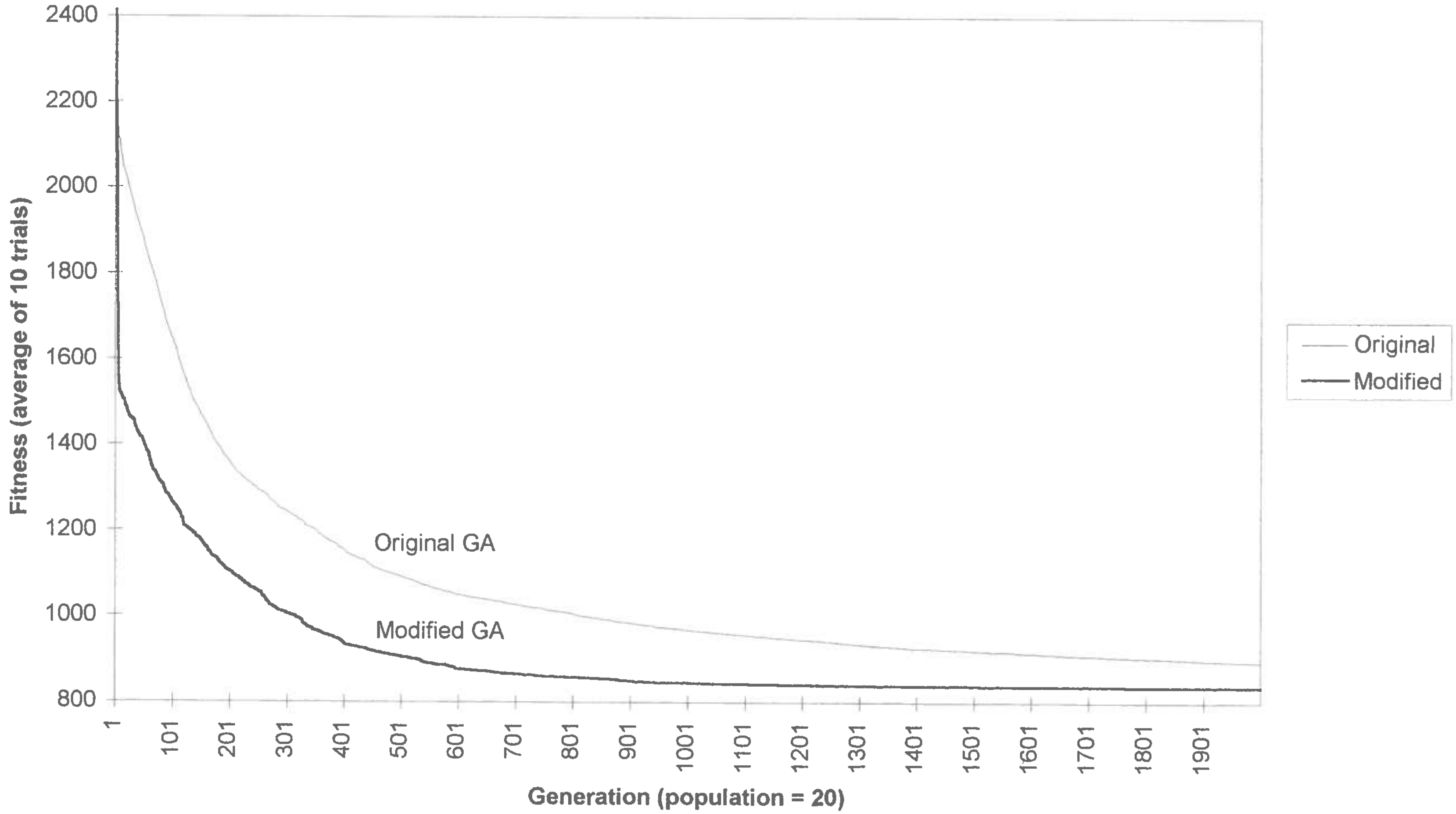
Figure 3.19
Comparision of Original vs Modified Genetic Algorithms Performance



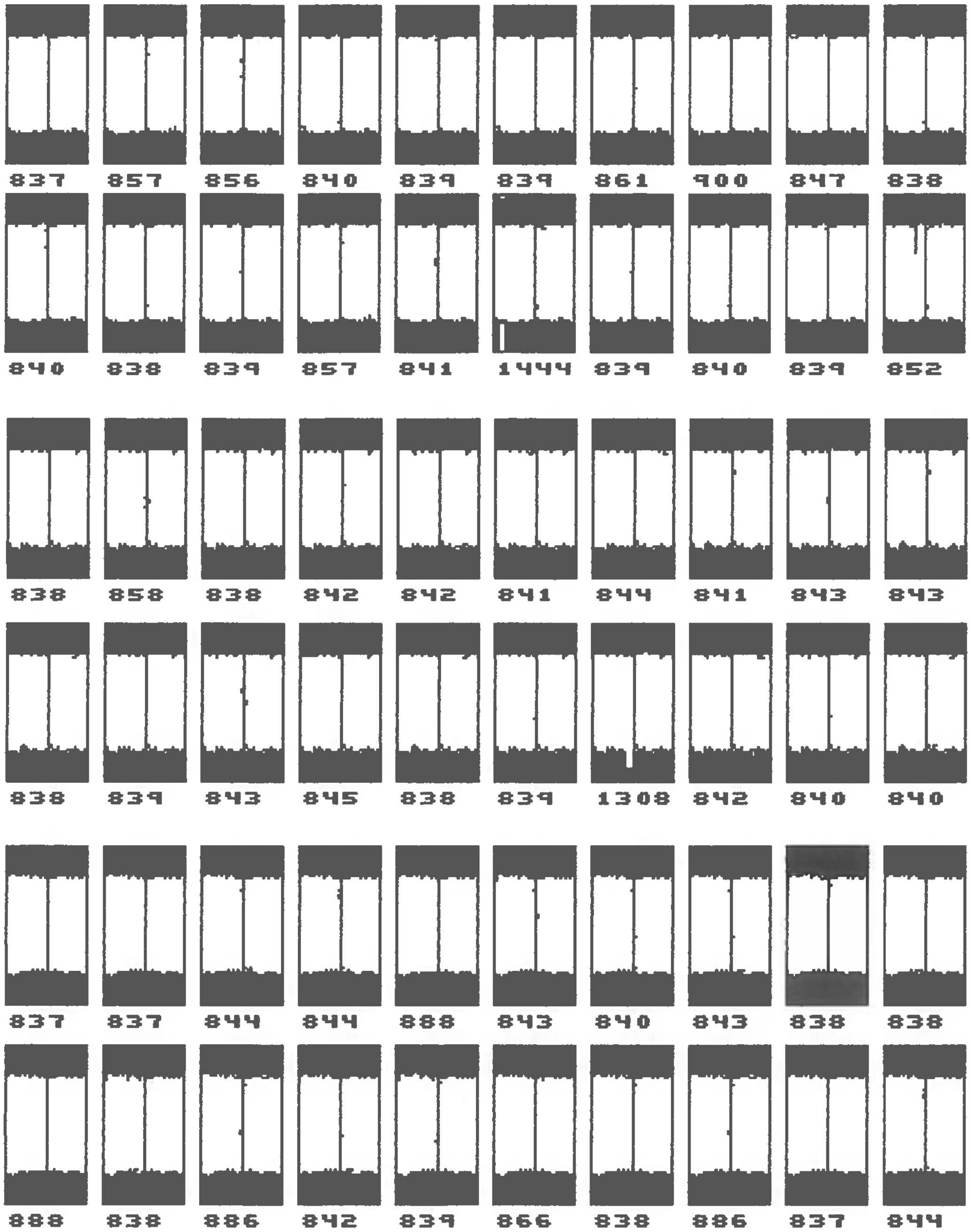
first ten generations and producing final results which are about 11% better than those found by the naïve GA. Also significant is the fact that the modified GA found almost no further improvements during the last thousand generations, the average best fitness over this time altered only from 846 to 838. Looking at the final results produced from this run in Figure 3.20, it is clear that they are very close to the theoretical global optimum for this problem.

3.4 Conclusions about the simplified physics model experiments.

The experiments described in this chapter were intended to find new operators and better parameter settings for a naïve GA, thereby improving it's performance ready for application to a complicated real-world problem. The focus has not been on fine-tuning the parameter settings for this particular optimisation, but rather on determining the effects of the various parameters. Similarly the new operators were not designed to solve the *unique* difficulties uncovered by the experiments, but to address issues relating to a generalised n-dimensional voxel based representation.

The results have shown that although a naïve GA does indeed suffer from the problems suggested by [Watabe & Okino 93], a small selection of operators informed only by domain knowledge about the representation, will effectively solve each of these difficulties and permit the production of useful results.

The final system uses a normal bitwise mutation operator in addition to the two new mutation operators, smoothing and two by two. Both of the new mutation operators are applied using a time-varying probability in order to maximise their effects at the appropriate stages of the optimisation. The smoothing operator rapidly cuts away unwanted areas of material during the early stages of the optimisation and can help to smooth ragged edges and fill small holes later on. The two by two mutation operator is highly effective at both smoothing off ragged edges and at filling in small holes in the material if they occur in undesirable places. The two-point crossover operator has been replaced by the n-dimensional UNBLOX operator



Typical End of Run Results From Modified GA

Figure 3.20

described in work by [Cartwright & Harris 93], and this alteration directly answers the issue of whether the children produced by crossover will be valid with any degree of reliability. Finally the Genitor style ranked survivor selection [Whitley 89] permits easy control over the degree of exploitation versus exploration that occurs, and also separates the proportion of fit individuals selected from their actual fitness values, making the whole GA less sensitive to changes in the type of evaluation function being applied.

