λx.x.2;ps))
      ⇒ ¬↑(0 ∈b map(λx.x.2;qs))
      ⇒ ¬↑(0 ∈b map(λx.x.2;ps ++ qs))
*T omral_plus_wf2          2 2 ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ps ++ qs ∈ |omral(g;r)|
*T omral_plus_dom          2 2
      ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ↑(dom(ps ++ qs) ⊆b dom(ps) ∪ dom(qs))
*T lookup_omral_plus      2 2
      ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀ps,qs:|omral(g;r)|. (ps ++ qs)[k] = ps[k] +r qs[k]
*T omral_plus_comm        1 2 ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ps ++ qs = qs ++ ps
*T omral_plus_assoc       1 2
      ∀g:OCMon. ∀r:CRng. ∀ps,qs,rs:|omral(g;r)|. ps ++ (qs ++ rs) = (ps ++ qs) ++ rs
*C omral_zmi_com
=====
OMRAL ZERO, MINUS AND INJECTION FUNCTIONS
=====
All lifted from oal development.
*D omral_zero_df          00<g:g:*,<r:r:*>== omral_zero{<g>; <r>}
*A omral_zero             00g,r == 00
*T omral_zero_wf          0 2 ∀g:OCMon. ∀r:CRng. 00g,r ∈ |omral(g;r)|
*D omral_minus_df
  Parens ::Prec(preop)::
    --<g:g:L>,<r:r:L> <ps:ps:L>
    == omral_minus{<g>; <r>; <ps>}
  Parens ::Prec(preop):: --<ps:ps:L>== omral_minus{<g>; <r>; <ps>}
*A omral_minus            --ps == --ps
*T omral_minus_wf         0 4 ∀g:OCMon. ∀r:CRng. ∀ps:|omral(g;r)|. --ps ∈ |omral(g;r)|
*D omral_inj_df
  inj{<g:g:*,<r:r:*>}<k:k:*,<v:v:*>== omral_inj{<g>; <r>; <k>; <v>}
  inj{<k:k:*,<v:v:*>}== omral_inj{<g>; <r>; <k>; <v>}
*A omral_inj              inj(k,v) == inj(k,v)
*T omral_inj_wf           0 3 ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|. inj(k,v) ∈ |omral(g;r)|
*T omral_dom_inj         1 2
      ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|.
      dom(inj(k,v)) = if v =b 0 then 0{g↓oset} else mset_inj{g↓oset}(k) fi
*T lookup_omral_inj      1 2
      ∀g:OCMon. ∀r:CRng. ∀k,k':|g|. ∀v:|r|. inj(k,v)[k'] = when k =b k'. v
*T comb_for_omral_inj_wf 0 0
      (λg,r,k,v,z.inj(k,v)) ∈ g:OCMon
      → r:CRng
      → k:|g|
      → v:|r|
      → ↓True
      → |omral(g;r)|
*T omral_fact             1 2
      ∀g:OCMon. ∀r:CRng. ∀ps:|omral(g;r)|.
      ps = msFor{oal_mon(g↓oset;r↓+gp)} k' ∈ dom(ps). inj(k',ps[k'])
*T omral_fact_a           2 2
      ∀g:OCMon. ∀r:CRng. ∀ps:|omral(g;r)|.
      ps = msFor{omral_alg(g;r)↓grp} k' ∈ dom(ps). inj(k',ps[k'])
*C omral_scale_com
=====
OMRAL SCALING FUNCTION
=====
Scales keys and values of an omralist.

