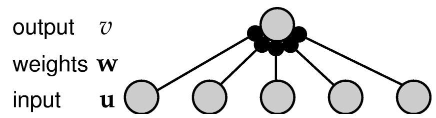
Neural Networks That Learn (Supervised Learning)

Readings: D&A, chapter 8.

- aka the prehistory of Deep Learning -

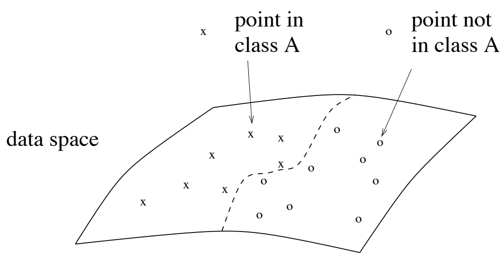
Supervised learning



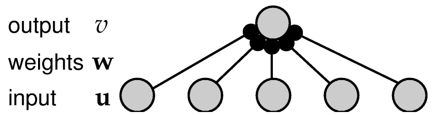
- we want to have a network which is able to do a classification of inputs into 2 categories (e.g. A or not A)
- we want to optimise the parameters = weights **w** of the network so that the network makes as few errors as possible;
- we assume that there exist a data base which contains examples (**u**,v) of the inputs to learn together with the correct (or 'target') output;
- Final task of the network: generalise to new data;

Supervised learning

- The task of learning and generalisation corresponds to finding a surface -- a.k.a discriminant function -- which separates the elements of class A from the others.



The Perceptron (1) - definition



$$v = \begin{cases} +1 & \text{if } \mathbf{w}\cdot\mathbf{u} - \gamma \geq 0 \\ -1 & \text{if } \mathbf{w}\cdot\mathbf{u} - \gamma < 0\,. \end{cases}$$

$$\mathbf{w}.\mathbf{u} = \sum_{i=1}^{i=N} w_i u_i$$

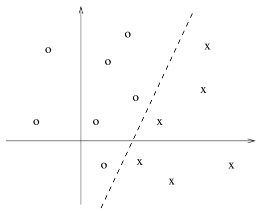
- simplest one-layer network (with binary output);
- the inputs are m input patterns **u;**
- **w** are the weights of the perceptron;
- Task of the perceptron : place each input pattern into one of two classes designated by the desired output v^m= -1 or v^m= +1;
- The threshold determines the dividing line between values of **w**.**u** that generate +1 and -1 outputs

The Perceptron (2) - Linear Separability

- The boundary between the 2 classes is given by:

$$\sum_{i=1}^{i=N} w_i u_i - \gamma = 0$$

- This is a linear equation and defines a hyper-plane in the input space.
- A simple perceptron can only solve problems which are linearly separable.



The Perceptron (3): how do we learn the weights?

- Test one data point $\mathbf{u}^m$ after the other, i.e. apply it at input layer and compare the output $v(\mathbf{u}^m)$ to the desired output v^m
- If output is correct, don't take any action;
- If output is incorrect, change **w**.
- The learning rule is [Rosenblatt, 1958]:

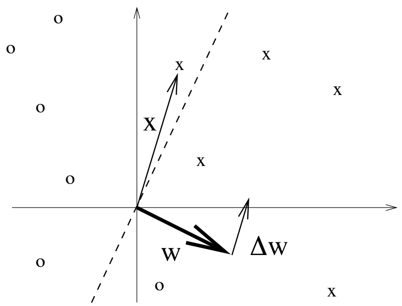
$$\Delta w_i = \eta(v^m - v(\mathbf{u}^m))u_i^m$$

- where η is the learning rate -- a small parameter.
- e.g. $v^m = +1$, $v(\mathbf{u}^m) = -1$

$$w_i \to w_i + 2\eta u_i$$

$$w_i.u_i \to w_i.u_i + 2\eta u_i^2$$

The Perceptron (4): Learning



- if a point x is misclassified, the weight vector is changed in direction of x. This rotates the separating line in the desired direction.

Batch Learning vs. *Online* Learning

- Two ways to apply the learning rule:
- Online: change the weights after presentation of each input data:

$$\Delta w_i = \eta(v^m - v(\mathbf{u}^m))u_i^m$$

- Batch: present all the data then change the weights:

$$\Delta w_i = \eta \sum_{m=1}^{m=Ns} (v^m - v(\mathbf{u}^m)){u_i}^m$$

- Batch learning is often more effective but a bit more prone to get stuck in local minima.
- Online learning is more plausible biologically, but error not guaranteed to go down at each step (optimising for a new pattern can result in unlearning the previous pattern).