Chapter Four.

Transferring the solutions to real problems.

The motivation for the experiments detailed in Chapter Three was to produce an effective GA which can quickly and efficiently optimise a two-dimensional shape optimisation problem represented by a voxel grid. The work so far has used an evaluation function based on a highly simplified physics model of the forces and stresses involved in a beam loading problem. The experiments in this chapter will detail how the same improved GA was then modified to use a commercial Finite Element Analysis (FEA) package called Ansys* as it's evaluation function, and how the combination of the optimised GA and Ansys worked to optimise an annulus design problem which has been under research by Rolls Royce. It will be seen that although several major difficulties were encountered in both the inter-package communications and in the optimisation itself, the difficulties were not insurmountable and the final system was capable of producing both interesting and useful results.

4.1 Motivation for this approach.

There were several different motivations for taking this approach to extending the experiments. It was considered important to ensure that the arguments put forwards in Section 3.3 were correct and that the optimisations made to the GA would still be applicable to a much harder real-world problem. Engineers do need to solve this type of shape optimisation problem and if this approach is successful then it could assist them to achieve better results. GA optimisations can easily be modified into hybrid systems [Tucson *et al.* 97] and in this case the computer would

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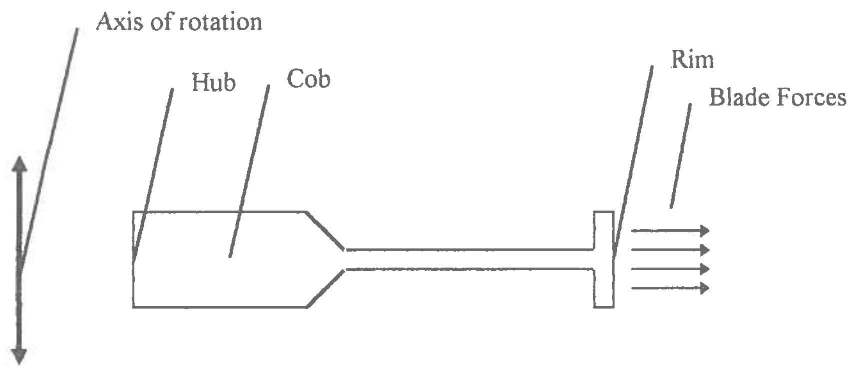
* Ansys is a registered trademark of SAS IP, Inc.

rely on an engineer's practical experience and knowledge of the problem domain to direct key choices in the optimisation process. Finally, GAs are also an ideal tool with which to address this type of problem, as the *population* of final solutions will allow engineers to choose the one they consider most appropriate using any further criteria which were not built into the fitness function.

4.2 The annulus design problem.

The annulus design problem comes from a specification supplied by Rolls Royce plc to Richard Smith during his time in the Mechanical Engineering Department at Edinburgh University. The full original specification of this problem is included as Appendix A. The problem is to design an annulus which is to function as the hub of a jet-engine turbine propeller. This part will be subject to extreme forces due to its high speed of rotation and the attachment of the turbine blades to its outer circumference.

The actual optimisation assumes that the part is axisymmetrical around the axis of rotation, and consequently it reduces to the two-dimensional shape optimisation problem shown as Figure 4.1.



Annulus Axisymmetric Cross-section.

Figure 4.1

The optimisation as performed for these experiments involved reducing the mass of the annulus whilst observing a series of four separate stress constraints. The constraints relate to the *hoop stresses* at the inner and outer circumferences and the *radial stresses* along the centre line of the annulus from the hub to the rim. Hoop stresses are those that go around the annulus, effectively into and out of the page given the cross-sectional picture in Figure 4.1. Radial stresses are those which go from the hub to the rim (or vice-versa) or left and right in Figure 4.1.

4.3 Implementation details.

The GA fitness function was defined as:

fit = -weight[b] + (1.0 / total_stress)

Equation 4.1

Constraint penalties were applied if any of the four constraints limits were broken, and the constraints were ordered in importance by using 4*K* for the most important, 3*K for the second most important, 2*K for the next and 1*K for the least important constraint.

In order to allow the GA program to use Ansys as its evaluation function, it was first necessary to find some way of communicating data about the shape to be optimised and the final evaluation value between the two programs. The Ansys manuals describe a method whereby it should be possible to link the entire Ansys program into a user supplied program as a library, however this relies on the use of a Fortran compiler and for the PC compatible version of Ansys it would need to be a Windows™ Fortran compiler. This approach was problematical due to the

* K = constraint penalty multiplier value.

difficulties involved in getting a suitable compiler and because the GA as written and optimised so far was entirely written in C.

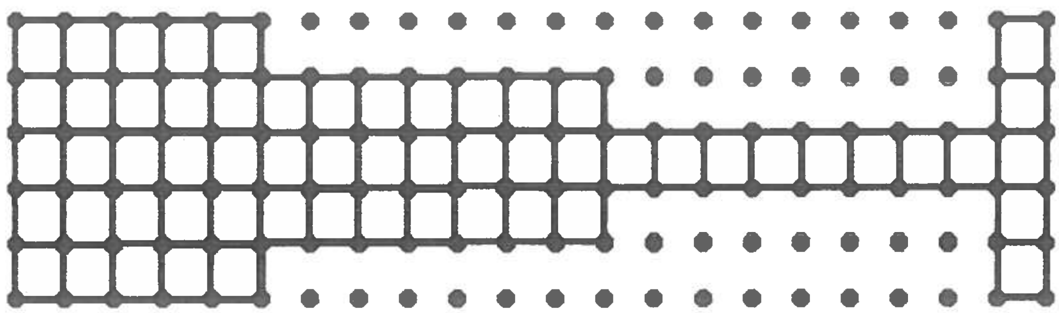
An alternative method of communication was attempted whereby the GA program would 'call-out' to Ansys using command line specifiers to control the Ansys program. It would also read and interpret the files that Ansys writes to preserve the results of an evaluation. This method was implemented and worked satisfactorily for very small problems, however the 'call-out' made use of the C 'system()' command which in turn uses a DOS™ type command box in order to interpret the command specified. This resulted in very slow evaluations as the DOS box takes two seconds (per evaluation) to open, and also permitted a memory leak from either the Windows DOS box or the Ansys program to gradually use up all available system resources which would eventually cause a crash after approximately ten generations with a population of twenty individuals.

After detailed study of the Ansys 'Programmer's Manual' a third alternative was discovered which involved the use of the Ansys internal macro programming language. This macro language whilst insufficiently powerful for the implementation of the full GA, does include instructions which perform counted loops and can 'call-out' to other programs. A memory leak problem was anticipated with the 'call-out' function, however by changing the orientation of the optimisation in this way, the 'call-out' would only be necessary once per generation instead of once per individual evaluation. Therefore if the memory leak was from the DOS box the program could reasonably expect to evaluate twenty times as many generations before crashing from the leak, which might be acceptable if no data is lost when the program stops. The reduced frequency of 'call-outs' also meant that the speed lost due to the start-up time for the DOS box would not be such a problem.

A simple macro was devised which looped a user-specified number of times and performed a 'call-out' to a small C program on each iteration. This worked extremely well, and did not suffer from memory leak problems even after one-thousand iterations. The memory leak was therefore in Ansys itself, which would

not be a problem as the new approach only involves starting Ansys once and then permitting the macro to control the optimisation process.

Ansys requires two different data files in order to define the shape which it is attempting to optimise; the nodal position file and the element connection file. The nodal position file specifies where in the optimisation space the nodal points are located. Each node has a number which is used in the element connection file to specify which nodes are connected together to form elements. Connected nodes form the corners of the voxels which combine to form the shape to be optimised. Figure 4.2 shows how these files combine to define the optimisation shape. The nodes are represented by the circles, and the lines represent the connections between them. In this implementation, the node position file is constant which allows it to be set up during the initialisation phase and thereby saves valuable processor time. The element files are created from the chromosomes just before the GA program releases control back to the Ansys macro.



Nodes are Connected Together to Form Elements.

Figure 4.2

Interpreting the files required by Ansys was achieved by detailed reference to the Programmer's manual and cross-checking with the output produced from a very simple analysis performed on a fully occupied rectilinear grid. The appropriate routines were added to the existing GA code and a command-line interpreting capability was also added so that the Ansys macro could easily command the GA

program to initialise the population or produce the next generation, as appropriate. Other functions added to the GA code included the ability to convert the chromosomes into the representation expected by the Ansys program and write these out in a series of standard files used by the macro. With these modifications in place the evaluation macro which would control the Ansys evaluations was written.

A listing of the full Ansys macro can be found in Appendix B as Listing B.1 followed by an abbreviated macro which was originally used to check that the evaluation would perform as desired. This abbreviated macro, included as Listing B.2, performs the following operations:

- Set the evaluation mode to Batch processing
- Restrict the amount of excess data that Ansys produces to an absolute minimum
- Instruct Ansys to output data about the stresses produced
- Set up the properties of the materials to be used
- Establish the element types to be used to represent the voxels
- Read the file specifying the locations of all of the corner point nodes
- Define the type of analysis to be performed
- Specify the rotation speed of the annulus
- Set the level of accuracy of analysis required
- Read the file specifying the nodal connections which form the voxel elements
- Select a subset of the nodes defining the extreme right-hand edge of the annulus
- Apply the force which simulates the attachment of the turbine blades
- Instruct Ansys to perform the evaluation
- Output the relevant results for both Bore and Hoop stresses to standard files
- Exit

The complete analysis macro commences by calling the GA program (called 'stress.exe') with command line options to initialise the population. It encloses the above evaluation commands with a loop that iterates a user specified number of

times, calling the GA program at the end of each loop in order to generate the next generation of chromosomes. Due to restrictions in the macro language it was not possible to *construct* suitable filenames for the output files for each individual member of the population, so the analysis portion of the macro was simply repeated twenty times inside the loop with a different set of filenames for each repetition. This obviously fixes the population size to twenty individuals, however the GA program is capable of modifying the macro instruction file before the analysis commences. This permits a simple addition to the program to create the appropriate number of evaluation instructions if a variable population size is desired.