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*D omral_scale_df
  Parens ::Prec(preop)::
    <<k:k:*,<v:v:*>>*<g:mon:L>,<r:rng:L> <ps:ps:E>
    == omral_scale{<g>; <r>; <k>; <v>; <ps>}
  Parens ::Prec(preop)::
    <<k:k:*,<v:v:*>>* <ps:ps:E>
    == omral_scale{<g>; <r>; <k>; <v>; <ps>}

*M omral_scale_ml
  <k,v>* ps
  ==r case ps of
    [] => []
    p::ps' => if (v * p.2) =b 0
      then <k,v>* ps'
      else <k * p.1, v * p.2>::(<k,v>* ps')
    fi
  esac

*M omral_scale_eval
  let omral_scale_nilC =
    FwdMacroC 'omral_scale_nilC'
    (RecEvalC 'omral_scale' ' ' [ <k,v>* [] ] );
  let omral_scale_cons_prC =
    FwdMacroC 'omral_scale_cons_prC'
    (RecEvalC 'omral_scale' ' ' [ <k,v>* (<k', v'>::ps) ] );
  add_AbReduce_conv 'omral_scale'
    (omral_scale_nilC ORELSEC omral_scale_cons_prC) ;;

*T omral_scale_wf      3 2
  ∀g:GrpSig. ∀r:RngSig. ∀k:|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  <k,v>* ps ∈ (|g| × |r|) List

*T omral_scale_dom_pred  3 4
  ∀g:OCMon. ∀r:CRng. ∀Q:|g| → ℤ. ∀k:|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  ↑(∀bx:(|g|) ∈ map(λz.z.1;ps). Q[k * x])
  ⇒ ↑(∀bx:(|g|) ∈ map(λz.z.1;<k,v>* ps). Q[x])

*T omral_dom_scale      4 6
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|. ∀ps:|omral(g;r)|.
  ↑(dom(<k,v>* ps) ⊆b fs-map(λk'.k' * k, dom(ps)))

*T omral_scale_dom_bound  3 5
  ∀g:OCMon. ∀r:CRng. ∀bound,k:|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  ↑(∀bx:(|g|) ∈ map(λz.z.1;ps). x <b bound)
  ⇒ ↑(∀bx:(|g|) ∈ map(λz.z.1;<k,v>* ps). x <b k * bound)

*C omral_scale_sd_ordered_com
  The proof here needs some cleaning up.
  Probably, worth pulling out the second
  induction and generalizing it a bit.

*T omral_scale_sd_ordered 5 6
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  ↑sd_ordered(map(λz.z.1;ps)) ⇒ ↑sd_ordered(map(λz.z.1;<k,v>* ps))

*T omral_scale_non_zero_vals 3 5
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  ¬↑(0 ∈b map(λx.x.2;ps)) ⇒ ¬↑(0 ∈b map(λx.x.2;<k,v>* ps))

*T omral_scale_wf2      2 4
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v:|r|. ∀ps:|omral(g;r)|. <k,v>* ps ∈ |omral(g;r)|

*T lookup_omral_scale_a  4 6
  ∀g:OCMon. ∀r:CRng. ∀k,k':|g|. ∀v:|r|. ∀ps:|omral(g;r)|.
  (<k,v>* ps)[k * k'] = v * ps[k']

*T lookup_omral_scale_b  3 6
  ∀g:OCMon. ∀r:CRng. ∀k,k':|g|. ∀v:|r|. ∀ps:(|g| × |r|) List.
  ¬(∃d:|g|. ↑(d ∈b dom(ps)) ∧ k * d = k') ⇒ (<k,v>* ps)[k'] = 0

