A Generative Grammar for Jazz Chord Sequences

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A Generative Grammar for Jazz Chord Sequences

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The recursive character of musical chord sequences makes generative grammar a suitable formalism for describing the rules that constrain such sequences. A small number of rules are presented which generate the members of a large class of complex chord sequences that are generally recognised to be closely related, namely the set of jazz 12-bar blues. The rules are illustrated using a testing corpus of jazz chord sequences, and certain extensions are considered.

"The Blues and the Abstract Truth"
(Oliver Nelson)

"Take care of the sevenths
and the sounds will take care of themselves"
(Traditional)

Introduction

People find the difference between random arrangements of notes and the music of their native culture as obvious as the difference between random arrangements of words and meaningful sentences of their native language. The reason must be that there are rules that govern musical forms, analogous to rules of grammar for languages. The present study attempts to characterize the rules that underlie the comprehension of one aspect of musical form in the musical culture of western tonal harmony[1] in much the same way that a linguistic study might try to characterise the rules underlying the comprehension of a language like English.

Musical comprehension is as many sided as its linguistic counterpart and includes many aspects that will not be considered here at all. But one

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1. This musical culture includes most "classical" music, and most current popular music and jazz, but is also a first cousin to certain nonharmonic or modal musics, such as the Indian classical tradition. However, it excludes certain modern Western musical innovations such as (some kinds of) atonality. The question of how far the conclusions of the present study generalize to these other domains will not be pursued here.

particularly important characteristic of a tonal melody is the sequence of chords that is used to accompany it, which reflects its underlying harmonic structure. In improvising on a theme, musicians know that whatever other liberties they may take with the original melody, any variation must have the same underlying chords, or a sequence that (in a sense to be investigated below) preserves the underlying harmonic structure. Nonmusicians "know" this as well: if an improvisation does not conform to the chord sequence, the audience is likely to notice and to reject it. They may well feel that the performer has not "understood" the melody and that the improvisation does not "make sense" in the context of that theme. (Of course, most of them will not be able to say what the chords should actually *be*, unless they happen to have been musically trained, any more than nonlinguists can state the grammatical rules of their language. We are talking about unconscious rules here, not conscious articulate knowledge.)

Chord sequences are therefore an important characteristic of melodies. They also present a more manageable problem for analysis than the entire class of accompanied melodies, since there are many fewer chord sequences than there are possible melodies. (The many possible variations that may be produced during improvisation on a theme are only one example of the fact that quite different melodies may share the same sequence of chords.) Nevertheless, to characterize the full set of all possible chord sequences within the western tonal idiom would still be a huge and rather ill-defined task. It is fortunate, therefore, that certain sets of chord sequences are considered by musicians to be closely related and to be harmonically equivalent in a sense which is close to the linguistic idea of "paraphrase." A case in point is the set of chord sequences that jazz musicians refer to as "12-bar blues."

The 12-bar blues (or 12-bar for short) has been a common musical form in the jazz and popular music of the last 50 years. It has undergone a rapid evolution in that time, so that the set of variations on the basic sequence is now extremely large and varied. Nevertheless, jazz musicians recognize that all 12-bars are equivalent in the sense that they are all related to a single, underlying simpler chord sequence that is still found in the black American folk music from which jazz mainly derives. A representative sample of modern jazz 12-bar chord sequences is shown in Figure 1 below (adapted from Coker, 1964). For the moment, most of the detail in the figure can be ignored. It will be sufficient merely to note that each line (a) to (i) represents a possible 12-bar chord sequence, that the symbols between slashes represent the chord or chord sequence played in each successive bar, that the first example (a) is one of the most basic folk blues sequences, and that the remaining examples (b) to (i) represent successively more and more elaborate variations on the basic form.

Coker's book *Improvising Jazz*, from which Figure 1 is adapted, is a widely respected manual for novice jazz musicians, and it can be assumed

<table>
<tr><td>(a)</td><td>I(M7)</td><td>/IV(7′)</td><td>/I(M7)</td><td>/I7</td><td>||IV(7′)/IV(7′)</td><td>/I(M7)</td><td>/I(M7)</td><td>||V7</td><td>/V7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(b)</td><td>I(M7)</td><td>/IV(7′)</td><td>/I(M7)</td><td>/Vm7,I7</td><td>||IV(7′)/♯IV°7</td><td>/I(M7)</td><td>/VI7</td><td>||IIm7</td><td>/V7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(c)</td><td>I(M7)</td><td>/IV(7′)</td><td>/I(M7)</td><td>/Vm7,I7</td><td>||IV(M7)/IVm7</td><td>/IIIm7</td><td>/VI7</td><td>||IIm7</td><td>/V7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(d)</td><td>I(M7)</td><td>/IIm(7′),♯II°7</td><td>/IIIm(7′)</td><td>/Vm7,I7</td><td>||IV(M7)/IVm7,♭VII7</td><td>/IIIm7</td><td>/♭IIIm7</td><td>||IIm7</td><td>/V7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(e)</td><td>I(M7)</td><td>/VIIΦ7,III7</td><td>/VIm7,II7</td><td>/Vm7,I7</td><td>||IV(M7)/IVm7,♭VII7</td><td>/♭III(M7)</td><td>/♭IIIm7,♭VI(7′)</td><td>||IIm7</td><td>/V7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(f)</td><td>I(M7)</td><td>/IV(7′)</td><td>/I(M7)</td><td>/♭IIm7,♭V7</td><td>||IV(M7)/♭Vm7,VII7</td><td>/IIIm7</td><td>/VI7</td><td>||IIm7,V7</td><td>/♭VIm7,♭II7</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(g)</td><td>♭II7,♭V7</td><td>/VII7,III7</td><td>/VI7,II7</td><td>/V7,I7</td><td>||IV(7′)/♯IV°7</td><td>/IIIm7</td><td>/♭III(M7)</td><td>||VI(M7)</td><td>/♭II(M7)</td><td>/I(M7)/I(M7)</td></tr>
<tr><td>(h)</td><td>Im(6)</td><td>/IIΦ7,V7(+5)</td><td>/Im(6)</td><td>/I7</td><td>||IVm(6)/IVm(6)</td><td>/Im(6)</td><td>/Im(6)</td><td>||IIΦ7</td><td>/V7(+5)</td><td>/Im(6)/Im(6)</td></tr>
<tr><td>(i)</td><td>Im(6)</td><td>/IIΦ7,V7(+5)</td><td>/Im</td><td>/VΦ7,I7</td><td>||IV(7′)/♯IV°7</td><td>/Im(6)</td><td>/♭IIIm7,♭VI(7′)</td><td>||IIΦ7</td><td>/V7(+5)</td><td>/Im(6)/Im(6)</td></tr>
</table>

Fig. 1. The corpus of paradigm jazz o12-bars, adapted from Coker (1964).

that these examples illustrate a wide and representative range of the permissible variations. It will be noted that the more elaborate examples, such as (e), (f), and (g), share very few bars in common with each other and the most basic form (a). Defining the full set of legal 12-bar chord sequences is therefore not a trivial problem.

The present study will attempt to provide a small set of rules that characterize this musical "sublanguage."[2] It will then be argued that the rules that are advanced to account for the restricted domain are in fact quite general and are part of a larger set of rules which characterize a much more comprehensive set of chord sequences that are similarly musically coherent within the tradition of western tonal harmony.