4.4 Results from the basic system.

The purpose of these first experiments was to determine if the Ansys macro-language/GA hybrid system would work as intended and also to see how well it performs on the annulus optimisation problem.

The settings used for the GA were:

population size	= 20
cross-over rate	= 0.3
mutation rate	= 0.8
smoothing rate	= 0.8
rank selection pressure	= 1.7
horizontal size of area	= 62 voxels
vertical size of area	= 27 voxels
constraint penalty	= 0.00005

The settings used for the annulus were:

horizontal size of area	= 0.25 metres
vertical size of area	= 0.05 metres
radius of hole	= 0.10 metres
blade force	= 10e5 Newton/radian
Young's modulus	= 2.238e11 Pascal
material density	= 8.221e3 kg/m^3
revolution speed	= 1571.0 radian/second

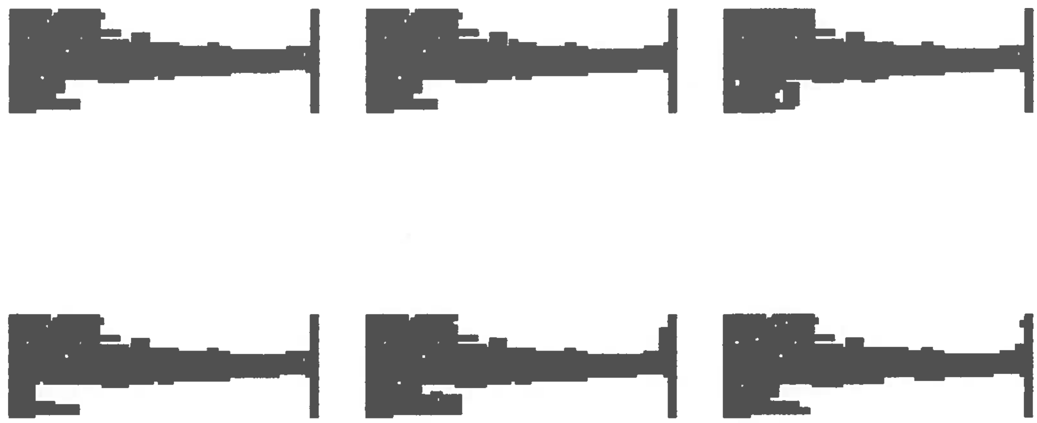
The constraints to be observed were:

hub hoop stress	< 1330 MPa
rim hoop stress	< 396 MPa
inner radial stress	< 741 MPa
outer radial stress	< 334 MPa

The basic system was applied without further modifications to the annulus optimisation, however the problem as specified by Rolls Royce is extremely tightly constrained which meant that the attempts to solve this problem using random population initialisation violated all of the stress constraints by large amounts, and the rate of improvement in the population, when extrapolated beyond the time period allocated to the experiments, indicated that a valid solution would not be found for some considerable number of generations still to come.

To circumvent this problem the population was instead initialised with a selection of variations on the annulus design supplied by Rolls Royce, which were modified further by an intensive random mutation operator which added and removed small areas of material over the surface of the annulus design. This kind of

intelligent initialisation is reasonable as a user will often want to start the GA with the existing designs in order to see what improvements can be made. Even when a totally new shape is being designed, the user will normally have some expectation about the final form, and this could be drawn by hand and scanned into the program. The intelligent initialisation approach meant that the initial population was not unreasonably far outside of the stress constraints, yet supplied the optimisation with sufficient variation that the population did not rapidly converge onto a single solution. Some of the results from this basic system can be seen in Figure 4.3 which shows six members of the population after seventy-five generations.



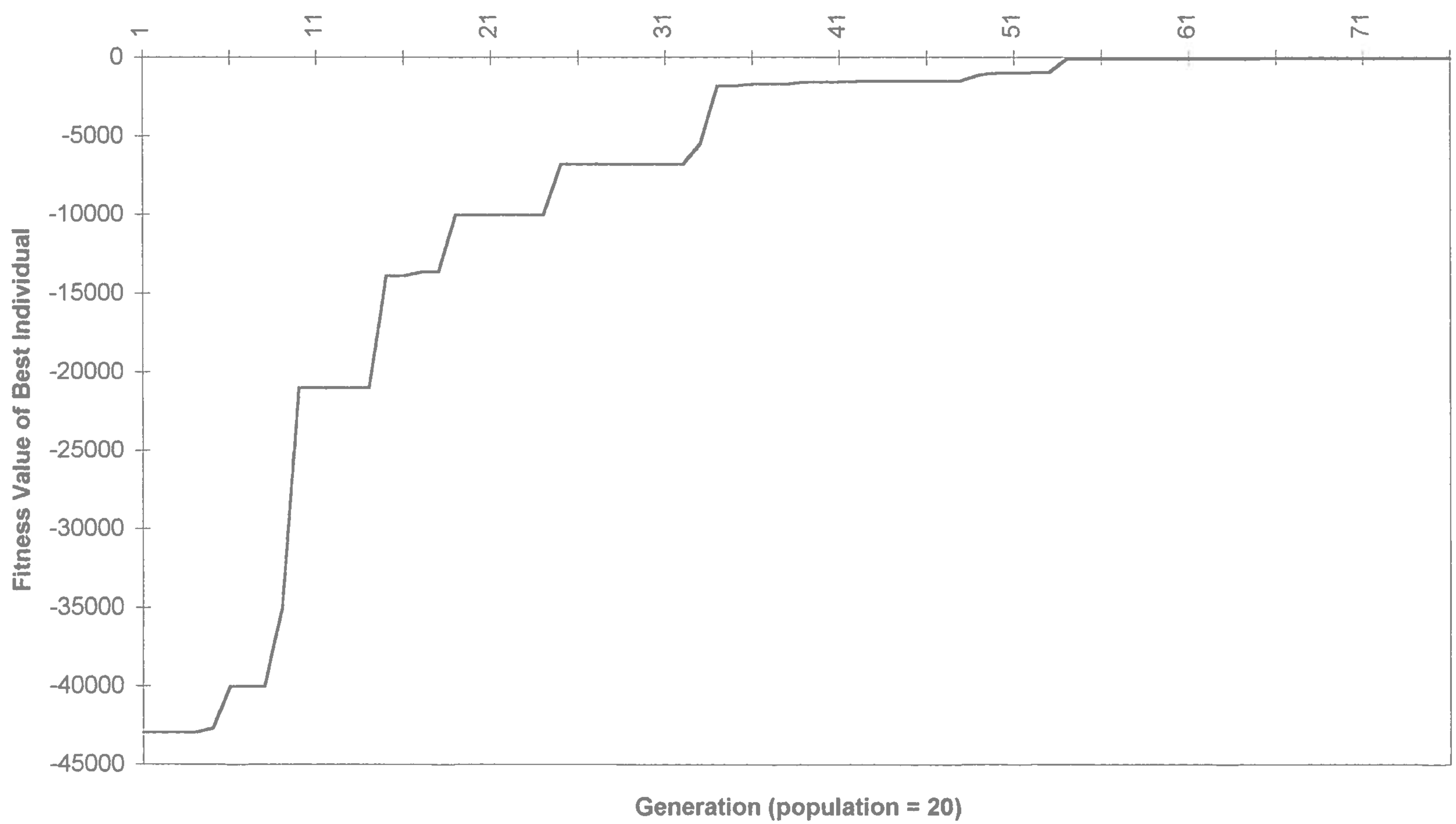
Results of the Basic Annulus Optimisation After 75 Generations.

Figure 4.3

The results shown in Figure 4.3 are extremely poor, the lack of symmetry around the horizontal axis and the uneven surfaces are just the most visible failings in this set of results. Another problem which becomes visible when using Ansys to analyse an individual was that extremely high stress values were present in the corners of many of the voxel areas.

The graph shown as Figure 4.4 shows the fitness curve for these results, with the generation represented on the horizontal axis and the fitness value of the best

Figure 4.4
Fitness Curve for Basic GA Annulus Optimisation



individual of the population shown as the vertical axis. Due to the fact that the fitness values returned by this evaluation are negative, the curve is vertically inverted compared with the curves produced for Chapter Three.

Although the first thirty-five generations show a substantial rate of improvement this is mainly due to the violations of the stress constraint becoming less severe. At the fifty-second generation the first valid solution is found and the curve from there on is almost level, indicating that no further significant improvements were found. The graph indicates that the GA is performing the optimisation correctly and is capable of producing a valid solution when supplied with an initial population that breaks the stress constraints, however it also shows that the rate of optimisation is very poor.

The results of this initial experiment are only moderately encouraging; without any special operators being designed for the new problem domain, the GA has managed to take a population of poor annulus designs and improve them. The first stage in this process was to obtain a viable solution, and the GA achieved this during the first fifty-two generations. The second stage should involve improving the solution to try to find a near-optimal answer, but the GA only managed to improve the fitness value from -108.245 at generation 52 to -107.156 at generation 75.

A further investigation into the reasons for this failure was made, and the fitness log file produced during the optimisation was examined to try to find an explanation. The log file is included in Appendix C as Listing C.1 and it is clear from the predominance of extremely large negative numbers in the final twenty generations that the majority of the population is violating at least one and probably several of the stress constraints. This indicates that the stress constraints are so tight that even minor changes to a valid chromosome can result in extreme violations of them.