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*T lookup_omral_scale_c 4 6
  ∀g:OCMon. ∀r:CRng. ∀z,k:|g|. ∀v:|r|. ∀ps:|omral(g;r)|.
  (<k,v>* ps)[z] = msFor{r↓+gp} y ∈ dom(ps). when (k * y) =b z. v * ps[y]
*T lookup_omral_scale_d 1 0
  ∀g:OCMon. ∀r:CRng. ∀z,k:|g|. ∀v:|r|. ∀ps:|omral(g;r)|.
  (<k,v>* ps)[z] = (∑y ∈ dom(ps). when (k * y) =b z. v * ps[y])
*C omral_times_com
  =====
  OMRAL TIMES FUNCTION
  =====
*D omral_times_df
  Parens ::Prec(inop)::
  <ps:ps:L> **<g:g:L>,<r:r:L> <qs:qs:L>
  == omral_times{<g>; <r>; <ps>; <qs>}
  Parens ::Prec(inop)::
  <ps:ps:L> ** <qs:qs:L>
  == omral_times{<g>; <r>; <ps>; <qs>}
*M omral_times_ml
  ps ** qs==r case ps of [] => [] | p::ps' => <p.1,p.2>* qs ++ (ps' ** qs) esac
*M omral_times_eval
  let omral_times_nilC =
    FwdMacroC 'omral_times_nilC'
    (RecEvalC 'omral_times' '[] ** qs' ;;
  let omral_times_cons_prC =
    FwdMacroC 'omral_times_cons_prC'
    (RecEvalC 'omral_times' '(<k, v>::ps) ** qs'
  ;;
  add_AbReduce_conv 'omral_times'
    (omral_times_nilC ORELSEC omral_times_cons_prC)
  ;;
*T omral_times_wf 2 2
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List. ps ** qs ∈ (|g| × |r|) List
*T omral_times_sd_ordered 2 3
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List.
  ↑sd_ordered(map(λz.z.1;qs)) ⇒ ↑sd_ordered(map(λz.z.1;ps ** qs))
*T omral_times_non_zero_vals 2 3
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:(|g| × |r|) List.
  ¬↑(0 ∈b map(λx.x.2;qs)) ⇒ ¬↑(0 ∈b map(λx.x.2;ps ** qs))
*T omral_times_wf2 2 4 ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ps ** qs ∈ |omral(g;r)|
*T lookup_omral_times 4 6
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ∀z:|g|.
  (ps ** qs)[z]
  = msFor{r↓+gp} x ∈ dom(ps)
    msFor{r↓+gp} y ∈ dom(qs). when (x * y) =b z. ps[x] * qs[y]
*T lookup_omral_times_a 0 0
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ∀z:|g|.
  (ps ** qs)[z] = (∑x ∈ dom(ps). ∑y ∈ dom(qs). when (x * y) =b z. ps[x] * qs[y])
*T mset_on_grp_eq 2 0 ∀g:OCMon. MSet{g↓set} = MSet{g↓oset}
*T mset_inc 2 0 ∀g:OCMon. MSet{g↓set} ⊆ MSet{g↓oset}
*T mset_inc_a 2 1 ∀g:OCMon. MSet{g↓oset} ⊆ MSet{g↓set}
*T omral_times_dom 5 8
  ∀g:OCMon. ∀r:CRng. ∀ps,qs:|omral(g;r)|. ↑(dom(ps ** qs) ⊆b dom(ps) × dom(qs))
*T omral_times_assoc 5 9 ∀g:OCMon. ∀a:CRng. Assoc(|omral(g;a)|;λps,qs.ps ** qs)
*T omral_times_assoc_a 1 0
  ∀g:OCMon. ∀a:CRng. ∀ps,qs,rs:|omral(g;a)|. ps ** (qs ** rs) = (ps ** qs) ** rs
#T omral_times_assoc_b 4 8
  ∀g:OCMon. ∀a:CRng. ∀ps,qs,rs:|omral(g;a)|. ps ** (qs ** rs) = (ps ** qs) ** rs