The rules will be presented in the form of a "generative grammar," a device that was originally developed for defining the formal languages of mathematics and logic and that has more recently been applied with some success to natural languages. At the heart of any formal grammar lies a set of syntactic rules that specify the set of all and only those strings of symbols that constitute legal or "well-formed" expressions in the language. In a generative grammar, these rules take the form of instructions that derive or "generate" strings of symbols from other strings of symbols.[3] Such grammars typically exploit properties like recursion in their rules to specify generatively infinitely large sets of strings (such as the well-formed formulae of a logic or the sentences of English) using only finite numbers of rules. This property of generative grammar is exploited in the present study, since the set of blues chord sequences also seems in principle infinite. Winograd

2. The fact that we are dealing with the *artistic* use of musical language raises problems for the analogy with linguistics that will recur throughout this study. It means that we are dealing with a corpus that is more like poetry than the mere well-formed strings of a language. One problem arises from the fact that it is almost a definition of a work of art that it will *break* the established rules in some way, as in the later example of a piece that exploits the harmonic disconnectedness of a move from C to F♯ and back again, or as in E. E. Cummings' poem that begins "Anyone lived in a pretty how town." However, works of art are still constrained by the need for the violations of the rules to be clearly perceived to *be* deliberate, rather than random errors. For this reason, as in Cummings' poem, the difference between the base of rules and the violation is usually quite clear. Studies like the present one can therefore confine themselves to the base rules, and exclude such usages for principled reasons.

A second problem is that a *good* chord sequence involves considerations that go beyond mere musical coherence, just as a good poem demands more than mere syntactic and semantic coherence. Again, the present study will confine itself to questions of coherence. It will attempt to specify the set of *possible* 12-bars—good, bad, and indifferent—rather than the set that musicians actually play. On occasion the rules will allow sequences that are too complex or bizarre for anyone to want to play or be able to understand, just as a formal grammar of English will allow sentences that are impossibly complex or whose meaning is absurd.

3. Although this description may appear to imply that generative grammar is an account of production, the formalism is in fact neutral with respect to processes of production and analysis.

(1968), Sundberg and Lindblom (1976), Lerdahl and Jackendoff (1977), Ulrich (1977), Keiler (1978, 1981), and Perlman and Greenblatt (1981) have also approached the study of musical form in terms of generative grammar and related formalisms.

The grammar that is presented in the sections that follow will generatively specify the set of possible blues chord sequences by including certain rules called "substitution rules," which allow the more elaborate sequences to be derived or generated from the most basic form by the *replacement* of the original chords by substitutes.

The rules will have to meet two important criteria. First, they must conform to the basic requirement of a generative grammar, that is, they must generatively specify the set of all and only the chord sequences that are recognized by those familiar with the music as possible jazz 12-bars. (However, for reasons discussed in footnote 2, the rules are not required to specify the set of *good* jazz 12-bars, merely the set of "potential" 12-bars, whether good, bad, or indifferent). Second (and more important), each rule must have what in a language grammar would be called a clearly defined *semantics*. In the present study, the domain that corresponds to semantics—that is, the "meaning" of chord sequences—is the domain of *harmony*, in which, for example, a given pair of chords played in succession may convey the meaning of a "cadence," where the second is perceived as a point of rest and the first as preparing for it. While the present paper will not be concerned with the formalization of harmonic "semantics," it is nevertheless vital that, where a rule of the grammar says that one sequence of chords may replace another, musicians should agree that the substitution is a possible expression of such aspects of the musical meaning as the underlying cadential sequence.[4] To that extent, the article will not really be entirely comprehensible unless the reader plays through the examples on some instrument such as piano or guitar (however haltingly) and, in particular, *plays the rules* to assure him or herself that they "make sense" musically.[5]

The argument will proceed as follows. The next two sections present some further observations about chord sequences and chord notation and define the most basic forms of the blues in terms of some trivial phrase structure rules. The next section presents the first and most powerful of the

4. Longuet-Higgins (1962, 1979) offers the basis of a formal theory of harmony. The concept of cadence is discussed later.

5. It follows that much of this article will escape nonmusicians. Nevertheless, I hope that they will be able to follow the logic of the argument and to this end have spelled out certain matters of musical terminology and notation to an extent that the musicians will want to skip.

substitution rules, from which most of the descriptive power of the grammar derives. The next two sections present some further rules and some refinements to do with distinctions between major and minor chords. A further section brings the rules together and examines the details of their operation using the corpus illustrated in Figure 1. The concluding sections consider some further extensions and implications.

Simple Chord Sequences

There are some chord sequences that *never* occur as the accompaniment to a tune. For example, it is hard to find any context in which a chord of C can be followed by one of F♯, and then by another chord of C (unless the music is actually making a point of being fragmented).[6] If one hears such a sequence in an accompaniment it just sounds like a mistake. Similarly, there are some chord sequences that are particularly simple and are understood by practically everyone. These are the ones that contain only three different chords, and they are fundamental to all tonal music including the blues. For any given key, a vast number of simple nonmodulating melodies, such as nursery rhymes, folk songs, and the most basic forms of the blues, can be accompanied by such chord sequences, a fact which is widely known among musicians as "the three-chord trick." In the key of C major the chords are C major, F major, and G major, and in general for any key the chords are the three whose roots or principal notes are the key-note (or tonic), the note four scale steps above the tonic (the subdominant), and finally the note five scale steps above the tonic (the dominant). The particular key used to accompany a melody is largely a matter of convenience, but the appropriate sequence of tonic, subdominant, and dominant chords is a fixed aspect of its meaning.

There is a common notation that conveniently reflects chord relationships independently of any particular key. It uses Roman numerals to represent chords, where the numeral identifies the degree of the chord root in the scale. This notation is used in Figure 1 and throughout this article. In this notation the chords used in the three-chord trick are written I (tonic), IV (subdominant), and V (dominant).

In addition to the Roman numerals indicating the degree of the chord relative to the key-note, the notation used in Figure 1 uses a system of prefixes and suffixes. This standard but possibly unfamiliar notation is

6. See footnote 2.

described in an appendix to the paper.[7] In general the suffixes specify certain additional notes that are played with the notes of the root chord and that modify the harmonic function of the chord, often raising an expectation of another chord. Those that appear in brackets are relatively unimportant and will be ignored until the section on the grammar. But one of the affixes is particularly important for the rules that follow. The suffix 7 (as distinct from M7 or 7′) means that the note a flattened or "dominant" seventh above the root is played with the other notes of the chord. For reasons that will be discussed elsewhere, this modification has the effect of making the listener expect a further chord related to the dominant seventh chord by having its root a fifth below or a fourth above. (Hence its name—the modification makes the modified chord act as or "mean" a dominant.) So, for example, the chord written I7 creates the expectation of IV, while V7 creates an expectation of I, and every novice musician who has mastered the three-chord trick knows that adding the dominant seventh note to the V chord when it is succeeded by the tonic I, making V7, makes the trick work even better.

Example (a) in Figure 1 shows that the simplest versions of the 12-bar blues may be entirely composed of I, IV, and V chords, with the appropriate dominant seventh 7 notation on the I of bar 4 and the V of bars 9 and 10 creating expectations of the IV in bar 5 and the final I, respectively. But to show how the remaining examples are related to it, it will be necessary first to look a little more deeply into the 12-bar blues form itself.

The 12-Bar Blues

A standard 12-bar in four–four time has as a metric or temporal framework a structure of three four-bar parts, each subdivided into two two-bar halves, each of which is divided again into two parts (the bars). Each of these is further subdivided into two half-bar units, themselves divided into two beats. The metric framework just described can be represented as the tree structure shown in Figure 2, where the numbers attached to the nodes identify the number of bars associated with the corresponding units of the meter. Twelve-bars can also be in triple time-signatures such as $\frac{3}{4}$ and $\frac{6}{8}$, just

7. Figure 1 draws one distinction that is not drawn by Coker or in the standard notation. There are *two* distinct harmonic functions that can be performed by the chord which on the keyboard is played with a note a semitone below the seventh of the key note. Besides the "leading" dominant seventh function just defined, it may have the function of a minor seventh chord, which does not lead anywhere in particular. In just intonation, there is a difference in frequency between the dominant seventh note and the minor seventh note, but in equal tempered tuning (for example, on the keyboard) they are the *same* note. In the standard notation both are, therefore, written with the suffix 7. Since the present rules treat these two "homophones" differently, the nonleading chords with the minor seventh are distinguished in Figure 1 with the nonstandard suffix 7′.