The experiments with the basic system operating on the annulus problem have highlighted several major problems which the optimisation must overcome in order to be successful:

- Random initialisation of the population was found to produce results that violated all of the constraints so badly that it would take a very long run just to get a valid solution.
- It takes a very long time to find a valid solution even when initialised with reasonable designs.
- Valid solutions when found contained small holes and protuberances and were not symmetrical.
- The constraints imposed are extremely tight and cause a proliferation of invalid solutions from even small changes in the chromosome.
- Extremely high stress values for both hoop and radial stresses are present in the corners of many of the voxel areas.

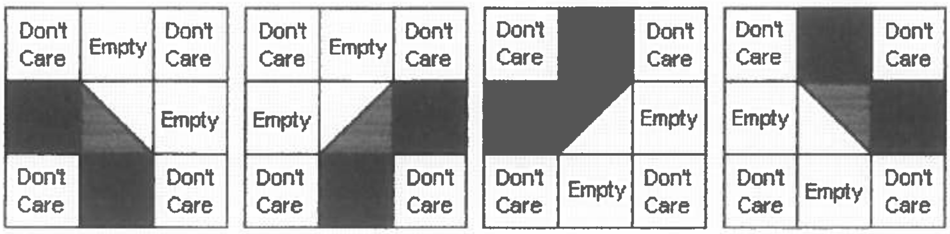
These problems are addressed in Section 4.5.

4.5 Improvements made to the system.

In order to improve the efficiency of the optimisation process several improvements were made to the GA and to the form of the analysis. Firstly, in order to reduce the search space of the problem being optimised, a symmetry constraint was imposed, with the axis of reflection defined to be horizontal through the middle of the shape. The GA was modified to reconstruct the final shape in its entirety only when producing the element definition files to be accessed by Ansys. This simple modification reduces the search space from a typical size of 2^{2542} for a 62 voxel by 41 voxel grid, to 2^{1302} which represents a 62 voxel by 21 voxel half-grid. The centre line along the axis of symmetry is not mirrored as it is now enforced by the GA to be

always turned on. This also provides a guaranteed central line of elements for the stress measurements to be taken from.

One of the problems that was highlighted in the first set of experiments is that the FE package does not deal very well with sharp corners. This is a general failing in FE packages and not a particular weakness of Ansys, but it constitutes a very major problem given the use of a voxel based representation. In an attempt to alleviate this problem a second type of element was defined by the optimisation macro. The new element type is triangular and is created by specifying connections between groups of three nodes in the element connection file. These triangular elements are added to the shape at all suitable 'steps', which are identified by convolving the voxels in the shape against a series of four matching template masks . If each square in the mask matches the value of the voxels surrounding an empty voxel then the appropriate triangular element is created in the 'step'. The convolution masks and the triangles which they cause to be inserted are shown in Figure 4.5.



Convolution Masks for Triangle Insertion Process.

Figure 4.5

The triangular elements were applied in order to smooth off the corners left at voxel 'steps', so it was expected that the artificially high stress levels reported by the FE evaluation at the corners of the voxel areas would be removed.

The small holes appearing in the material may be due to one or more of several different causes:

- The holes are appearing in low stress areas and are a valid part of the solution.
- They are caused by the mutation operators and would be removed if the optimisation was allowed to progress further.
- They are an attempt by the GA to represent a lower density material indicating that although material is needed at that location, it need not be as strong as the basic material being used.

Regardless of which of these explanations is accepted, the reality is that current manufacturing processes would find it extremely difficult to produce a solid annulus with these small holes present, so it is therefore desirable to modify the operators to try to eliminate them, or at least reduce the number of them present. This was accomplished very easily by modifying the two by two mutation operator (the operator which can either fix holes or cause them to appear) to only mutate areas where, as well as at least one voxel being turned on, at least one of the four voxels is also turned off. The result of this modification is that the two by two mutation operator can now only mutate at the boundaries of the shapes being formed, and consequently it should also help reduce the number of small protuberances.

The improved GA for annulus optimisation used the same settings as the basic system for all parameters except that the chromosomal grid was set to 21 voxels high, which is mirrored due to the symmetry constraint to produce a voxel grid height of 41 voxels.

The analysis was permitted to continue for 114 generations and this took approximately twenty-four hours in total. The fitness curve for this analysis is shown as Figure 4.6 which shows the generation number on the horizontal axis and fitness on the vertical axis, the data line represents the fitness of the best member of the population at each generation. This graph shows a considerable improvement in the GA performance over the basic system. The population once again starts without

