- Why did you choose this course?
- Why did you choose the clothes you're wearing?
- Why are you sitting where you are?
- Why are you reading this?
- Who or what made the decision???

Theoretical framework: statistical inference

- decision making can be thought of as a form of statistical inference.
- decide = select among competing hypotheses h₁, h₂ (and may be more)
- elements of this decision process:
- * priors $P(h_1)$ = Probability that h_1 is correct before collecting any evidence = a bias (or prejudice)
- * evidence (e) = information we can collect in factor of h_1 . This evidence is only useful when we know how likely it is to be true if the hypothesis is true i.e. if we have conditional probabilities such as $P(el h_1)$ = the likelihood
- * value(v) = subjective costs and benefits for each outcome.

Decision Making

Readings: Gold and Shadlen, the neural basis of decision making, 2007

Bayes' Theorem

- Bayes' theorem is a result in probability theory that relates conditional probabilities P(AIB) and P(BIA)
- Given the likelihood and the prior, we can compute the posterior.

$$P(h_1|e) = \frac{P(e|h_1)P(h_1)}{P(e)}$$

$$posterior = \frac{likelihood \times prior}{normalizing\ constant}$$



Reverend Thomas Bayes, 1702- 1761

To decide, compare probabilities of each hypothesis

Choose h₁ if:

$$P(h_1|e) = \frac{P(e|h_1)P(h_1)}{P(e)}$$
>
$$P(h_2|e) = \frac{P(e|h_2)P(h_2)}{P(e)}$$



Values (1)

- It might be that the costs and benefits associated with the various outcomes are very different.
- benefit of choosing h₁ =
 value of choosing h₁ if h₁ is true (V₁₁)
 + value of choosing h₁ if h₁ is wrong
 (V₁₂) given the evidence.
- benefit of choosing h₂ =
 value of choosing h₂ if h₂ is true (V₂₂)
 + value of choosing h₂ if h₂ is wrong
 (V₂₁) given the evidence.



run or not?

• So we now want to compare:

$$V_{11}P(h_1|e) + V_{12}P(h_2|e)$$
 with $V_{22}P(h_2|e) + V_{21}P(h_1|e)$

Likelihood ratio test

• Just re-organizing the terms of this inequality: - choose h₁ if:

$$\frac{P(e|h_1)}{P(e|h_2)} > \frac{P(h_2)}{P(h_1)}$$

- This is known as the likelihood ratio test = optimal decision rule.
- If the prior probabilities are equal (0.5), choose h₁ if

$$LR = \frac{P(e|h_1)}{P(e|h_2)} > 1$$



Values (2)

• rewriting this gives the general (optimal) rule: choose h₁ if:

$$\frac{P(e|h_1)}{P(e|h_2)} > \frac{(V_{22} - V_{12})P(h_2)}{(V_{11} - V_{12})P(h_1)}$$

- which has also the form of comparing the LR with a threshold.
- Signal detection theory: LR (or any monotonic function of it e.g. LOG) provides an optimal 'decision variable'.

Sequential Analysis

- This framework can be extended to the situation where we have multiple pieces of evidence e₁, e₂, ..e_n observed over time.
- Here we allow the decision variable to 'accumulate the evidence' in time: $P(e_1, e_2, \dots, e_n | h_1)$

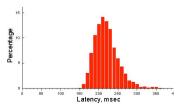
$$\log LR_{12} \equiv \log \frac{P(e_1, e_2, \dots, e_n | h_1)}{P(e_1, e_2, \dots, e_n | h_2)}$$
$$= \sum_{i=1}^n \log \frac{P(e_i | h_1)}{P(e_i | h_2)}.$$

- When the DV reaches a threshold (which possibly reflects priors and values), a decision is made.
- This is known as the sequential probability ratio test (optimal rule).

$$\begin{array}{ccc} e_{\scriptscriptstyle 0} & \rightarrow f_{\scriptscriptstyle 0} \left(e_{\scriptscriptstyle 0} \right) & \Rightarrow \underset{or}{^{Slop}} \\ & \swarrow \\ & e_{\scriptscriptstyle 1} \rightarrow & f_{\scriptscriptstyle 1} \left(e_{\scriptscriptstyle 0}, e_{\scriptscriptstyle 1} \right) & \Rightarrow \underset{or}{^{Slop}} \\ & \swarrow \end{array}$$

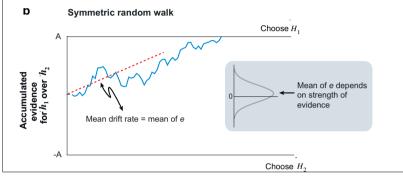
Random Walk model (2)

- Well-studied mathematically (diffusion processes)
- many variants (discrete time, continuous time, leaky integration)
- These models have been compared systematically and shown to successfully account for [Smith & Ratcliff, 2004]:
- Distribution of Reaction Times
- Speed-accuracy tradeoff: decreasing the boundary has the effect of increasing speed and decreasing accuracy.
- Error response RTs (sometimes error responses can be very quick..).



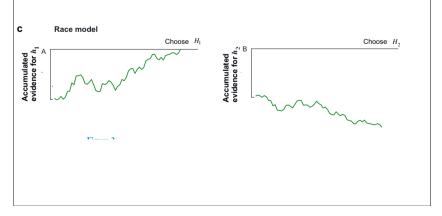
Random Walk model (1)

- Related to this framework are the random walk and race models of decision making developed by psychologists to explain behavioral data.
- The Decision Variable is the cumulated sum of the evidence. The bounds represent the stopping rule.
- If e is log LR, then this model = sequential prob ratio test.