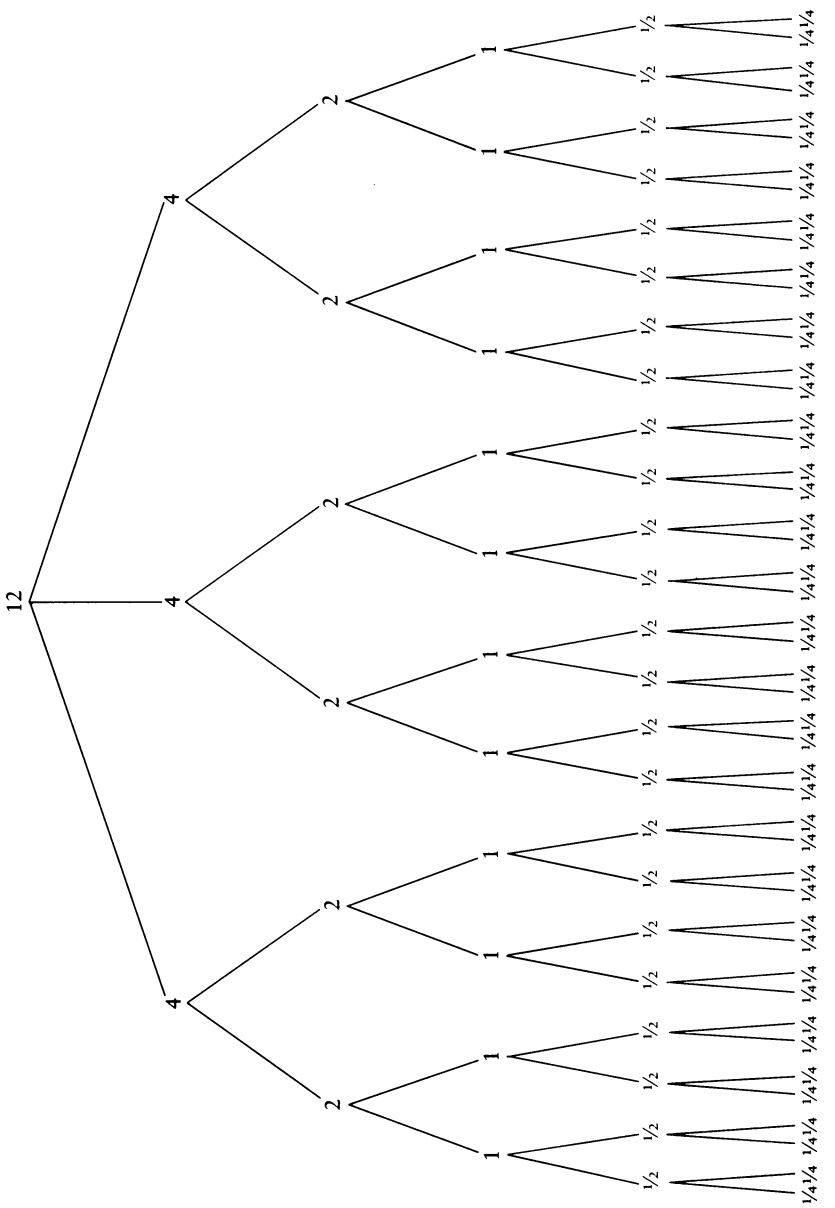


Fig. 2. The metric structure of a 12-bar in 4 time.

so long as they have a total of twelve bars. (Indeed, it is virtually impossible to find a blues that does not have a triplet feel somewhere in its metric structure, usually below the level of the beat.)

In its simplest and presumably original form, the first 4-bar phrase of a 12-bar consists entirely of tonic (I) chords, the second consists of a "plagal cadence" composed of two chords of the subdominant (IV) followed by two bars of the tonic (I), and the third consists of an "authentic cadence," composed of two bars of the dominant (V), followed by two of the tonic (I).[8] Since the I in the fourth bar is followed by IV, the chord of which it is the dominant, it is commonly realized as a dominant seventh chord I7. For the same reason, the V chords in bars 9 and 10 are also dominant seventh chords V7. The 12-bar in this simplest manifestation (an example of which is *Digging My Potatoes*, by Washboard Sam) can be written as follows, where slashes separate the bars, and double vertical slashes separate the three main phrases:

(1) I/I/I/I7||IV/IV/I/I||V7/V7/I/I

There are a variety of other chord sequences which, while containing more chord changes, intuitively seem to be remarkably similar and are also generally recognized to be examples of the 12-bar. For example, nothing much is changed if the second bar tonic (I) chord is replaced by a chord of its subdominant (IV): the sequence is still a 12-bar blues, equivalent (apart from insignificant details) to Coker's example (a) in Figure 1:

(2) I/*IV*/I/I7||IV/IV/I/I||V7/V7/I/I

Trivially different though this variant may be, it reveals an important characteristic of the rules that define the form when compared with some close relatives. It seems that a similar substitution of a chord's subdominant may occur for chords other than I and at the level of the individual beat rather than the whole bar—cf. *Betty and Dupree* (Dave van Ronk), of which the first four bars can be written:

(3) I, *IV*, I, I/IV, ♭*VII*, IV, IV/I, *IV*, I,I/I,*IV*, I, I7||IV . . . . . .

—where this time the bar is subdivided into the individual beats that make it up, separated by commas, and substitutions are again italicized. This example and the previous one suggest that the replacement of a chord by its subdominant can apply to any chord at any level of the metric hierarchy

8. A cadence is a sequence of chords that is perceived as a unified progression with the last chord as a destination or point of rest. The cadence plays an important part in defining the end of a phrase or chord sequence. The authentic and plagal cadences are the two most important kinds. They can be realized by more elaborate sequences than these. Indeed, this whole article can be seen as an attempt to formalize the various ways in which an authentic cadence can be expressed.

illustrated in Figure 2—to a bar, half-bar, or quarter-bar beat. However, it is not the case that any and every chord may be substituted in this way. If the substitution were applied to Bar 1 of Example (1), giving a bar of the subdominant IV, the result would *not* be a 12-bar.

(4) *IV*/I/I/I7‖IV/IV/I/I‖V7/V7/I/I

The difference appears to be that in the previous examples, the replacement of a chord by its subdominant occurred on a *right* branch of the metric tree illustrated in Figure 2, but this example involves a substitution on a *left* branch. Not surprisingly, since the leftmost branches of the hierarchy are the metrically stressed ones, such a substitution changes the harmonic character of the sequence.

It follows that a few simple "rewrite rules" of musical grammar will serve to generate Examples (1) to (3), together with a number of other similar simple blues sequences, while excluding nonblues sequences like (4). The first rule merely defines the basic 12-bar framework of tonic, followed by plagal cadence, followed by authentic cadence. Since it is entirely ad hoc and merely serves to provide a standard input to the other rules, it is called "Rule 0."

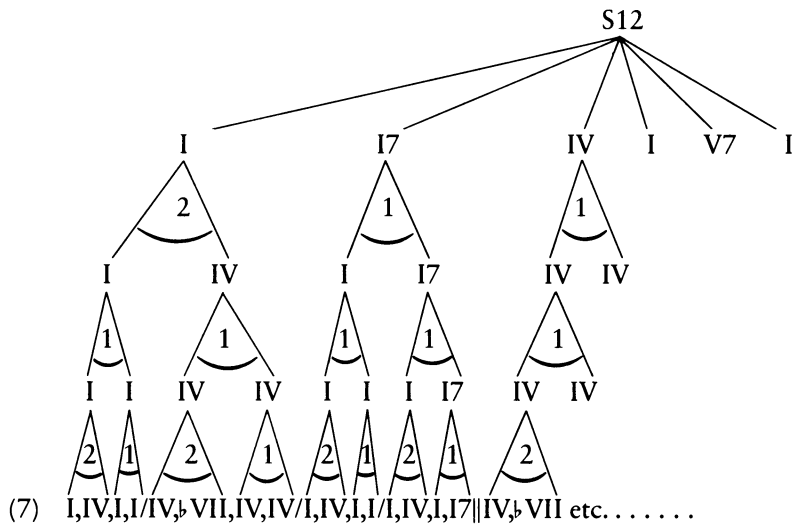
(5) Rule 0: S12 → I I7 IV I V7 I

In this and the other rules that follow, the arrow means "the thing on the left is made up of the things on the right." S12 stands for a sequence occupying 12 bars, the chord symbols mean what they always mean and there is a convention in all rules that the thing(s) on the right occupy the same total amount of time as the thing(s) on the left—in this case, each of the six chords occupies two bars. The next two rules are a little more interesting:

(6) Rule 1: $x(7) \rightarrow x \quad x(7)$
Rule 2: $x(7) \rightarrow x(7)\ \mathrm{Sd}_x$

They are to be read as follows: x is a variable over the set of chord roots—that is, over I, II, III, IV, V, etc. As usual 7 is the suffix indicating a dominant seventh chord on that root. The fact that the 7 is in brackets on the left-hand side of both rules means that they may apply to *either* plain chords or dominant seventh chords. The brackets around the 7 suffix of one chord on the right-hand side of each rule means "if the rule did in fact apply to a dominant seventh chord, then this chord must also be a dominant seventh chord." The symbol Sd_x in Rule 2 stands for the chord with the subdominant of x as its root—that is, the chord whose root is related to x by four scale steps upward or five scale steps downward. (For example, if x is I then Sd_x is IV. If x is IV then Sd_x is ♭VII.) Again, the convention is that the things on the right-hand side occupy the same total amount of time as the thing on the

left—that is, they each last half as long. Rule 1 can therefore be paraphrased "a chord can be replaced by two copies of itself, each lasting half as long. If the original chord was a dominant seventh chord, then the rightmost of its offspring is too." Rule 2 can be paraphrased "a chord can be replaced by a copy of itself, followed by its subdominant each lasting half as long. If the original chord was a dominant seventh chord, then the leftmost of its offspring is too."[9] These rules (which linguists would recognize as being of a particularly simple variety known as "context-free phrase structure" rules) generatively specify a set of blues sequences including examples (1), (2), and (3).[10] Example (3) is generated as follows:



Extending the Authentic Cadence

In the classic early period of evolution of the blues, a common site for variation upon the basic form was in the authentic cadence of the final four bars. The simplest such variation was the replacement of the first of the two bars of the dominant V7 chord with a bar of *its* dominant, namely the

9. The reason for the non-inheritance of the 7 affix by certain chords on the right-hand side is that these chords are not acting as dominant sevenths. For example, they are not followed by the chord of which they are the dominant, so in the terms of the introduction they do not bear the "meaning" of a dominant seventh that the affix implies.

10. It was noted earlier that blues can involve triple time signatures, such as $\frac{3}{4}$. It follows that there should really be two more rules, like Rules 1 and 2, but expanding a chord as *three* subunits each occupying a third as much time as the parent. Whether the binary or the ternary version is used in a derivation then depends on the overall metric structure that is in force. However, in the remainder of this article only binary structures will be considered.

supertonic, II7, as in the following example (*St. Louis Blues*, W. C. Handy) and as in the last four bars of Coker's Example (h) in Figure 1.

(8) I/IV/I/I7||IV/IV/I/I||*II7*/V7/I/I

This example can be generated from the skeleton resulting from Rule 0 (Example 5) by the earlier rules plus a rule of a new kind called a "substitution" rule. The new rule replaces a chord with the dominant seventh of the one that follows. For example, the V of Bar 9 in Example (1) can be replaced by the dominant seventh of the V in Bar 10, namely II7, as in (8) above. Like Rules 1 and 2, the new rule can apply to any level of the meter, for example at the half-bar level as the following example illustrates [cf. first four bars of Example (b) in Figure 1].

(9) I,I/IV,IV/I,I/*V7*,*I7*||IV etc., etc.

But it must be restricted to certain contexts, since for example, if it were to apply to the I of bar 7, the result, although perfectly musical, would not count as a 12-bar.

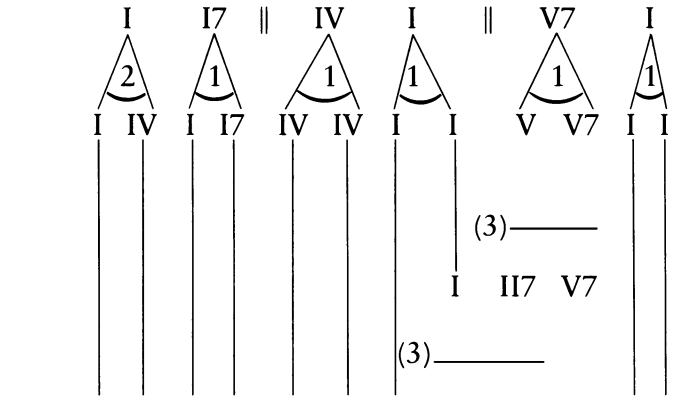
(10) I/IV/I/I7||IV/IV/*V7*/I||V7/V7/I/I

It is therefore necessary to place the following context restriction on the rule: the chord that is to be replaced must be followed by a dominant seventh chord, a chord that the rules so far imply will itself always be the beginning of an "authentic" cadence. Thus the rule has the effect of "extending" an authentic cadence backward in the sequence. It can be written:

(11) Rule 3: $w\ x7 \rightarrow D_x7\ x7$

where D_x stands for the chord of the dominant of x. The absence of brackets around the 7 annotations means that these chords *must* be dominant sevenths. The symbol w is a variable over chords, like x. The chord that it represents may be the same as or different from x. However, there must be similarly stringent restrictions upon the chord that w may match. For example, it may not be D_x7, or the rule might recursively apply to its own output, replacing D_x7 by D_x7 for ever. The restriction is partly accomplished by the fact that the w bears no annotation, which means it cannot match a dominant seventh introduced by Rule 3 or any other rule. There must be a further restriction that *w may not match a chord that has had its root changed by the previous application of a substitution rule.* (A generative linguist would recognize that this restriction is a necessary condition for the rule to be reversible and for a processing algorithm to exist for the grammar.)

However, there is another way in which the rule can quite correctly apply recursively to its own results (since it produces a dominant seventh chord) to produce one of the commonest of the classic blues variations, as shown in



Example (12) [e.g., *Move Up To the Country And Paint My Mailbox Blue*, T. Mahal, and Example (b) in Figure 1]. (In derivations like the above, the operation of all substitution rules is indicated by a horizontal line, indexed with a number of the rule, with the input to the rule above it and the result below. The II7 chord in this example would normally [cf., Example (b) in Figure 1] be a *minor* chord IIm7, a detail that will be returned to later.) The II7 produced by the first application of the rule is a dominant seventh chord. Therefore, the preceding chord can in turn be replaced by the dominant seventh chord with the dominant of II as its root, namely VI7. (This recursive aspect of the rules that characterize the set of well-formed 12-bars is of course the primary reason for using the formalism of phrase-structure grammar.)

Rule 3 can be applied recursively yet again to Bars 7 and 8 of (12), to produce the following [cf. Example (c), Figure 1]:

(13) I/IV/I/I7||IV/IV/*III7*/*VI7*||II7/V7/I/I

(A detail that we will continue to ignore is that the III7 and II7 would normally be realized as the *minor* chords IIIm7 and IIm7.) There seems to be no particular limit to this recursion. Further applications of the rule to Bars 6 and 7 of the above and then to 5 and 6 would produce:

(14) I/IV/I/I7||IV/*VII7*/*III7*/VI7||II7/V7/I/I
tritone

(15) I/IV/I/I7||*#IV7*/*VII7*/III7/VI7||II7/V7/I/I
tritone

—which, although less usual (perhaps because of the harmonically remote jumps of a tritone), seem permissible. [Examples of exceptionally deep recursion of Rule 3 are to be found in Figure 1, (f) and (g). they are discussed in the section on the grammar.]

Such sequences as these, which are quite typical of the later development of the 12-bar in jazz after the Second World War, are radically different from the original skeleton, yet are still recognized as examples of the same form.