Figure 4.6
Fitness Curve for Improved GA Annulus Optimisation

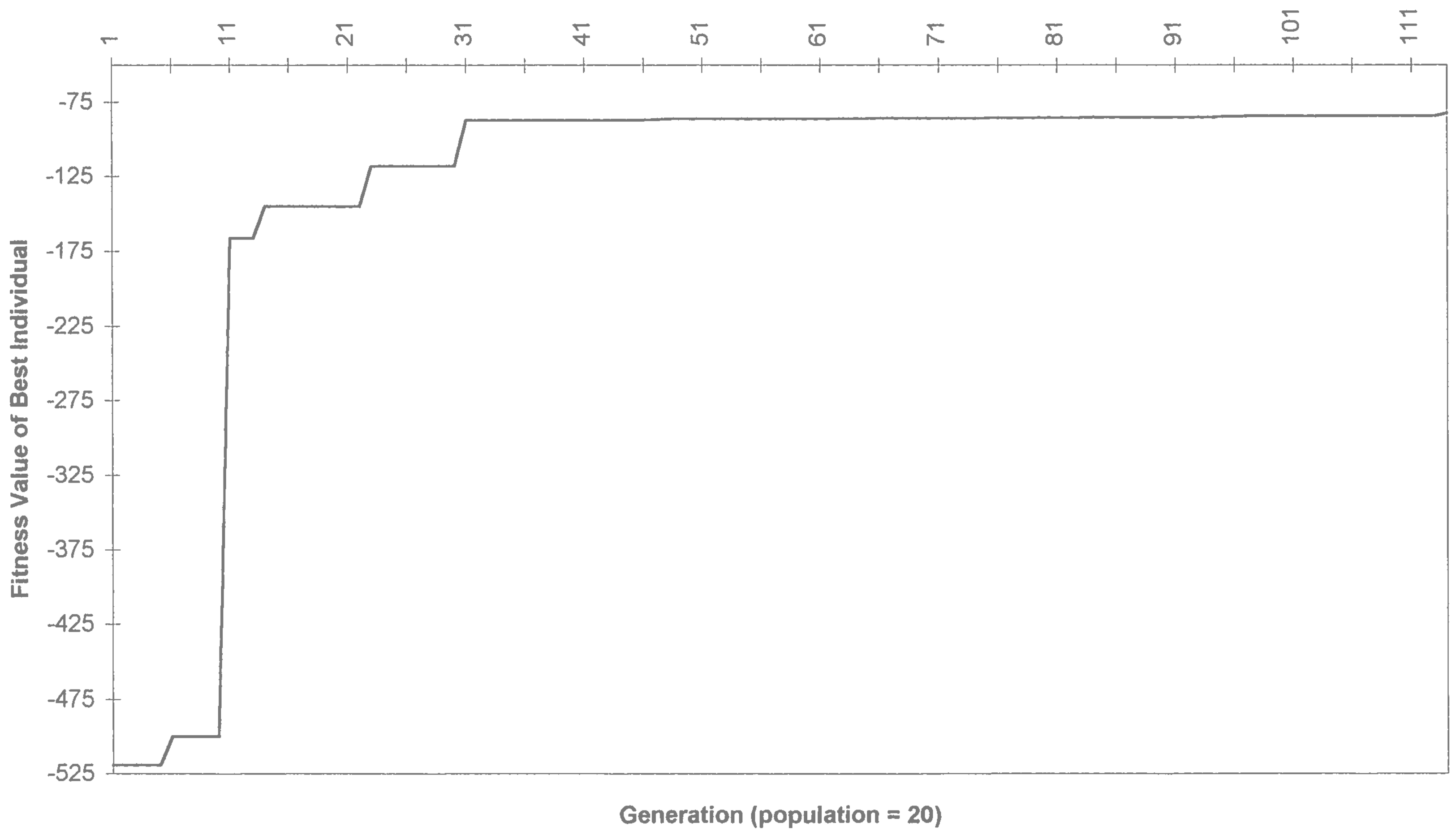
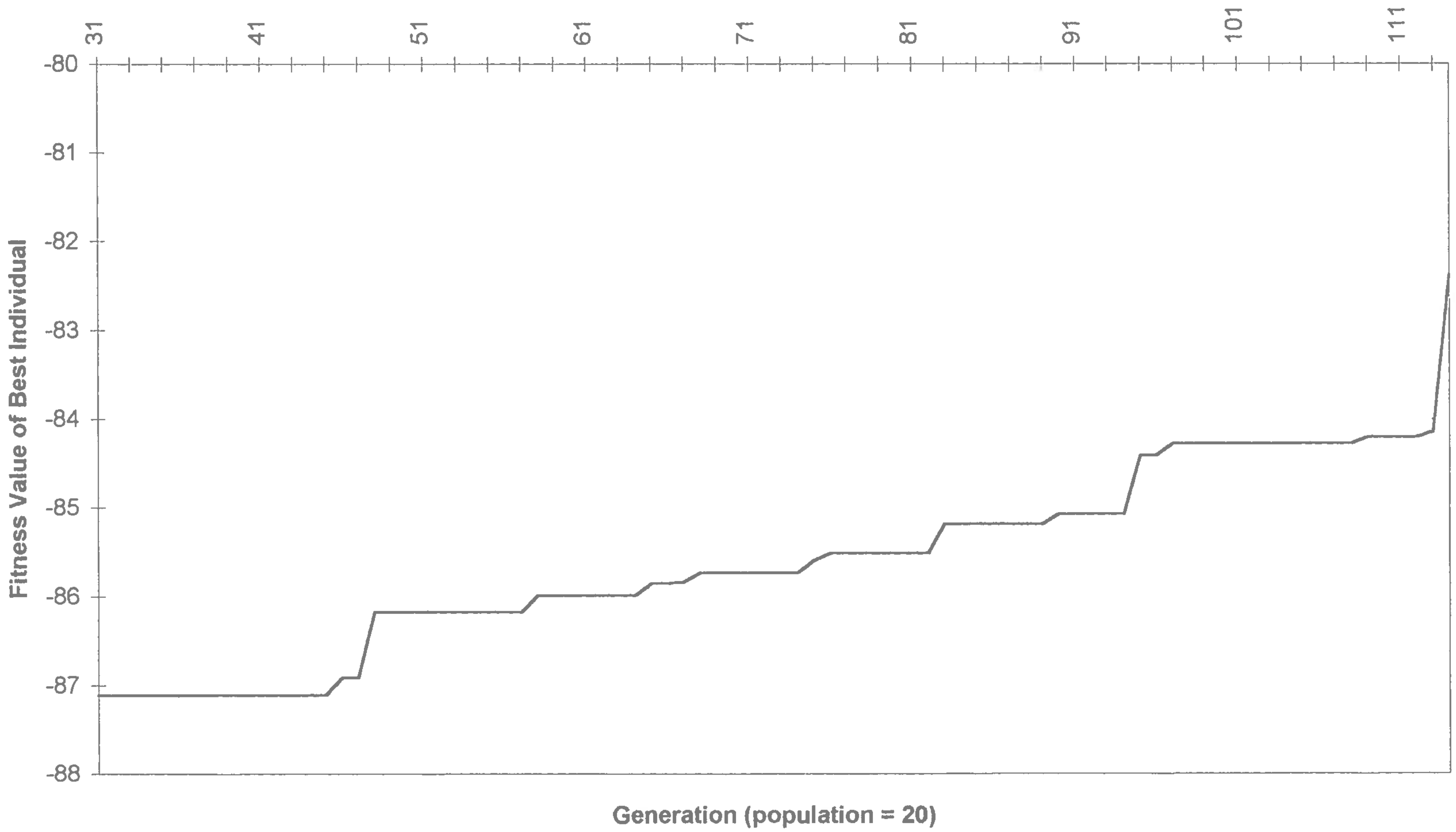
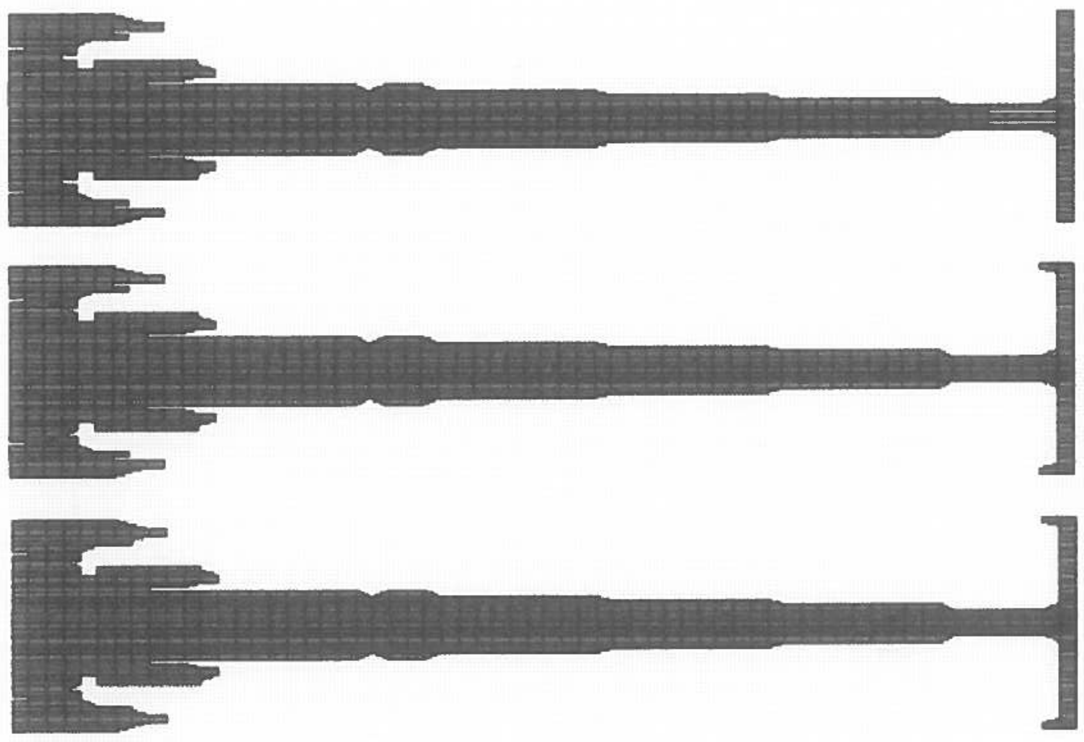


Figure 4.7
Fitness Curve for Improved GA Annulus Optimisation (Detail)



any valid individuals but some extremely rapid early improvements followed by regular small advancements generate the first valid member after only thirty-one passes. Once again the GA has difficulty improving upon this first valid individual, however examining the detail of the portion of the graph where valid individuals were created (Figure 4.7) reveals constant small improvements being made with occasional larger changes. The fitness log for this run is included in Appendix C as Listing C.2.

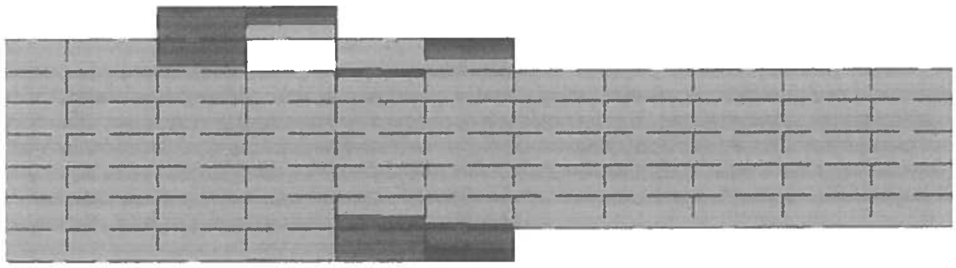
Some of the final population created by the improved GA is shown in Figure 4.8 which displays three of the twenty individuals and shows a clear improvement in quality over the results generated previously. The small protuberances have been totally eliminated and only a few members of the population contain small holes. The rate at which a valid solution was found is considerably faster than the basic implementation, and once found, the GA continued to improve upon this solution even to the very last pass of this trial.



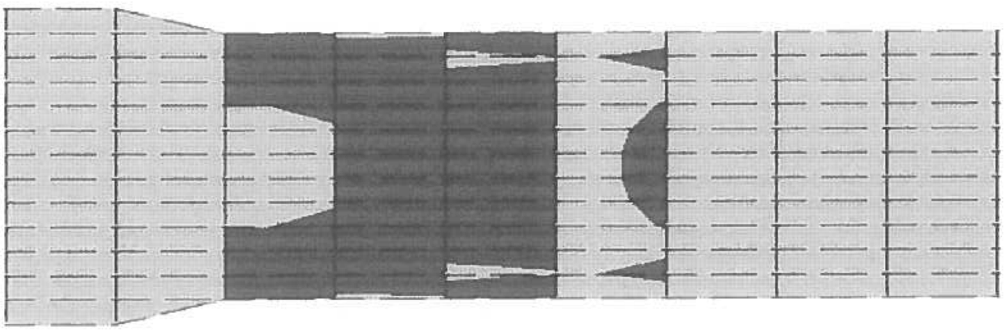
Final Annulus Cross-Sections From Improved GA

Figure 4.8

After using Ansys to examine the solutions produced by this optimisation, it was possible to confirm that the triangles are indeed working as expected and the amount of stress in the regions immediately surrounding a step which has been smoothed is far less than for the steps which have not been smoothed. Figure 4.9 shows the stress values calculated by Ansys for the voxels surrounding steps in two typical runs and clearly shows how the triangles permit the excess stress to be distributed in a more even pattern. Darker shades indicate higher stress levels in both of these pictures.



Without Smoothing Triangles.