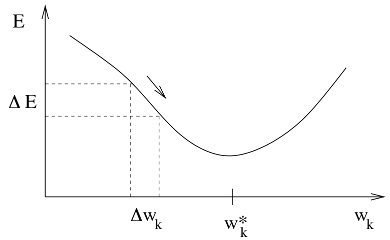
Gradient descent (1)

- The perceptron is a simple case. More generally, we consider a continuous output function: $$v(\mathbf{u}^m) = g(\mathbf{w}.\mathbf{u}^\mathbf{m} - \gamma)$$

- The total quadratic error is: $$E(\mathbf{w}) = \frac{1}{2}\sum_m (v^m - v(\mathbf{u}^m))^2$$
= a function of **w**

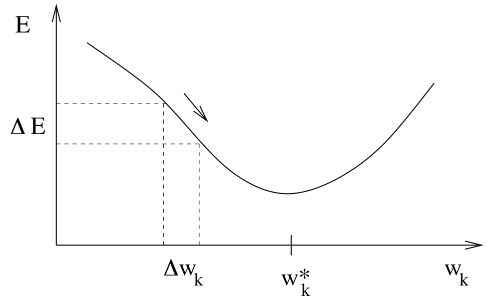
- We want to change the weights such that the error decreases = in direction of the negative gradient:

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$



Gradient descent (2)

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$



- if we are in a region where the slope of $E(w_k)$ is negative, we want to increase w_k;
- if we are in a region where the slope is positive, we want to decrease w_k;
- the steepest the slope, the more we want to change the weights.

Gradient descent (3)

- The gradient can be easily calculated, we get:

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i} = \eta \sum_{m=1}^{m=Ns} g'(\mathbf{w}.\mathbf{u}^\mathbf{m})(v^m - v(\mathbf{u}^m)).{u_i}^m$$

- This is known as the delta rule [Widrow & Hoff, 1960].
- The perceptron rule is a particular case of this, where g'=1.
- `delta' refers to:

$$\delta_m = g'(\mathbf{w}.\mathbf{u}^\mathbf{m})(v^m - v(\mathbf{u}^m))$$

- Online rule:

$$\Delta w_i = \eta \delta_m u_i^m$$

Perceptron cannot solve XOR

- perceptron can compute AND and OR but not XOR -- since XOR is not linearly separable.

X	Y	O
0	0	0
0	1	1
1	0	1
1	1	0

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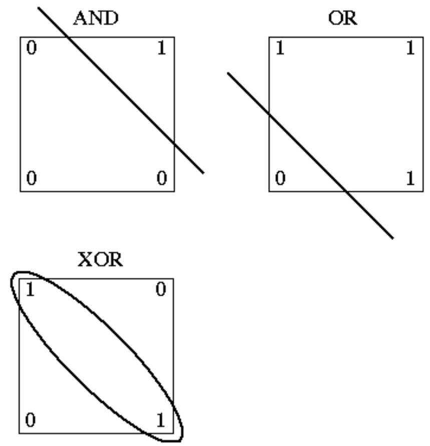
XOR



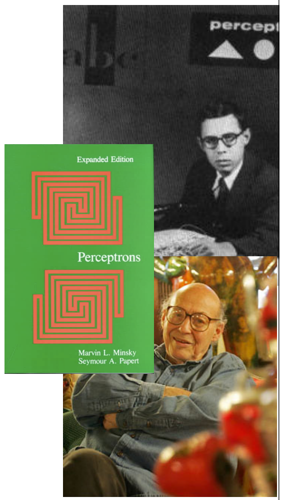
History of AI -- Perceptrons and the dark age of connectionism

• A perceptron was introduced in 1958 by Frank Rosenblatt -- a schoolmate of Marvin Minsky.

• He predicted that **"perceptron may eventually be able to learn, make decisions, and translate languages."**

• An active research program into the paradigm was carried out throughout the 60s but came to a sudden halt with the publication of Minsky & Papert's 1969 book *Perceptrons*. They showed that there were severe limitations to what perceptrons could do and that Frank Rosenblatt's claims had been grossly exaggerated.

• The effect of the book was devastating: virtually no research was done in connectionism for 10 years. Rosenblatt died in a boating accident shortly after the book was published.



http://en.wikipedia.org/wiki/History_of_artificial_intelligence

Towards Multi-Layer networks

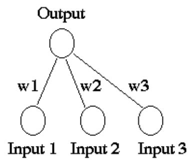
• What to do when the problem is not linearly separable?

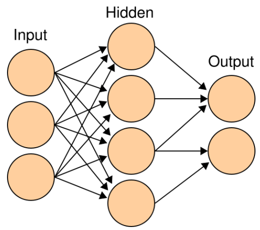
• 1) preprocess to make the problem separable (e.g. by mapping to a higher dimension space) -- cf Support Vector Machines ; or

• 2) use a multi-layer network.

• The most important learning rule for multi-layer networks is the (error) back-propagation algorithm. = Generalization of the delta rule [Chauvin & Rumelhart, 1985].