Further Elaboration of the Authentic Cadence

The jazz of the late forties and early fifties involved a further kind of elaboration. Besides the recursive extension of the authentic cadence, certain "passing chords" could be substituted for the chords introduced by the earlier rules. A passing chord is a chord whose function is to lead "chromatically," via semitones, on to the following one, rather than to stand alone, or lead harmonically, as the dominant seventh chord does. The rules that introduce them into the sequence can also be expressed using substitution rules.

A chord that is the dominant seventh of the following chord can always be replaced with a passing chord whose root is a diatonic semitone above that following chord—that is, its flattened supertonic. The substitute need not be a dominant seventh chord, but if the following chord itself is a dominant seventh then the substitute normally will be as well. This rule can be written:

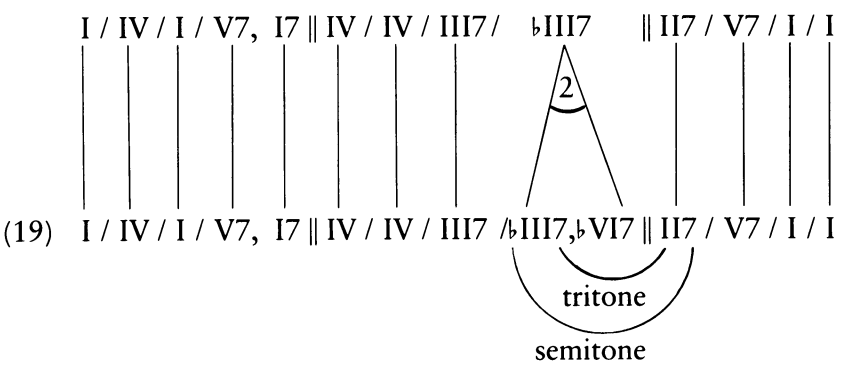
(16) Rule 4: $D_x7\ \ x(7) \rightarrow {\flat}St_x(7)\ \ x(7)$

where ${\flat}St_x$ is the chord of the flattened supertonic of x. Rule 4 generates sequences such as the following, when applied to Example (13), generated by the earlier rules [cf. Figure 1, (d) and (g)].

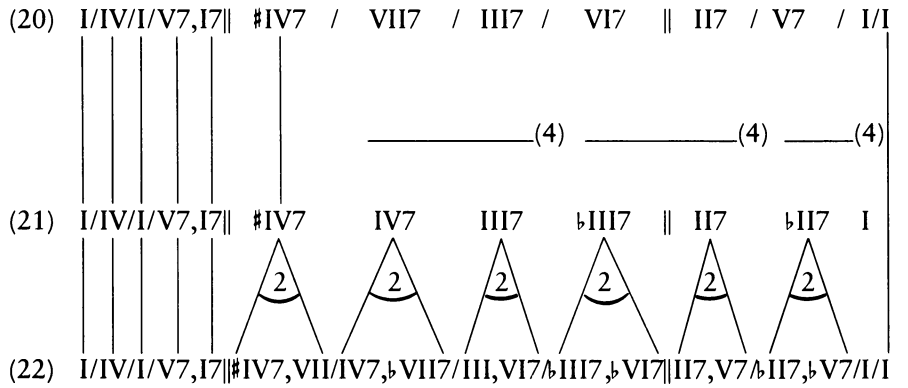
(17) I/IV/I/V7,I7‖IV/IV/III7/♭*III7*‖*II7*/V7/I/I

(18) I/IV/I/V7,I7‖IV/IV/III7/♭*III7*‖*II7*/♭*II*/*I*/I

In conjunction with Rule 4, the earlier Rule 2 (which introduces subdominants) produces a very frequently used kind of sequence. For example, if Rule 2 is used to expand the III7 chord that results from the application of Rule 4 in Example (17), we get the sequence shown in Example (19).



Rule 2 introduces a step of a tritone between the VI7 and the II7—a harmonically remote interval. But the ♭III7 is still related to the succeeding II7 by the interval of a diatonic semitone,, a harmonically close and chromatically "leading" interval, the reason why the substitution still "means" the same as the underlying dominant cadence. If Rules 2 and 4 are applied repeatedly to Example (15), repeated here as (20),the result of an extremely deep recursion of Rule 3, the following sequence emerges:



Strictly speaking, some steps in the above derivation involve "enharmonic changes" in the interpretations of certain chords. These details are ignored here, but cf. Example (34) below.] Examples (21) and (22) may not be very *good* 12-bars, any more than (15) is. However, they do seem to *be* 12-bars, albeit of a rather fringe variety.[11]

One further substitution rule provides a way of elaborating a chord that is *not* part of a dominant cadence and in fact is not "leading" to any other chord in particular. It is seen in the first three bars of Example (d) of Figure 1, which read as follows (ignoring for the moment the diminished seventh chord in the second half of Bar 2):

(23) *I/IIm/IIIm*/V7,I7|| . . .

Again, the substitution can apply to other chords, and at other metric levels:

(24) I/IV/I/V7,I7||IV,*IV/Vm,VIm*/I/VI7|| . . .

Accordingly, it can be introduced by the following rule:

(25) Rule 5: $x \ x \ x \rightarrow x$ $St_x m$ $M_x m$

The substitute chords in this variation, which is widely used in jazz, are always *minor* chords on the root of the supertonic St_x and mediant M_x respectively, as the notation $St_x m$ and $M_x m$ indicates.

11. Cf. footnote 2 for a discussion of reasons why such examples should be generated by the rules.

Major Chords, Minor Chords, and Diminished Seventh Chords

A matter that has been alluded to a couple of times in the above account is that the sequence of dominant sevenths that are introduced by Rule 3 typically alternate major and minor dominant seventh chords. [The reason for such an alternation is partly that the minor third of a minor dominant seventh chord (such as Dm7) is the same note (at least on the equally tempered keyboard) as the dominant seventh (in this case, F) of the chord (in this case G7) that it resolves onto. The dominant seventh (C) of the minor chord (Dm7) is also related to the major third (B) of the major chord (G7) by a chromatically leading semitone.] The V7 of the skeleton is never a minor chord Vm7, since it does not resolve onto a dominant seventh chord, but rather onto a simple I. But if a II7 is substituted for a V7 in Bar 9, via Rule 3, then since it is followed by a (major) dominant seventh chord it *can* be (and normally would be) the minor version. For the same reason, the dominant seventh on V in the first half of the fourth bar of Example (b) of Figure 1 is also minor. The correct alternation can be induced by a simple modification to Rule 3, the rule that introduces new dominant sevenths. Rule 3 splits into two, and becomes

(26) Rule 3a: $w\ \ x7 \rightarrow D_x(m)7\ \ x7$

Rule 3b: $w\ \ x\text{m}7 \rightarrow D_x7\ \ \ x\text{m}7$

Rule 3a says that a chord w followed by a major dominant seventh chord on x may be replaced by a dominant seventh on the dominant of x which may optionally be minor. Rule 3b says that a chord followed by a minor dominant seventh of x may be replaced by the corresponding *major* chord only. A similar modification is required for Rule 4, which replaces dominant seventh chords with flattened supertonic sevenths. When the chord onto which the supertonic resolves is a *minor* chord then the supertonic should be as well. The rule can now be written

(27) Rule 4: $D_x7\ \ x(m)(7) \rightarrow \flat St_x(m)7\ \ x(m)(7)$

Such modifications imply that the other rules should be specified as to their applicability to major and minor chords. Rules 1 and 2 can apply to either minor or major chords, so can be written as follows:[12]

(28) Rule 1: $x(m)(7) \rightarrow x(m)\ \ \ x(m)(7)$

Rule 2: $x(m)(7) \rightarrow x(m)(7)\ \ Sd_x$

12. The subdominant chord Sd_x appears always to be a major, irrespective of whether its parent was major or minor.

Rule 5, however, appears only to apply to *major* chords, so remains as written in (25). Since blues may be either in a minor or major overall key, the initial rule, Rule 0, can be written:

(29) Rule 0: S12(m) → I(m) I7 IV(m) I(m) V7 I(m)

One more rule is required to complete the grammar. Another kind of passing chord that is common in jazz (as in other western harmonic music) is the "diminished seventh chord," written with the °7 affix. (See the appendix for details of its composition.) Such chords occur in Bar 6 of (b), Bar 2 of (d), and Bar 6 of (g) and (i) in Figure 1. They can be introduced with the following rule (the brace indicates alternatives):

(30) Rule 6: x(m) x(m) $\left\{\begin{matrix} D_x \\ St_x m \\ L_x m7 \end{matrix}\right\}$ → x(m) ♯x°7 $\left\{\begin{matrix} D_x \\ St_x m \\ L_x m7 \end{matrix}\right\}$

(Although this is in effect *three* rules, they are more generally applicable than the few examples in Figure 1 might suggest.)