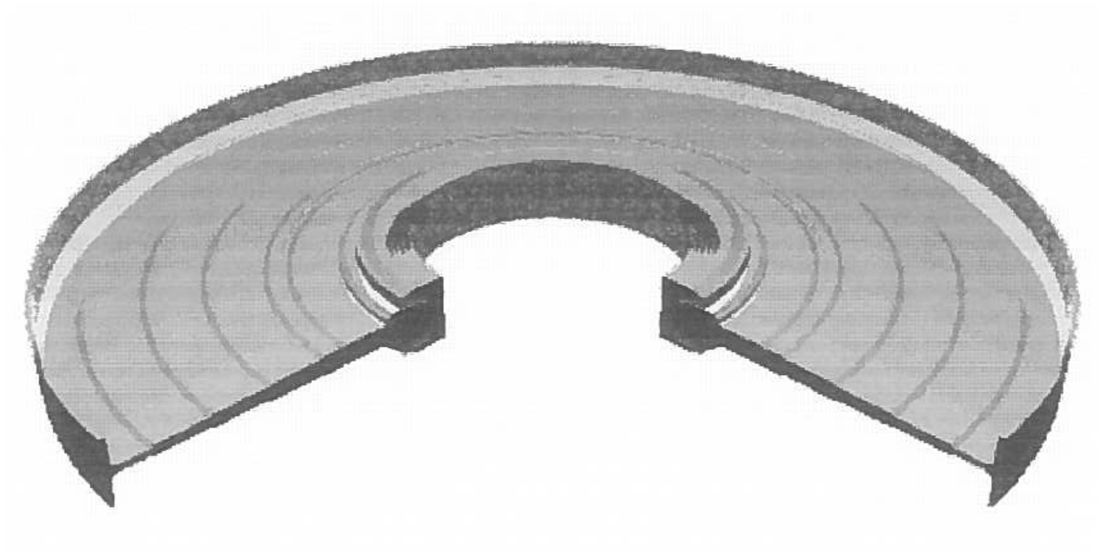
With Smoothing Triangles.

Figure 4.9

Examination of the final results of these experiments reveals several unexpected features, in particular the overhanging component in the cob of the annulus is extremely interesting. A trial analysis using Ansys to evaluate the shape with the overhang replaced with a more conventional sloping solid cob resulted in violations of both the maximum radial stress constraint and the inner hoop stress constraint. Several different adjustments were made but the violations could not be easily repaired. These results imply that the overhang is a genuine benefit to the annulus design, permitting a lower mass solution than is possible without it's presence. A parametric representation of the annulus would probably not have discovered this result because there is no obvious reason to suspect that an overhang will be beneficial, and so there would be no incentive to provide variable parameters which would allow the overhang to form. This unexpected discovery thoroughly vindicates the choice of a voxel representation, and validates the points made in Section 2.3.3

Another interesting result is the presence in the best individual of the population of two short horizontal holes near the point where the neck of the annulus connects to the rim. This is unexpected as the operators have been heavily biased against producing such holes which implies that they too offer genuine advantages to the annulus design. The holes appeared just before the end of the trial run and it would be interesting to see if they remain after many further generations or whether they are just fore-runners of some larger change in the shape.

4.6 Conclusions about the real-world system.



The Best Annulus Design From the Final Set of Experiments

Figure 4.10

This Chapter has described the experiments which extended the work from Chapter Three to deal with a real-world problem using a full commercial Finite Element Analysis package.

Two sets of experiments were performed: the first set took all of the previous work and applied it directly to the new problem domain. As was expected, this was not very successful due to problems with population initialisation, holes and protuberances, lack of symmetry and extremely high stress values due to the sharp corners caused by the voxel representation.

The next set of experiments applied the same principle of directly addressing the problems that was used successfully in Chapter Three, this time via operator modifications and additions to the representation used. These experiments were considerably more successful, providing much faster early improvements leading to valid individuals very quickly. The rate of further improvements after discovery of a

valid individual was also much greater which lead to continual progress throughout the whole optimisation run.

The final results gained from the improved optimisation process contained several interesting features and should be viable annulus designs according to the stress figures produced by Ansys.

Chapter Five.

Conclusion:

The work described in this thesis was intended to find a set of suitable operators to permit effective optimisation of a voxel based representation using an evolutionary algorithm. The experiments then attempted to discover whether the combination of the GA coupled with the voxel based shape representation could perform a real-world analysis task with worthwhile results.

5.1 Summary of Conclusions.

The choice of a voxel based representation for this problem was contrary to some of the arguments in the literature [Watabe & Okino 93], but the results showed that with a combination of good operator design and a careful, principled and informed choice of the domain knowledge to be applied, the problems of small holes, long chromosomes, meaningless crossover and uneven edges can be avoided. The advantages to be gained from the voxel representation make the additional effort required to get good solutions a worthwhile endeavour.

The experiments which were performed on the beam cross-section optimisation were evaluated by a fitness function which makes simplifying assumptions about the physics of the situation in order to permit rapid investigations into the design and implementation of better operators. They included: implementation of the Genitor rank selection algorithm due to [Whitley 89]; discovering the effect of changing the constraint penalty multiplier; experimenting with a dynamically changing constraint penalty multiplier; development of the new mutation operator called 'smoothing'; implementation of the UNBLOX crossover algorithm [Cartwright & Harris 93]; creation of another new mutation operator

called 'two by two mutation' and experimentation with dynamically changing mutation probabilities. These experiments resulted in considerable improvements in the effectiveness of the GA at searching for solutions to the beam optimisation problem, eventually reaching the point where the GA was finding a near optimal solution in less than a thousand generations (Section 3.3.10).

The annulus design experiments used the knowledge gained from this first stage of operator design and testing, and applied it to the real-world problem of optimisation of a turbine annulus for minimum mass within multiple stress constraints, using the Ansys commercial FE analysis package to perform the evaluations. These experiments were quite unsuccessful initially, with very poor results being generated and several difficulties with the interactions between voxels and the FE package being discovered. However, using the knowledge about the GA gained during the beam analysis and by directly addressing the problems as they manifested themselves in the voxel grid, it was possible to solve these difficulties and achieve a successful optimisation system. The final system was able to optimise an annulus design which initially failed several of the stress constraints and was quite heavy, into a light-weight annulus which met all of the constraints.

One of the key issues involved in taking an approach that involved two separate optimisation stages, was whether the operators designed for the simple beam optimisation problem would work effectively on a much harder problem like the annulus design? The results gained from the annulus design experiments seem to indicate that this is the case, which is an important result for researchers who are dealing with similar long evaluation period GAs and who may wish to design operators in a more interactive way.

A very important result was gained from the final set of annulus design experiments; the generation of an annulus with an overhanging ledge in the cob was totally unexpected and argues strongly for the use of unbounded representational approaches such as voxels. It is highly unlikely that a person designing a parametric representation for the annulus optimisation would include parameters to represent such a ledge, unless they already expected that it would be required.

The real-world optimisation system used to evolve an annulus design was not directed towards that problem in particular. By modifying the macro which controls Ansys it should be possible to use the same system to evolve designs for many different two-dimensional components, provided that Ansys is capable of evaluating the fitness of the individuals. The restriction to two-dimensions is not necessary either, however no work was done using these operators on a three-dimensional optimisation so it is not known whether they will work as desired (although it would appear to be likely).

5.2 Further work.

Further developments and improvements to this optimisation program should address the weaknesses of the system as it stands, some simple code additions should also be mentioned. These will now be considered in turn:

1. The GA requires a better means of initialising the population to a prepared design shape. It should be a relatively easy step to add code that will take a scanned image of a design and allow the user to clean it up and use it for initialisation of the population. Such a modification would simplify the incorporation of preconceived solutions into the design space, and would also be extremely useful for design modifications if the program was developed into a hybrid system.
2. The system has not yet been tested on three-dimensional problems and no experiments have yet been performed to test the effectiveness of the n-dimensional operators in more than two-dimensions. The key issue in such experiments would be to see if the operators do extend to three-dimensions as easily and effectively as was predicted in Chapter Three.
3. The major weakness of the voxel representation was not due to the long chromosomes or any of the other predicted problem areas. Rather it was due to the fact that finite element analyses do not deal very well with the sharp corners or the

voxels. It may be possible to rewrite the Ansys macro to use the 'auto-meshing' capabilities of the package; auto-meshing permits the Ansys program to subdivide the outline of a shape into suitable elements for analysis itself. This approach would involve a further change of representation from voxels to the auto-meshed outline of the shape under consideration, but this should be irrelevant as the two representations would be functionally equivalent.

4. Finally, on a technical note, the optimisation fails after a large number of generations due to the creation and constant expansion of a 'page' file maintained by Ansys. It would aid both the execution speed and the usefulness of the system if some way of preventing its' growth or destroying this file between generations could be found.

In summary, with only a few modifications to the GA program, it should be possible to allow this system to use human input and create a hybrid system which will work with an engineer present to assist in the design process. Such systems are becoming increasingly popular as they combine the search power of a GA with the knowledge and design intuition of experienced engineers [Tucson *et al.* 97].

Appendix A ~ Rolls Royce annulus problem specification.

internal memorandum

ROLLS-ROYCE plc - BRISTOL
Mail Code -
Mechanical Science Dept

from: A S Squires

ref: E/ASS/40002

date: 11 June 1992

tel: 95337

to: Phy
T M Edmunds
R G Payne
B Banes

c: JBW,CJM,Murf/ACT
E Prempeh
D C Uppington
J B Wynne

Disc optimisation problem

Consider a simplified disc profile without details such as arms. The shape can be defined in terms of twelve different dimensions. Such a proposal is illustrated by Figure 1. Using the shorthand $\underline{x} = (x_1, x_2, \ldots, x_{12})$ the problem is to find

$$f(\underline{x}^*) < f(\underline{x}) \qquad \underline{x} \in R^{12}$$

such that

$$C_1(\underline{x}^*) < 0$$

$$C_m(\underline{x}^*) < 0$$

Where f, C_j are functions of stress and mass, and $\underline{x}^*$ denotes the optimal values of the design variables

Variable	**Description**	**Initial value** $\underline{x}^o$
x_1	Cob width	50 mm
x_2	Cob height	70 mm
x_3	Rim depth	5 mm
x_4	Rim width	50 mm
x_5	Bore radius	100 mm
x_6	Rim radius	350 mm
x_7	Diaphragm min thickness	10.5 mm
x_8	Diaphragm angle	4°
x_9	Shoulder diaphragm angle	43°
x_{10}	Shoulder - diaphragm fillet radius	25 mm
x_{11}	Rim - diaphragm angle	72°
x_{12}	Rim - diaphragm fillet radius	10 mm

Objective/constraint functions

		σ ($\underline{x}^o$)
σ_1	Bore hoop stress	1330 MPa
σ_2	Maximum radial stress base of diaphragm	741 MPa
σ_3	Maximum radial stress top of diaphragm	334 MPa
σ_4	Rim hoop stress	396 MPa
M	Mass of disc	68.6 Kg

Assumptions

Material = QDY (Waspalloy)

Disc rotation = 1571 rad/s

Temperature = 500°C

Youngs modulus = 2.238E5 MPa

Poissons ratio = 0.3

Density = 8.221E-9 Mg/mm^3

Alpha = 1.37E-5

Blade load = 10E5 N/rad

Point restraint at base

2D axisymmetric

Elastic

'Safety net' type constraints may have to be imposed such as radii >0 to avoid infeasible solutions. These should be added where necessary but should not be active at the solution.

