2D Pose Estimation from Lines

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2D Pose Estimation

Goal: find flat object object pose (eliminate some invalid matches)

Given a set $\{(m_i, d_{j_i})\}, i = 1..L$ of compatible pairs

Find the rotation $\mathbf{R}$ and translation $\vec{t}$ that transforms the model onto the data features.

This is the 'pose' or 'position'

Let $\mathbf{R} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$ be the rotation matrix

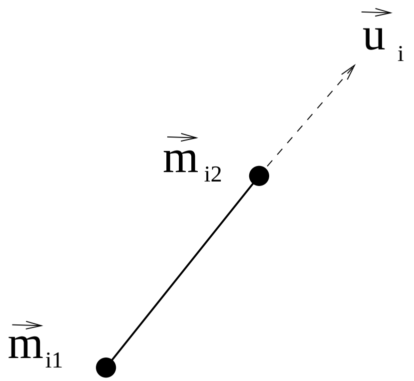
If $\vec{p}$ is a model point, then $\mathbf{R}\vec{p} + \vec{t}$ is the transformed model point

Usually estimate rotation $\mathbf{R}$ first and then translation $\vec{t}$

Estimating Rotation

Given model line i endpoints $\{(\vec{m}_{i1}, \vec{m}_{i2})\}$

Corresponding data line endpoints $\{(\vec{d}_{i1}, \vec{d}_{i2})\}$



Model line direction unit vector:

$$\vec{u}_i = \frac{\vec{m}_{i2} - \vec{m}_{i1}}{|| \vec{m}_{i2} - \vec{m}_{i1} ||}$$

Data line direction unit vector:

$$\vec{v}_i = \frac{\vec{d}_{i2} - \vec{d}_{i1}}{|| \vec{d}_{i2} - \vec{d}_{i1} ||}$$

If no data errors, want $\mathbf{R}$ such that

$$\vec{v}_i = \pm \mathrm{R} \vec{u}_i$$

($\pm$ as don't know if endpoints are in same order)

But, as we have errors $\rightarrow$ least squares solution

Step 1: compute vectors perpendicular to $\vec{v}_i$
If $\vec{v}_i = (v_{x1}, v_{y1})$, then perpendicular is $(-v_{yi}, v_{xi})$

Step 2: compute error between $\vec{v}_i$ and $\mathrm{R}\vec{u}_i$
Use dot product of $\mathrm{R}\vec{u}_i$ and perpendicular, which equals sin() of angular error, which is small, so sin(error) $\doteq$ error

$$\epsilon_i = (-v_{yi}, v_{xi}) \mathbf{R} (u_{xi}, u_{yi})'$$

Step 3: Reformulate error

Let $\mathbf{R} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$

Multiplying out and grouping terms:

$$\epsilon_i = (v_{xi}u_{yi} - v_{yi}u_{xi}, v_{yi}u_{yi} + v_{xi}u_{xi})(cos(\theta), sin(\theta))'$$

Make a matrix equation

$$\vec{\epsilon} = \mathrm{D}(cos(\theta), sin(\theta))'$$

Each row of L vector $\vec{\epsilon}$ is ϵ_i and each row of $L \times 2$ matrix $\mathbf{D}$ is $(v_{xi}u_{yi} - v_{yi}u_{xi}, v_{yi}u_{yi} + v_{xi}u_{xi})$

The least square error is $\vec{\epsilon}'\vec{\epsilon} = (cos(\theta), sin(\theta))\mathbf{D}'\mathbf{D}(cos(\theta), sin(\theta))'$

Step 4: Finding rotation that minimizes least square error

Let $\mathbf{D}'\mathbf{D} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

Then, we minimize $(cos(\theta), sin(\theta)) \begin{bmatrix} e & f \\ g & h \end{bmatrix} (cos(\theta), sin(\theta))' =$
$ecos(\theta)^2 + (f+g)cos(\theta)sin(\theta) + hsin(\theta)^2$

Differentiate wrt θ and set equal to 0 gives:

$$(f+g)cos(\theta)^2 + 2(h-e)cos(\theta)sin(\theta) - (f+g)sin(\theta)^2 = 0$$

Divide by $-cos(\theta)^2$ (if $cos(\theta) = 0$ then use special case) gives:

$$(f+g)tan(\theta)^2 + 2(e-h)tan(\theta) - (f+g) = 0$$

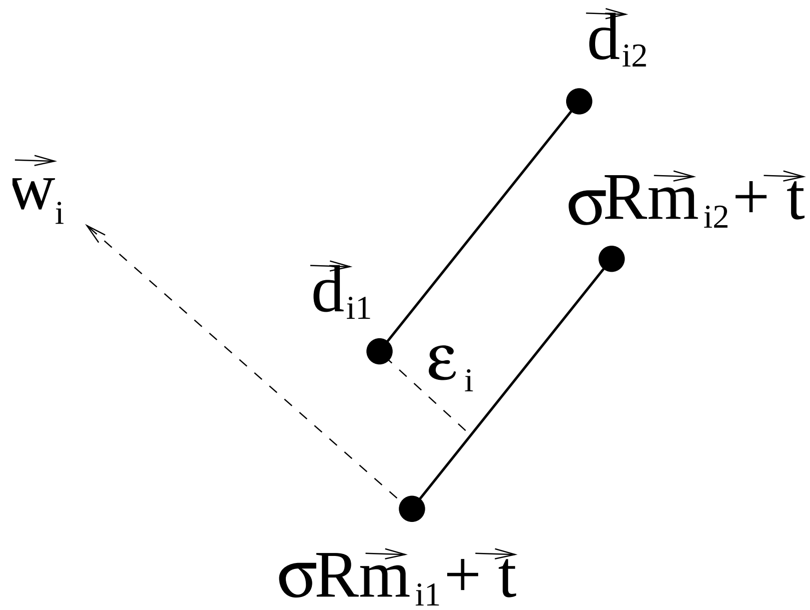
Solving gives:

$$tan(\theta) = \frac{(h-e) \pm \sqrt{(e-h)^2 + (f+g)^2}}{(f+g)}$$

Four θ solutions (2 for $\pm$, 2 for $tan(\theta) = tan(\pi + \theta)$).

Try to verify all 4.

Estimating Translation By Least Squares



$\vec{w}_i$ is perpendicular to rotated model line i

Offset error $\epsilon_i = (\vec{d}_{i1} - \sigma \mathbf{R} \vec{m}_{i1} - \vec{t})' \vec{w}_i$

Differentiate $\sum_i \epsilon_i^2$ wrt $\vec{t}$, set equal to $\vec{0}$ and solve for $\vec{t}$ gives:

$$\vec{t} = (\sum \vec{w}_i \vec{w}_i')^{-1} \sum \vec{w}_i \vec{w}_i' (d_{i1} - \sigma \mathbf{R} \vec{m}_{i1})$$

What Have We Learned?

- 2D Least Squares rotation and translation estimation algorithms