The Grammar and Examples

At this point it is time to collect the rules of the grammar and to examine them using the corpus of Figure 1.

The grammar now contains the following rules:

(31)

0:			S12(m) →	I(m)	I7	IV(m)	I(m)	V7 I(m
1:			x(m)(7) →	x(m)		x(m)(7)		
2:			x(m)(7) →	x(m)(7)		Sd_x		
3a:		w	x7 →	D_x(m)7		x7		
3b:		w	xm7 →	D_x7		xm7		
4:		D_x7	x(m)(7) →	♭St_x(m)(7)		x(m)(7)		
5:	x	x	x →	x		St_xm	M_xm	
6:	x(m)	x(m)	$\left\{\begin{matrix} D_x \\ St_x m7 \\ L_x m7 \end{matrix}\right\}$ →	x(m)		♯x°7	$\left\{\begin{matrix} D_x \\ St_x m \\ L_x m7 \end{matrix}\right\}$	

[It will be recalled that the convention here is that an affix (such as 7) in brackets on the left of the arrow is optional. If there is such a bracket on the left, a bracketed affix on the right is obligatory if, and only allowed if, the item on the left did in fact bear the affix. Otherwise such an item is optional. It will also be recalled that to ensure processability for Rules 3a and 3b, the

variable *w* can only match a chord whose root has not been changed by the previous application of a substitution rule.]

The six rules generate from the "skeleton" produced by Rule 0 every bar of Coker's nine paradigm jazz 12-bars, shown in Figure 1, with the correct root and specification of the presence or absence of the dominant seventh and diminished seventh notes, with the sole exception of the absent dominant sevenths in Bars 8, 9, and 10 of Example (g), an omission that is discussed below. They also correctly specify the appropriate chords as minor. The remaining annotations of chords (included in brackets in Figure 1) as minor chords and chords with the sixth, major and minor sevenths, ninth, flattened tenth, augmented and diminished fifth, and so on can be added by including the following trivial optional rules.

(32) (a) $x \rightarrow \{xM7, \quad x7', \quad x9, \quad x13\}$

(b) $x7 \rightarrow \{x\flat 9, \quad x\flat 10, \quad x7+5\}$

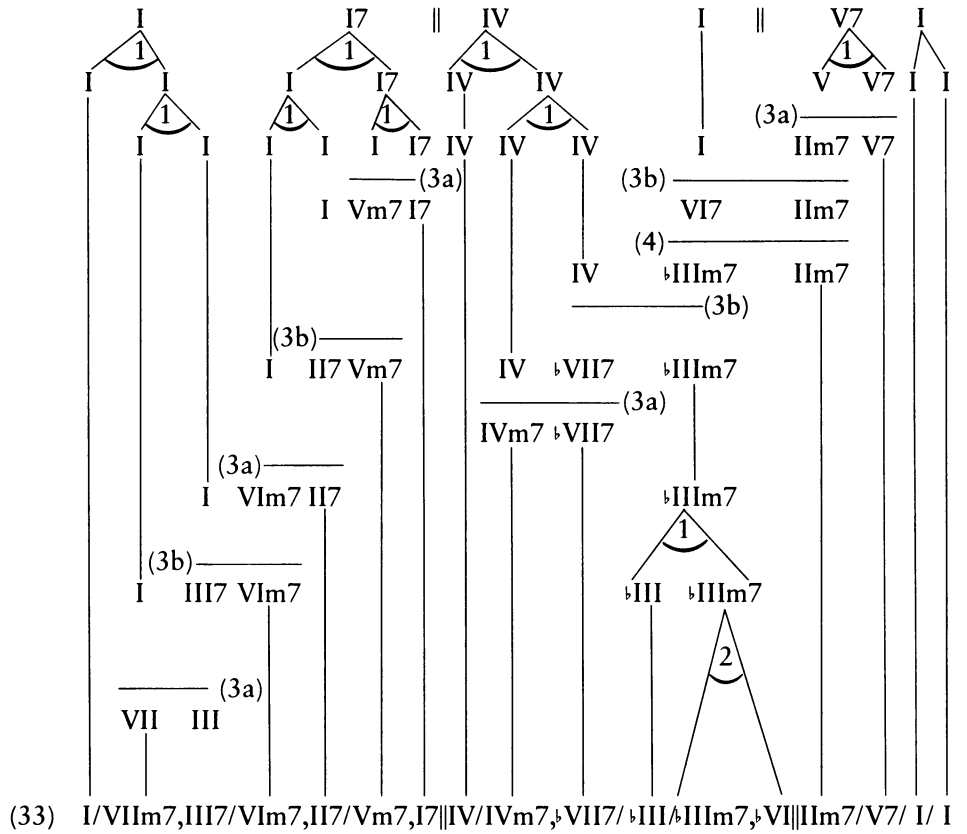
(c) $xm \rightarrow \{xm7', \quad xm6\}$

(d) $xm7 \rightarrow \{xm9, \quad x\phi 7\}$

(The braces mean "replace the thing on the left by any of these." However, the rules cannot be applied indiscriminately. In any given blues if *one* I chord is a IM7 then in general all will be.)

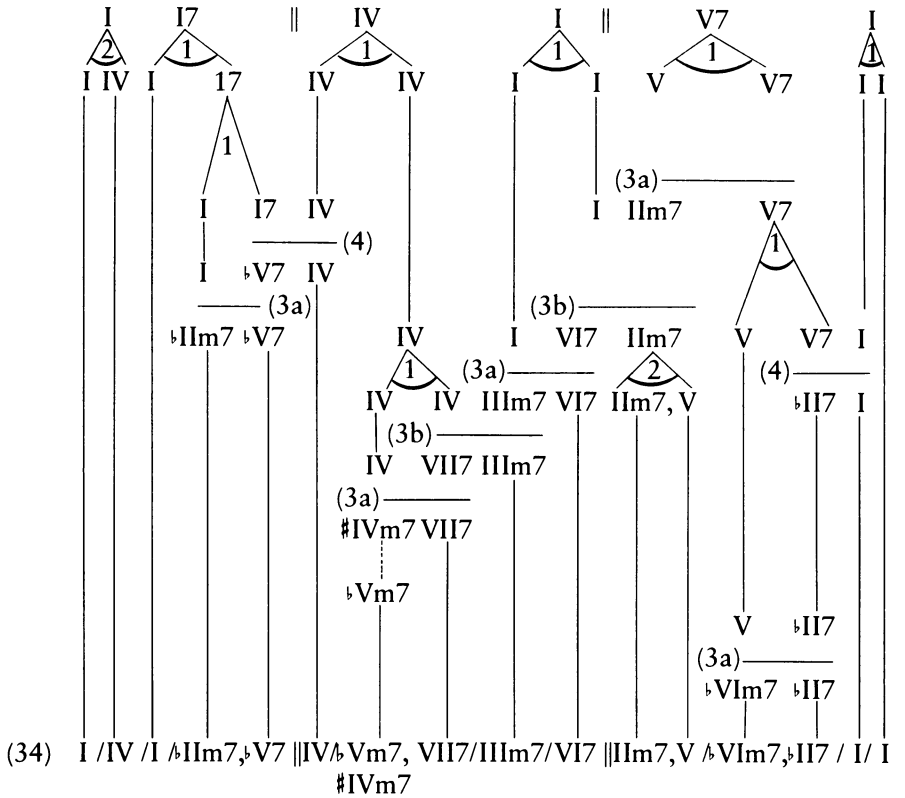
The three most complicated derivations in Coker's corpus—Examples (e), (f), and (g)—are given below by way of illustrating the rules and their operation. Figure 1(e) is derived as in Example (33). The I and IV in Bars 1 and 5, the VIII in Bar 7, and Is in Bars 11 and 12 do not receive the full annotation as given in Figure 1(e), but they are all realized as M7 chords, one of the alternatives offered in (32) for non-dominant-seventh chords. The $\phi 7$ chord on VII in the first half of Bar 2, which is generated as M7, is also covered by an alternative in (32), as is the minor seventh 7′ chord on $\flat$VI in the second half of Bar 8. Figure 1(f) is derived as in Example (34). The example requires further explication. The recursive application of Rule 3 goes exceptionally deep in this blues, to produce Bar 6, containing $\flat$Vm7,VII7. This is possible because the rule first introduces a VII7, then replaces the preceding chord with $\sharp$IVm7. However, strictly speaking, $\sharp$IV is even more harmonically remote from the IV that precedes it than the tritone. It is only by allowing it to be reinterpreted as the chord $\flat$Vm7 (which on the keyboard is played with the same notes) that this derivation can be allowed. This is an "enharmonic change" and is indicated by the dotted line in the derivation. Apart from that, the rules again produce all the correct chord notations with the exceptions of the major seventh M7 and the minor seventh 7′ chords, which can be added by the extra rule (32).