Race Model

- Another variant is the race model
- •Two or more decision processes represent the accumulated evidence for each alternative.

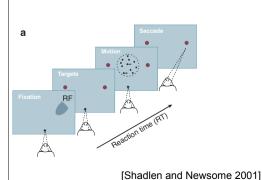


• Anything like that in the brain?



Random Dots Motion Direction Task

- monkey decides between 2 possible opposite directions, and saccade to signal his choice whenever he is ready.
- task difficulty is controlled by varying coherence level
- decision = problem of movement selection





• study decision on perceptual tasks

yes

Mike Shadlen, Paul Glimcher (and others)

neural basis of the perceptual decision?

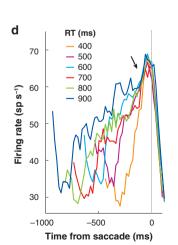
- a sensory stage where the evidence is collected. MT seems to fulfill the role.
- a decision stage 'reading-out' the sensory stage.
- These neurons must accumulate the information over time to explain performance accuracy
- A sustained activity is needed to compare alternatives presented successively in time.
- neurons in parietal and frontal 'association' cortex
- possibly the neurons that are linked to the specific behavioral response (= the preparation of the saccade)

Accumulation of Evidence in LIP (1)

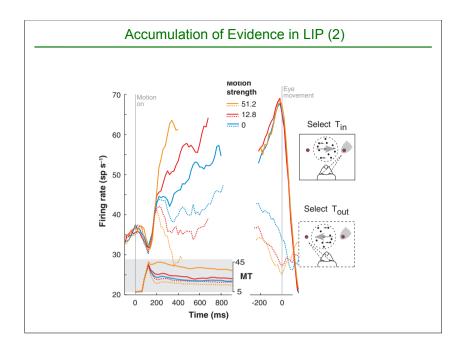
- LIP receives inputs from MT and MST, outputs in FEF and SC (generation of saccades)
- LIP is implicated in selection of saccade targets, working memory, intention etc..
- FEF LIP MT SC
- Record neurons which have one of the choice targets in the response field and the other outside.
- After ~ 220 ms, response reflects decision faster rise for easier choices, decrease for opposite direction.
- \bullet Aligning responses to saccade initiation reveals correlate of commitment: a threshold rate at which the decision is made, \sim 70 msec before saccade initiation.

Accumulation of Evidence in LIP (3)

- Responses grouped by RT
- Responses achieve a common level of activity ~ 70 msec before saccade initiation
- When the monkey chooses other direction, another set of neurons (with chosen target in their RFs) behave similarly
- as if the fact that they reach a threshold value 'determines the termination of the decision process'



[Gold and Shadlen 2007]



Accumulation of Evidence in LIP (4)

- pattern of LIP activity matches prediction of diffusion/race models.
- rise of activity appears to reflect accumulation of evidence
- evidence could come from a difference in activity of pools of MT neurons with opposite direction preferences, which was suggested to approximate the LogLR (Gold & Shadlen, 2001)
- suggests that LIP neurons represent the decision variable?
- implements a logLR test?
- How is the criterion / threshold set and what happens when it is reached?

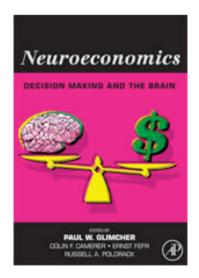
Platt & Glimcher 1999 (1)

- monkeys cued by a color of a fixation stimulus to saccade on of 2 targets
- change the reward associated with each target (value)
- vary the probability that a saccade to a target will be required (prior)
- offset of the responses of LIP neurons before and during presentation of the saccade target
- suggests that behavioral outcome and priors are also encoded.

"understand the processes that connect sensation and action by revealing the neurobiological mechanisms by which decisions are made"

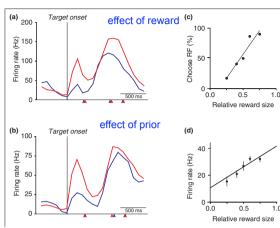
...

"an emerging transdisciplinary field that uses neuroscientific measurement techniques to identify the neural substrates associated with economic decisions"



Platt & Glimcher 1999 (2)

Neural correlates of behavioral value (a) Average firing rate of a single LIP neuron plotted as a function of time, on trials in which a saccade in the preferred direction (RF) of the neuron was cued. Neuronal activity was greater when a large reward was associated with the cued saccade (red curve) than when a small reward was associated with the same movement (blue curve). Arrows indicate, successively mean times of instruction due onset central fivation. stimulus offset, and saccade onset in high (red) and low (blue) reward blocks. (b) Neuronal activity for a second LIP neuron was greater when the cued movement was more probable (red curve) than when the same movement was less probable (blue curve). Conventions as in (a). (c) When free to choose, monkeys shift gaze to the target associated with the larger reward. Relative reward size reflects the volume of juice available for a saccade in the neuron's preferred direction, divided by the total volume of juice available from both possible saccades, within a block of trials. Data are from a single experiment. (d) Average activity (+ standard error) of a single LIP neuron measured after target onset and plotted as a function of relative reward size, for trials in which the monkey shifted gaze in the neuron's preferred direction. Data are from the same experiment as in (c). Adapted with permission from [60], RF, response field,



Summary

- a decision = process that weights priors, evidence, and value to generate a commitment
- Signal detection theory and sequential analysis provide a theoretical framework for understanding how decisions are formed
- Studies that combine behavior and neurophysiology have begun to uncover how the elements of decision formation are implemented in the brain
- Perceptual tasks are used to distinguish evidence and decision variable.
- comparing a decision variable to a given threshold seems to be the basic mechanism of decision making
- Many open questions though ...