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*T omral_times_comm      3 5  $\forall g:\text{OCMon}. \forall a:\text{CRng}. \text{Comm}(|\text{omral}(g;a)|; \lambda ps, qs. ps ** qs)$ 
*T omral_times_comm_a   1 0  $\forall g:\text{OCMon}. \forall a:\text{CRng}. \forall ps, qs: |\text{omral}(g;a)|. ps ** qs = qs ** ps$ 
*T omral_bilinear        5 7
       $\forall g:\text{OCMon}. \forall a:\text{CRng}. \text{BiLinear}(|\text{omral}(g;a)|; \lambda ps, qs. ps ++ qs; \lambda ps, qs. ps ** qs)$ 
*T omral_bilinear_a     1 0
       $\forall g:\text{OCMon}. \forall a:\text{CRng}. \forall ps, qs, rs: |\text{omral}(g;a)|.$ 
       $ps ** (qs ++ rs) = (ps ** qs) ++ (ps ** rs)$ 
       $\wedge (qs ++ rs) ** ps = (qs ** ps) ++ (rs ** ps)$ 
*C omral_one_act_com
      =====
      OMRAL ONE AND ACTION
      =====
*D omral_one_df
      Parends ::Prec(preop)::  $11\langle g:g:L\rangle, \langle r:r:L\rangle == \text{omral\_one}\{\}\langle g\rangle; \langle r\rangle$ 
       $11 == \text{omral\_one}\{\}\langle g\rangle; \langle r\rangle$ 
*A omral_one            11 == inj(e,1)
*T omral_one_wf          0 0  $\forall g:\text{OCMon}. \forall r:\text{CRng}. 11 \in |\text{omral}(g;r)|$ 
*T omral_dom_one        2 3  $\forall g:\text{OCMon}. \forall r:\text{CRng}. \neg(0 = 1) \Rightarrow \text{dom}(11) = \text{mset\_inj}\{g\downarrow\text{oset}\}(e)$ 
*D omral_action_df
      Parends ::Prec(inop)::
       $\langle v:v:L\rangle .. \langle g:g:L\rangle, \langle r:r:L\rangle \langle ps:ps:L\rangle$ 
       $== \text{omral\_action}\{\}\langle g\rangle; \langle r\rangle; \langle v\rangle; \langle ps\rangle$ 
      Parends ::Prec(inop)::
       $\langle v:v:L\rangle .. \langle ps:ps:L\rangle$ 
       $== \text{omral\_action}\{\}\langle g\rangle; \langle r\rangle; \langle v\rangle; \langle ps\rangle$ 
*A omral_action          v .. ps ==  $\langle e, v\rangle * ps$ 
*T omral_action_wf      0 0
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall v:|r|. \forall ps:|\text{omral}(g;r)|. v .. ps \in |\text{omral}(g;r)|$ 
*T comb_for_omral_action_wf 0 0
       $(\lambda g, r, v, ps, z. v .. ps) \in g:\text{OCMon}$ 
       $\rightarrow r:\text{CRng}$ 
       $\rightarrow v:|r|$ 
       $\rightarrow ps:|\text{omral}(g;r)|$ 
       $\rightarrow \downarrow \text{True}$ 
       $\rightarrow |\text{omral}(g;r)|$ 
*C omral_dom_action_com
      Nice simple example of monotonicity
      reasoning here.
*T omral_dom_action      3 4
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall v:|r|. \forall ps:|\text{omral}(g;r)|. \uparrow(\text{dom}(v .. ps) \subseteq_b \text{dom}(ps))$ 
*T lookup_omral_action  2 3
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall k:|g|. \forall v:|r|. \forall ps:|\text{omral}(g;r)|. (v .. ps)[k] = v * ps[k]$ 
*C omral_alg_com
      =====
      ASSEMBLY OF OMRAL FREE MONOID ALGEBRA
      =====
*T omral_times_ident_r  2 1  $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall ps:|\text{omral}(g;r)|. ps ** 11 = ps$ 
*T omral_times_ident_l  4 6  $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall ps:|\text{omral}(g;r)|. 11 ** ps = ps$ 
*T omral_action_one     1 3  $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall ps:|\text{omral}(g;r)|. 1 .. ps = ps$ 
*T omral_action_times   1 3
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall v, w:|r|. \forall ps:|\text{omral}(g;r)|. (v * w) .. ps = v .. (w .. ps)$ 
*T omral_action_times_r1 4 7
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall v:|r|. \forall ps, qs:|\text{omral}(g;r)|.$ 
       $v .. (ps ** qs) = (v .. ps) ** qs$ 
*T omral_action_times_r2 1 2
       $\forall g:\text{OCMon}. \forall r:\text{CRng}. \forall v:|r|. \forall ps, qs:|\text{omral}(g;r)|.$ 
       $v .. (ps ** qs) = ps ** (v .. qs)$ 