Figure 1(g) is generated as in Example (35). (The extended recursions of



Rule 3 are abbreviated using brackets. The first of these goes exceptionally deep, and again involves an enharmonic change.) In the first recursion the rules allow (although they do not demand) the alternating minors as shown. This is not as in Coker's example but it seems to constitute an improvement, without changing the basic musical "meaning" of his extended dominant cadence. The second recursion applies to the output of the application of Rule 4 to the V7 in Bar 10. However, the recursion predicts that the resulting elaborated cadence should also consist of the usual alternating major and minor dominant sevenths. In fact, inspection of Example (g) in the figure will show the chords in Bars 8, 9, and 10 to be *major* seventh M7 chords ♭IIIM7, ♭VIM7, and ♭IIM7. The underlying sense of the two sequences does seem the same, but this time Coker's annotation seems to be an improvement and is *not* allowed by the current rules, which even with the additional rules (32) will not allow a dominant seventh $x7$ to be realized as xM7. (An alternative with major dominant sevenths throughout *is* allowed.)

A grammar must do more than generate all the sequences of some "lan-

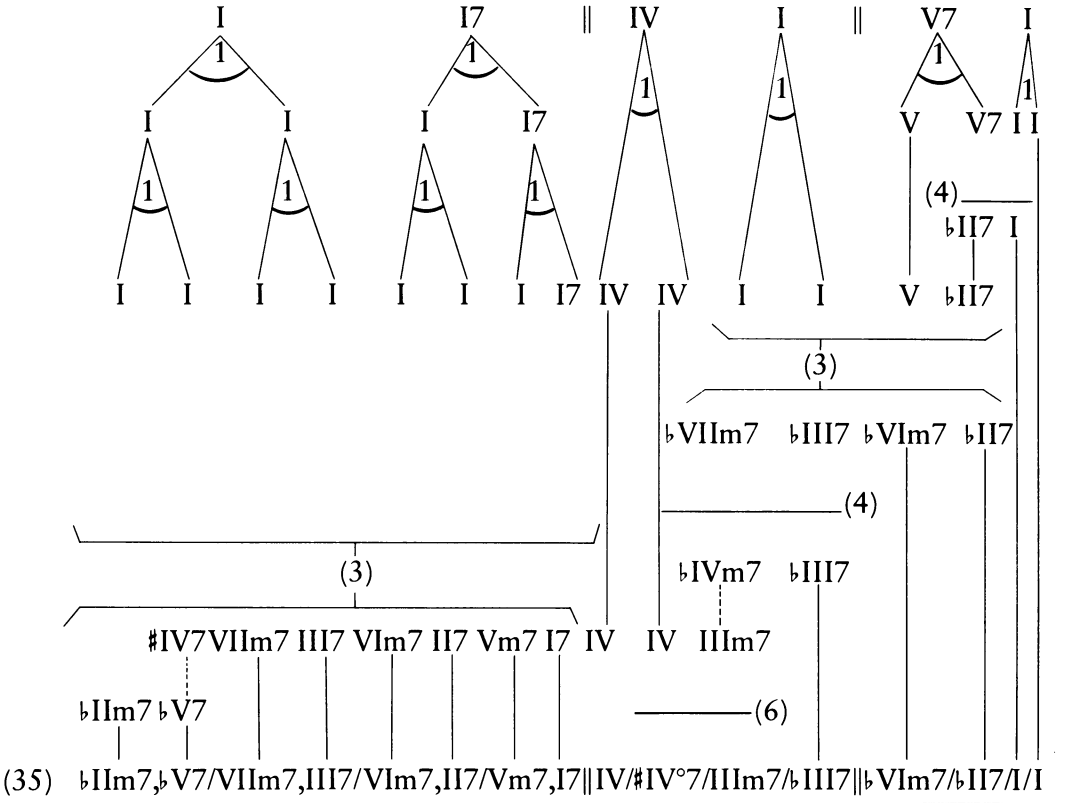


guage." In particular, it should generate *only* those sequences. It is, therefore, important to show that the rules do not generate any ill-formed chord sequences. The most obvious danger arises from the recursive character of such rules as Rule 3a and 3b.

(36) Rule 3a: $w\ x7 \rightarrow D_x(m)7\ x7$

(37) Rule 3b: $w\ x\text{m}7 \rightarrow D_x7\ \ x\text{m}7$

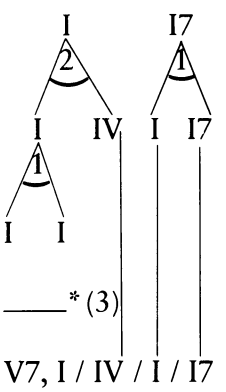
These rules, together with Rule 4, introduce new dominant seventh chords to which they may then recursively apply. It is not easy to show that the rules never generate anything that is not a potential 12-bar, especially since it is not quite clear what that class includes (cf. footnote 2). But the restriction of *w* to non-dominant seventh chords and to chords whose root has not been changed by a previous application of a substitution rule, and the restriction of the *x* chord to chords that *are* dominant sevenths prohibits a lot of ill-formed sequences that would otherwise arise. (Another rule to



whose output Rule 3 might otherwise apply, namely Rule 2, does not induce any dominant seventh affixes that were not there already.)

(38) Rule 2: $x(\mathrm{m})(7) \rightarrow x(\mathrm{m})(7)\ \mathrm{Sd}_x(\mathrm{m})$

For example, the following successive applications of Rule 2 and Rule 3 in the first bars of a 12-bar (which would change the meaning) are not allowed because the second chord in bar 1 is not a dominant seventh.



There *is* a possibility of unreasonably deep recursion of Rule 3, even within the limits of the last eight bars. For example, an extra "layer" of expansion to the level of the half bar by Rule 1 allows Rule 3 to recursively elaborate the final cadence to an extreme degree, yielding (given the usual enharmonic change halfway):

(40)
I/IV/I/IVm7,I7‖♭VIIm7,♭III7♭VIm7,♭II7/♯IVm7,VII7/IIIm7,VI7‖IIm7/V7/I/I
♭Vm7

The jump of a descending tone seems unacceptably abrupt. With additional elaboration by a monomaniacal application of Rule 4 there results:

(41)
I/IV/I/V7,I7‖♭VIIm7,VIm7♭VIm7,Vm7/♯IVm7,IVm7/IIIm7,♭IIIm7‖IIm7/V7/I/I
♭Vm7

(which at least seems no *more* absurd). The following is also possible:

(42) I/IV/I/V7,I7‖♯IVm7/IVm7,♭VII/IIIm7♭IIIm7,♭VI‖IIm7♭IIm,♭V/I/I
♭Vm7

Such sequences do not seem actually ill formed and so may be allowed. (It will be recalled that the present attempt is to account for legal or "well-formed" 12-bars, not *good* 12-bars.)