Input 1	Input 2	Input 3	Output
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0





Back-propagation algorithm

• initialize weights to small random values

• apply a sample input pattern r^{in} to the input nodes

• propagate input through the network by calculating the rate of nodes in successive layers *l*

$$r_i^l = g(h_i^{l-1}) = g(\sum_j w_{ij}^l r_j^{l-1})$$

• Compute the delta term for the output layer

$$\delta_i^{out} = g'(h_i^{out})(v_i^{out} - r_i^{out})$$

• Back-propagate delta terms through the network

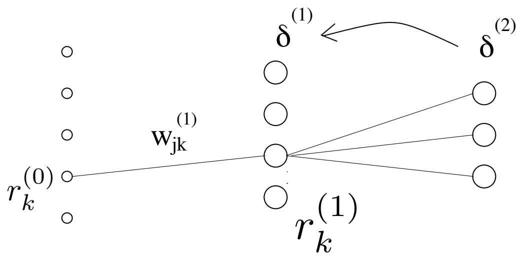
$$\delta_i^{l-1} = g'(h_i^{l-1}) \sum_j w_{ij}^l \delta_j^l$$

• Update weight matrix by adding the term

$$\Delta w_{ij}^l = k \delta_i^l r_j^{l-1}$$

• Repeat until error is sufficiently small.

Back-propagation algorithm

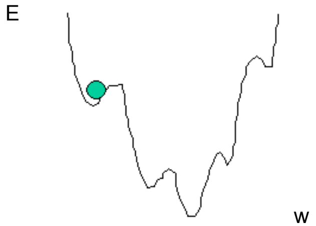


• At output layer, same as delta rule

• At hidden layer, each units receives the weighted sum of the delta-terms of the units it connects to (in output layer). This serves as delta-term in this layer.

$$\delta_i^{l-1} = g'(h_i^{l-1}) \sum_j w_{ij}^l \delta_j^l$$

Stuck in a Local Minimum ?



• a general limitation of pure gradient descent methods is the possibility that the network gets trapped in a local minimum of the Error surface.

• Solution: include some stochastic process that enable random search

• simulated annealing: add some noise to the weights values. the noise level is then gradually reduced to unsure convergence.

Over-fitting and Generalization

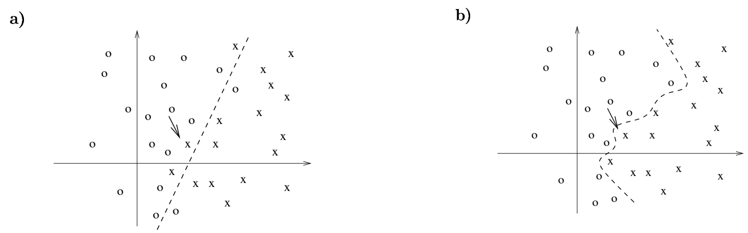
• is it so good to have a very flexible network?

• in some cases it is better to have a network which doesn't perform perfectly on the training data set

• learning the noise in the data = over-fitting. This happens when the number of free parameters (weights) in the model is too large.

• stopping the training when the error on the testing data set increases is one way to prevent over-fitting (regularization by early-stopping)

• having lots of data is another.



History of AI: revival of connectionnism

The introduction of the Hopfield nets (1982) by John Hopfield and of the backpropagation algorithm by David Rumelhart (1985) revived the field of connectionism which had been abandoned since 1970.

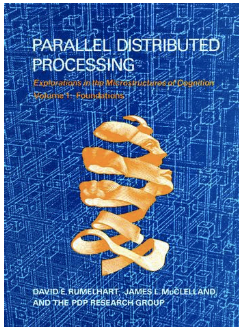
The new field was unified and inspired by the appearance of Parallel Distributed Processing in 1986—a two volume collection of papers edited by Rumelhart and psychologist James McClelland.

Neural networks would become commercially successful in the 1990s, when they began to be used as the engines driving programs like optical character recognition and speech recognition. Nowadays, deep learning.

Deep Learning
use of big data, graphic cards (GPUs) and improved algorithms to train multi-layer networks.
Application in voice recognition (2010), image processing (2013) and more ..

Models of the brain?

- Still used as models of the brain, and disease
- but controversial.
- Supervised learning is a better model of learning for some systems (e.g. motor learning -- visual feedback) than for others (e.g. development)
- Back-propagation of error signals is the most problematic feature;
- Inclusion of derivative terms;
- Different authors have proposed more biologically plausible implementations of back-propagation (O Reilly (1996), Roelsfema & Van Ooyen (2005))



PDP, 1986

Can Deep Learning inform neuroscience?

- maybe …

frontiers in Computational Neuroscience

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Toward an Integration of Deep Learning and Neuroscience

Adam H. Marblestone[1*], *Greg Wayne*[2] *and Konrad P. Kording*[3]

[1] *Synthetic Neurobiology Group, Massachusetts Institute of Technology, Media Lab, Cambridge, MA, USA,* [2] *Google Deepmind, London, UK,* [3] *Rehabilitation Institute of Chicago, Northwestern University, Chicago, IL, USA*

Neuroscience has focused on the detailed implementation of computation, studying neural codes, dynamics and circuits. In machine learning, however, artificial neural networks tend to eschew precisely designed codes, dynamics or circuits in favor of brute force optimization of a cost function, often using simple and relatively uniform initial architectures. Two recent developments have emerged within machine learning