Problem definition

In practice the rim radius is fixed, so either delete as variable or constrain x_6 = 350, minimise the mass of the disc such that none of the stress constraint functions increases.

Find $\underline{x}^*$ such that

$M(\underline{x}^*) < M(\underline{x}) \quad \underline{x} \in R^n$

$\sigma_1 (\underline{x}^*) < \sigma_1 (\underline{x}^o)$

$\sigma_2 (\underline{x}^*) < \sigma_2 (\underline{x}^o)$

$\sigma_3 \ (\underline{x}^*) < \sigma_3 \ (\underline{x}^\circ)$

$\sigma_4 \ (\underline{x}^*) < \sigma_4 \ (\underline{x}^\circ)$

$X_6 = 350$

At the solution the stress constraints should all be active so there are probably seven degrees of freedom at the solution.

An alternative problem is to find $\underline{x}^*$ such that

$\sigma_1 \ (\underline{x}^*) < \sigma_1(\underline{x}) \quad \underline{x} \in R^n$

such that

$\sigma_2 \ (\underline{x}^*) < \sigma_2 \ (\underline{x}^\circ)$

$\sigma_3 \ (\underline{x}^*) < \sigma_3 \ (\underline{x}^\circ)$

$\sigma_4 \ (\underline{x}^*) < \sigma_4 \ (\underline{x}^\circ)$

$M(\underline{x}^*) < M(\underline{x}^\circ)$

The first problem attempts to find the lowest weight disc with the stress (life) of the current design, and the latter attempts to maximise the life (minimise stress) for a given mass and other stresses. Both are valid objectives that might be invoked during design iterations.

Problem solution

It is proposed that this problem be solved by as many techniques as possible at this early stage to give comparison data. At present four methods are envisaged:-

1. Generic algorithms. R G Payne

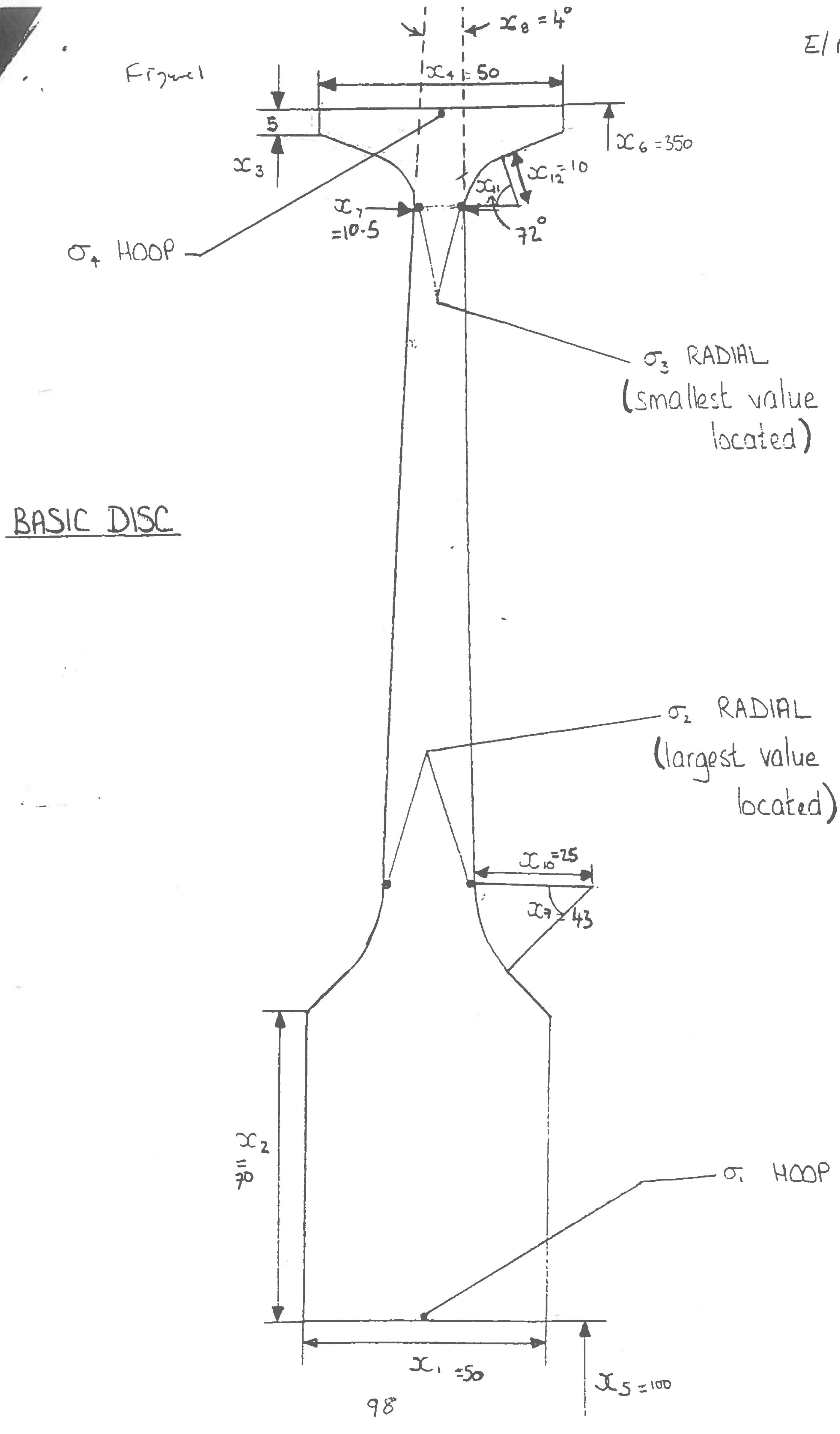
2. SC03 sequential recursive quadratic programming.
   T M Edmunds

3. RASNA sequential recursive quadratic programming.
   D G Pashley

4. IMSL solution of linearized scheme.
   B Banes/A Squires

A. Squires

Figure 1



x8 = 4°
x4 = 50
5
x3
x6 = 350
x12 = 10
x11
x7 = 10.5
72°
σ4 HOOP
σ3 RADIAL (smallest value located)
BASIC DISC
σ2 RADIAL (largest value located)
x10 = 25
x9 = 43
x2 = 70
σ1 HOOP
x1 = 50
x5 = 100