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*T omral_action_plus_l 1 3
  ∀g:OCMon. ∀r:CRng. ∀v,w:|r|. ∀ps:|omral(g;r)|.
  (v +r w) .. ps = (v .. ps) ++ (w .. ps)
*T omral_action_plus_r 1 4
  ∀g:OCMon. ∀r:CRng. ∀v:|r|. ∀ps,qs:|omral(g;r)|.
  v .. (ps ++ qs) = (v .. ps) ++ (v .. qs)
*T omral_action_inj 2 2
  ∀g:OCMon. ∀r:CRng. ∀k:|g|. ∀v,v':|r|. v .. inj(k,v') = inj(k,v * v')
*D omral_alg_df
  omral_alg(<g:g:*>;<r:r:*>)== omral_alg{}(<g>; <r>)
*A omral_alg omral_alg(g;r) ==
  <|omral(g;r)|
  , =b
  , λx,y.tt
  , λx,y.x ++ y
  , 00g,r
  , λx.--x
  , λx,y.x ** y
  , 11
  , λx,y.(inr . )
  , λa,x.a .. x>
*T omral_alg_wf 0 4 ∀g:OCMon. ∀r:CRng. omral_alg(g;r) ∈ AlgebraSig(|r|)
*T omral_alg_wf2 5 5 ∀g:OCMon. ∀r:CRng. omral_alg(g;r) ∈ r-CAlgebra
*T omral_inj_mon_op 3 6
  ∀g:OCMon. ∀r:CRng. ∀k,k':|g|. inj(k * k',1) = inj(k,1) ** inj(k',1)
*D omral_alg_umap_df
  alg_umap{<g:mon:*>,<a:rng:*>}(<n:alg:*>,<f:mon->alg:*>)
  == omral_alg_umap{}(<g>; <a>; <n>; <f>)
  alg_umap(<n:alg:*>,<f:mon->alg:*>)== omral_alg_umap{}(<g>; <a>; <n>; <f>)
*A omral_alg_umap alg_umap(n,f) == λps:|omral(g;a)|. Σk ∈ dom(ps). ps[k] ·n (f k)
*T omral_alg_umap_wf 0 3
  ∀g:OCMon. ∀a:CRng. ∀n:a-Algebra. ∀f:|g| → |n|.
  alg_umap(n,f) ∈ |omral(g;a)| → |n|
*T omral_alg_umap_is_hom 6 8
  ∀g:OCMon. ∀a:CRng. ∀n:a-Algebra. ∀f:MonHom(g,n|rg|xmn).
  IsAlgHom{a,omral_alg(g;a),n}(alg_umap(n,f))
*T omral_alg_umap_tri_comm 3 5
  ∀g:OCMon. ∀a:CRng. ∀n:a-Algebra. ∀f:|g| → |n|.
  alg_umap(n,f) o (λk.inj(k,1)) = f
*T omral_alg_umap_unique 4 7
  ∀g:OCMon. ∀a:CRng. ∀n:a-Algebra. ∀f:|g| → |n|.
  ∀f':a-AlgebraHom(omral_alg(g;a);n).
  f' o (λk:|g|. inj(k,1)) = f ⇒ f' = alg_umap(n,f)
*D omral_fma_df
  omral_fma(<g:g:*>;<a:a:*>)== omral_fma{}(<g>; <a>)
*A omral_fma omral_fma(g;a) == <omral_alg(g;a), λk.inj(k,1), λn,f.alg_umap(n,f)>
*T omral_fma_wf 0 4 ∀g:OCMon. ∀a:CRng. omral_fma(g;a) ∈ FMASig(g;a)
*T omral_fma_wf2 4 5 ∀g:OCMon. ∀a:CRng. omral_fma(g;a) ∈ FMonAlg(g;a)
*C polynom_3_end
  *****
Thm stats: <log2 (# pscript lines)> <log2 (expansion time in sec)>

```