The above rules are only a first attempt at a formal grammar for a fragment of tonal music. It is inevitable that further rules, albeit of a rather minor nature, will be needed even for the 12-bar blues.[13] It is also likely that the attempt to keep the rules as few and as simple as possible has meant that they will generate some sequences that they should not. But a more important criterion than overgeneration or undergeneration remains the extent to which the rules and the descriptions that they ascribe to the sequences accord with the intuitions of those who know the musical "language" involved. This question can only be answered by the reader and other musicians, but each of the individual rules seems to make good sense musically. Where the derivations identify a chord as related to an underlying cadence, or an often quite harmonically distant "destination" or resolution, this does indeed seem to be the character of the chord in question.

Beyond the 12-Bar Blues

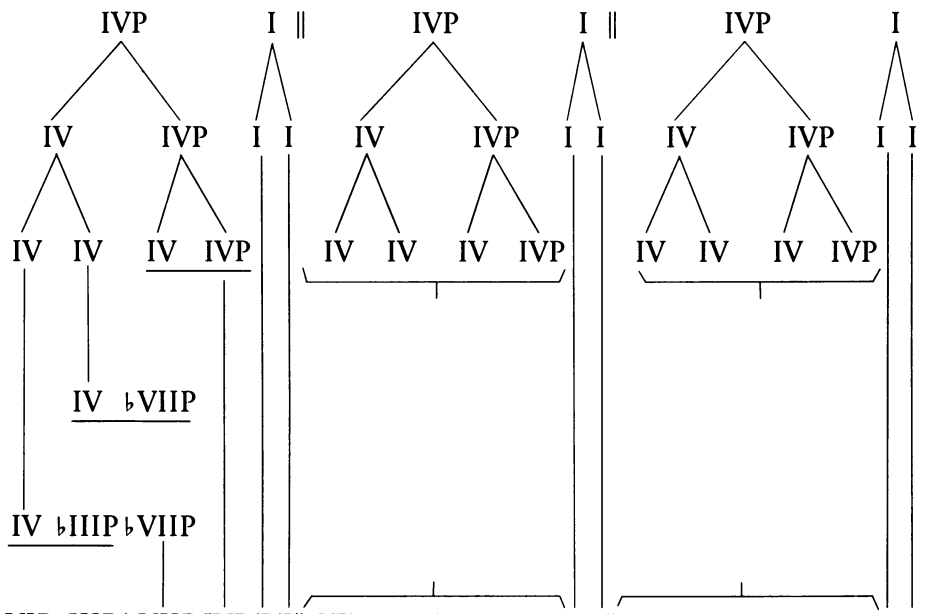
Although the present concern is only to generatively describe the set of all and only the chord sequences that count as 12-bar blues, the rules clearly have much wider applicability. For example, they seem to generate appro-

13. For example, a rule to allow the sequence V7 IV7 I I (an "interrupted" cadence) for V7 V7 I I in the final four bars has been omitted, because it appears to be almost entirely restricted to that particular context.

priate variations upon the chords of *I Got Rhythm* (Gershwin), a sequence almost as common in jazz as the 12-bar, and most other sequences given by Coker (1964), given an appropriate skeleton of tonics, dominants, and subdominants in place of the one provided here by "Rule 0." The fact that they do so is mainly the result of the central place that elaboration of the authentic V-I cadence plays in modern jazz. But there is an obvious extension to the grammar that should be considered if it is to be taken beyond the domain of jazz and blues. Since there is a rule, number 3, which recursively expands the authentic cadence, a similar rule might be expected to expand the plagal (IV-I) cadence. And of course, there is one. Such a rule would be written as follows, by analogy with Rule 3:

(43) $w\ \ x\mathrm{P} \rightarrow \mathrm{Sd}_x\mathrm{P}\ \ x\mathrm{P}$

—where Sd_x is (as usual) the subdominant of x and the suffix P performs the analogous function to the dominant seventh suffix 7. The extended plagal cadence can be seen in its full glory in *Hey Joe* (Jimi Hendrix) whose derivation is given below:



(44) ♭VIP,♭IIIP/♭VIIP,IVP/I/I‖♭VIP,♭IIIP/♭VIIP,IVP/I/I‖♭VIP,♭IIIP/♭VIIP,IVP/I/I

This is a close relative of the 12-bar, as its structure and its lyric show (although it is doubtful whether anyone would refer to it as such, since it does not end in an authentic cadence). So why is it that jazz concentrates almost exclusively on the authentic or dominant cadence? Would we expect one day to find a "looking glass" jazz that had a grammar elaborating the plagal cadence to a comparable degree of complexity? It is possible, but

probably unlikely. Whereas the addition of the dominant seventh to a chord provides a way of making a chord behave as a dominant, even the corresponding addition of a sixth to the minor chord does not make it as convincingly plagal. The above example may for this reason come close to the limits of elaboration of the plagal cadence.

Conclusion

A small number of rules have been presented in the form of a grammar that seems to go some way toward a specification of a recognizable and coherent subset of harmonically "meaningful" chord sequences. The rules appear to be quite generally applicable and not to be merely local to the jazz 12-bar. But generative rules are only really interesting when they can be used to drive a model of human performance on a task that involves understanding. What task of musical understanding could use these rules?

Earlier work (Longuet-Higgins & Steedman, 1970; Steedman, 1973, 1977; Longuet-Higgins, 1976) was concerned with modeling the musical understanding involved in inferring the meter and the relative key of unaccompanied melodies. While it is possible to specify rules for metrical inference without appeal to an extensive harmonic analysis of the piece (Steedman, 1977), rules for key analysis proposed by Longuet-Higgins and Steedman (1970) and Steedman (1973) made mistakes that clearly indicated the lack of an account of the chord structure that underlay the melodies in question. Rules such as those offered in this article, which define "meaningful" sequences of chords that may accompany or underlie melodies, may finally allow an explanation of this simple but so far elusive aspect of human musical understanding.

Appendix

Conventions Used in Figure 1

The sequences (a) to (i) represent the 12-bar chord sequences. Oblique strokes separate the bars. Where only one chord symbol occurs in a bar it is to be understood to last for all four beats of the bar. Where there are two symbols, they each occupy *two* beats.

The root of each chord is identified by a Roman numeral from I to VII. This indicates a degree in the major scale of the keynote of the piece, I being the tonic and VII the seventh. The prefixes ♭ and ♯ identify the root of the chord in question as being one diatonic semitone above or below the degree in question. For example, ♭III indicates a chord whose root is the *minor* third of I.

All chords are understood to be based on the major chord unless explicit

indication is given that they are based on the minor by a small m immediately following the Roman numeral, as in ♭IIIm.

Further numerical suffixes indicate that additional "passing" notes are to be included with the notes of the basic minor or major chord. [The ones in brackets are those accounted for by subsidiary rules (32).] They indicate this in a rather obscure (but standard) way. The suffix 7 means that the "dominant" seventh note, a tone below the root, is to be included, as in ♭III7 and IIIm7. (The nonstandard suffix 7′ also means that a note a keyboard tone below the root is added. However, in these chords the additional note functions as the *minor* seventh, rather than the dominant seventh—cf., footnote 7.) The suffix M7, in contrast, indicates the inclusion of the leading note or *major* seventh, a semitone below the root, as in IVM7. The suffix +5 indicates the addition of the note an augmented fifth above the root (G♯ for the chord of C). It often occurs in combination with the dominant seventh, as in V7 + 5.

The suffix 6 indicates that the major sixth is added. The suffix ϕ7 indicates that the minor third, the diminished fifth (G♭ for the chord of C), and the dominant seventh are included. The suffix°7 indicates that the minor third, the diminished fifth, and the *diminished* seventh (B𝄫 for the chord of C°7) are all included—this is the so-called diminished seventh chord.[14]

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