Appendix B Ansys macro listings

Listing B.1: Full optimisation macro mask listing

This appendix contains the macro mask that is used to build the input file to Ansys to perform the full optimisation.

```
/batch                  !put Ansys into batch processing mode
/uis,msgpop,3           !limit the number of warning messages produced
/prep7                  !enter the pre-processor mode
/nolist
/nopr
/nop                    !turn off most of the text file outputs
outres,all,none
outres,strs,last        !set the output mode to dump stresses only
time,1                  !set the time stage for the optimisation to occur at
uimp,1,ex,,,****        !mask statement for the material properties
uimp,1,dens,,,****      !mask statement for the material density
et,1,plane42,0,0,1,,2,0 !element type 1 is rectangles
et,2,plane2,,,1,,2,0    !element type 2 is squares
finish

/syp,"f:\stress",1      !make the stress.exe program initialise the population

/prep7
nread,nodes,dat,f:\     !read in the node position data file
antype,STATIC           !specify the type of analysis to be performed
omega,0,****,0,0        !mask statement to set the rotational velocity
eqslv,iter,1e-005,0     !set the equation solver limits of accuracy
finish

*do,pass,0,loops,1      !for a user specified number of loops do

/prep7
numcmp,elem             !compress the element numbers down to start from 1
eread,elems00,dat,f:\   !read the element connection file
nsel,s,loc,x,****,**** !mask statement to select the rightmost column of nodes
f,all,fx,****           !mask statement to apply the blade load force to selected
nodes
nsel,all                !select all nodes again
/solv
solve                   !solve the analysis
/post1
/output,xsol_00,log,f:\ !specify the output file
etable,table,s,x        !dump the radial stresses into the output file
```

```
pretab,table
/output,zsol_00,log,f:\ !specify the output file
etable,table,s,z         !dump the hoop stresses into the output file
pretab,table
/output
finish                   !finish output processing mode

/prep7                   !do it all again for the next 19 members of the population…
esel,all
edele,all
numcmp,elem
eread,elems01,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_01,log,f:\
etable,table,s,x
pretab,table
/output,zsol_01,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems02,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_02,log,f:\
etable,table,s,x
pretab,table
/output,zsol_02,log,f:\
etable,table,s,z
pretab,table
/output
finish
```

```
/prep7
esel,all
edele,all
numcmp,elem
eread,elems03,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_03,log,f:\
etable,table,s,x
pretab,table
/output,zsol_03,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems04,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_04,log,f:\
etable,table,s,x
pretab,table
/output,zsol_04,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems05,dat,f:\
```

```
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_05,log,f:\
etable,table,s,x
pretab,table
/output,zsol_05,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems06,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_06,log,f:\
etable,table,s,x
pretab,table
/output,zsol_06,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems07,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
```

```
/output,xsol_07,log,f:\
etable,table,s,x
pretab,table
/output,zsol_07,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems08,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_08,log,f:\
etable,table,s,x
pretab,table
/output,zsol_08,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems09,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_09,log,f:\
etable,table,s,x
pretab,table
/output,zsol_09,log,f:\
etable,table,s,z
pretab,table
```

```
/output
finish


/prep7
esel,all
edele,all
numcmp,elem
eread,elems10,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_10,log,f:\
etable,table,s,x
pretab,table
/output,zsol_10,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems11,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_11,log,f:\
etable,table,s,x
pretab,table
/output,zsol_11,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
```

```
edele,all
numcmp,elem
eread,elems12,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_12,log,f:\
etable,table,s,x
pretab,table
/output,zsol_12,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems13,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_13,log,f:\
etable,table,s,x
pretab,table
/output,zsol_13,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems14,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
```

```
/solv
solve
/post1
/output,xsol_14,log,f:\
etable,table,s,x
pretab,table
/output,zsol_14,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems15,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_15,log,f:\
etable,table,s,x
pretab,table
/output,zsol_15,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems16,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_16,log,f:\
etable,table,s,x
pretab,table
```

```
/output,zsol_16,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems17,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_17,log,f:\
etable,table,s,x
pretab,table
/output,zsol_17,log,f:\
etable,table,s,z
pretab,table
/output
finish

/prep7
esel,all
edele,all
numcmp,elem
eread,elems18,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_18,log,f:\
etable,table,s,x
pretab,table
/output,zsol_18,log,f:\
etable,table,s,z
pretab,table
/output
finish
```

```
/prep7
esel,all
edele,all
numcmp,elem
eread,elems19,dat,f:\
nsel,s,loc,x,****,****
f,all,fx,****
nsel,all
/solv
solve
/post1
/output,xsol_19,log,f:\
etable,table,s,x
pretab,table
/output,zsol_19,log,f:\
etable,table,s,z
pretab,table
/output
finish

/syp,"f:\stress"          !call the stress.exe program to generate the next generation

/prep7
esel,all
edele,all                 !delete all the elements to leave the system clean

*enddo                    !end of for loop
```

Listing B.2 Single optimisation macro listing

```
/batch                  !put Ansys into batch processing mode
/uis,msgpop,3           !limit the number of warning messages produced
/prep7                  !enter the pre-processor mode
/nolist
/nopr
/nop                    !turn off most of the text file outputs
outres,all,none
outres,strs,last        !set the output mode to dump stresses only
time,1                  !set the time stage for the optimisation to occur at
uimp,1,ex,,,223800000000.000000   !set the material's Youngs modulus value
uimp,1,dens,,,8221.000000         !set the material's density value
et,1,plane42,0,0,1,,2,0 !element type 1 is rectangles
et,2,plane2,,,1,,2,0    !element type 2 is squares
finish

/syp,"f:\stress",1      !make the stress.exe program initialise the population

/prep7
nread,nodes,dat,f:\     !read in the node position data file
antype,STATIC           !specify the type of analysis to be performed
omega,0,1571.000000,0,0    !set the rotational velocity of the annulus
eqslv,iter,1e-005,0     !set the equation solver limits of accuracy
finish

/prep7
numcmp,elem             !compress the element numbers down to start from 1
eread,elems00,dat,f:\   !read the element connection file
nsel,s,loc,x,0.347984,0.352016    !select the nodes to apply force to
f,all,fx,224399.478571            !apply the force to the nodes
nsel,all                !select all nodes again
/solv
solve                   !solve the analysis
/post1
/output,xsol_00,log,f:\ !specify the output file
etable,table,s,x        !dump the radial stresses into the output file
pretab,table
/output,zsol_00,log,f:\ !specify the output file
etable,table,s,z        !dump the hoop stresses into the output file
pretab,table
/output
finish                  !finish output processing mode
```

Appendix C Results log files

Listing C.1 Log of basic GA results on annulus problem

The format of these files is:

Generation Number : fitness 0 fitness 1 fitness 2 ... fitness 19

1 : -42932.375355 -209052.043278 -366222.725516 -177921.441817 -290383.502632 -204717.638115 -383428.754670 -149488.406312 -210767.469526 -274748.083995 -299505.669458 -120901.038435 -252522.230075 -377815.136390 -390323.154032 -192801.527640 -158886.531250 -261009.410987 -253844.248821 -255119.580400
2 : -42932.375355 -192801.527640 -210767.469526 -192801.527640 -290383.502632 -196517.325499 -149294.406311 -253969.098581 -206613.760103 -211263.427786 -161835.724010 -252030.590813 -249666.813568 -258225.203667 -410058.101044 -115619.653922 -205433.928061 -256991.346884 -156798.467123 -207461.492518
3 : -42932.375355 -44122.208632 -250060.025003 -177708.965719 -263667.765633 -78624.839133 -258296.914046 -211263.427786 -106925.388304 -202542.499690 -259175.200558 -160472.183664 -42932.375355 -158686.967882 -146031.188406 -188856.054211 -156798.467123 -108204.632130 -225885.370146 -144272.084068
4 : -42932.375355 -43558.811498 -225515.726612 -145914.420455 -154900.736163 -376506.668266 -324827.820724 -273849.672397 -106925.388304 -199532.264419 -88522.798834 -42677.194010 -268729.756541 -55287.905629 -144272.084068 -144272.084068 -319877.895688 -76804.905379 -43642.997481 -218997.164334
5 : -42677.194010 -108745.288764 -41165.448989 -144272.084068 -40011.338148 -379005.216762 -43558.811498 -388031.637565 -55287.905629 -44198.294526 -58864.539302 -269974.331478 -218997.164334 -43558.811498 -141282.379258 -42677.194010 -146044.623574 -43642.997481 -106965.544765 -157113.183425
6 : -40011.338148 -52849.520737 -42076.767242 -148885.889172 -211045.631199 -173141.170431 -44098.723471 -388031.637565 -40336.240788 -40011.338148 -41165.448989 -146108.866156 -41044.708056 -43383.245081 -107440.458598 -41544.530026 -315581.782211 -40011.338148 -409644.958301 -144272.084068
7 : -40011.338148 -146108.866156 -41544.530026 -398423.700875 -41544.530026 -41044.708056 -42076.767242 -402264.330353 -133035.602144 -317505.435520 -170945.798095 -145659.837706 -145024.582836 -124929.602305 -151672.666770 -114599.417809 -41044.768089 -135768.564719 -180816.657632 -118605.761249
8 : -40011.338148 -44135.823197 -41105.530798 -73891.477348 -100343.623077 -41044.708056 -35048.991381 -116415.012551 -504957.670094 -41044.768089 -46339.790514 -154947.405396 -114704.734773 -118605.761249 -153203.003157 -41044.768089 -317505.435520 -58324.407626 -41044.708056 -41812.723425
9 : -35048.991381 -316254.609307 -73831.164427 -40890.652761 -41074.874779 -42169.085328 -41105.530798 -41105.530798 -54563.126489 -441843.303268 -76247.062142 -46339.790514 -113703.504565 -241599.264192 -21018.368779 -117361.720546 -314676.670188 -145559.788427 -101048.531939 -71480.653982
10 : -21018.368779 -490525.281716 -45847.837620 -44716.433491 -113703.504565 -54563.126489 -33457.099977 -41059.973388 -41260.050089 -46173.668269 -71445.406383 -74746.085732 -114023.256966 -317319.290293 -310157.374655 -74702.381589 -75735.889772 -240461.078222 -44658.271307 -84856.692010
11 : -21018.368779 -54563.126489 -44658.271307 -45847.837620 -264496.696514 -23774.718548 -111812.956484 -73666.909096 -114108.187600 -45847.837620 -490004.720262 -240530.583645 -80454.671345 -74141.974418 -402077.893855 -79614.737010 -47364.845086 -41260.050089 -318888.751744 -84856.692010

12 : -21018.368779 -302491.442518 -128942.190072 -41113.411986 -264496.696514 -44658.271307 -48120.085981 -25287.183891 -81124.050792 -74562.634477 -52857.961825 -404302.353515 -86616.330567 -264496.696514 -42205.315422 -54563.126489 -402729.634974 -23774.718548 -46508.553673 -48171.263281
13 : -21018.368779 -21018.368779 -23774.718548 -53888.897282 -42582.699273 -53188.791522 -61427.313107 -53331.786775 -167001.740976 -23774.718548 -109731.433664 -164398.268449 -48628.172143 -42190.123814 -45278.177605 -52857.961825 -264496.696514 -81124.050792 -73346.927757 -45457.802473
14 : -21018.368779 -44602.705110 -61746.628617 -53188.791522 -128833.050545 -13895.732149 -162113.533775 -42582.699273 -18712.630467 -30821.444523 -42582.699273 -24341.533161 -45278.177605 -45457.802473 -73346.927757 -53331.786775 -21018.368779 -53061.157822 -254946.350827 -42190.123814
15 : -13895.732149 -44602.705110 -45310.094091 -20887.174779 -30720.845961 -162113.533775 -42582.699273 -53188.791522 -21018.368779 -29248.952301 -52115.832458 -35312.744528 -42159.894878 -13895.732149 -42054.321316 -44602.705110 -42190.123814 -148731.101534 -60121.363268 -36324.185273
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Listing C.2 Log of improved GA results on annulus problem

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